Semiparametric inference for impulse response functions using double/debiased machine learning

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Introduction

Impulse response function (IRF) for horizon $h \in \mathbb{N}_0$:

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where we observe $\{(Y_t, X_t, D_t) \,:\, t \in \mathcal{T}\}$ and D_t is a discrete policy variable.

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- Effect of monetary policy decision on macroeconomic aggregates; $D_t \in \{-0.50, -0.25, 0.00, +0.25, +0.50\}$
- Price impact of order submission in financial markets; $D_t \in \{$"USD 1M sell", "no order submission", "USD 1M buy"$\}$

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Our contribution: bring ML-based estimation approaches for *cross-sectional* data (Chernozhukov et al., 2018) to a *time series* setting.

▸ Further references

Plug-in estimator

1. Estimate nuisance functions (conditional expectations) using an appropriate ML algorithm: $\hat{\mathbb{E}}[Y_{t+h}|D_t = d, X_t]$
2. Estimate the impulse response function:

$$\hat{\theta}_{RA}^{(h)} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \hat{\mathbb{E}}[Y_{t+h}|D_t = 1, X_t] - \hat{\mathbb{E}}[Y_{t+h}|D_t = 0, X_t]$$

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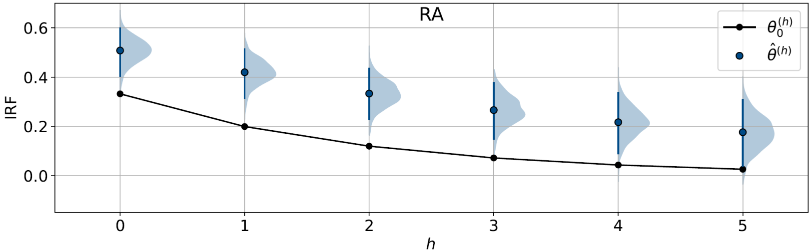
Figure: True and estimated IRF across 1'000 simulated samples using Random Forest



Element 1 - Correct for plug-in bias

Recover the quantity of interest as

$$\theta_0^{(h)} = \mathbb{E}\left[g\left(Z_t^{(h)}; \Gamma_0\right)\right]$$

where $Z_t^{(h)} = (Y_{t+h}, X_t, D_t)$ and

$$g\left(Z_t^{(h)}; \Gamma_0\right) = \mathbb{E}[Y_{t+h}|D_t = 1, X_t] - \mathbb{E}[Y_{t+h}|D_t = 0, X_t]$$

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$$\begin{aligned} g\left(Z_t^{(h)};\Gamma_0\right) =& \mathbb{E}[Y_{t+h}|D_t = 1, X_t] - \mathbb{E}[Y_{t+h}|D_t = 0, X_t] \\ &+ \frac{D_t}{\Pr[D_t = 1|X_t]}(Y_{t+h} - \mathbb{E}[Y_{t+h}|D_t = 1, X_t]) \\ &- \frac{1 - D_t}{1 - \Pr[D_t = 1|X_t]}(Y_{t+h} - \mathbb{E}[Y_{t+h}|D_t = 0, X_t]) \end{aligned}$$

Neyman orthogonality: small deviations from the true nuisance functions (cond. expectations and probabilities) have no first-order effect on the estimation of $\theta_0^{(h)}$ ▸ Neyman orthogonality

Neyman orthogonal estimator

1. Estimate nuisance functions using appropriate ML algorithms: $\hat{\Gamma} = \left(\hat{\mathbb{E}}[Y_{t+h}|D_t = d, X_t], \widehat{\Pr}[D_t = 1|X_t]\right)$
2. Estimate the impulse response function:

$$\hat{\theta}_{DR}^{(h)} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} g\left(Z_t^{(h)}; \hat{\Gamma}\right)$$

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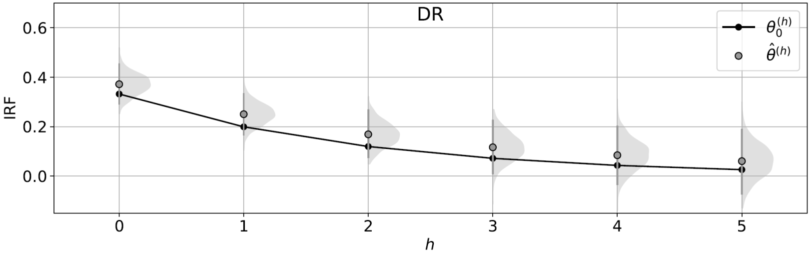
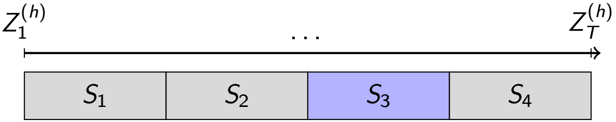


