Global Trade, Tariff Uncertainty and the U.S. Dollar*

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Abstract

We study the effects of tariff uncertainty on exchange rates, with a focus on the recent depreciation of the U.S. dollar following the 2025 tariff announcements. While standard macro–trade models predict that unilateral tariffs appreciate the imposing country's currency, we show that uncertainty about future tariff policy can reverse this prediction. We develop a two-country general equilibrium model with risk-averse agents and segmented financial markets in which tariff volatility enters the uncovered interest parity condition as a risk-premium wedge. In this framework, increases in tariff uncertainty raise precautionary savings and risk premia, generating contemporaneous currency depreciation even when tariff levels rise. Quantitatively, the model matches the observed magnitude and timing of the dollar's depreciation following the 2025 tariff announcements. Our findings highlight the importance of policy uncertainty in shaping exchange rate dynamics during episodes of disruptive trade policies.

JEL Codes: E2, E3, E6, F1, F4
Keywords: Tariffs, tariff uncertainty, exchange rate volatility, UIP.

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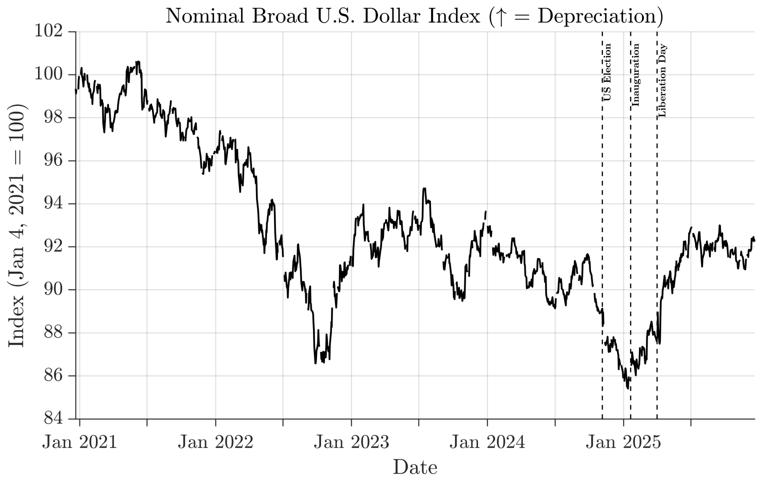
1 Introduction

We extend Kalemli-Özcan, Soylu and Yildirim (2025) (KSY) to study the impact of tariffs on the U.S. dollar exchange rate. KSY analyze the macroeconomic effects of trade distortions within a global dynamic general equilibrium framework featuring multi-sector, multi-country production networks with full input–output linkages and nominal rigidities under incomplete markets. Analytically, they show that tariffs generate global inflation and lower output, accompanied by an appreciation of the home currency. In contrast, the tariff 'threats' employed by the U.S. administration in 2025 generate deflationary pressures, higher unemployment, and a modest depreciation of the dollar in the quantitative version of their model.

In reality, the dollar has depreciated substantially. Figure 1 depicts the nominal Broad U.S. Dollar Index. The dashed vertical lines mark the U.S. election, the inauguration, and Liberation Day, respectively. As the figure shows, the dollar began to depreciate following the inauguration, losing 1.52% of its value between the inauguration and April 1, 2025 (immediately prior to Liberation Day). It then depreciated by an additional 1.04% between April 1 and April 10, 2025.[1] Figure 2 and Table 1 shows that the scale of USD depreciation depends on reference currency and reference time frame. The U.S. dollar's depreciation is larger when one focuses on the USDEUR exchange rate compared to the nominal Broad U.S. Dollar Index. While month-on-month measures indicate depreciation after Liberation Day, year-on-year changes in the broad dollar index suggest only a modest reversal of the substantial appreciation accumulated during 2024. By contrast, the USD–EUR exchange rate exhibits a clearer and more persistent depreciation. This suggests that there could be an element of correcting for 2024's appreciation as well as a currency-pair–specific adjustment.

[1] The Liberation Day announcement took place on April 2, 2025. Over the following week, several additional announcements were made, and a pause was announced on April 9. We compare the value of the nominal broad U.S. dollar index on April 1, 2025 and April 10, 2025 to capture the cumulative impact of announcements made between April 2 and April 9, 2025.

Figure 1. Nominal Broad US Dollar Index



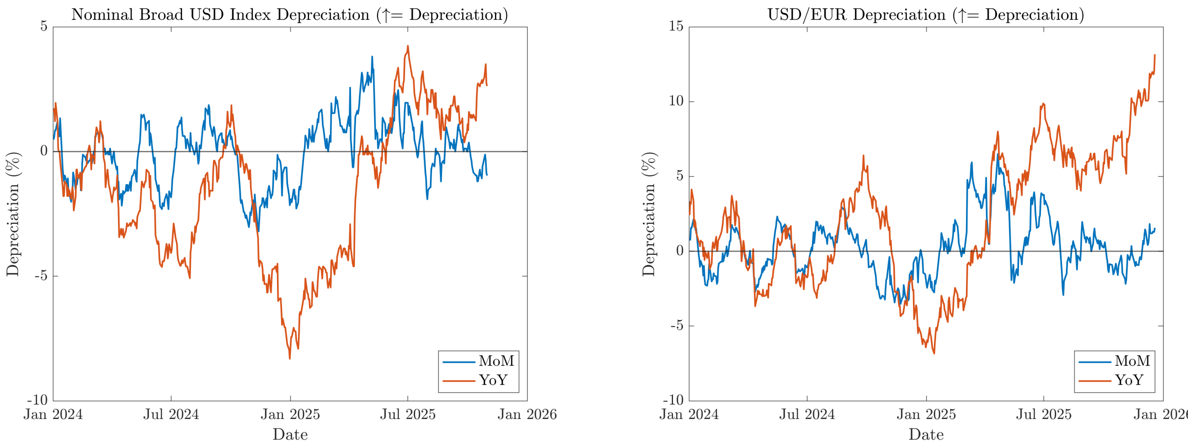
Note: The broad U.S. Dollar index since January 1, 2021. Source: FRED.

Table 1. U.S. Dollar Depreciation/Appreciation

	Broad US Dollar Index		**USD–EUR**	
Date	MoM (30d, %)	YoY (365d, %)	MoM (30d, %)	YoY (365d, %)
2025-04-01	1.4	-3.8	3.8	0.6
2025-04-02	1.0	-3.9	3.5	0.9
2025-04-03	2.6	-2.9	4.9	2.1
2025-04-04	0.5	-3.9	2.3	1.4
2025-04-07	-0.6	-4.5	0.5	0.7
2025-04-08	-0.6	-4.6	0.5	0.5
2025-04-09	-0.3	-4.6	1.9	1.7
2025-04-10	0.8	-2.7	2.4	4.2

Note: Positive values indicate depreciation of the US dollar; negative values indicate appreciation. Source: FRED.

Figure 2. US Dollar Depreciation Measures



(a) Broad US Dollar Index **(b)** USD–EUR Exchange Rate

Note: Positive values indicate depreciation of the US dollar and negative values indicate appreciation. The left panel shows month-on-month and year-on-year changes in the broad US dollar index, highlighting the partial reversal of the strong 2024 appreciation in 2025. The right panel shows USD–EUR depreciation, where the depreciation pattern is more pronounced. Source: FRED.

In this paper, we analytically derive the conditions under which home currency depreciation can arise following announcements of home tariffs that are uncertain in nature—that is, when agents do not know the future level of tariffs and the range of possible future tariff outcomes widens today. When the home country imposes import tariffs under such uncertainty, and when announced tariff rates may deviate from ultimately implemented rates, the risk-premium wedge in the Uncovered Interest Parity (UIP) condition can become sizable. Specifically, the UIP condition can be written as:

$$R_{H,t} = R_{F,t} + \mathbb{E}_t \left[\mathcal{E}_{t+1} / \mathcal{E}_t \right],$$

where $R_{H,t}$ and $R_{F,t}$ denote the home and foreign interest rates at time t, respectively, $\mathcal{E}_t$ denotes the nominal exchange rate, defined as the price of foreign currency in units of home currency and $\mathbb{E}_t[\mathcal{E}_{t+1}/\mathcal{E}_t]$ is the expected change in the nominal exchange rate between t and

$t + 1$.

The risk-premium wedge is typically assumed to be zero under first-order log-linear approximations of the UIP condition. The workhorse first-order log-linear approximation of UIP can be written as:[2]

$$i_{H,t} - i_{F,t} = \mathbb{E}_t \left[\mathcal{E}_{t+1} - \mathcal{E}_t \right], \tag{1}$$

which implies that, in logs, a higher home interest rate relative to the foreign rate predicts an expected depreciation of the home currency when UIP holds.

A second-order log approximation to Equation 1, however, introduces an additional variance term:

$$i_{H,t} - i_{F,t} = \mathbb{E}_t[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t] + \frac{1}{2}\text{Var}(\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t), \tag{2}$$

where $\hat{\mathcal{E}}$ denotes the change of the exchange rate. More compactly, we can write:

$$i_{H,t} - i_{F,t} = \mathbb{E}_t[\mathcal{E}_{t+1} - \mathcal{E}_t] + \rho_t, \tag{3}$$

where ρ_t is commonly interpreted as a time-varying currency risk premium that compensates investors for exchange rate uncertainty.

We show that this risk premium can become quantitatively large when exchange rate volatility is linked to tariff uncertainty. Under constant absolute risk aversion (CARA) utility for households and a risk-averse financial intermediary that solves a mean–variance portfolio problem over home and foreign bonds in segmented financial markets,[3] a second-order equilibrium approximation allows tariff volatility to enter the UIP condition as a risk-premium wedge. The quantitative version of this stylized model can generate a depreciation of the dollar between 1.5 and 2 percent following the Liberation Day tariff announcements,

[2] $i_{H,t}$ ($i_{F,t}$) denotes the log of the home (foreign) gross interest rate, $R_{H,t}$ ($R_{F,t}$).

