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Session: Environment, Innovation & Agriculture
"Minimum subsistence requirement and the EKC"
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Presentation outline

Presentation outline

Summary of the Paper

- **Main research question:**
  - How do subsistence constraints (poverty) affect capital accumulation and the income–emissions relationship captured by the EKC?
- **Data:**
  - Unbalanced panel of developing countries.
  - CO_2 emissions, GDP per capita, investment rates, and poverty headcount from the World Development Indicators.
  - Capital stock and human capital measures from the Penn World Table.
- **Methods:**
  - Poverty headcount used as a reduced-form proxy for minimum subsistence requirements.
  - Empirical analysis structured around investment, capital accumulation, and EKC specifications.
- **Main findings:**
  - Higher poverty is associated with lower investment rates.
  - Reduced investment slows capital accumulation.
  - The EKC holds, but poverty shifts the income–emissions relationship, delaying the turning point.

Motivation

- Most growth models employed in economic analysis overlook critical characteristics of developing countries.
- Poverty levels of different countries is rarely taken into account.
- **Steger (2000)** presents four stylized facts of economic growth in developing countries among which:
  1. A big diversity in the growth rates of per capita income.
  2. β-divergence for the lower range of per capita income and β-convergence for the upper range of per capita income $\Rightarrow$ a hump-shaped pattern of growth.
  3. **Positive correlation** between the **growth rate** and the level of **per capita income**$\Rightarrow$ β-divergence.
  4. **Positive correlation** between the **saving rate** and **per capita income.**
- According to Steger (2000) the facts (3) and (4) can be reproduced by a simple linear growth model.
  - **Subsistence as a mode of production**: defined as (mostly agricultural) production for home-consumption.
  - **Subsistence as a mode of consumption**: denotes a standard of living that allows for the satisfaction of the minimum (physical and mental) basic needs of life.

Motivation Cont'd

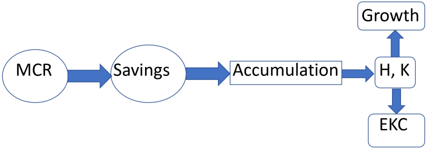
- Steger (2000) shows that simple endogenous growth models incorporate **Stone−Geary preferences:**

$$U[C] = \int_0^\infty e^{-\rho t} \left(\frac{[C - \bar{C}]^{1-\sigma} - 1}{1 - \sigma} \right) dt$$

- This utility function has the following characteristics:
  - $\rho > 0$; $\sigma > 0$
  - Twice continuously differentiable and strictly concave.
  - Consumption is additively separable.
  - CIES function ⇒ special case with $\bar{C} = 0$.
  - SGP ⇒ a variable IES for two immediate points in time IES $= \theta(C) = -\frac{u\prime(C)}{u\prime\prime(C)C} = -\frac{C-\bar{C}}{\sigma C}$
    - $\theta(C) = 0$ if $C = \bar{C}$
    - $\frac{\partial \theta(C)}{\partial C} > 0$
    - $\lim_{C \to \infty} \theta(C) \to \sigma^{-1}$

Mechanism

- EKC -an inverted U-shape relationship between income and some types of pollution ⇒ the pollution levels increase as poor economies begin to develop and then decrease as economies become rich.
- Preferences: CIES vs SGP
  - Constant inter-temporal elasticity of substitution (CIES): smooth intertemporal substitution at all income levels;
  - Stone–Geary (SGP): subsistence first, limited substitution when consumption is near the minimum.



- **Research Question:** What is the impact of having a minimum subsistence requirement on the evolution of pollution in an endogenous growth model setting?
- **Hypothesis:** With a minimum consumption requirement, the turning point of the EKC is delayed.