Element 2 - Cross-fitting to mitigate overfitting



1. Estimate nuisance functions using appropriate ML algorithms on S_1, S_2, S_4: $\hat{\Gamma} = \left(\hat{\mathbb{E}}[Y_{t+h}|D_t = d, X_t], \widehat{\Pr}[D_t = 1|X_t]\right)$
2. Estimate the impulse response function on S_3:

$$\hat{\theta}_{S_3}^{(h)} = \frac{1}{|\mathcal{T}_3|} \sum_{t \in \mathcal{T}_3} g\left(Z_t^{(h)}; \hat{\Gamma}\right)$$

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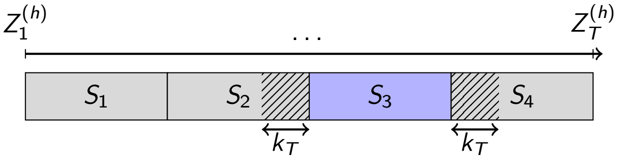

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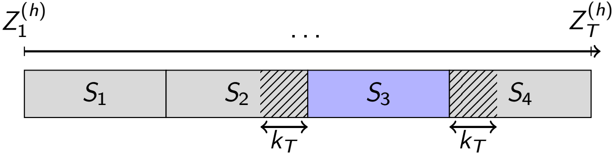

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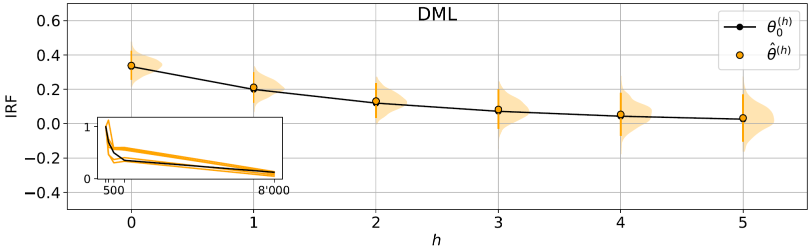
3. Repeat steps 1. and 2. and obtain IRF estimate: $\frac{1}{4}\sum_{i=1}^{4} \hat{\theta}_{S_i}^{(h)}$

Double machine learning estimator

Consistent and asymptotic normally distributed IRF estimator converging at parametric rate when combining:

- Element 1: Neyman orthogonality
- Element 2: K-fold cross-fitting

Figure: True and estimated IRF across 1'000 simulated samples using Random Forest



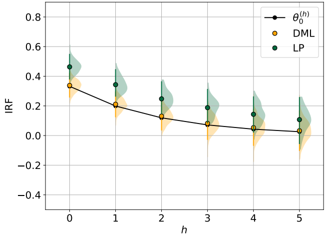
Valid inference across different sample sizes

Table: Simulation results for horizon $h = 0$ for different sample sizes

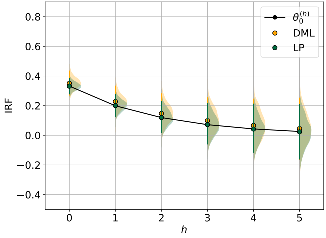
T	Neyman orthogonal + Cross-fitting				Plug-in				Neyman orthogonal			
	Bias	std($\hat{\theta}_h$)	RMSE	C_b(95%)	Bias	std($\hat{\theta}_h$)	RMSE	C_b(95%)	Bias	std($\hat{\theta}_h$)	RMSE	C_b(95%)
125	**0.053**	0.797	0.799	**0.945**	0.360	0.455	0.580	0.511	0.263	0.413	0.490	0.707
250	**0.060**	0.398	0.402	**0.949**	0.285	0.300	0.413	0.514	0.208	0.278	0.347	0.732
500	**0.030**	0.220	0.222	**0.954**	0.219	0.196	0.294	0.464	0.142	0.183	0.232	0.776
1'000	**0.029**	0.144	0.147	**0.931**	0.182	0.139	0.229	0.422	0.103	0.131	0.167	0.784
8'000	**0.006**	0.042	0.042	**0.966**	0.176	0.048	0.182	0.016	0.040	0.042	0.058	0.853

Details simulation

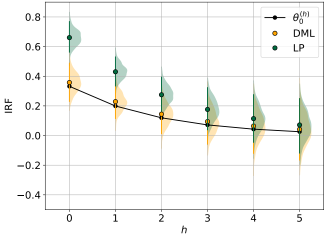
Comaprison to local projections



(a) Nonlinear DGP



(b) Linear DGP



(c) Linear DGP with interactions

Summary of key assumptions

- $g\left(Z_t^{(h)}; \Gamma_0\right)$ satisfies assumptions for the central limit theorem for α-mixing processes.
- Machine learners converge fast enough to the true nuisance functions Γ_0: product of their L_2-norm approximation error converges at rate $\sqrt{T}$.
- $Z_t^{(h)}$ is α-mixing such that after removing k_T observations between training and inference sample, they become asymptotically independent.