[3] Note that these features—CARA utility and segmented financial markets—are absent in KSY, although their framework features incomplete financial markets.

closely matching the observed dynamics shown above.[4]

In macro–trade general equilibrium models, such as KSY, unilateral tariffs induce an appreciation of the home currency–the currency of the country imposing the tariffs. The intuition behind this standard result is straightforward. Tariffs reduce demand for foreign goods while increasing demand for domestically produced goods; as relative demand for home goods rises, their relative price increases, leading to an improvement in the terms of trade and an appreciation of the nominal exchange rate (In the absence of non-tradable goods and when domestic inflation remains below foreign inflation, this also implies an appreciation of the real exchange rate). In settings with production networks, such as KSY, the appreciation of the home currency is further amplified. In these environments, tariffs act as global cost-push shocks that disrupt input linkages and raise production costs, thereby reducing global output and making the home country's goods relatively scarcer.

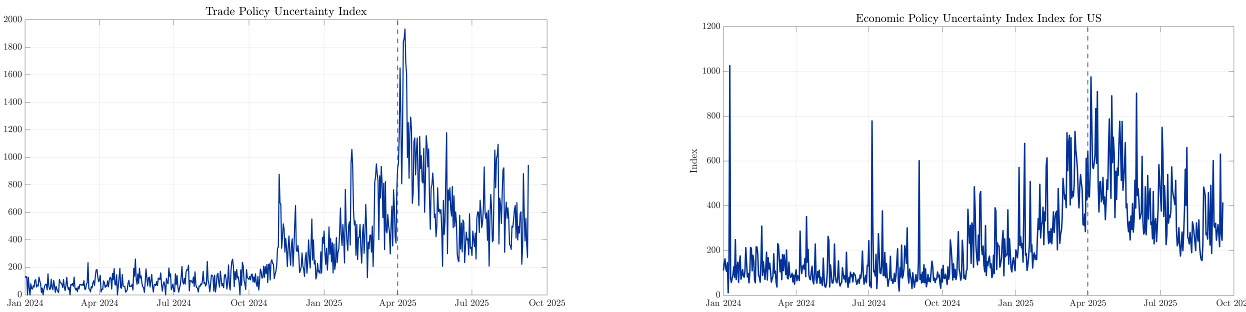
This paper also features a flexible-price macro–trade general equilibrium model, but with a novel twist on the financial side. By introducing risk-averse financial intermediation and allowing households to be sensitive to consumption risk, we incorporate an additional source of fluctuations beyond tariff levels—namely, tariff volatility. This modeling approach is well suited to the empirical observation that the Liberation Day announcements constituted not only a shock to the level of tariffs, but also a shock to their second moment, reflecting heightened policy uncertainty.

Figure 3 illustrates that both trade policy uncertainty and broader economic policy uncertainty peaked around the Liberation Day announcement. Such uncertainty can generate currency depreciation through two complementary channels. First, heightened uncertainty suppresses home consumption demand via a precautionary saving motive. Second, it widens the UIP risk premium by increasing expected excess returns on domestic assets. Both mech-

[4] Although the dollar depreciated by approximately 1–2 percent within a week—from April 1 to April 10—the broad dollar index depreciated by 5% (9% against the euro) between April 1 and June 30, by 4.8% (8.5% against the euro) between April 1 and October 1, and by 4% (7.5% against the euro) between April 1 and December 1.

anisms reduce contemporaneous demand for the home currency, potentially leading to depreciation at the time of the announcement.[5]

Figure 3. Measures of Policy Uncertainty



(a) Trade Policy Uncertainty **(b)** U.S. Economic Policy Uncertainty

NOTE: Trade and economic policy uncertainty indices of Baker, Bloom and Davis (2016) and Caldara et al. (2020) from January 2024 to October 2025. Both series rise sharply around the Liberation Day tariff announcements and remain elevated afterward. Although some uncertainty dissipates, each measure stays above its level from the prior year.

2 Model

We develop a simplified two-country, one-good version of KSY to illustrate the mechanism through which tariff uncertainty can generate a depreciation of the dollar. Relative to KSY, our framework features an endowment economy rather than a full production network, and monetary policy is simplified by fixing the aggregate price level (CPI). In addition, we assume symmetry and set both the elasticity of substitution across goods and the intertemporal elasticity of substitution equal to one, following Cole and Obstfeld (1991).

We further assume that only home households exhibit CARA utility, which introduces

[5] A decline in demand for the dollar is also consistent with hedging behavior observed around the Liberation Day announcement, as documented by Jiang et al. (2025) and BIS (2025). An alternative mechanism is proposed by Itskhoki and Mukhin (2025), who argue that when the tariff-imposing country has a negative net foreign asset position (as in the case of the United States) and its liabilities are denominated in domestic currency while its assets are denominated in foreign currency, an improvement in the trade balance requires a depreciation of the domestic currency. Such a depreciation reduces the real value of liabilities while increasing the value of foreign assets.

sensitivity to consumption variance in the Euler equation. Finally, we introduce financial intermediaries that solve a mean–variance portfolio optimization problem. Under these assumptions, the five-equation system in Kalemli-Özcan, Soylu and Yildirim (2025) collapses to a tractable set of equilibrium conditions that capture the key mechanisms of interest. These conditions, which are also closely related to those in Obstfeld and Rogoff (1995), Itskhoki and Mukhin (2021), and Kekre and Lenel (2024), are presented below. Full derivations are provided in Appendix A.

When a shock occurs, we track changes in variables as percent deviations from their steady-state values, denoted by a caret (ˆ). Unconditional moments (e.g., variances) are evaluated at the ergodic distribution, i.e., under non-zero volatility. The model variables are defined as follows. $p_{H,t}$ denotes the price of home goods produced and consumed domestically, while $p_{F,t}$ denotes the price of foreign goods produced and consumed abroad. $\hat{\mathcal{E}}_t$ is the nominal exchange rate and real exchange rate (since aggregate price levels are fixed) where an increase corresponds to a depreciation of the home currency. $R_{H,t}$ and $R_{F,t}$ denote the nominal interest rates in the home and foreign economies, respectively. $V_{H,t}$ is the net debt position of the home country, inclusive of interest payments. $C_{H,t}$ and $C_{F,t}$ denote consumption by home and foreign households. Finally, $(1-\gamma_H)$ captures home bias in consumption.

Definition 1. An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}\hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the equations (4)-(11) hold:

- Euler equations with Home country exhibiting CARA utility:

$$(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \underbrace{\frac{1}{2}\text{Var}_t(\hat{C}_{t+1})}_{\eta\sigma_t^2} \tag{4}$$

$$(\mathbb{E}_t\hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t}, \tag{5}$$

where η is a constant obtained through second-order approximation.

- Defition of the aggregate price level with policy stabilizing aggregate price levels in both countries:

$$0 = (1-\gamma_H)\,\hat{p}_{H,t} + \gamma_H\,(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \tag{6}$$

$$0 = (1-\gamma_H)\,\hat{p}_{F,t} + \gamma(\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{7}$$

- UIP condition holds with a wedge that depends on the variance of the exchange rate:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t] + \underbrace{\text{Var}_t(\hat{\mathcal{E}}_{t+1})}_{\kappa\sigma_t^2} \tag{8}$$

where κ is a constant obtained through second-order approximation.

- Goods market clears for each country in both periods:

$$0 = (1-\gamma_H)\left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma_H\left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) \tag{9}$$

$$0 = \gamma_H\left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1-\gamma_H)\left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) \tag{10}$$

- Balance of payments equation is given by:

$$R_H^{-1}\hat{V}_{H,t} = \hat{V}_{H,t-1} - \gamma_H\left[(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] \tag{11}$$

The model features two shock variables: $\hat{\tau}_t$ and $\hat{\sigma}_t^2$. The first is a one-time shock to the level of tariffs, expressed as a deviation from the steady state, with $\hat{\tau}_t \sim \mathcal{N}(0, \sigma_{t-1}^2)$. We allow the variance of $\hat{\tau}_t$ to vary exogenously over time, which constitutes the second shock. Importantly, our timing convention assumes that the variance of shocks at time $t+1$ is known and determined at time t. This structure captures tariff uncertainty: the range of possible future tariff realizations widens today. We consider one-time shocks to both the level and the variance of tariffs. As shown in the Appendix, this setup allows us to approximate the variance of the response of endogenous variables, such as $\hat{C}_{H,t+1}$, as proportional to the variance of tariffs, scaled by a constant.

As detailed in the Appendix, our model is largely linear, with the exception of terms involving variances. We therefore perform a second-order approximation and simplify cross terms that are quantitatively negligible in our setting. This yields a system that is linear in the shock variables when σ_t^2 is treated as a state variable (rather than σ_t itself). Under this formulation, we solve the model using the method of undetermined coefficients, following Kalemli-Özcan, Soylu and Yildirim (2025). When the model is solved, tariff level shocks are appreciationary (captured by the first term in Equation 12), whereas shocks to tariff volatility are depreciationary (captured by the second term in Equation 12).

$$\hat{\mathcal{E}}_t = \underbrace{\left(\left(R_H^{-1} - \tfrac{1}{2}\right)(1-2\gamma)^2 - \tfrac{1}{2}\right)}_{<0} \hat{\tau}_t + \underbrace{R_H^{-1}(1-2\gamma)^2(\eta+\kappa)}_{>0} \sigma_t^2 + \underbrace{\frac{(1-R_H^{-1})(1-2\gamma)^2}{\gamma}}_{>0} \hat{V}_{H,t-1} \tag{12}$$

Higher tariff uncertainty induces precautionary saving behavior, as it effectively serves as an Euler equation shock (e.g., similar to a patience shock). This increase in precautionary savings reduces current demand and leads to a depreciation of the home currency. Simultaneously, higher policy volatility induces financial intermediaries to demand a higher risk premium, (e.g., similar to a country risk shock or financial intermediation shock that widens the UIP premium). All else equal, this generates depreciation pressure on the exchange rate while reducing aggregate consumption in the home country relative to the foreign country. Under home bias, where each country consumes a larger share of its own goods, this decline in domestic demand further contributes to currency depreciation. The mechanism is analogous to the precautionary saving channel induced by volatility shocks.

