Euro Area Output Gaps and the Transmission of Common Shocks

Jointly modeling output gaps using a multicycle Beveridge-Nelson decomposition.

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Summary

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Why does it matter?

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How are we doing it?

Propose multicycle trend-cycle decomposition using a prior on the signal-to-noise ratio.

Stabilization Policy and the Output Gap

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But how do we measure potential output and the output gap?

Beveridge Nelson Decomposition

- Assume we have a nonstationary time series y_t with a trend component τ_t that follows random walk with constant drift μ and a cyclical component c_t.

$$y_t = \tau_t + c_t \tag{1}$$

- Beveridge and Nelson (1981) define trend as

$$\tau_t = \lim_{j \to \infty} E_t[y_{t+j} - j \cdot \mu] \tag{2}$$

$$= y_t + \lim_{J \to \infty} \sum_{j=1}^{J} E[\Delta y_{t+j} - \mu]. \tag{3}$$

and cycle as

$$c_t = y_t - \tau_t \tag{4}$$

- We need to specify a **forecasting model** to estimate the long-horizon conditional expectation.

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- We separate choice of degree of shrinkage for full VAR system from targeting multiple cycles
- **Approach:** Specify a prior on the signal-to-noise ratio of the respective BN cycle of interest.
- **Intuition:** If included variables do not add information, system is shrunk towards univariate BN decomposition with a smooth trend

Data and Estimation

- Specify joint multicountry model[1] for four largest euro area economies[2]
- includes *real GDP*, *hours worked*, the *unemployment rate* and *inflation* for each economy from 2000Q1 to 2025Q2.
- Prior on signal-to-noise ratio is set to $\delta_0 = 10\%$.

▸ Shrinkage prior

[1] 16-variable BVAR(4)
[2] Germany, France, Italy, and Spain.

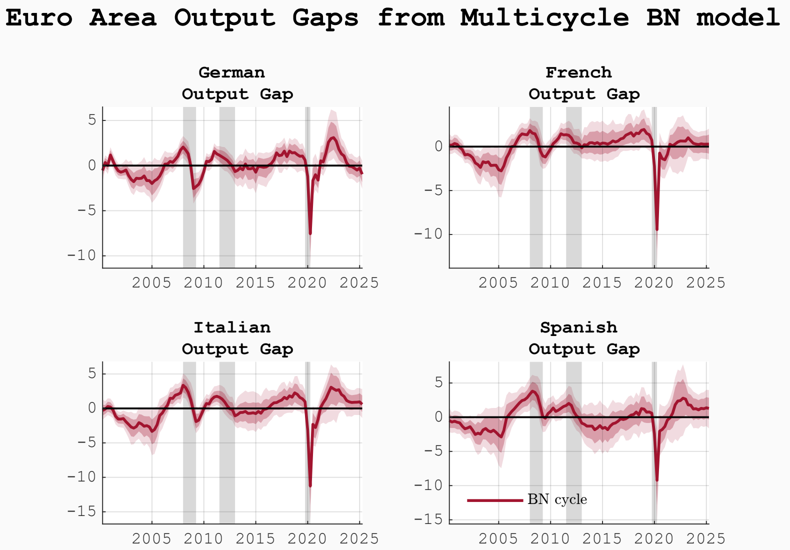

Figure 1: The euro-area output gaps. 90% credible sets given by dotted lines.