▸ Assumptions

Theoretical result

Theorem

Given the stochastic process $S = \{Z_t^{(h)} : t \in \mathcal{T}\}$*, define* $K \geq 2$ *sub-sequences* $S_i = \left\{Z_t^{(h)} : t \in \mathcal{T}_i\right\}$*. Define the estimator as*

$$\hat{\theta}^{(h)} = \sum_{i=1}^{K} \frac{|\mathcal{T}_i|}{T} \hat{\theta}_{S_i}^{(h)} \quad \textit{with}$$

$$\hat{\theta}_{S_i}^{(h)} = \frac{1}{|\mathcal{T}_i|} \sum_{t \in \mathcal{T}_i} g\left(Z_t^{(h)}, h; \hat{\Gamma}_{S_{-i}}\right),$$

where the nuisance functions $\hat{\Gamma}_{S_{-i}}$ *are estimated using the sub-sequence* $S_{-i} = \{Z_t^{(h)} : t \in \mathcal{T}_{-i}\}$*. It holds that*

$$\sqrt{T}(\hat{\theta}^{(h)} - \theta_0^{(h)}) \xrightarrow{\mathrm{d}} \mathcal{N}\left(0, V_0^{(h)}\right).$$

Conclusion and outlook

- Semiparametric estimator that opens up the possibility to use machine learning approaches to estimate IRFs
- Provide theoretical results of the estimator's asymptotic consistency and normality
- Simulation results and empirical application provide evidence of finite sample performance of the estimator
- Restriction: estimator requires a discrete policy variable D_t (or appropriate discretization)
- Work-in-progress: partially linear local projections estimator that relaxes this restriction

$$Y_{t+h} = \theta_0^{(h)} D_t + m(X_t) + \epsilon_{t+h}$$

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Appendix

Further references

- Adamek, Smeekes, and Wilms (2024): inference for high-dimensional penalized local projections
- Paranhos (2025): generalized random forest for locally linear local projections in a panel setting
- Hauzenberger, Huber, Klieber, and Marcellino (2025): impulse response functions computed from Bayesian neural networks
- Angrist, Jordà, and Kuersteiner (2018) and Jordà and Taylor (2016): use inverse probability weighting to estimate IRFs
- Coulombe and Klieber (2025): plug-in estimator with random forests

Back to Introduction

Plug-in bias

Illustrative decomposition of the plug-in estimator:

$$
\begin{aligned}
\sqrt{T}\left(\hat{\theta}-\theta_0\right) = & \frac{1}{\sqrt{T}}\sum_{t\in\mathcal{T}}\left(g(Z_t;\Gamma_0)-\theta_0\right) \\
& -\frac{1}{\sqrt{T}}\sum_{t\in\mathcal{T}}\Bigg(\frac{D_t}{\widehat{\Pr}[D_t=1|X_t]}(Y_t-\hat{\mathbb{E}}[Y_t|D_t=1,X_t]) \\
& \qquad -\frac{1-D_t}{1-\widehat{\Pr}[D_t=1|X_t]}(Y_t-\hat{\mathbb{E}}[Y_t|D_t=0,X_t])\Bigg) \\
& + \text{“empirical process”} + \text{“reminder”}
\end{aligned}
$$

Neyman orthogonality: $\partial_r \mathbb{E}[g(Z_t;\Gamma_0+r(\hat{\Gamma}-\Gamma_0))]\Big|_{r=0}=0$

Back

Simulation study data generating process

$$Y_t = b(X_t) + (D_t - 0.5)\,\tau(X_t) + \gamma Y_{t-1} + \epsilon_t,$$

with

$$\begin{aligned}
e_0(X_t) &= \left(1 + e^{-X_{1,t}} + e^{-X_{2,t}}\right)^{-1} \\
b(X_t) &= 0.5\left((X_{1,t} + X_{2,t} + X_{3,t})^{+} + (X_{4,t} + X_{5,t})^{+}\right) \\
\tau(X_t) &= (X_{1,t} + X_{2,t} + X_{3,t})^{+} - (X_{4,t} + X_{5,t})^{+}
\end{aligned}$$

and

$$\begin{aligned}
\epsilon_t &= \sigma_t \zeta_t, \qquad \zeta_t \sim \mathcal{N}(0,1) \quad \text{and} \quad \sigma_t^2 = \omega + \beta_1 \zeta_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \\
X_t &= \sum_{i=1}^{p} A_i X_{t-i} + \sum_{j=1}^{q} M_j u_{t-j} + u_t, \qquad u_t \sim \mathcal{N}(0, \sigma_u^2 I_n) \\
D_t | X_t &\sim \mathrm{Ber}\left(e_0(X_t)\right)
\end{aligned}$$

Back

Assumptions I

Assumption

For some $\beta > 2$, the following conditions hold.