Previous Studies

- **John and Pecchenino (1994); Jones and Manuelli (1995)**: OLG models that include environmental considerations.
- **Bovenberg and Smulders (1995)**: Model uses endogenous pollution-reducing technological progress. They also discuss government policies to implement the optimum.
- **Elbasha and Roe (1996)**: Endogenous models of technical change.
- **Stokey (1998)**: AK and neoclassical growth model extensions of endogenous growth. A pollution tax or a voucher system can implement the optimum for either AK or NCG models.
  - More demand oriented $\Rightarrow$relies on assumptions about preferences to obtain the environmental Kuznets curve.
- **Andreoni and Levinson (2001)**: With relatively weak assumptions about preferences, they model increasing returns to scale pollution abatement technology that also generates an EKC .
  - Generating an EKC from the supply side in that **they make weak assumptions about preferences** but have a well-developed **pollution abatement technology.**
- **Jeffords and Thompson (2019)** modifies Andreoni and Levinson's (2001) EKC model to include Stone-Geary preferences.

Previous Studies Cont'd

- Jeffords and Thompson (2019): at each level of income, **an increase in the MCR is associated with higher levels of pollution**, and the threshold level of income at which the EKC inverts changes with a change in the MCR.
- **Ikefuji and Horii (2012)** Uses an endogenous growth model with K and H accumulation $\Rightarrow$considers the sustainability of economic growth when the use of a polluting input (e.g., fossil fuels) intensifies the risk of capital destruction through natural disasters.
  - $\Rightarrow$ **growth is sustainable only if the tax rate on the polluting input increases over time.**

Presentation outline

Household Preferences with Minimum Consumption Requirements

- **Assumptions:**
  - Consider an economy with stocks of human capital and physical capital.
  - The production of physical output generates pollution but that human capital is produced without pollution.
  - Neither type of capital depreciates.
  - **It is possible to reduce the amount of pollution that is generated from production by using some physical capital for pollution control.**
  - The instantaneous utility function has a non-homothetic form and RI preferences are given by:

    $$U[C, X] = e^{-\rho t} \left(\int_0^{\infty} \frac{[C - \bar{C}]^{1-\sigma} - 1}{1 - \sigma} - \phi \frac{X^{\gamma}}{\gamma} \right) dt$$

    $\rho > 0$; $\sigma > 0$; $\phi > 0$; and $\gamma > 1$
  - The marginal cost of pollution in terms of consumption, $\frac{\phi X^{\gamma-1}}{(C-\bar{C})^{-\sigma}}$.
  - The instantaneous utility function is increasing in C , decreasing in X , and jointly concave in C and X , but it is not homothetic.

Production, Capital Accumulation, and Pollution

- $Y = (zK)^{\eta}(uH)^{1-\eta}$, where $0 < \eta < 1$; $0 \leq u \leq 1$; $0 \leq z \leq 1$.
- $Y = C + \dot{K}$
- Evolution of Human capital $\dot{H} = \delta(1-u)H$, where $\delta > \rho$
- Flow of Pollution: $X = z^{\beta\eta}Y$ where $\beta > 0$ and $1 - z$ is fraction of stock of physical capital used for pollution control $\Rightarrow Y = X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}}$.
- There is an upper limit on the amount of pollution that can be used as an input because the constraint $z \leq 1$ requires $X \leq (zK)^{\eta}(uK)^{1-\eta}$
- Following Hartman and Kwon (2005) I model two versions of the model:
  1. Instantaneous utility function depends on consumption and the flow of contemporaneously generated pollution.
     - Focus is on **flow** of pollution.
     - Appropriate for the types of pollution that dissipate rapidly.
  2. Pollution accumulates and the instantaneous utility function depends on consumption and the stock of pollution.
     - Focus is on **stock** of pollution.

Model 1: Pollution as a flow

-
$$\begin{aligned}
\underset{C,X}{\text{maximize}} \quad & U(C,X) = \int_0^{\infty} e^{-\rho t}\left(\frac{[C-\bar{C}]^{1-\sigma}-1}{1-\sigma} - \phi\frac{X^{\gamma}}{\gamma}\right) dt \\
\text{subject to} \quad & \dot{K} = X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}} - C, \\
& \dot{H} = \delta(1-u)H, \\
& z \leq 1, \\
& K(0) \equiv K_0 > 0, \text{is given}, \\
& H(0) \equiv H_0 > 0, \text{is given}, \\
& \lim_{t\to\infty} e^{\rho t} K \geq 0, \\
& \lim_{t\to\infty} e^{\rho t} H \geq 0
\end{aligned} \tag{1}$$

$$H = \frac{(C-\bar{C})^{1-\sigma}-1}{1-\sigma} - \phi\frac{X^{\gamma}}{\gamma} + \lambda_1[X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}} - C] + \lambda_2[\delta(1-u)H]$$