Informational decomposition of German Output Gap

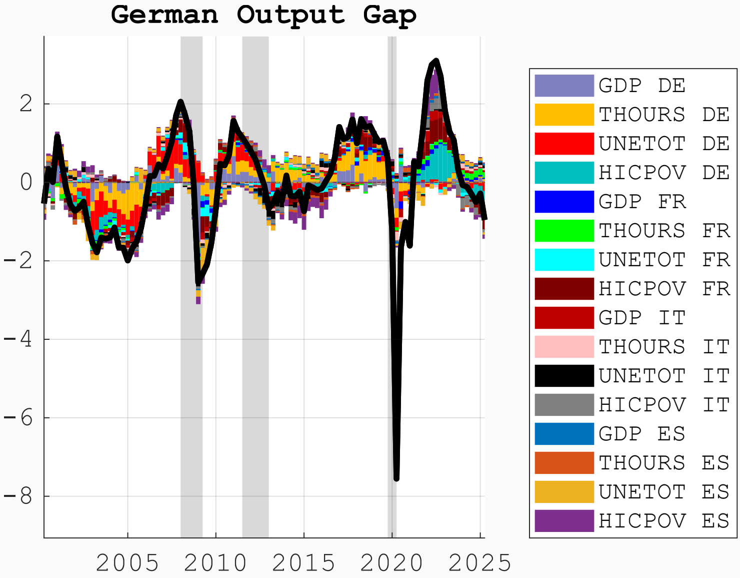

Figure 2: Information drivers of the German output gap

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- By writing each output gap $c_{i,t}$ as function of structural shocks, we can attribute dynamic causal effects

$$c_{ij,t} = -\sum_{l=0}^{t-1} \mathbf{s_k F^{l+1}(I - F)^{-1} H A s_j' s_j} \epsilon_{\mathbf{t-l}}. \qquad (5)$$

where $\mathbf{e_t} = \mathbf{A}\epsilon_{\mathbf{t}}$ with $\mathbf{AA'} = \mathbf{\Sigma}$.

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- Obtain cross-correlations of euro area output gaps conditional on a common shock.
- Estimate impulse responses of euro area output gaps to common shocks.

Conclusion

Contribution

- Propose a multicycle extension to the BN decomposition.
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Thank you for your attention!

Data

The empirical application makes use of the large macroeconomic euro area dataset *EA_MD_QD* by Barigozzi and Lissona (2024). Table 1 gives an overview on the series used in the different specification and the respective data transformation. The *baseline* model only GDP growth and hours worked.

Table 1: Transformations. 1 - log level 2 - log differenced 3 - differenced.

Variable	Economy	Model	Transformation
Real GDP	EA9 economies	*baseline*, *robustness*	2
Empl	EA9 economies	*robustness*	2
Hours	EA9 economies	*baseline*	1
Unempl	EA9 economies	*baseline*	2
HICP	EA9 economies	*baseline*	2
Interest rate	Aggregate EA	*robustness*	0

Output gap measurement

Different approaches in the literature

- Estimation of potential output using production functions (Havik et al., 2014)
- Trend-cycle decompositions using statistical filter or state-space models (Hodrick and Prescott, 1997, Hamilton, 2018, Morley and Piger, 2012, Barigozzi and Luciani, 2023)

Our approach

- Potential output is output that should prevail in an economy in absence of any cyclical shocks
- The BN decomposition assumes the long-horizon conditional expectation of a time series only reflects trend

BN decomposition

Consider forecasting model

$$(\mathbf{\Delta X_t} - \mu) = \mathbf{F}(\mathbf{\Delta X_{t-1}} - \mu) + \mathbf{He_t} \tag{6}$$

Derive long-run forecast

$$\begin{aligned} E_t(\mathbf{\Delta X_{t+1}} - \mu) &= \mathbf{F}(\mathbf{\Delta X_t} - \mu) \\ E_t(\mathbf{\Delta X_{t+2}} - \mu) &= \mathbf{F^2}(\mathbf{\Delta X_t} - \mu) \\ \vdots \quad & \quad \vdots \\ E_t(\mathbf{\Delta X_{t+j}} - \mu) &= \mathbf{F^j}(\mathbf{\Delta X_t} - \mu) \end{aligned}$$

Given stationarity and eq. (6), we can write the cumulative sum at time t of expected future deviations of the vector process from its unconditional mean as

$$E_t \sum_{h=1}^{\infty} (\mathbf{\Delta X_{t+h}} - \mu) = \mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}(\mathbf{\Delta X_t} - \mu). \tag{7}$$