To formalize this intuition, let us define the perfect risk sharing benchmark and endogenous deviations from it, denoted by $\hat{w}_t$, as follows: $\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t$. Under complete markets (e.g., with Arrow-Debreu securities), there would be no wedge $\hat{w}_t = 0$.[6] In this benchmark case, perfect ex-ante insurance by households ensures that capital flows in the direction of the country, whose consumption basket is cheaper. The risk-sharing wedge, $\hat{w}_t$ captures deviations from this benchmark. When $\hat{w}_t > 0$, ($\hat{w}_t < 0$) all else being equal, this pushes the nominal exchange rate to depreciate (appreciate) and it reduces (increases) the consumption of the home country relative to the foreign country.

When the model is solved, the risk-sharing wedge can be expressed as a function of the state variables, as shown in equation (13). Tariff level shocks reduce $\hat{w}_t$, which tends to lower $\hat{\mathcal{E}}_t$ (an appreciation) while increasing $(\hat{C}_{H,t} - \hat{C}_{F,t})$, implying higher relative home consumption. In this sense, tariff level shocks transfer wealth to the country that is imposing unilateral tariffs; compared to the perfect risk sharing benchmark, the home country's relative consumption is higher and its currency is more valuable. In contrast, shocks to tariff volatility generate a positive wedge between the exchange rate and relative consumption. They push $\hat{\mathcal{E}}_t$ upward (a depreciation) while reducing $(\hat{C}_{H,t} - \hat{C}_{F,t})$, thereby worsening risk sharing. By the same logic as above, volatility shocks transfer wealth away from the home country. Formally,

$$\hat{w}_t = \underbrace{R_H^{-1}(\eta + \kappa)\sigma_t^2 + \frac{\left(1 - R_H^{-1}\right)}{\gamma}\hat{V}_{H,t-1}}_{>0} + \underbrace{\left(R_H^{-1} - 1\right)\hat{\tau}_t}_{<0} \tag{13}$$

When both shocks (tariff levels and tariff volatility) are present, the overall effect on the exchange rate is ambiguous. When the sensitivity to tariff volatility in the Euler equation, captured by η, the sensitivity of investors to tariff-related volatility κ, and the underlying volatility of tariffs captured by σ_t are all large, it is possible for tariff volatility shocks to yield depreciation even at the same time as a tariff hike. Thus, whether appreciationary forces

[6] This would be a special case of the Backus-Smith condition often denoted as $\hat{Q}_t = \hat{P}_{F,t} + \hat{\mathcal{E}}_t - \hat{P}_{H,t} = \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$, where $\hat{Q}_t$ is the real exchange rate. In our model because the aggregate price level in both countries are stabilized, the real exchange rate is the same as the nominal exchange rate and with $\sigma = 1$ we arrive at the expression above.

from tariff levels or depreciationary forces from tariff volatility dominate is a quantitative question, which we turn to next.

3 Data and Construction of Shocks

We obtain all tariff data from WTO and IMF (2025). The final implemented tariffs are measured as of October 17, 2025 and differ substantially from those announced on Liberation Day. Subsequent changes in implemented tariffs after this date are negligible (see KSY).

To compute the time-varying standard deviation of tariffs, we compile all tariff-related events—including announced and threatened tariffs—from the Trade Compliance Resource Hub (Lowell et al., 2025). Using these data, we construct a time series of tariff volatility. Because announcements and threats reflect the full range of tariff levels under consideration by policymakers, this measure captures the uncertainty surrounding future trade policy. In particular, it incorporates erratic announcements, such as announced 125% tariffs on China or the 250% tariffs threatened against the Canadian dairy industry, which we believe played an important role in amplifying tariff uncertainty.

Specifically, for each date, we compute the standard deviation of all tariffs that have been announced or threatened up to that point in time. On average, tariff volatility is 72% during the first quarter of 2025, declines to 60% by the end of the second quarter, and falls further to 49% by the end of the third quarter. Over the full sample period, the standard deviation of tariffs is 43%. Our tariff volatility measure follows a similar path to the Trade Policy Uncertainty Caldara et al. (2020) Index in our period of interest from 4Q2024-3Q2025.

Exchange rate volatility also varies considerably across time horizons and currency measures. For example, between April 1 and June 30, the volatility of the dollar–euro exchange rate is 36%, while the corresponding volatility of the broad dollar index is 16%. Between April 1 and October 1, dollar–euro volatility is 27%, compared with 11% for the broad dollar.

Finally, between April 1 and December 1, dollar–euro volatility reaches 23%, whereas the broad dollar exhibits a volatility of 10%.

Table 2. U.S. Weighted Average Implemented Tariff Rate

Date	Tariff (%)	Date	Tariff (%)
1-Jan-25	2.5	23-Jun-25	15.6
4-Feb-25	3.9	30-Jun-25	15.6
4-Mar-25	11.7	1-Aug-25	15.6
7-Mar-25	6.3	6-Aug-25	15.8
12-Mar-25	7.4	7-Aug-25	17.0
3-Apr-25	8.5	27-Aug-25	17.4
5-Apr-25	12.2	1-Sep-25	17.4
9-Apr-25	21.4	8-Sep-25	17.3
3-May-25	23.0	16-Sep-25	17.2
14-May-25	13.8	1-Oct-25	17.2
4-Jun-25	15.5		

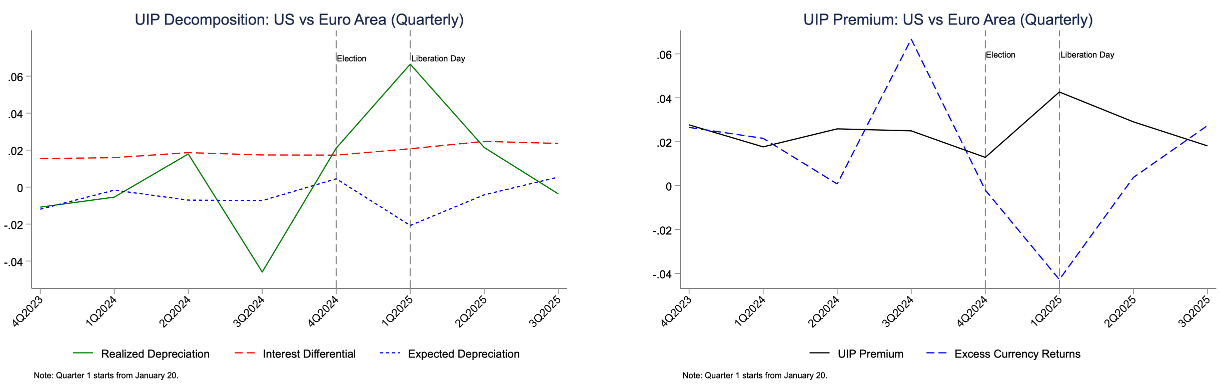
NOTE: Table 2 reports the average U.S. tariff rates between January 1, 2025 and October 1, 2025. We downloaded the tariff data from WTO and IMF (2025) as of October 17, 2025, at the HS-6 level of classification. To compute the average U.S. tariff rate, we use 2024 import weights to aggregate tariff rates across products.

Finally, to provide empirical basis for our quantitative exercise, we calculate the UIP risk-premium wedge, following Kalemli-Özcan and Varela (2021). Figure 4 plots the UIP risk-premium wedge alongside realized excess currency returns on the dollar (second panel), together with their individual components (first panel). To align the data with the timing of tariff policy changes, this figure and the rest of the paper defines quarterly values as follows: we assume the first quarter of 2025 starts on January 20, 2025 with the inauguration of the president and all other quarters are shifted by 20 days accordingly.[7] Using Consensus

[7]With this definition of quarters, we are able to capture both Liberation Day announcements and the

survey data on exchange rate expectations, we compute the UIP premium. Figure 4a shows that following the election, both spot depreciation and future expected appreciation widened substantially, implying a higher UIP premium as shown in panel b. This pattern is consistent with the elevated trade and economic policy uncertainty documented by Baker, Bloom and Davis (2016) and Caldara et al. (2020), shown in Figure 3. As uncertainty increased and the UIP premium rose, the U.S. dollar depreciated. Because the depreciation occurred contemporaneously, agents subsequently expected an appreciation of the dollar. The first major tariff increases were announced in March 2025, which coincides with the peak of the UIP premium. The subsequent survey wave aligns with the week in which tariff pauses were announced (April 9th), after which the UIP premium begins to decline and realized excess currency returns increase.

Figure 4. Decomposing the UIP Condition and Excess Currency Returns



(a) Interest Differential and Depreciation

(b) UIP and Excess Currency Returns

NOTE: Figure 4a plots the components, whereas Figure 4b plots the UIP premium calculated with Consensus survey data and realized excess currency returns. All calculations are for dollar vs euro. To capture the timeline of tariff policy changes, in this figure and throughout the paper, we calculate quarterly figures with the first quarter of 2025 starting on January 20, 2025 with the inauguration and move each quarter by 20 days accordingly. With this definition of quarters, we are able to capture both Liberation Day announcements and the major tariff rate increases in February and March 2025 in the same quarter.

major tariff rate increases in February and March 2025 in the same quarter. In the quantitative model, this will correspond to the impact period of the shock.

4 Quantitative Exercise

Next, we feed two shocks into the model: a level shock to tariffs and a volatility shock. Specifically, the level of tariffs increases by 18.9 percentage points, while the variance of tariffs rises by 72.1 percentage points relative to their steady-state values.[8] Both shocks are introduced as one-time innovations. Based on the KSY dataset, which relies on the OECD Inter-Country Input–Output (ICIO) tables (Yamano and et al., 2023) for 2019, we calibrate the home-bias parameter to $\gamma = 0.0708$, equal to the foreign expenditure share in U.S. final consumption of goods and services. The analytical solution yields exact coefficients for κ and η dependent on model primitives and these are calibrated accordingly. As shown in the Appendix, the UIP wedge contains a risk sensitivity parameter, χ; we calibrate this to match the deviation of the UIP wedge from the last quarter of 2024, which we treat as the steady state.