1. *The stochastic process $\mathcal{G}$ is weakly stationary.*
2. *For the variance it holds that* $0 < V_0^{(h)} = \lim_{T\to\infty} \mathrm{Var}\left[\frac{1}{\sqrt{T}} \sum_{t\in\mathcal{T}} g\left(Z_t^{(h)}, h; \Gamma_0\right)\right]$.
3. *$\mathcal{G}$ is uniformly L_β-bounded, i.e.* $\sup_{t\in\mathcal{T}} \mathbb{E}\left[\left|g\left(Z_t^{(h)}, h; \Gamma_0\right)\right|^\beta\right] < \infty$.
4. *$\mathcal{G}$ is α-mixing, with coefficients $\alpha(s)$, $s \in \mathbb{N}$, satisfying $\sum_{s=1}^{\infty} \alpha(s)^{(\beta-2)/\beta} < \infty$.*

Assumptions II

Assumption

Let the realization set be $\Xi_T^{(h)}$, which is a shrinking neighborhood of the true nuisance functions $\Gamma_0 = (\mu_0(d, x, h), e_0(x))$. Let $\{\Delta_T\}_{T\geq 1}$ and $\{\delta_T\}_{T\geq 1}$ be sequences of positive constants converging to zero.
Define the statistical rates $r_{\mu,T} = \sup_{t\in\mathcal{T}} \sup_{\mu\in\Xi_T^{(h)}} \|\mu(D_t, X_t, h) - \mu_0(D_t, X_t, h)\|_2$ and $r_{e,T} = \sup_{t\in\mathcal{T}} \sup_{e\in\Xi_T^{(h)}} \|e(X_t) - e_0(X_t)\|_2$. Let C be a fixed strictly positive constant. For all $i = 1, ..., K$ and $h \in \mathbb{N}_0$, the following conditions hold.

1. *The nuisance function estimators $\hat{\Gamma}_{S_{-i}}$ belong to $\Xi_T^{(h)}$ with probability at least $1 - \Delta_T$.*
2. *For $q > 2$, we have $\sup_{t\in\mathcal{T}} \sup_{\mu\in\Xi_T^{(h)}} \|\mu(D_t, X_t, h) - \mu_0(D_t, X_t, h)\|_q \leq C < \infty$ and $\sup_{t\in\mathcal{T}} \sup_{e\in\Xi_T^{(h)}} \|e(X_t) - e_0(X_t)\|_q \leq C < \infty$.*
3. $r_{\mu,T} \leq \delta_T$, $r_{e,T} \leq \delta_T$ *and* $r_{\mu,T} \cdot r_{e,T} \leq T^{-1/2}\delta_T$.
4. $\sup_{t\in\mathcal{T}} \sup_{e\in\Xi_T^{(h)}} \|e(X_t) - 1/2\|_\infty \leq 1/2 - \eta$ *for* $0 < \eta < 1$.
5. $\sup_{t\in\mathcal{T}} \mathbb{E}\left[(Y_{t+h} - \mu_0(d, X_t, h))^2 \,|X_t, D_t = d\right] \leq \epsilon_d^2 < \infty$

Assumptions III

Assumption

For all $t \in \mathcal{T}$ *and* $h \in \mathbb{N}_0$ *we have that*
$\mathbb{E}[Y_{t+h}|X_t, D_t = d, \{Z_u^{(0)} : u \in \mathcal{T}, u < t\}] = \mathbb{E}[Y_{t+h}|X_t, D_t = d]$ *and*
$\mathbb{E}[D_t|X_t, \{Z_u^{(0)} : u \in \mathcal{T}, u < t\}] = \mathbb{E}[D_t|X_t]$.

Assumptions IV

Assumption

For $d \in \{0, 1\}$, $h \in \mathbb{N}_0$ and some scalar constant $p \geq 1$ the following conditions hold.

