Just Energy Transition in India: Policy Modelling and Implementation

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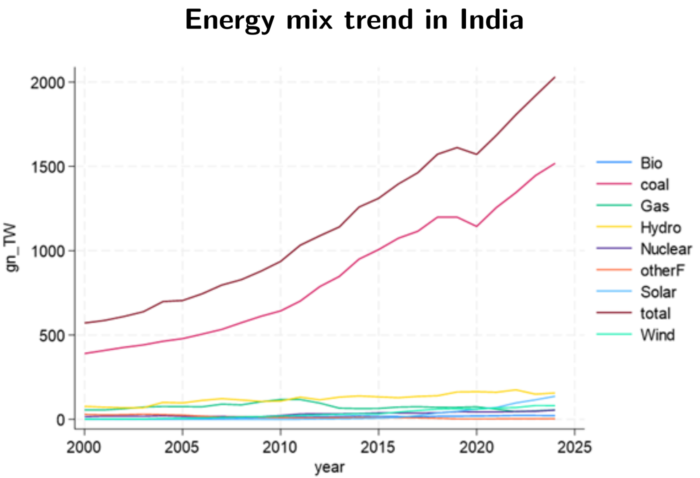
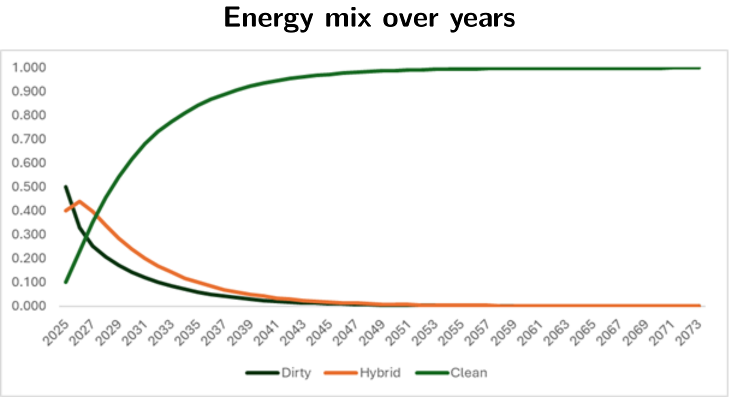
Energy-mix trend in India in the last two decades



- **Key Insight:** Fossil fuel-based energy is growing fast and has been the dominant source of energy in India in last two decades

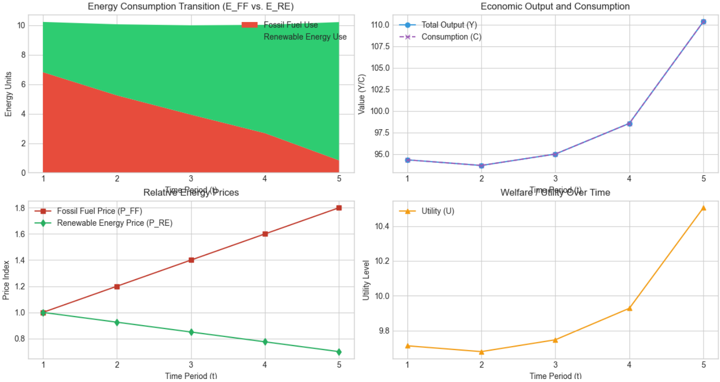
Visualisation transition process to full renewable state



- **Key Insight:** Renewable energy will replace fossil energy in 40 years.

Main results of the dynamic CGE

Macro impacts of energy transition



- **Key Insight:** Fossil fuels are a lot more expensive than renewables; both demand and supply factors lead to full transition to renewables.

1. JET in India: Context and Dual Mandate

- **The Core Challenge:** Decarbonize the energy system while meeting a rapidly **growing** energy demand.
- **The 'Just' Component:** Ensure the transition does not exacerbate existing regional or socio-economic inequalities.
- **India's Stance:** A transition rooted in national development goals, emphasizing energy security and affordability (The E^3 Trilemma).

The Dual Mandate

Scale-up Renewables — Support Coal-Dependent Regions

The Solar Revolution (2005–2025)

- **2010:** $\approx$ \$0.38/kWh
- **2024:** $\approx$ \$0.04/kWh
- **Decline:** $> 89\%$ in 14 years.