Consistent with the model's predictions, Figure 5a shows that tariff level shocks are appreciationary, whereas increases in tariff volatility are depreciationary. In particular, the level shock generates a 2.6% appreciation of the exchange rate, while the volatility shock leads to a 2.3% depreciation. These numbers are highly sensitive to parametrization and assumptions; the model here is a parsimonious one.[9] With that caveat this exercise shows that there are two competing pressures from the introduction of tariffs in the first quarter of 2025. One that raised the level of tariffs, and thereby created appreciationary pressure and another that widened the range of possible future tariffs leading to depreciationary pressure. These pressures are large enough that under different parametrizations and with a more detailed model, one can match more closely the path of the observed exchange rate. For example, in this exercise $\theta = 1$ and we abstract away from higher order terms and endoge-

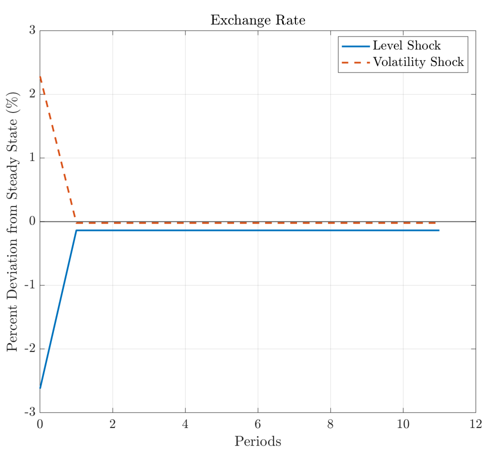
[8] From Table 2, the 18.9 percentage point tariff level shock is calculated as the difference between April 9 and January 1 effective tariff rates.

[9] For example, persistence (and perceived persistence) of the shocks matter significantly, as explored in KSY. If agents expect tariffs not to persist, the appreciationary effect of tariff levels can be dampened. Conversely, if tariff volatility is persistent rather than transitory, its depreciationary effect can be amplified.

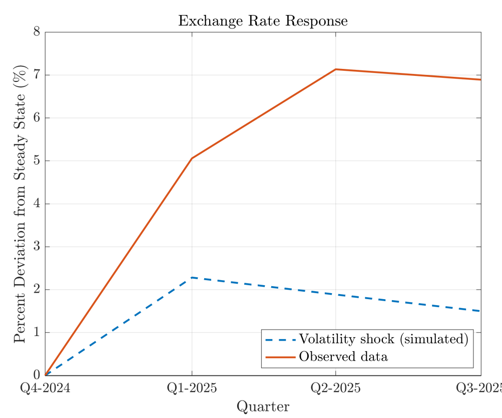
nously time-varying portion of the risk premium. With differing elasticities of substitution and parameter asymmetry across countries introduced, as in KSY, the appreciationary impact of tariffs can also be muted to the point that the volatility shock's depreciationary impact sufficiently exceeds the appreciationary impact of tariffs.

Figure 5b presents an alternative thought experiment. Our baseline model features one-time volatility shocks with constant loadings. Here, we consider the impulse responses resulting from a series of volatility shocks that reveal themselves in successive periods. We use our time-varying tariff volatility series and feed this series into the model as a sequence of one-time volatility shocks, in the spirit of the tariff-threat shocks studied by KSY. We construct a cumulative IRF from these successive shocks and compare the model-implied dynamics to the percent deviation of the quarterly Nominal Broad U.S. Dollar Index from its 4Q2024 level. Figure 5b depicts the path of the exchange rate following these successive volatility shocks. In terms of scale, nearly half of the exchange rate response on impact in the first quarter of 2025 can be explained with the volatility shock.

Figure 5. Exchange Rate Responses to Tariff Level and Volatility Shocks



(a) Exchange Rate Responses to Level and Volatility Shocks



(b) Comparing Model Response to Data: Evolving Tariff Volatility

NOTE: Figure 5 shows the model-implied responses to shocks in $\hat{\tau}_t$ and σ_t^2. Panel (b) additionally compares the model-implied dynamics with the observed exchange rate response under evolving tariff volatility.

5 Conclusion

This paper studies how tariff uncertainty affects exchange rates, with a particular focus on the depreciation of the U.S. dollar following the 2025 tariff announcements. While standard trade models predict that tariffs lead to currency appreciation, we show that this result can be overturned when tariff policy is uncertain. By introducing risk-averse households and financial intermediaries and allowing tariff volatility to affect the uncovered interest parity condition, we demonstrate that uncertainty reduces demand and generates a risk-premium wedge that weakens the currency of the tariff-imposing country.

Our quantitative results indicate that plausible increases in tariff uncertainty can account for a sizable share of the observed dollar depreciation, even in the presence of rising tariff levels. The mechanism operates through both precautionary savings behavior and increased compensation for exchange rate risk, reducing demand for domestic assets at the time of heightened uncertainty. This framework helps reconcile the observed exchange rate dynamics during the 2025 tariff episode with standard macro-trade theory.

Our findings carry important policy implications, suggesting that the dollar's "exorbitant privilege" may not be permanent. Dollar dominance rests on its functional advantages (trade invoicing, medium of exchange, unit of account, and lower borrowing costs), incumbency through network effects, and its perceived safety as a store of value. However, as argued by Rogoff (2025), this safety is increasingly in question due to persistent U.S. fiscal excesses and rising public debt, which under the current administration also intersect with concerns about Federal Reserve independence. In this context, previously unthinkable outcomes—such as a loss of investor confidence in the dollar—become conceivable. Major policy errors, including trade policies that heighten uncertainty as documented in this paper, may accelerate an erosion of confidence that is already underway as argued by Rogoff (2025).

In fact, our findings emphasize that trade policy uncertainty is not merely a second-order feature of tariff policy but a central determinant of macro-financial outcomes. Incorporating uncertainty into macro–trade models is therefore essential for understanding exchange rate movements during periods of geopolitical and policy instability. Future work could extend this framework to richer production networks, endogenous policy formation, or heterogeneous exposure across sectors and countries.

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A Model Appendix

A.1 Set-up

Timing and countries. Home (subscript H) and Foreign (subscript F) have representative households. Each country is endowed with its own tradable good, $y_{H,t}$ and $y_{F,t}$. One-period nominal discount bonds in each currency are traded internationally.

Preferences and intratemporal aggregator. Each household consumes a CES bundle of Home and Foreign Goods:

$$\max_{\{C_{H,t}, c_{H,H,t}, c_{H,F,t}\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(C_{H,t})\right], \qquad C_{H,t} = \left[(1-\gamma_H)\, c_{H,H,t}^{\frac{\theta-1}{\theta}} + \gamma_H\, c_{H,F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \;\; \theta > 0,\; \theta \neq 1.$$

(Home bias parameter $1 - \gamma_H \in (0,1)$; Foreign reliance on Home is $1 - \gamma_F$.)

Given home-currency good prices $p_{H,H,t} = p_{H,t}, p_{H,F,t}$, the unit-expenditure (CPI) index and Hicksian demands are

$$P_{H,t} = \left[(1-\gamma_H)\, p_{H,t}^{1-\theta} + \gamma_H\, p_{H,F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \quad c_{H,t} = (1-\gamma_H)\left(\frac{p_{H,t}}{P_{H,t}}\right)^{-\theta} C_{H,t}, \quad c_{F,t} = \gamma_H \left(\frac{p_{H,F,t}}{P_{H,t}}\right)^{-\theta} C_{H,t}.$$

Foreign utility maximization, CPI $P_{F,t}$ and demands $c_{F,H,t}, c_{F,F,t}$ are defined analogously with prices in Foreign currency $p_{F,H,t}, p_{F,t}$.

Law of One Price, tariffs and exchange rates. Let $\mathcal{E}_t$ be the nominal exchange rate (Home currency per unit of Foreign currency). For each good,

$$p_{F,H,t} = \frac{p_{H,t}}{\mathcal{E}_t} \qquad p_{H,F,t} = \mathcal{E}_t\, p_{F,t} \tau_t$$

where τ_t is a gross import tariff rate imposed by the home country on the foreign country.

The gross tariff rate is a random variable, whereby $\tau_t = e^{\hat{\tau}_t}$ and $\hat{\tau}_t$ is independently and identically distributed following a normal distribution: $\hat{\tau}_t \sim \mathcal{N}(0, \sigma_{t-1}^2)$. We allow for the variance of $\hat{\tau}_t$ to exogenously vary across time. Notably, we choose the following timing convention. The next period's tariff shock depends on a variance term that is known today: $\text{Var}_t(\hat{\tau}_{t+1}) = \sigma_t^2$. We do this for notational ease in capturing uncertainty. In our stylized model, to capture uncertainty, we will consider one-time changes in σ_t^2, which will impact next period's state variable, $\hat{\tau}_t$. That is today uncertainty increases about tomorrow's tariffs.

Define the real exchange rate $Q_t \equiv \mathcal{E}_t \, P_{F,t}/P_{H,t}$.

Home and foreign budget constraints with nominal bonds.

$$P_{H,t}C_{H,t} + V_{H,t-1} = p_{H,t}y_{H,t} + \frac{V_{H,t}}{R_{H,t}}$$
$$P_{F,t}C_{F,t} + V_{F,t-1} = p_{F,t}y_{F,t} + \frac{V_{F,t}}{R_{F,t}}$$

where $V_{H,t}$ and $V_{F,t}$ are nominal bonds denominated in local currency.

Policy. Let us assume monetary policy in the two countries, perfectly stabilizes the aggregate price level such that:

$$P_{H,t} = P_{F,t} = 1 \; \forall t$$

A.2 Households' Problem

The Home household maximizes expected discounted utility over an infinite horizon subject to a sequence of nominal budget constraints:

$$\max_{\{C_{H,t}, V_{H,t}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t u(C_{H,t})$$

subject to

$$P_{H,t}C_{H,t} + V_{H,t-1} = p_{H,t}y_{H,t} + \frac{V_{H,t}}{R_{H,t}}$$

for all $t \geq 0$, given initial debt $V_{H,-1}$.