- Link to FONC

Model 1: Optimal Solution and EKC

- $C = Y - \dot{K} \Rightarrow C = X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}} - \dot{K} \Rightarrow \frac{\partial C}{\partial X}$**=MB of pollution in terms of consumption.**
- The amount of X available is limited by the constraint $X \leq P \Rightarrow$ If the level of pollution is freely variable at the margin, then it is optimal to choose it so that its marginal beneïňĄt equals its marginal cost $\Rightarrow \frac{1}{\beta+1} X^{\frac{1}{\beta+1}-1} P^{\frac{\beta}{\beta+1}} = \frac{\phi X^{\gamma-1}}{C^{-\sigma}}$

$$z = \begin{cases} \dfrac{(C-\bar{C})^{-\sigma} P^{1-\gamma}}{(\beta+1)\phi}, & \text{if } (C-\bar{C})^{-\sigma} < (\beta+1)\phi \ P^{\gamma-1} \\ 1, & \text{if } (C-\bar{C})^{-\sigma} \geq (\beta+1)\phi \ P^{\gamma-1} \end{cases} \tag{2}$$

- Intuitively:
  1. Consider a growing economy for which K and H both are initially small $\Rightarrow 0 \leq u \leq 1$ and $\gamma > 1 \Rightarrow$ there is an interval of time during which $P^{\gamma-1}$ and C are small and the MU of consumption ($(C-\bar{C})^{-\sigma}$) is large $\Rightarrow$
     - **The economy does not control pollution $\Rightarrow$ Pollution increases with output so long as there is no pollution control.**

Model 1: Optimal Solution and EKC

- 2. Next consider an economy that is in the stage where pollution is controlled and the condition MB=MC holds.
  - As the economy becomes more wealthy, both $P^{\gamma-1}$ and $(C-\bar{C})^{\sigma}$ increase, and optimality requires that z falls $\Rightarrow$ **the fraction of the stock of physical capital that is devoted to pollution control increases in a growing economy once it has begun to control pollution.**

Long Run Growth

- Consumption and output growth are sustainable in the long run.
- Production of all final output generates pollution and that this pollution can be alleviated by using physical capital for pollution control.
- Cleanly produced human capital can be substituted for physical capital in the production of final output thereby freeing up more physical capital for pollution control.
- Fraction of stock of physical capital used to produce final output decreases to zero, $\Rightarrow 1 - z$ increases to one.
- Whether the amount of pollution increases, decreases, or remains constant in the long run depends on the magnitude of σ
    - If $\sigma > 1 \Rightarrow$ amount of pollution decreases at a constant rate in the long run $\Rightarrow$ **environmental Kuznets curve for an economy that begins with H and K small.**
    - If $\sigma > 1$ then this elasticity is 'small' and the representative individual is relatively **unwilling to substitute current consumption for future consumption.**
    - Link to Competitive Equilibrium: Pigouvian Tax or Government Voucher System

Model 2: Pollution Accumulates

- In which **pollution accumulates in the environment.**
- The variables C , Y , z , K, u, and H are defined as before; X continues to be a flow of newly generated pollution.
- The stock of pollution in the environment=Q; $\dot{Q} = X - \epsilon Q$ where $\epsilon > 0$ is the rate at which pollution decays.
- The instantaneous utility function has a non-homothetic form, and RI preferences are given by:

$$U[C, Q] = e^{-\rho t} \left(\int_0^\infty \frac{(C - \bar{C})^{1-\sigma} - 1}{1 - \sigma} - \phi \frac{Q^\gamma}{\gamma} \right) dt$$