Wright's Law: Learning rate of $\approx 20\%$ for every doubling of cumulative capacity.

LCOE Comparison & Projections

Technology	2010	2024	2045 (Est)
Solar PV (Utility)	$0.38	$0.04	$0.018
Onshore Wind	$0.09	$0.03	$0.022
Offshore Wind	$0.18	$0.07	$0.040
Battery Storage	$1,200	$130	$< $60

Table: LCOE per kWh (Storage per kWh capacity)

The Core Metric: Understanding LCOE

Levelized Cost of Electricity (LCOE)

The average net present cost of electricity generation for a generating plant over its lifetime.

$$LCOE = \frac{\sum_{t=1}^{n} \frac{I_t + M_t + F_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{E_t}{(1+r)^t}}$$

- **Notations** I, M, F, E and r represent investment cost; M, material or maintenance cost, F is fuel cost; E electricity and r discount rate
- **CAPEX-heavy:** Renewables have high upfront costs but near-zero marginal costs.
- **Fuel Cost:** $0 for Solar, Wind, and Hydro.
- **Sensitivity:** Highly dependent on the "Cost of Capital."

Wind Power: Scaling Upward

- **Onshore Wind:** 70% cost reduction since 2010. Currently $\approx$ \$0.033/kWh.
- **Offshore Wind:** 60% cost reduction; benefited from massive 15MW+ turbines.
- **Innovation:** Larger rotor diameters and higher hub heights capture more consistent, high-velocity winds.

Reference Links

Here are some links to websites of renewable companies in India:

- **Direct URL:** `https://www.tatapower.com/renewables`
- **Hyperlink Text:** ReNew
- **Beamer Button:** ▸ Adani Energy Group
- **Hyperlink Text:** Infosys
- **Hyperlink Text:** Reliance
- **Hyperlink Text:** Mahinda and Mahindra
- **Hyperlink Text:** Bajaj
- **Hyperlink Text:** Niti Ayog Energy Dashboard

Hydro and Bioenergy: The Mature Baselines

- **Hydropower:**
  - Most stable renewable cost profile (\$0.04–\$0.06/kWh).
  - High barriers to entry: Geography and environmental permits.
- **Bioenergy:**
  - Sensitive to local feedstock supply chains.
  - Crucial for providing "dispatchable" renewable power.

Why have costs collapsed?

1. **Technological Efficiency:** Higher conversion rates (e.g., Bifacial solar, PERC cells).
2. **Economies of Scale:** "Gigafactories" and automated manufacturing pipelines.
3. **Capital De-risking:** Renewables are now viewed as low-risk infrastructure by institutional investors, lowering interest rates.

2025–2045: The Next Frontier

- **Perovskite Tandem Cells:** Pushing solar efficiency limits beyond 30%.
- **Floating Offshore Wind:** Accessing deeper waters with higher wind potential.
- **Projected Floor:** Solar LCOE targeting **$0.015/kWh** in high-insolation regions.

Storage: Solving Intermittency

- **Battery Price Drop:** Lithium-ion pack prices down 90% since 2010.
- **Next-Gen Storage:** Solid-state batteries and Green Hydrogen (H_2).
- **Grid Parity:** Storage costs are the final hurdle to a 100% renewable grid.

Technology and cost: A New Energy Paradigm

- **Economic Reality:** Renewables are now the "default" choice for new energy capacity.
- **The 2045 Vision:** A grid dominated by zero-marginal-cost energy.
- **Strategic Shift:** Global competition is now focused on the **speed** of the transition, not just the cost.

End of fossil fuels soon??

Literature Review: Social and Global Perspectives

- **Agbaitoro and Ekhator (2025):** Focuses on social inclusion and environmental rights-based imperatives in Africa's energy transition[cite: 394, 411].
- **Ahluwalia and Patel (2025):** Discusses strategies for India's decarbonisation through the Centre for Social and Economic Progress[cite: 396, 413].
- **Aklin (2025):** Analyzes the political logic behind just transition policies[cite: 414].
- **Baker et al. (2020):** Highlights the need for coal phase-out policies to align with local socio-economic realities to prevent job losses in South Africa[cite: 176].