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t \Big[u(C_{H,t}) + \lambda_{H,t} \Big(p_{H,t}y_{H,t} - P_{H,t}C_{H,t} - V_{H,t-1} + \tfrac{V_{H,t}}{R_{H,t}} \Big) \Big].$$

FOCs with respect to $C_{H,t}$ and $V_{H,t}$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_{H,t}} &: \quad u'(C_{H,t}) - \lambda_{H,t}P_{H,t} = 0, \\ \frac{\partial \mathcal{L}}{\partial V_{H,t}} &: \quad \frac{\lambda_{H,t}}{R_{H,t}} - \beta_H \mathbb{E}_t[\lambda_{H,t+1}] = 0. \end{aligned}$$

Since policy perfectly stabilizes the aggregate price level such that:

$$P_{H,t} = P_{F,t} = 1 \quad \forall t,$$

we obtain the Euler equation:

$$u'(C_{H,t}) = \beta_H R_{H,t} \, \mathbb{E}_t[u'(C_{H,t+1})] \, .$$

Using the CARA form, $u'(c) = e^{-\alpha c}$,

$$e^{-\alpha C_{H,t}} = \beta_H R_{H,t} \, \mathbb{E}_t \big[e^{-\alpha C_{H,t+1}} \big] \, .$$

Since uncertainty is driven by tariff shocks and $C_{H,t+1}$ is conditionally normal,

$$C_{H,t+1} \mid \mathcal{F}_t \sim \mathcal{N}(\mathbb{E}_t C_{H,t+1},\ \mathrm{Var}_t(C_{H,t+1}))\,.$$

Then

$$\mathbb{E}_t\big[e^{-\alpha C_{H,t+1}}\big] = \exp\big(-\alpha\,\mathbb{E}_t C_{H,t+1} + \tfrac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\big)\,.$$

So the Euler equation is:

$$\begin{aligned} e^{-\alpha C_{H,t}} &= \beta_H R_{H,t} e^{\left(-\alpha\,\mathbb{E}_t C_{H,t+1} + \frac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\right)} \\ e^{\left(\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) - \frac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\right)} &= \beta_H R_{H,t}. \end{aligned}$$

Taking logs:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \tfrac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1}).$$

We shall assume that the foreign household, has CRRA utility instead of CARA utility, so that yields (with its discount factor):

$$\left(\frac{\mathbb{E}_t C_{F,t+1}}{C_{F,t}}\right)^{\alpha} = \beta_F R_{F,t}$$

A.3 Relative Demand Conditions

Standard CES structure yields:

$$\begin{aligned} \frac{c_{H,H,t}}{c_{H,F,t}} &= \frac{1-\gamma_H}{\gamma_H}\left(\frac{p_{H,t}}{p_{H,F,t}}\right)^{-\theta} \\ \frac{c_{F,H,t}}{c_{F,F,t}} &= \frac{\gamma_F}{1-\gamma_F}\left(\frac{p_{F,H,t}}{p_{F,t}}\right)^{-\theta} \end{aligned}$$

A.4 Intermediaries' Problem

Redefine bond variable inclusive of interest payments. Intermediaries solve a mean-variance optimization problem:

$$\max_{V_{H,t},V_{F,t}} \quad \mathbb{E}_t \pi_{H,t+1} - \frac{\chi_t}{2} \operatorname{Var}_t(\pi_{H,t+1})$$

subject to the resource constraint at t,

$$0 = \mathcal{E}_t \frac{V_{F,t}}{R_{F,t}} + \frac{V_{H,t}}{R_{H,t}},$$

and with profits (in Home currency) realized at $t+1$,

$$\pi_{H,t+1} = V_{H,t} + \mathcal{E}_{t+1} V_{F,t}.$$

Using the constraint to substitute out $V_{H,t} = -R_{H,t}\mathcal{E}_t \frac{V_{F,t}}{R_{F,t}}$, we obtain

$$\pi_{H,t+1} = V_{F,t}\Big(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}}\Big) \tag{14}$$

Therefore, the (unconstrained) Lagrangian/objective can be written as

$$\mathcal{L}_t = \mathbb{E}_t\Big[V_{F,t}\Big(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}}\Big)\Big] - \frac{\chi_t}{2}(V_{F,t})^2 \operatorname{Var}_t\Big(\mathcal{E}_{t+1}\Big),$$

where $\chi_t > 0$ is an aversion to risk. First order condition (interior) yields:

$$0 = \mathbb{E}_t\Big[\Big(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}}\Big)\Big] - \chi_t(V_{F,t}) \operatorname{Var}_t(\mathcal{E}_{t+1}), \tag{15}$$

$$\iff \quad \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}} = \mathbb{E}_t[\mathcal{E}_{t+1}] - \chi_t(V_{F,t}) \operatorname{Var}_t(\mathcal{E}_{t+1}) \tag{16}$$

Dividing both sides by $\mathcal{E}_t$:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t \frac{V_{F,t}}{\mathcal{E}_t} \text{Var}_t(\mathcal{E}_{t+1})$$

Rearranging:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t \underbrace{V_{F,t}\mathcal{E}_t}_{Quantity\ of\ Risk} \underbrace{\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)}_{Price\ of\ Risk}$$

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi_t V_{H,t} \frac{R_{F,t}}{R_{H,t}} \text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$$

This yields a UIP condition with a deviation based on quantity of risk times price of risk (variance of depreciation). In the usual interpretation, with $R_{H,t} > R_{F,t}$, $V_{F,t} < 0$ as intermediaries go long in the home currency and short the foreign currency, so as variance increases the UIP wedge increases.

Let us suppose risk aversion is such that intermediaries are not sensitive to quantity of risk:

$$\chi_t = \chi \frac{R_{H,t}}{R_{F,t}} \frac{1}{V_{H,t}}$$

This allows a convenient risk premium expression, ρ_t where

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \underbrace{\chi \text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)}_{\rho_t}$$

We assume that the intermediation profits are transferred to the foreign household. We make this simplification for analytical simplicity.

A.5 Equilibrium Definition

An equilibrium comprises 8 sequences $\{p_{H,t}, p_{F,t}, \mathcal{E}_t, R_{H,t}, R_{F,t}, V_{H,t}, C_{H,t}, C_{F,t}\}_{t=0}^{\infty}$ such that, exogenous variables (τ_t, y_H, y_F), equations (17)-(24) hold:

- Euler equations:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \frac{1}{2}\alpha^2 \text{Var}_t(C_{H,t+1}) \tag{17}$$

$$\left(\frac{\mathbb{E}_t C_{F,t+1}}{C_{F,t}}\right)^{\alpha} = \beta_F R_{F,t} \tag{18}$$

- CPI equations with price level substituted out:

$$1 = \left[(1-\gamma_H)\, p_{H,t}^{1-\theta} + \gamma_H \left(\mathcal{E}_t\, p_{F,t}\tau_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{19}$$

$$1 = \left[(1-\gamma_F)\, p_{F,t}{}^{1-\theta} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{20}$$

- UIP condition:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi \text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \tag{21}$$

- Goods Market Clears for each country in both periods

$$y_{H,t} = (1-\gamma_H)\left(\frac{p_{H,t}}{1}\right)^{-\theta} C_{H,t} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} \tag{22}$$

$$y_{F,t} = \gamma_H \left(\frac{\mathcal{E}_t\, p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t} + (1-\gamma_F)\left(\frac{p_{F,t}}{1}\right)^{-\theta} C_{F,t} \tag{23}$$

- Balance of payments equation

$$\frac{V_{H,t}}{R_{H,t}} = V_{H,t-1} - \text{NX}_t$$

$$= V_{H,t-1} - p_{H,t}c_{F,H,t} + p_{F,t}\mathcal{E}_t c_{H,F,t}$$

$$= V_{H,t-1} - p_{H,t}\gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} + p_{F,t}\mathcal{E}_t\gamma_F \left(\frac{\mathcal{E}_t\, p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t} \tag{24}$$

A.6 2nd Order Approximation

Let us assume $\gamma_H = \gamma_F$ and use the Cole and Obstfeld (1991) parametrization $\alpha = \theta = 1$.[10] The two countries will be different in size, so even as home bias parameters are the same trade will be unbalanced at the steady state. To achieve this we normalize home country's consumption $C_H = 1$ and let C_F be some constant (i.e., the rest of the world can be larger).

We will conduct a second-order approximation around a steady state with unconditional moments evaluated at the ergodic distribution. Hat variables denote deviation from this steady state:

A.6.1 Steady State

- Euler equations:

$$\ln(\beta R_H) = -\frac{1}{2}\sigma_c^2 \to R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}$$

$$R_F = \beta_F^{-1}$$

  We'll assume in our work that volatility is small enough that the steady-state gross interest rate is greater than 1: $R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2} > 1$.

- All prices and exchange rate will be 1 at the steady state:

$$P_H = P_F = \mathcal{E} = 1$$

[10] When $\theta = 1$, the consumption price basket becomes a Cobb-Douglas function that is exactly log-linear.

- UIP condition:

$$\frac{R_H}{R_F} = 1 + \ \sigma_{\mathcal{E}}^2$$

Plugging in what we know from the Euler equation we can show how the endogenous volatility of the exchange rate is related to the endogenous volatility of consumption:

$$\frac{R_H}{R_F} = 1 + \ \sigma_{\mathcal{E}}^2 = \frac{\beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}}{\beta_F^{-1}}$$

Importantly, σ_c^2 and $\sigma_{\mathcal{E}}^2$ are a function of the volatility of tariffs, σ^2, at the steady state. For the existence of a steady state, with a constant exchange rate, we assume that β_H and β_F are such that the equality above holds.

- Goods Market Clears for each country in both periods

$$y_H = 1 + \underbrace{\gamma_H(C_F - 1)}_{\bar{\text{NX}}} = 1 + \bar{\text{NX}}$$
$$y_H = 1 - \gamma_H(C_F - 1) = 1 - \bar{\text{NX}}$$

- Balance of payments equation

$$\begin{aligned}
\frac{V_H}{R_H} &= V_H - \gamma_H C_{F,t} + \gamma_H \\
&= V_H - \underbrace{\gamma_H(C_{F,t} - 1)}_{\bar{\text{NX}}} \\
\frac{V_H}{R_H} &= V_H - \bar{\text{NX}} \\
V_H &= \frac{R_H}{R_H - 1} \bar{\text{NX}}
\end{aligned}$$

In essence, these expressions demonstrate that setting the relative size of the countries with C_F is equivalent to setting net exports, which helps determine size of steady-state endowment and debt. If two countries both allocate 10% of their consumption to foreign goods, but one of them is twice the size of the other one, the larger country will be a net importer.