Model 2: Pollution Accumulates

- $$\begin{aligned}
\underset{C,\,Q}{\text{maximize}} \quad & U(C,Q) = \int_0^\infty e^{-\rho t}\left(\frac{[C-\bar{C}]^{1-\sigma}-1}{1-\sigma} - \phi\frac{Q^\gamma}{\gamma}\right)dt \\
\text{subject to} \quad & \dot{K} = X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}} - C, \\
& \dot{H} = \delta(1-u)H, \\
& \dot{Q} = X - \epsilon Q, \\
& z \leq 1, \\
& K(0) \equiv K_0 > 0, \text{is given}, \\
& H(0) \equiv H_0 > 0, \text{is given}, \\
& Q(0) \equiv Q_0 > 0, \text{is given}, \\
& \lim_{t\to\infty} e^{\rho t} K \geq 0, \\
& \lim_{t\to\infty} e^{\rho t} H \geq 0, \\
& \lim_{t\to\infty} e^{\rho t} Q \geq 0
\end{aligned} \tag{3}$$

$$H = \frac{(C-\bar{C})^{1-\sigma}-1}{1-\sigma} - \phi\frac{Q^\gamma}{\gamma} + \lambda_1[X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} (uH)^{\frac{\beta(1-\eta)}{\beta+1}} - C] + \lambda_2[\delta(1-u)H] + \lambda_3[X - \epsilon Q]$$

Link to FONC

Model 2: Optimal Solution

- Optimality is obtained when the **marginal benefit equals the marginal cost.**

$$z = \begin{cases} \left(\frac{(C-\bar{C})^{-\sigma}}{(\beta+1)(-\lambda_3)}\right)^{\frac{1}{\beta\eta}}, & \text{if } (C-\bar{C})^{-\sigma} < (\beta+1)\phi\,(-\lambda_3) \\ 1, & \text{if } (C-\bar{C})^{-\sigma} \geq (\beta+1)\phi\,(-\lambda_3) \end{cases} \tag{4}$$

- The system of differential equations governing optimal behavior in this version of the model is much more complicated.
- Output and consumption growth are sustainable in the long run, and the flow of pollution decreases in the long run if $\sigma > 1$.
  - For model 2, the additional result is that the **stock of pollution also declines in the long run if $\sigma > 1$.**
  - MCR delays this turning point at which the stock of pollution begins to decline.

Presentation outline

Data

- The dataset covers 110 countries over 40 years, yielding 4,359 unbalanced panel observations of developing countries (annual).
- **Data Sources:**
  1. World Bank Development Indicators
     - CO_2 emissions per capita, i.e., $\ln(\mathrm{CO_2})$.
     - GDP per capita (constant USD), i.e., $\ln(\mathrm{GDPPC})$.
     - Poverty headcount at $2.15/day (2017 PPP).
     - Investment rate (% of GDP).
  2. Penn World Tables
     - Capital stock growth and
     - Human capital index.

Data: Summary Statistics

Table 1. Summary Statistics

	mean	sd	p25	p50	p75
ln CO_2 emissions per capita	-0.276	1.479	-1.424	-0.128	0.929
GDP per capita	2778.289	2507.998	862.478	1866.079	3949.289
Poverty headcount ($2.15/day$)	0.225	0.238	0.039	0.138	0.331
Investment rate (% of GDP)	23.640	9.609	17.607	22.350	28.305
Capital growth	4.196	4.056	1.769	3.823	6.136
Human capital index	1.930	0.569	1.474	1.823	2.332

Data: Summary Statistics

Table 2. Summary Statistics

Variable	Obs.	Countries	$\bar{T}$	Overall Mean	Between SD	Within SD
CO_2 emissions PC	4280	107	40.00	-0.276	1.405	0.481
ln(GDP per capita)	4189	110	38.08	7.515	0.877	0.357
Poverty ($2.15/day)	1023	106	9.65	0.225	0.247	0.115
Investment rate	3286	98	33.53	23.640	7.129	6.970
Capital growth	2782	76	36.61	4.196	2.347	3.388
Human capital index	3361	86	39.08	1.930	0.530	0.257

Empirical Strategy

- **Poverty, Investment, and the Capital Accumulation Channel**
  1. Poverty and Investment

$$\text{Investment}_{it} = \gamma_1 \text{POV}_{i,t-1} + \gamma_2 \ln y_{i,t-1} + \alpha_i + \lambda_t + u_{it} \tag{5}$$

  2. Capital accumulation equation

$$\Delta K_{it} = \theta_1 \text{Investment}_{it} + \theta_2 \text{POV}_{i,t-1} + \alpha_i + \lambda_t + u_{it} \tag{6}$$

- The models are descriptive and not designed to identify causal mediation.