Literature Review: Justice and Ethics

- **Foley et al. (2024):** Proposes adopting just transition ethics to restore trust in ESG investing[cite: 440].
- **Heffron and McCauley (2018):** Examines the core concept of energy justice across various academic disciplines[cite: 441].
- **Wang and Lo (2021):** Provides a conceptual review of just transition, categorizing it into themes like labor, governance, and perception[cite: 202, 478].
- **Newell and Mulvaney (2013):** Cautions that without accountability mechanisms, transitions may perpetuate power imbalances[cite: 175].

Literature Review: Economic and Institutional Insights

- **Jha and Leslie (2025):** Investigates start-up costs and market power dynamics within the renewable energy transition[cite: 444, 445].
- **Mitra et al. (2025):** Analyzes the global economic impact of technological decoupling, specifically for Asia[cite: 195].
- **Sovacool et al. (2021):** Identifies dozens of injustices across different spatial scales through mixed-methods research[cite: 169, 171].
- **Yang et al. (2024):** Explores the connotations, mechanisms, and effects of energy transitions[cite: 477].

International Reports and Frameworks

- **IEA (2023):** Provides a global outlook on energy shifts and strategic investments[cite: 442, 443].
- **IRENA (2023):** Emphasizes international cooperation and funding for large-scale projects in the Global South[cite: 192, 443].
- **European Commission (2020):** Outlines mechanisms within the European Green Deal to support regions reliant on fossil fuels[cite: 173].

Just Transition to Net zero Target: India's Dual Challenge

- **Climate Goals:** India has committed to ambitious climate targets (e.g., Net Zero by 2070, 50% cumulative electric power installed capacity from non-fossil sources by 2030).
- **Energy Transition:** Assessing the shift from the current $\sim$80% Fossil Fuel (FF) / 20% Renewable Energy (RE) mix to a $\sim$2% FF / 98% RE scenario.
- **Economic Development:** The transition must be achieved while maintaining high rates of GDP growth and alleviating poverty.

Problem Statement

How does a policy-driven, rapid energy transition impact key macroeconomic variables (GDP, Consumption, Investment) and inter-temporal welfare?

Model Framework: Dynamic CGE

Focus on Inter-temporal Optimization

- **Households:** Inter-temporally optimize utility, deciding on consumption vs. saving (investment).
- **Firms:** Minimize costs subject to production technology.
- **Dynamics:** Capital accumulation links periods, and the discount factor governs time preference.

Key Functions

- **Utility (Household):** Cobb-Douglas $U_t = C_t^{\gamma} L_t^{1-\gamma}$
- **Production (Firm):** Nested CES/Cobb-Douglas
- **Energy Aggregate:** CES substitution between FF and RE.

Transition Mechanism

- **Carbon Tax:** Increases the relative price of P_{FF}.
- **Tech Spillovers:** Exogenous fall in the relative price of P_{RE}.
- **Policy Quota:** Mandated shift in energy input shares $(\omega_{ff,t}, \omega_{re,t})$.

Model Core: Market Clearing and Capital

Ensuring Consistency and Dynamics

1. Capital Accumulation (Linking the Future) The dynamic equation for capital stock (K) determines the potential productive capacity for the next period.

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- δ: Depreciation rate of capital.
- I_t: Investment in period t, determined by household savings (S) and foreign borrowing/lending.

2. Goods Market Clearing (The Macro Identity) Total supply must equal total demand for the single composite good (Y).

$$Y_t = C_t + I_t + G_t + X_t - M_t$$

- C_t: Consumption (Household demand).
- G_t: Government expenditure (Exogenous policy variable).
- $X_t - M_t$: Net Exports (Trade balance).

Model Core: Market Clearing and Capital (Cont.)

3. Factor Market Clearing (Labor and Energy)

- **Labor:** Supply ($\bar{L}$) must equal demand (L^d). Since labor is usually fixed/exogenous in CGE: $\bar{L} = L^d$. The wage rate (w_t) adjusts.
- **Energy:** Demand for the aggregate energy input (E^d_{agg}) derived from production must equal the supply of the composite energy good (E^s_{agg}).

$$E^d_{agg} = E^s_{agg}$$

- **FF/RE Sub-Market:** Within the energy sector, the price ratio P_{ff}/P_{re} is determined by the cost minimization FOC, linking the shares ω and the substitution elasticity σ.