A.6.2 Approximation

Given the parametrization, with $\alpha = \theta = 1$, the model is already highly linear except for the variance terms, so we shall focus on those. [11]

There are two key variances in the model $\text{Var}_t(C_{t+1})$ and $\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$. Any variable can be expressed as the steady-state value multiplied by percent deviation: $X_t = X(1 + \hat{X}_t)$. Then we can write:

$$
\begin{aligned}
\text{Var}(C_{t+1}) &= \text{Var}(C(1 + \hat{C}_{t+1})) \\
&= \text{Var}(\hat{C}_{t+1})
\end{aligned}
$$

Similarly, since $log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right) \approx \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t$ and $e^x \approx 1 + x$, so we have:

$$
\begin{aligned}
\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) &= \text{Var}_t\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right) \\
&= \text{Var}_t\left(e^{log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right)}\right) \\
&\approx \text{Var}_t\left(1 + \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right) \\
&= \text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)
\end{aligned}
$$

[11] We assume near equivalence of log deviation and percent deviation from the steady state. Additionally with consumption and prices normalized to 1, many terms can be converted into hat variables by taking logs.

With these the Euler equation and the UIP condition read as follows:

$$(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \frac{1}{2}\text{Var}(\hat{C}_{t+1})$$
$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t[\hat{\mathcal{E}}_{t+1}] - \hat{\mathcal{E}}_t + \chi\text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)$$

We have a system with three state variables: $\hat{\tau}_t, \hat{V}_{t-1}, \sigma_t^2$. With a second order approximation, every endogenous variable, $\hat{x}_t$ (e.g., including $\hat{\mathcal{E}}_{t+1}$ and $\hat{C}_{t+1}$) will be some linear and quadratic function of the state variables:

$$\hat{x}_t = a_1\hat{\tau}_t + a_2\hat{V}_{H,t-1} + a_3\sigma_t^2 + a_4\hat{\tau}_t^2 + a_5\hat{V}_{H,t-1}^2 + a_6(\sigma_t^2)^2$$
$$+ a_7\hat{\tau}_t\hat{V}_{H,t-1} + a_8\hat{\tau}_t\sigma_t^2 + a_9\hat{V}_{H,t-1}\sigma_t^2.$$

Iterating one period forward:

$$\hat{x}_{t+1} = a_1\hat{\tau}_{t+1} + a_2\hat{V}_{H,t} + a_3\sigma_{t+1}^2 + a_4\hat{\tau}_{t+1}^2 + a_5\hat{V}_{H,t}^2 + a_6(\sigma_{t+1}^2)^2$$
$$+ a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma_{t+1}^2 + a_9\hat{V}_{H,t}\sigma_{t+1}^2.$$

Let us assume that we are interested in one-time increases in uncertainty, so $\sigma_{t+j} = 0 \forall j > 0$ and this is known by agents. If the variance of $\hat{\tau}_{t+1}$ is σ_t^2, then

$$\begin{aligned}\text{Var}_t(\hat{x}_{t+1}) &= \text{Var}_t\Big(a_1\hat{\tau}_{t+1} + a_4\hat{\tau}_{t+1}^2 + a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma_{t+1}^2\Big)\\ &= \text{Var}_t\Big(\big(a_1 + a_7\hat{V}_{H,t}\big)\hat{\tau}_{t+1} + a_4\hat{\tau}_{t+1}^2\Big)\\ &= \big(a_1 + a_7\hat{V}_{H,t}\big)^2\sigma_t^2 \ + \ 2a_4^2\sigma_t^4\end{aligned}$$

We shall assume $\sigma_t < 1$. Given this, for small shocks, higher order terms will be near zero (i.e. $\big(a_7\hat{V}_{H,t}\big)^2\sigma_t^2 \ + \ 2a_4^2\sigma_t^4 \approx 0$. That is we choose to focus on a variance that is only a function of the exogenous variance of $\hat{\tau}_t$, and simplify away the endogenous time-varying component.[12] With that, the variance term will be directly a function of the dependence of the endogenous variable on $\hat{\tau}_t$:

$$\text{Var}_t(\hat{x}_{t+1}) \approx a_1^2 \sigma_t^2$$

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2)\}_{t=0}^{\infty}$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta \sigma_t^2 \tag{25}$$

$$(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \tag{26}$$

- CPI equations with price level substituted out:

$$0 = (1 - \gamma_H)\, \hat{p}_{H,t} + \gamma_H \,(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \tag{27}$$

$$0 = (1 - \gamma_F)\, \hat{p}_{F,t} + \gamma_F (\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{28}$$

- UIP condition with a wedge that endogenously widens as outstanding debt increases and as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa \sigma_t^2 \tag{29}$$

[12] That is via the $a_7 \hat{V}_{H,t}$ term, there could otherwise be endogenous time variation in how the variance of $\hat{C}_{t+1}$ and $\hat{\mathcal{E}}_{t+1}$ depends on the underlying variance of tariffs. We assume that in our model and context of shocks, this is small enough to simplify away.

- Goods market clears for each country in both periods

$$0 = (1-\gamma_H)\left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma_H C_F \left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) \tag{30}$$

$$0 = \gamma_F \left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1-\gamma_F) C_F \left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) \tag{31}$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:[13]

$$\hat{V}_{H,t} = R_H \hat{V}_{H,t-1} - \Xi \left[C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] + \hat{i}_{H,t} \tag{36}$$

For analytical tractability let us additionally assume symmetry (i.e. setting $\gamma_H = \gamma_F$) and an initial position of zero debt. Then equilibrium conditions read as follows:

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta \sigma_t^2 \tag{37}$$

$$(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \tag{38}$$

[13] The derivation is as follows:

$$\frac{V_{H,t}}{R_{H,t}} = V_{t-1} - p_{H,t}\gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} + p_{F,t}\mathcal{E}_t\gamma_H \left(\frac{\mathcal{E}_t\, p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t} \tag{32}$$

$$\frac{V_H}{R_H}(\hat{V}_{H,t} - \hat{i}_{H,t}) = V_H \hat{V}_{H,t-1} - \left(\gamma_H C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H(\hat{C}_{H,t} - \hat{\tau}_t)\right) \tag{33}$$

$$(\hat{V}_{H,t} - \hat{i}_{H,t}) = R_H \hat{V}_{H,t-1} - \frac{R_H}{V_H}\left(\gamma_H C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H(\hat{C}_{H,t} - \hat{\tau}_t)\right) \tag{34}$$

$$\hat{V}_{H,t} = R_H \hat{V}_{H,t-1} - \underbrace{\frac{R_H - 1}{\bar{\text{NX}}}\gamma_H}_{\Xi}\left(C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right) + \hat{i}_{H,t} \tag{35}$$

where the last two lines follow from $V_H = \frac{R_H}{R_H - 1}\bar{\text{NX}}$.

- CPI equations with price level substituted out:

$$0 = (1 - \gamma_H)\, \hat{p}_{H,t} + \gamma_H \left(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t\right) \tag{39}$$

$$0 = (1 - \gamma_H)\, \hat{p}_{F,t} + \gamma(\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{40}$$

- UIP condition with a wedge that endogenously widens as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa \sigma_t^2 \tag{41}$$

- Goods Market Clears for each country in both periods

$$0 = (1 - \gamma_H) \left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma_H \left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) \tag{42}$$

$$0 = \gamma_H \left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1 - \gamma_H) \left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) \tag{43}$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:

$$R_H^{-1} \hat{V}_{H,t} = \hat{V}_{H,t-1} - \gamma_H \left[(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] \tag{44}$$

Global Networks, Monetary Policy and Trade*

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Abstract

We develop a New Keynesian open-economy model with incomplete financial markets to study the macroeconomic impact of trade distortions. Our global general equilibrium framework features multi-sector-country production networks with full input–output linkages and nominal rigidities. Tariffs are simultaneous demand and supply shocks, leading to distortions in consumption, production, and international risk sharing. We show analytically that production networks are important both for dynamic propagation of shocks and transmission of distortions across sectors and countries, shaping the inflation–output trade-off at home. Heterogeneity in price stickiness and complementarities in production generate persistent inflation combined with larger output loss relative to canonical models. Overall impact of tariffs, theoretically and quantitatively, depends on the endogenous response of monetary policy, both at home and abroad, even without retaliation. Quantitatively, large U.S. tariffs with no monetary policy response induce stagflation worldwide, while U.S. tariff-threats generate deflationary pressures and a depreciation of the dollar.

JEL Codes: E2, E3, E6, F1, F4

Keywords: Tariffs, input-output linkages, inflation expectations, exchange rates, trade imbalances, monetary policy

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Online Appendix available here.

1 Introduction

We introduce a new framework to analyze the macroeconomic consequences of protectionist trade policies. Our global New Keynesian open-economy (NKOE) model incorporates realistic structural features—including full international input-output (I-O) linkages, sector-specific nominal rigidities, and cross-country heterogeneity in monetary policy preferences—to provide a comprehensive assessment of both domestic and global macro effects of tariffs. The model allows trade to be unbalanced and assumes incomplete financial markets. Our core contribution is to delineate how the macroeconomic impact of tariffs can differ from canonical models when tariffs act as simultaneous demand and supply shocks, create distortions both in consumption and production decisions, and impact international risk sharing when countries can borrow to smooth the impact of tariff shock.

We start by introducing five primitives:

(i) *Home Consumption Bias:* Consumers in each country make choices prior to the imposition of tariffs, revealing their biases toward home and foreign goods. This is captured by the *consumption share matrix* $\mathbf{\Gamma}$; to relate to standard small open economy (SOE) and two-country settings, we use scalar counterparts for home (H) and foreign (F) countries, denoted by γ_H and $\gamma_F = 1 - \gamma_H$.

(ii) *Production Interdependence:* Producers optimize their production by sourcing inputs globally. This is represented by the *I-O matrix* $\mathbf{\Omega}$, with scalar counterparts Ω_H and Ω_F capturing home and foreign input shares, respectively.

(iii) *Import Substitution:* Goods from any country can be substituted—both in consumption and production—by goods within the same sector or across sectors. This is modeled through nested CES bundles. The *elasticities of substitution* (EoS) are given by the vector $\boldsymbol{\theta}$, giving us flexibility in our assumptions, where EoS can differ (substitutes vs. complements) for upper vs. lower layers of the production/supply chain.

(iv) *Nominal rigidities:* The sluggish adjustment of prices captured by the frequency of price adjustment at the sectoral level, denoted by $\mathbf{\Lambda}$, allowing different rigidities across tradable (manufacturing) and non-tradable sectors (services).