Empirical Strategy Cont'd

- **Growth, Convergence, and Poverty**
  - 1. Do poorer countries grow faster, conditional on nothing else?

$$g_{it} = \beta_1 \ln y_{i,t-1} + \alpha_i + \lambda_t + u_{it} \quad (7)$$

  - 2. Does convergence survive once we control for factor accumulation?

$$g_{it} = \beta_1 \ln y_{i,t-1} + \beta_2 ky_{i,t-1} + \beta_3 h_{i,t-1} + \alpha_i + \lambda_t + u_{it} \quad (8)$$

  - 3. Does poverty constrain growth even after accounting for capital and human capital?

$$g_{it} = \beta_1 \ln y_{i,t-1} + \beta_2 ky_{i,t-1} + \beta_3 h_{i,t-1} + \beta_4 \mathrm{POV}_{i,t-1} + \alpha_i + \lambda_t + u_{it} \quad (9)$$

- **EKC and Poverty**

$$\ln(\mathrm{CO_2pc})_{it} = \beta_1 \ln y_{it} + \beta_2 (\ln y_{it})^2 + \beta_3 \mathrm{POV}_{i,t-1} + \alpha_i + \lambda_t + \varepsilon_{it} \quad (10)$$

Empirical Strategy & Data

- Subsistence constraints are not directly observable $\Rightarrow$ poverty headcount used as a reduced-form proxy.
- Identification uses **within-country changes over time**.
- All regressions include:
  - **Country fixed effects** (time-invariant heterogeneity)
  - **Year fixed effects** (global shocks/common trends)
  - **Clustered SE** at the country level

Presentation outline

Results 1: Theoretical Results

- Competitive equilibrium is not Pareto efficient $\Rightarrow$ optimum can be implemented with a pollution tax or with a voucher system.
- Both models are consistent with sustainable long run economic growth so long as the natural dissipation rate of pollution is not too small if pollution does accumulate.
- These results hold whether or not pollution accumulates.
- Both models are consistent with the EKC.
- However, the model shows that the turning point of the EKC is delayed when subsistence consumption is incorporated.
- This result is important for policy because recommendations can be made to step up efforts to curb pollution with the understanding that pollution is expected to last longer in developing countries.

Result 2: Poverty, Investment and Capital Accumulation Channel

$$\text{Investment}_{it} = \gamma_1 \text{POV}_{i,t-1} + \gamma_2 \ln y_{i,t-1} + \alpha_i + \lambda_t + u_{it}$$
$$\Delta K_{it} = \theta_1 \text{ Investment}_{it} + \theta_2 \text{POV}_{i,t-1} + \alpha_i + \lambda_t + u_{it}$$

	(1) Investment rate (% of GDP)	(2) Capital growth (%)
Lagged poverty headcount (2.15/*day*)	-14.347***	0.652
	(4.086)	(1.543)
Lagged ln GDP per capita	1.337	
	(1.955)	
Lagged investment rate (% of GDP)		0.223***
		(0.026)
Observations	932	773
Countries	96	73
Year FE	Yes	Yes
Country FE	Yes	Yes

Result 3: Growth, Convergence and Poverty

$$g_{it} = \alpha_i + \lambda_t + \beta_1 \ln y_{i,t-1} + \beta_2 ky_{i,t-1} + \beta_3 h_{i,t-1} + \beta_4 \text{POV}_{i,t-1} + u_{it}$$

	(1) Baseline	(2) Augmented Solow	(3) Solow & Poverty
Initial ln GDP per capita	-3.914***	-4.116***	-4.513***
	(0.678)	(1.263)	(1.113)
Capital-output ratio		2532.519	-30703.777
		(24847.807)	(31748.450)
Human capital index		-0.567	-0.614
		(1.410)	(1.707)
Lagged poverty headcount (2.15/*day*)			-1.745
			(1.562)
Observations	4079	2353	708
Countries	110	63	63
Year FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes

- The dependent variable is the Growth rate of GDP per capita.