The Dynamic Constraint (Transition)

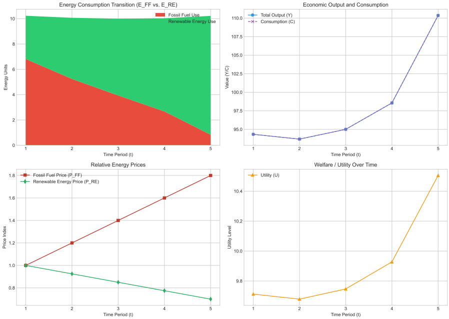
The policy target ensures $\frac{E_{ff,t}}{E_{re,t}} \rightarrow \frac{2}{98}$ by period T, which forces adjustments in relative prices, investment requirements, and hence K_{t+1}.

Results: Dynamic Paths

Visualizing the Transition Impact

Energy Mix and Output

- Initial dip in GDP due to resource reallocation cost.
- Sustained long-term growth driven by cheaper RE and technology boost (TFP).



Investment and Capital

- Investment (**I**) growth is higher than consumption, reflecting the large up-front need for new RE infrastructure and capacity (capital deepening).

(Refer to the lower-right panel of the chart on the left for K/I paths)

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Discussion: Relevance for India

The India Context

- **Investment Requirement:** The model highlights the huge investment need ($I \uparrow 15\%$) for RE infrastructure, which aligns with India's need for capital in the power sector.
- **Trade-Offs:** The model shows a short-term welfare cost (small C dip at $t = 1$) followed by long-term gains. Policy must manage this short-term strain.
- **Factor Mobility:** Unlike the simple model (fixed labor/capital), the CGE needs to account for labor transition from coal to RE sectors (Skill Mismatches).

Key Finding for Policy

The transition is **growth-enhancing** in the long run, provided that the exogenous fall in $\mathbf{P_{RE}}$ (technology imports/R&D) and successful **Carbon Pricing** are achieved.

Lessons

- **Model Success:** The Dynamic CGE framework successfully captures the inter-temporal trade-offs and factor substitution essential to modeling energy transition.
- **Policy Implication:** Rapid transition (80/20 to 2/98) requires massive **Investment** ($\mathbf{I_t}$) to expand $\mathbf{K_t}$ and is highly dependent on **Technology Diffusion** to keep energy costs low.
- **Welfare:** In the 'Green Growth' scenario ($\mathbf{P_{RE}}$ falls, $\mathbf{A_t}$ rises), the economy enjoys a net welfare gain.

Next Steps:

- Disaggregate the economy into multiple sectors (e.g., Coal, Power, Manufacturing).
- Introduce detailed climate damage functions.

Ultimatum Game: The Challenge of a Just Energy Transition (J.E.T.)

- **Global Imperative:** Rapid shift from fossil fuels to renewables is necessary for climate goals and future economic stability.
- **The Social Conflict:** Transition creates a conflict over the distribution of costs and benefits.
  - **Government (Proposer):** Seeks swift, efficient transition (large benefit R).
  - **Fossil Fuel Workers (Responder):** Face job displacement, demand fair compensation/security (justice).
- **Risk of Failure:** Worker resistance (strikes, protests) can halt the entire process, leading to **zero benefit** and high conflict costs.

Motivation: Why Use the Ultimatum Game?

- **Strategic Interaction:** The J.E.T. compensation negotiation is a two-stage strategic decision: Government proposes, Workers accept or reject.
- **Take-It-or-Leave-It:** The government's policy offer is often presented as a non-negotiable package (the "ultimatum").
- **Role of Bargaining Power:** The model explicitly incorporates the Workers' **minimum acceptable share (T)**, which captures their sense of justice and bargaining power.
- **Predictive Goal:** Predict the **Subgame Perfect Nash Equilibrium (SPNE)**—the stable outcome of the compensation share.