(v) *Policy:* Monetary policy determines the price level. Central banks can respond to price changes according to a Taylor rule, with response coefficients captured by the diagonal matrix $\mathbf{\Phi}$ (or scalar ϕ_π). Alternative forms of monetary policy are also possible, like stabilizing consumption with a real rate rule, stabilizing nominal demand, and/or fixed exchange rate.

Our model delivers two novel insights. The first arises from the New Keynesian (NK) block and demonstrates that both inflation persistence and the magnitude of output losses depend critically on production network linkages—specifically, on the presence of multiple intermediate input sectors. To establish this result, we introduce a dynamic general equilibrium object, the NKOE Leontief inverse, which characterizes the propagation of trade distortions across sectors, countries, and over time. This object provides a transparent framework for analyzing the dynamics of the domestic inflation–output trade-off. Inflation initially increases due to the direct effects of trade distortions on both consumer and producer prices. Under nominal rigidities, this increase can unfold gradually, generating persistent inflationary pressures. As a consequence, output contraction and rising inflation may occur simultaneously, with inflation persistence being amplified by the structure of production networks.

The second novel insight arises from the open-economy intertemporal optimization block and concerns the exchange rate response to a tariff shock. Although this mechanism does not depend directly on production networks per se, it is closely related to the larger output contraction—relative to a small open economy—that emerges from network interactions and incomplete import substitution. Because world consumption is endogenous in our framework, the exchange rate adjustment plays a central role in determining whether domestic consumption and output move in opposite directions. When the home country imposes import tariffs, the resulting appreciation of the home currency and the associated wealth transfer toward the domestic economy are stronger under incomplete markets than under complete markets. For transitory but persistent tariffs, the trade balance improves only on impact and subsequently converges back to its initial steady state, as the wealth transfer induced by exchange rate appreciation allows the home country to sustain a trade deficit in long-run equilibrium. This outcome is consistent with the well-established result in the literature that tariffs do not affect trade balances in the long run. Moreover, owing to complementarities within the production network, an exchange rate depreciation can temporarily worsen the trade balance in the short run. In this sense, the open-economy block of our model constitutes a network-based extension of Obstfeld and Rogoff (1995) with endogenous world consumption, yielding new insights into the joint responses of exchange rates, consumption, and output to changes in relative prices and demand.

We use portfolio adjustment costs (PAC) for well-behaved dynamics of net debt. Representative households in every country can save in a nominal local-currency bond (net zero supply), and also save/dissave in USD bonds. We obtain equilibrium trade imbalances due to country heterogeneity and linearize around a steady-state with non-zero debt that is consistent with these asymmetric primitives by country.[1] Our representative household in each country makes a consumption and saving decision that equalizes the expected ratio of marginal utilities, taking into account differences in the relative price of each country's consumption basket.

In fact, a central theme in our work is country asymmetry that helps us to initialize the trade imbalances. To make the intuition clear, take the standard two-country (H and F) one-industry example. Suppose H places tariffs on F without retaliation and asymmetry is in terms of country size, where H is large. Under flexible prices, with low home bias (γ_H) H is a relatively sizable buyer of F's goods. H's consumption increases and its exchange rate appreciates. Our model in its simplest form can be taught of keeping this structure and further adding production with endogenous labor supply and imported intermediate inputs. H's production dependence on F is given by Ω_H, that if high, tariffs act as a negative supply shock both for H and F. As wages increase and inputs become expensive, labor supply goes down as leisure becomes relatively cheaper. Elasticity of substitution is a key parameter here affecting both consumption and production side: if both final goods and production inputs are highly substitutable within and across borders (θ), then tariffs can be expansionary, whereas if there is sufficient complementarity even only on the production side then tariffs might be contractionary.

Adding nominal rigidity (Λ) and monetary policy (ϕ_π) changes the above flex-price intuition allowing a New Keynesian Phillips Curve where higher nominal rigidity (Λ) dampens the impact of tariffs on prices and amplifies the decline in output. If monetary policy targets inflation, domestic consumption can get hit, in spite of the boost from exchange rate appreciation, both due to higher domestic prices and higher interest rates. Generalizing to N countries and J industries, we show a five-equation Global New Keynesian representation of our model: (i) the New Keynesian IS (NKIS) equation; (ii) the New Keynesian Phillips Curve (NKPC) for producer prices derived with Rotemberg costs; (iii) a definition of the consumption price vector, which deviates from producer prices due to exchange rate movements and tariff distortions; (iv) an Uncovered Interest Parity (UIP) condition that nests international arbitrage conditions; and (v) an equation of motion for external debt, which also incorporates the market-clearing condition. Together, these equations characterize the equilibrium and nest a broad class of NKOE models. The last three can be further combined into a risk sharing wedge under the tariff shock.

The network literature has shown why having more than one sector ($J > 1$) matters in closed economy networks (e.g., Pasten et al. (2020) and Rubbo (2023)). Many sectors alter

[1] In the quantitative model, we discipline these steady-state net debt levels with real-life trade imbalances.

the slope of the aggregate Phillips Curve and sector-specific shocks propagate differently depending on sectoral rigidity and network centrality. Our framework identifies a third and novel channel: when the number of sectors exceeds one, lagged sectoral prices affect current inflation through the NKOE Leontief inverse, $\mathbf{\Psi}^{NKOE}$, because this object is the coefficient matrix multiplying the lagged price vector in the solution for producer prices. When the number of sectors $J = 1$, $\mathbf{\Psi}^{NKOE}$ equals the identity matrix ($\boldsymbol{I}$) and the lagged price vector has no impact on contemporaneous inflation. However, when $J > 1$ $\mathbf{\Psi}^{NKOE} - \boldsymbol{I} \neq \mathbf{0}$, implying that past sectoral prices feed into current producer price inflation even in the absence of new shocks, delivering persistent inflation out of a transitory tariff shock.

NKOE Leontief inverse also helps us to decompose the general equilibrium response to the tariff shock into channels. If a given sector is central to production—either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)—it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices—corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)—and is located in a country with relatively loose (or tight) monetary policy, the inflationary impact of a tariff on that sector will be amplified (or muted) by the network captured in the NKOE Leontief inverse.

Under producer currency pricing, we have full pass-through of tariffs to domestic prices. To analyze the quantitative impact of this assumption, we also introduce dollar pricing which dampens the impact of tariffs on inflation. Thus, pricing assumptions will not have much impact on positive implications of trade distortions that we focus on in theory but they of course change quantitative implications. Even though our model does not search for normative implications, we undertake a quantitative exercise to show the min value of tariffs when consumption switches from being negative to positive. These results are consistent with the optimal tariff literature where optimal tariff for a large country like the U.S. is not zero under significant terms of trade gains.

We contribute to both trade and NKOE literatures. Most of the papers in trade literature that study impact of tariffs on long-run production and welfare assume flexible prices, balanced trade with no international borrowing and use static models.[2] There are also small open economy models in the new Keynesian (NK) tradition, though most of these models are not used to study the impact of tariffs, and when they are used for that purpose then they omit intermediate input imports and supply chains.[3] We argue that, under both transitory

[2]Note that this literature treats tariffs as permanent and works with exact hat-algebra in two-period models, recently (see, for example, Costinot and Rodríguez-Clare, 2014).

[3]Early Keynesian literature studies the impact of transitory tariffs. See, for example, Mundell (1961), Eichengreen (1981), and Krugman (1982), without the micro foundations of the modern small open economy

and permanent tariffs, the dynamics of inflation-output trade-off critically depends on the network structure and input complementarity.[4] Our approach is directly linked to the closed economy production network literature and can be thought of as connecting two or more Rubbo (2023)[5] economies under incomplete markets with trade imbalances. Alternatively, our paper can be seen as dynamic incomplete market version of papers such as Baqaee and Farhi (2024), Rodríguez-Clare et al. (2020).

After solving the model analytically to first order, we additionally conduct a number of quantitative counterfactuals, where we solve the model non-linearly with MIT shocks. In these quantitative analyses, the sectoral heterogeneity in price setting is disciplined by estimates from Nakamura and Steinsson (2008) and steady state network before tariffs is calibrated using OECD's Inter-Country Input-Output (ICIO) tables (Yamano and et al., 2023), imposing no *a priori* assumptions on whether a good is purely final or tradable. Thus the quantitative results are not driven solely by the overall share of material inputs in marginal costs, as is often the case in conventional NKOE models. Instead, this relationship arises endogenously from the global I-O structure, nesting other models.[6]

In our quantitative work, we first road-test the model on 2018 tariffs. The model predicts a 3% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan on impact, eventually reaching 4.2% nominal appreciation in the long run. This aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018. Real GDP loss reaches 0.1 percentage points, in the range of the estimate of Fajgelbaum et al. (2020), which found an aggregate real income loss of 0.04% of GDP. Finally, the model predicts an inflation impact of 0.27 percentage points, which is close to the 0.1-0.2 percentage point estimate of Barbiero and Stein (2025).

Having validated the quantitative model, we test different scenarios for 2025's tariffs with 6 regions, U.S., Euro Area, China, Mexico, Canada and ROW, and 8 main sectors. Using WTO-IMF Tariff Tracker (WTO and IMF, 2025), we document both implemented

new Keynesian (SOE-NK) literature as in Galí and Monacelli (2005). Most of the modern SOE-NK literature focuses on optimal exchange rate and monetary policies in SOEs. The paper by Barattieri et al. (2021) is an example who studied macro impact of tariffs in a SOE-NK model.

[4]The essential role of intermediate inputs and cross-border production chains in trade is well established (e.g., di Giovanni and Levchenko, 2010; Johnson, 2014). A large literature in trade also shows the importance of connecting shocks to producers' marginal costs (e.g. Caliendo and Parro (2015)), for network amplification.

[5]As in other closed economy network papers Rubbo (2023) highlights the importance of relative price changes in understanding the behavior of aggregate inflation (e.g., Pasten et al., 2020, 2024; Rubbo, 2024; Afrouzi and Bhattarai, 2023). Similar to our work, Afrouzi et al. (2024) also implement network adjusted heterogeneity in price stickiness across sectors.