Recap: From Accumulation to the EKC

- Poverty reduces investment and slows capital accumulation.
- Slower capital deepening delays structural transformation.
- This shifts the income–emissions relationship, delaying the EKC turning point.

$$\textit{Poverty} \rightarrow \textit{Investment} \rightarrow \textit{Capital} \rightarrow \textit{Emissions}$$

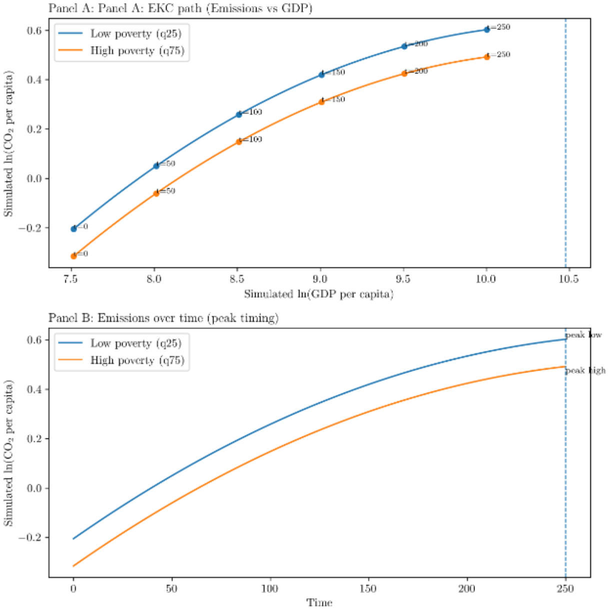
Result 4: EKC with Poverty

- Baseline EKC: $\ln(CO_2pc)_{it} = \beta_1 \ln y_{it} + \beta_2 (\ln y_{it})^2 + \alpha_i + \lambda_t + \varepsilon_{it}$

	(1) Baseline EKC	(2) With Poverty	(3) With Lagged Poverty
ln GDP per capita	1.451*	2.270***	2.531***
	(0.796)	(0.643)	(0.684)
(ln GDP per capita)^2	-0.038	-0.111***	-0.126***
	(0.054)	(0.039)	(0.041)
Poverty headcount (2.15/*day*)		-0.355***	
		(0.122)	
Lagged poverty headcount (2.15/day)			-0.323**
			(0.130)
Observations	4115	981	949
Countries	107	103	103
Year FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes

Link to Models with interactions

Result 5: Poverty and the Timing of Emissions Peaks



Presentation outline

Conclusion

- Minimum subsistence requirements reduce investment $\Rightarrow$ Slower capital accumulation delays income growth.
- The emissions peak occurs later, even with unchanged preferences or technology.
- The EKC relationship is present in the data.
- Higher poverty is associated with lower investment rates and slower capital growth.
- Countries with higher poverty reach peak emissions at later income levels.
- Poverty systematically delays the EKC.
- Higher poverty slows capital accumulation and income growth, postponing the transition to cleaner production.
- The EKC is not invalidated for developing economies, but this study argues that the turning point arrives later.

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Suggestions/Questions?

Graphical Illustration of the EKC

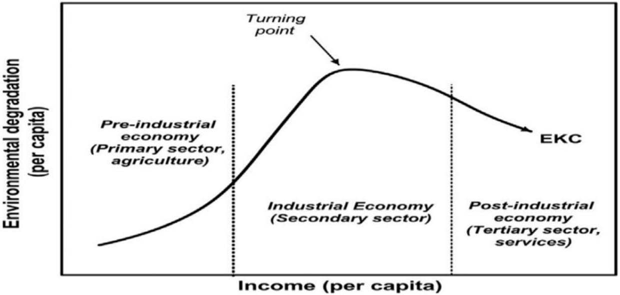

Figure: Source Gill, Viswanathan and Hassan (2018)