Approach: Game Formulation and Payoffs

Players and Resource ($R = 1$)

- **G (Proposer):** Offers $x \in [0, 1]$
- **W (Responder):** Accepts or Rejects
- **Key Parameter:** T (Workers' Minimum Acceptable Share)

Workers' Strategy

$$\text{Accept if } x \geq T$$

$$\text{Reject if } x < T$$

Payoff Structure

- **If Accepted:**

$$\pi_G(x) = 1 - x$$

$$\pi_W(x) = x$$

- **If Rejected (Conflict):**

$$\pi_G(\text{Reject}) = -C_G$$

$$\pi_W(\text{Reject}) = -C_W$$

Approach: Analytical Solution (SPNE)

1. **Backward Induction (Workers):** Workers accept if $x \geq T$.
2. **Government's Maximization:** G maximizes $\pi_G(x) = 1 - x$ subject to $x \geq T$.

The Optimal Proposal $\mathbf{x}^*$

$$x^* = T$$

SPNE Equilibrium Payoffs

- **Government Payoff:** $\pi_G^* = 1 - T$
- **Workers Payoff:** $\pi_W^* = T$

The distribution of benefit is entirely determined by the workers' bargaining threshold, T.

Results: Computational Analysis

Three Scenarios based on T

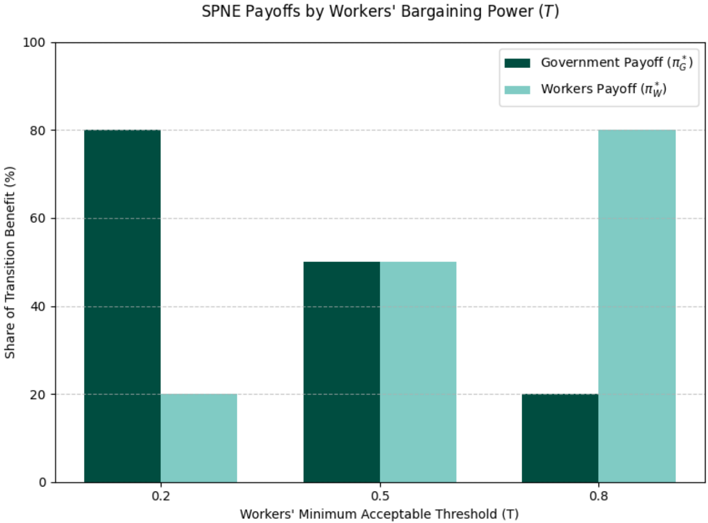
- $T = 0.2$: Weak Workers (Low bargaining power)
- $T = 0.5$: Fair Split (Equal demand)
- $T = 0.8$: Strong Workers (High bargaining power/high conflict risk)

Table: SPNE Results for J.E.T. Ultimatum Game

T (Threshold)	**Optimal Offer** $\mathbf{x}^*(\%)$	**Government** $\pi^*_{\mathbf{G}}(\%)$	**Workers** $\pi^*_{\mathbf{W}}(\%)$
0.2	20.0	80.0	20.0
0.5	50.0	50.0	50.0
0.8	80.0	20.0	80.0

Results: Payoff Distribution Visualization

SPNE Payoffs by Workers' Bargaining Power (T)



Policy Implications for a Successful J.E.T.

- **Accurate Assessment of T is Crucial:**
  - Offering less than $T \implies$ Rejection, conflict, zero benefit.
  - Offering exactly $T \implies$ Cooperation, maximum π_G^*.
- **The J.E.T. Problem is an Information Problem:** Governments must accurately gauge the compensation/security package (T) that satisfies workers' demands for justice.
- **Credible Commitment:** The Government must ensure its offer $x^* = T$ is a credible commitment, or workers may inflate their threshold.

Conclusion 1: Livelihood and Regional Disruption

- **The Coal Workforce:** Estimated **2.6 million** workers (direct/indirect) at risk, primarily informal and lacking social security benefits.
- **Stranded Assets & Communities:** Entire towns and district economies in states like Jharkhand and Chhattisgarh are reliant on coal royalties and mining activities.
- **Fiscal Shock:** State governments face significant loss of revenue (royalties, taxes) that fund essential services, requiring immediate replacement strategies.
- **The Skill Gap:** New RE jobs are geographically misaligned and require different skillsets than traditional coal mining.

Conclusion 2: The Critical Financing Gap

- **Investment Requirement:** Estimated $**200 billion annually** through 2030 for clean energy and related infrastructure.
- **Debt Burden Concern:** India is cautious about international JET-P models (e.g., South Africa) that rely heavily on concessional loans, prioritizing grants and technology transfer.
- **Cost of Reskilling:** Significant capital is needed for creating, scaling, and managing regional economic diversification funds and comprehensive social safety nets.
- **Risk Perception:** Clean energy projects in India still face high capital costs due to market risks and lack of fully matured, long-term financing instruments.