1. $k_T = O(T)$.
2. *The nuisance functions* $\mu_0(d, x, h)$, $e_0(x)$ *and the functions* $\mu(d, x, h), e(x) \in \Xi_T^{(h)}$ *are measurable.*
3. *For for* $r > p$ *and* $1/r = 1/r' + 1/r''$, *we have* $\sup_{t \in \mathcal{T}} \|\sup_{\mu \in \Xi_T^{(h)}} (\mu(d, X_t, h) - \mu_0(d, X_t, h))\|_{2r'} < \infty$ *and* $\sup_{t \in \mathcal{T}} \|e_0(X_t) - D_t\|_{2r''} < \infty$.
4. *For* $q > p$ *and* $1/q = 1/q' + 1/q''$, *we have* $\sup_{t \in \mathcal{T}} \|\sup_{e \in \Xi_T^{(h)}} (e(X_t) - e_0(X_t))\|_{2q'} < \infty$ *and* $\sup_{t \in \mathcal{T}} \|Y_t - \mu_0(D_t, X_t, h)\|_{2q''} < \infty$.
5. *The stochastic process* $S = \{Z_t^{(h)} : t \in \mathcal{T}\}$ *is* α*-mixing with coefficients* $\alpha(s)$, *satisfying for* $T \to \infty$ *that* $\alpha(k_T)^{\psi} = o(T^{-1})$, *where* $\psi = 1/p - 1/\min(r, q)$.

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Variance estimator I

Assumption

The following conditions hold.

1. *There are some fixed finite constants C and $r > 4$ such that* $\sup_{t \in \mathcal{T}} \mathbb{E}\left[\left|g\left(Z_t^{(h)}, h; \Gamma_0\right)\right|^r\right] < C$.
2. *There exists a measurable function $m(z)$ such that* $\sup_{\Gamma \in \Xi_T} |g(z, h; \Gamma)| < m(z)$, *where for some finite constant D,* $\sup_{t \in \mathcal{T}} \mathbb{E}[m(Z_t)^2] < D$.
3. *For $q > 2$ and some fixed strictly positive and finite constant C we have* $\sup_{t \in \mathcal{T}} \|Y_t\|_q \leq C$.
4. *For some scalar $0 < b_m < b_r \leq 1/2$ it holds that:*
   4.1 *The bandwidth m_T is a function of the sample size such that* $\lim_{T \to \infty} m_T = \infty$ *and for $T \to \infty$ it holds that $T^{-b_m} m_T = o(1)$.*
   4.2 $r_{\mu,T} \leq \delta_T T^{-b_r}$ *and* $r_{e,T} \leq \delta_T T^{-b_r}$.

Variance estimator II

Theorem

Given the stochastic process $S = \{Z_t^{(h)} : t \in \mathcal{T}\}$, define the sub-sequences S_i for $i = 1, ..., K \geq 2$. Furthermore, define $v_t^{(h)} = g\left(Z_t^{(h)}, h; \Gamma_0\right) - \theta_0^{(h)}$. Let $w(s, m_T) = 1 - s/(m_T + 1)$, where m_T is a bandwidth parameter. Moreover, define the following Newey and West (1987) type variance estimators for each sub-sequence as

$$\widehat{V}_{S_i}^{(h)} = \frac{1}{|\mathcal{T}_i|}\left(\sum_{t\in\mathcal{T}_i}(\hat{v}_{S_{-i},t}^{(h)})^2 + 2\sum_{s=1}^{m_T} w(s, m_T)\sum_{t\in\mathcal{T}_{i,s}} \hat{v}_{S_{-i},t}^{(h)}\hat{v}_{S_{-i},t-s}^{(h)}\right).$$

The variance estimator is finally defined as

$$\widehat{V}^{(h)} = \sum_{i=1}^{K}\frac{|\mathcal{T}_i|}{T}\widehat{V}_{S_i}^{(h)}$$

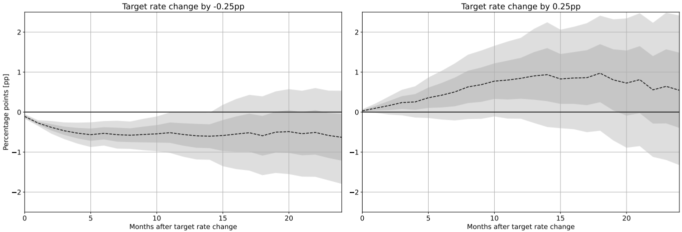
As $T \to \infty$ it holds that

$$\left|\widehat{V}^{(h)} - V_0^{(h)}\right| \xrightarrow{p} 0$$

with measure $\mathcal{P}$.

Empirical results (Angrist et al., 2018)

(a) Federal Funds Rate



(b) Unemployment Rate