Forecasting Model

- Set up medium scale Bayesian VAR as forecasting model

$$(\mathbf{\Delta X_t} - \mu) = \mathbf{F}(\mathbf{\Delta X_{t-1}} - \mu) + \mathbf{He_t} \tag{8}$$

- Hence, BN trend and cycle are given by (see Morley and Wong, 2020)

$$\boldsymbol{\tau_t} = y_t + \mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}(\mathbf{\Delta X_t} - \mu) \tag{9}$$

$$\boldsymbol{c_t} = -\mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}(\mathbf{\Delta X_t} - \mu) \tag{10}$$

- Note each cycle $c_{i,t}$ fully decomposes into the forecast errors of all the K variables, i.e. $c_{i,t} = \sum_{j=1}^{K} c_{ij,t}$, where

$$c_{ij,t} = -\sum_{l=0}^{t-1} \mathbf{s_k F^{l+1}} (\mathbf{I} - \mathbf{F})^{-1} \mathbf{H s_j' s_j e_{t-l}}. \tag{11}$$

Shrinkage priors for VAR coefficients

Estimate VAR coefficient $\mathbf{\Phi}_j$, $j = 1, \ldots, p$ using Minnesota-type shrinkage prior. Specifically, we set

$$E[\phi_j^{*ik}] = 0 \tag{12}$$

$$Var[\phi_j^{*ik}] = \begin{cases} \frac{\lambda^2}{j^2} & \text{, if i=k} \\ \frac{\lambda^2}{j^2}\frac{\sigma_i^2}{\sigma_k^2} & \text{, otherwise.} \end{cases} \tag{13}$$

The Signal-to-Noise Ratio and the Output Gap

- Signal-to-noise ratio $\delta = \sigma^2_{\Delta\tau}/\sigma^2_{\varepsilon}$ is key concept in trend-cycle decompositions

- Lets assume an AR(p) as a competing forecasting model for Δy_t

$$\Delta y_t = \mu + \sum_{j=1}^{p} \phi_j(\Delta y_{t-j} - \mu) + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2_{\varepsilon}) \tag{14}$$

- There exists a direct mapping from the sum of autoregressive coefficients $\phi(1)$ to δ (see Kamber et al., 2018)

$$\delta = (1 - \phi(1))^{-2}$$

Informational and structural decomposition

We can derive the share of forecast error $c_{ij,t}$ of the jth variable in $\mathbf{x_t}$ on the cycle $c_{i,t}$ while $\Delta y_{i,t}$ still has the kth position in $\mathbf{x_t}$

$$c_{ij,t} = -\sum_{l=0}^{t-1} \mathbf{s_k} \mathbf{F^{l+1}} (\mathbf{I} - \mathbf{F})^{-1} \mathbf{H} \mathbf{s_j'} \mathbf{s_j} \mathbf{e_{t-l}}. \tag{15}$$

Note that the cycle $c_{i,t}$ fully decomposes into the forecast errors of all the K variables contained in $\mathbf{x_t}$

$$c_{i,t} = \sum_{j=1}^{K} c_{ij,t} \tag{16}$$

To attribute dynamic causal effect in the interpretation of the cycle, we need to write $c_{i,t}$ as a function of orthogonal structural shocks

$$c_{ij,t} = -\sum_{l=0}^{t-1} \mathbf{s_k} \mathbf{F^{l+1}} (\mathbf{I} - \mathbf{F})^{-1} \mathbf{H} \mathbf{A} \mathbf{s_j'} \mathbf{s_j} \epsilon_{\mathbf{t-l}}. \tag{17}$$

where $\mathbf{e_t} = \mathbf{A} \epsilon_{\mathbf{t}}$ with $\mathbf{A}\mathbf{A}' = \mathbf{\Sigma}$.