Conclusion 3: Technical and Grid Integration

- **Grid Modernization:** The current grid structure, optimized for centralized coal power, must be fundamentally upgraded to handle intermittent, distributed Renewable Energy (RE).
- **Storage Imperative:** Massive investment in **Battery Energy Storage Systems (BESS)** and Pumped Hydro is necessary for grid stability and peak-load management.
- **Transmission Bottlenecks:** Urgent need for high-capacity Green Energy Corridors to transport power from RE-rich regions (e.g., deserts) to demand centres.
- **Manufacturing vs. Cost:** Balancing the push for 'Make in India' domestic RE manufacturing with the need for rapid, cost-effective deployment often creates supply chain and cost trade-offs.

Solutions: A Way Forward

- **Just Transition Authority:** Establish a dedicated national body to coordinate funding, reskilling, and infrastructure development in coal-affected regions.
- **Global Equity Demand:** Advocate for a new global climate finance architecture that provides grants and affordable technology, treating the JET as a collective global investment.
- **Focus on Decentralization:** Promote decentralized RE solutions and microgrids to enhance energy access and reduce pressure on the national transmission grid.
- **Repurposing Assets:** Transition former coal infrastructure and land into centers for green energy storage or industrial parks for new, clean industries.

Transition probability model

- The Challenge: Understanding Transition Dynamics
- The Tool: Markov Chain Fundamentals
- Model Setup: Defining States
- The Core: Transition Probability Matrix
- **Numerical Example**
- Conclusion: Calculating Time to Net-Zero

The Challenge: Modeling Complex Transitions

Goal of the Model

To estimate the probability and expected time required for an economy to move from a state of high pollution to a **Net-Zero Emission** state.

- **Complexity:** Transition relies on policy, technology, economics, and political will.
- **Stochastic Process:** Outcomes are uncertain, making probabilistic models ideal.

The Tool: Markov Chain Fundamentals

Definition (Markov Chain)

A stochastic process where the probability of transitioning to any future state depends **only** on the current state, and not on the sequence of events that preceded it (the "memoryless" property).

- **Discrete Time:** Steps are measured annually, biennially, etc.
- **Finite States:** We use a limited, measurable set of emission reduction levels.

Model Setup: Defining the States

We define a finite set of states S representing emission reduction thresholds. Let $N = 2$ for a simplified example.

$$S = \{S_0, S_1, S_2\}$$

- S_0: **Current Pollution State** (Baseline/High Emissions).
- S_1: **Intermediate State** (Partial Emission Reduction, e.g., 50% target achieved).
- S_2: **Net-Zero Emission State** (The Absorbing Target).

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Absorbing State

S_2 is an **Absorbing State**, meaning once reached, the system remains there.

$$\text{Probability } P(S_2 \to S_2) = p_{22} = 1$$

The Core: Transition Probability Matrix (**P**)

The matrix **P** contains the probabilities of moving from state i (rows) to state j (columns) in one time step.

General Transition Matrix for $N = 2$:

$$\mathbf{P} = \begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ 0 & 0 & 1 \end{pmatrix}$$

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- p_{01}: Probability of **successful initial policy implementation**.
- p_{10}: Probability of **reversing course** (failure to maintain momentum).
- p_{ii}: Probability of **stagnation** (remaining in the current state).

Numerical Example: One-Step Transition

Let's assume the following probabilities (per 5-year period):

Example Transition Matrix **P**

$$\mathbf{P} = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}$$

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Key Calculation: Future Probability

Let $\pi^{(t)}$ be the probability vector at time t.

$$\pi^{(t)} = \begin{pmatrix} \pi_0^{(t)} & \pi_1^{(t)} & \pi_2^{(t)} \end{pmatrix}$$

Evolution of the System

The state distribution after T time steps is given by:

$$\pi^{(T)} = \pi^{(0)} \mathbf{P}^T$$

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The Goal: Find the probability of reaching Net-Zero (S_2) after T time steps:

$$\pi_2^{(T)}$$

(Requires calculating $\mathbf{P}^T$)