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Model 1: FONC

- The first order and necessary conditions are:

$$\frac{\partial H}{\partial C} = 0 \Rightarrow (C - \bar{C})^{-\sigma} = \lambda_1 \tag{11}$$

$$\frac{\partial H}{\partial X} = 0 \Rightarrow \frac{(1-\eta)(\lambda_1 Y - \phi X^{\gamma})}{u} = \lambda_2 \delta H \tag{12}$$

$$\frac{\partial H}{\partial K} = \rho\lambda_1 - \dot{\lambda_1} \Rightarrow \dot{\lambda_1} = \rho\lambda_1 - \frac{\eta(\lambda_1 Y - \phi X^{\gamma})}{K} \tag{13}$$

$$\frac{\partial H}{\partial H} = \rho\lambda_2 - \dot{\lambda_2} \Rightarrow \dot{\lambda_2} = \rho\lambda_2 - \frac{(1-\eta)(\lambda_1 Y - \phi X^{\gamma})}{H} - \lambda_2\delta(1-u) \tag{14}$$

$$\frac{\partial H}{\partial z} \geq 0 \Rightarrow \frac{\eta(\lambda_1 Y - (\beta+1)\phi X^{\gamma})}{z} \geq 0 \tag{15}$$

With the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1 K = 0$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2 H = 0$$

back

Implementing a Competitive Equilibrium

- 1. **Pigouvian Tax:**
  - Assume that the economy consists of a **representative ïňĄrm,** a **representative individual**, and a **government** that levies taxes and distributes subsidies.
  - Both the ïňĄrm and the individual act as price takers.
  - The ïňĄrm rents human and physical capital from the individual, and the government levies a tax on the pollution produced by the ïňĄrm.
  - The ïňĄrm chooses how much of the physical capital that it rents is used for pollution control subject to the constraint that $z \leq 1$
  - Let $L = uH \Rightarrow Y = X^{\frac{1}{\beta+1}} K^{\frac{\beta\eta}{\beta+1}} L^{\frac{\beta(1-eta)}{\beta+1}} \Rightarrow$ homog(1) $\Rightarrow$ EulerâĂŹs Theorem applies $\Rightarrow$ Optimized proïňĄt=Zero.
  - **Proposition:** Optimal pollution tax: $\Rightarrow \tau = \frac{\phi X^{\gamma-1}}{(C-\bar{C})^{-\sigma}}$. back

Implementing a Competitive Equilibrium

- 2. **Government Voucher System:**
  - Assume: **one individual, two firms, and a government** whose only activity is distributing pollution vouchers to one of the firms.
  - The individual and both firms act as price takers.
  - Let τ be the price of a pollution voucher & the flow of vouchers is X $\Rightarrow$ evolves as in the planner's optimum.
  - The firm that receives the vouchers sells them to the other firm and engages in no other activity $\Rightarrow$ profit$=\tau X$.
  - The firm that purchases the vouchers produces output using human and physical capital that it rents from the individual.
  - Clearly, this problem is to the pollution tax. back

Model 2: FONC

- The first order and necessary conditions are:

$$\frac{\partial H}{\partial z} \geq 0 \Rightarrow \frac{\eta(\lambda_1 Y - (\beta+1)\lambda_3 X}{z} \geq 0 \tag{16}$$

$$\frac{\partial H}{\partial X} = 0 \Rightarrow \frac{(1-\eta)(\lambda_1 Y - \lambda_3 X)}{u} = \lambda_2 \delta H \tag{17}$$

$$\frac{\partial H}{\partial K} = \rho\lambda_1 - \dot{\lambda_1} \Rightarrow \dot{\lambda_1} = \rho\lambda_1 - \frac{\eta(\lambda_1 Y - \lambda_3 X)}{K} \tag{18}$$

$$\frac{\partial H}{\partial H} = \rho\lambda_2 - \dot{\lambda_2} \Rightarrow \dot{\lambda_2} = \rho\lambda_2 - \frac{(1-\eta)(\lambda_1 Y - \lambda_3 X}{H} - \lambda_2\delta(1-u) \tag{19}$$

$$\frac{\partial H}{\partial Q} = \rho\lambda_3 - \dot{\lambda_3} \Rightarrow \dot{\lambda_3} = \rho\lambda_3(\rho+\epsilon) + \phi Q^{\gamma-1} \tag{20}$$

With the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_3 Q = 0$$

back