Cross-correlations of Output Gaps

- Recall the VAR model and the definition of the BN cycle

$$(\mathbf{\Delta X_t} - \mu) = \mathbf{F}(\mathbf{\Delta X_{t-1}} - \mu) + \mathbf{He_t}$$
$$c_{i,t} = -\mathbf{s_k}\mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}(\mathbf{\Delta X_t} - \mu)$$

- We know from Morley (2002) that $\mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}(\mathbf{\Delta X_t} - \mu)$ contains the estimated BN cycles
- Berger et al. (2022) show that the variances of estimated BN cycles can be calculated as

$$\mathbf{\Psi} = \mathbf{F}(\mathbf{I} - \mathbf{F})^{-1}\mathbf{\Omega}[(\mathbf{I} - \mathbf{F})^{-1}]'\mathbf{F}'$$

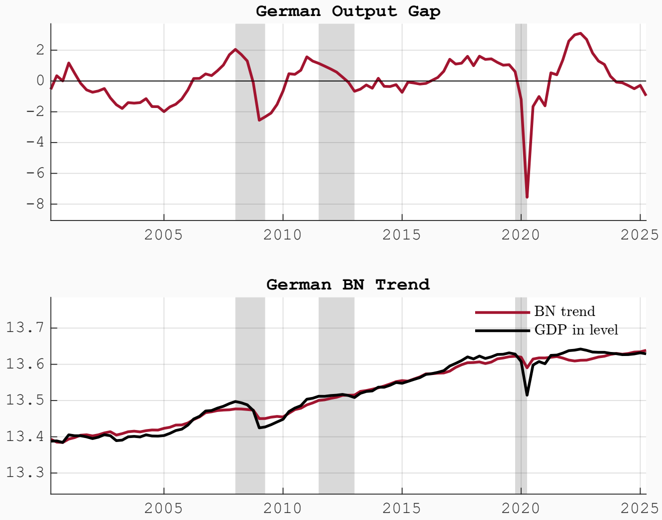
where $\mathbf{\Omega}$ is the variance of $\mathbf{\Delta X_t}$ and $vec(\mathbf{\Omega}) = [\mathbf{I} - \mathbf{F} \otimes \mathbf{F}]^{-1} vec(\mathbf{Q})$, where

$$\mathbf{Q} = \begin{bmatrix} \mathbf{\Sigma} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}$$

- finally, normalize cross-covariance matrix $\mathbf{\Psi}$ into cross-correlation matrix ψ

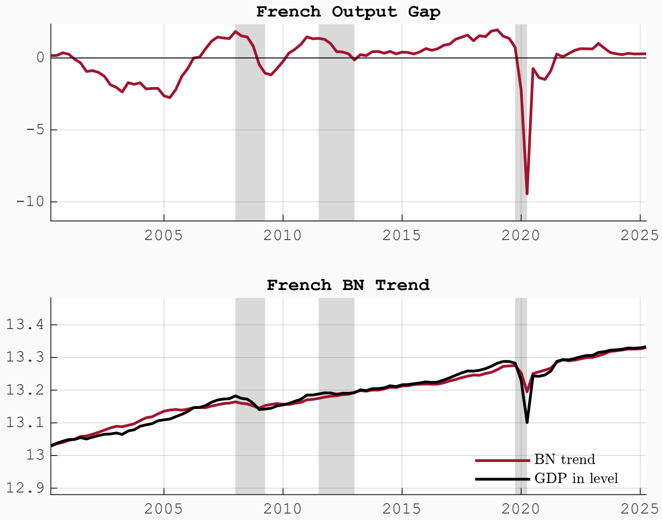
German Output Gap

Figure 3: German baseline results.



French Output Gap

Figure 4: French baseline results.



Italian Output Gap

Figure 5: Italian baseline results.

Spanish Output Gap

Figure 6: Spanish baseline results.