[6]For example, a model without intermediate inputs—where tariffs affect only demand—can be represented by collapsing the I-O matrix $\mathbf{\Omega}$. Likewise, a model with a single imported intermediate input and a final consumption good corresponds to a structure in which the columns of $\mathbf{\Omega}$ associated with final goods are zero vectors.

and announced but not implemented tariffs at sector-country level. The former motivates our tariff level shocks and the latter case motivates what we call "reversed tariff threats."

The next exercises study, first, reversed tariff threats—situations in which tariffs are announced in the current period, inducing expectations of future retaliation, but are fully withdrawn before implementation—and, second, the quantitative effects of the 2025 tariffs and related counterfactuals. In a perfect foresight setting, agents optimize over the full sequence of announcements, so a temporary tariff threat that is expected to become permanent and trigger a trade war leads the exchange rate to adjust immediately, front-loading anticipated changes in consumption behavior. In this scenario, the U.S. NEER appreciates by 1.7 percent on impact and subsequently depreciates, real GDP falls by 0.4 percent, and deflation occurs despite no contemporaneous change in monetary policy; these effects are driven by expectations, as agents price in lower future imports by the U.S., a net importer, generating exchange-rate adjustment and deflation even before any mechanical tariff price effects arise, while the trade balance worsens as imports are front-loaded ahead of anticipated tariffs. Turning to the 2025 tariffs, the model predicts a small increase in U.S. real GDP of 0.1 percent alongside a 0.2 percent decline in consumption, a 0.6 percentage point rise in inflation, a 0.3 percent fall in real wages, and an improvement in the trade balance of 0.5 percentage points of steady-state GDP; abroad, output contracts most sharply in China (–2.1 percent), Mexico (–1.3 percent), and Canada (–0.9 percent), with larger inflation increases than in the U.S., while the euro area and the rest of the world also contract and experience sizable real wage declines.

Two additional counterfactuals highlight policy interactions: holding nominal interest rates constant globally amplifies and prolongs inflation with similar output and employment losses, while symmetric retaliation by non-U.S. countries produces a larger global slowdown and a larger rise in inflation. Finally, we incorporate additional realism by allowing for gradual tariff pass-through through domestic retail importers, which generates more persistent inflation dynamics, and by introducing a widening UIP premium at tariff implementation, which can induce short-run dollar depreciation despite the appreciationary nature of tariffs. This last case is motivated by the fact that the UIP wedge increased by 2.98 percentage points from 4Q2025 to 1Q2025 and that this can be modeled with a UIP wedge that widens with the volatility of tariffs as it impacts the variance of the exchange rate, as shown in Kalemli-Özcan et al. (2026). Together, these extensions help reconcile the model with observed inflation persistence and exchange rate movements following the introduction of tariffs.

Overall, our work shows the importance of global trade and production networks for analyzing the macro impact of trade distortions. Models without networks can under-estimate unemployment and overestimate inflation, missing entirely inflation persistence. Without networks, tariffs lead to a one-time price jump and to higher output under perfect import substitution, whereas with networks, even there is full import substitution, there can still be persistent inflation under large tariffs.

Roadmap. The remainder of this paper is organized as follows. Section 2 reviews the literature. Section 3 outlines our baseline New Keynesian open-economy model and Section 4 develops the flexible price solution. In Section 5, we introduce nominal rigidities and monetary policy. Section 6 show why networks matter. Data and quantitative analysis are featured in Section 7. Finally, Section 8 concludes.

2 Literature

The literature on tariffs organizes around two key concepts: Terms of trade manipulation and Lerner symmetry (Lerner, 1936).[7] Recently, Werning et al. (2025) argues that tariffs are cost-push shocks with deviations from efficient steady state inducing welfare loss. Jeanne and Son (2024) shows theoretically that exchange rate appreciations can offset import restrictions by limiting exports, however, there is a large empirical literature that shows the impact of imports tariffs is not fully offset by exchange rate movements. This literature also demonstrates that exchange rate pass-through to prices is much lower than tariff pass-through, although the latter can also be less than full.

The extent of tariff pass-through to border prices versus retail prices is subject of an extensive debate. There is an active empirical debate on how much of the tariff is in the retail price faced by the consumer and how much of it impacts the marginal costs of both foreign and domestic firms. For example, for the 2018 tariffs, Amiti et al. (2019), Fajgelbaum et al. (2020), and Fajgelbaum and Khandelwal (2022), find complete pass-through of tariffs to consumer prices, whereas Cavallo et al. (2021) finds that the degree of pass-through from border to retailers and consumers is not complete. For categories like washing machines, the pass-through can be high (e.g., Flaaen et al., 2019). However, for more aggregated price indices that combine goods that are affected and unaffected by the tariffs, the pass-through is less clear-cut. Thus, the retailers absorbing a significant share of the cost, those raising their prices on goods competing with imports, and those increasing their prices on goods not directly exposed are hard to separate. Inventory "front-running," moving supply chains away, or studying the early months with sticky prices can also blur the picture on aggregate price increases and inflationary impulse. In addition, it is well-known in the two-country

[7] See Erceg et al. (2018), Lindé and Pescatori (2019) and Costinot and Werning (2019) for modern treatments.

NKOE literature (e.g., Obstfeld and Rogoff, 1995; Clarida et al., 2002) that if exchange rate pass-through is less than full, domestic inflation (PPI) in open economies can differ from CPI inflation that includes imported goods.

Several SOE models study short-run impact of trade barriers on the macroeconomy focusing on normative implications, driving optimal trade and/or monetary policy, such as Auray et al. (2024a,b), Ambrosino et al. (2024). These papers highlight the importance of both demand and supply side and the former, like us, argue that if labor supply and intermediate inputs are added, the tariff outcome depends critically on the monetary policy stance. Bianchi and Coulibaly (2025) finds that optimal monetary policy is expansionary as long as households do not internalize the impact of rebated tariff revenues. Bergin and Corsetti (2023) finds the opposite that optimal policy is contractionary due to higher inflation. Auclert et al. (2025) argues that it is important to add intermediate inputs to these standard SOE models studying tariffs macro impact. They argue that without taking the recession into account optimal tariffs cannot be calculated. Monacelli (2025), adds intermediate inputs and finds optimal monetary policy is expansionary due to declining output. Our work differs from Monacelli (2025), as we focus on positive implications in global general equilibrium, that we show how other countries' monetary policy responses are also important in shaping the home country inflation-output trade-off.[8]

There is also renewed interest in studying optimal tariffs and trade imbalances. Similar to Auray et al. (2024a,b), Itskhoki and Mukhin (2025) highlight the importance of valuation effects for the determination of changes in steady state trade imbalances with tariffs, especially when gross assets and liabilities are in different currencies (in terms of nominal values). Costinot and Werning (2025), on the other hand, changes in real trade deficits with tariffs depend on the extensive margin of trade.

3 Modeling Framework

We develop a multi-country multi-sector New Keynesian model that incorporates nominal rigidities via Rotemberg costs, standard open-economy features such as portfolio adjustment costs, trade distortions and production networks.[9]

Households optimize intertemporally, allocating consumption and labor supply while facing portfolio adjustment costs when holding foreign bonds. The production side follows a

[8]Few papers also study SOEs with networks in quantitative models such as Qiu et al. (2025) and quantitatively as in Cuba-Borda et al. (2025), Ho et al. (2022).

[9]The world will be a closed economy, similar to frameworks such as Long and Plosser (1983); Acemoglu et al. (2012); Atalay (2017); Liu (2019); Baqaee and Farhi (2019, 2022); Baqaee (2018); Carvalho et al. (2021b); Carvalho and Tahbaz-Salehi (2019), among others.

nested CES structure, with goods classified by sector and origin, and firms producing using labor and intermediate inputs. Prices are set in the producer's currency (PCP) and are subject to revenue-neutral tariffs. Monetary policy follows a Taylor rule (although we also solve the model under alternative rules). Exchange rates are endogenous. There are also endogenous deviations from Uncovered Interest Parity (UIP) arise due to portfolio adjustment costs; as a country's net debt increases, the effective interest rate it pays also rises.

3.1 Intertemporal problem.

The household in country n maximizes the present value of lifetime utility:

$$\max_{\{C_{n,t},L_{n,t},B^{US}_{n,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \chi \frac{L_{n,t}^{1+\eta}}{1+\eta} \right]$$

subject to:

$$P^C_{n,t}C_{n,t} + T_{n,t} - B_{n,t} - \mathcal{E}^{US}_{n,t}B^{US}_{n,t} + \mathcal{E}^{US}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t}) \leq$$
$$W_{n,t}L_{n,t} + \sum_i \Pi_{ni,t} - (1+i_{n,t-1})B_{n,t-1} - \mathcal{E}^{US}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}$$

where $P^C_{n,t}$ is the price of the consumption bundle ($C_{n,t}$) at time t, β is the discount factor, σ is the intertemporal elasticity of substitution, χ denotes labor disutility weight and η captures the elasticity of labor. $\mathcal{E}^{US}_{n,t}$ is the exchange rate between country n and the U.S. An increase in $\mathcal{E}^{US}_{n,t}$ implies a depreciation of the local currency relative to the U.S. dollar. $W_{n,t}$ is the wage in country n at time t, $L_{n,t}$ is the quantity of labor supplied in country n, $i_{n,t}$ is the nominal interest rate in local currency bond $B_{n,t}$, and $i_{US,t}$ is the interest rate on the U.S. bond $B^{US}_{n,t}$, where these bonds are net foreign liabilities. The term $\psi(B^{US}_{n,t}/P^{US}_{n,t})$ represents a stationarity-inducing portfolio adjustment cost that ensures a unique steady-state level of real debt (i.e., debt denominated in USD, deflated by the U.S. consumer price level). Taxes and transfers are denoted by $T_{n,t}$. In our model, tariffs are revenue-neutral; tariff revenue is rebated back to domestic households in a lump-sum manner through the $T_{n,t}$ term.

Maximizing the household's lifetime utility subject to the present and future budget constraints yields the following standard first-order conditions (see Appendix A.1):

$$1 = \beta E_t \left[\left(\frac{C_{n,t+1}}{C_{n,t}} \right)^{-\sigma} \frac{P^C_{n,t}}{P^C_{n,t+1}} (1+i_{n,t}) \right] \forall n \in N, \forall t \qquad \text{(Euler Equation)}, \quad (1)$$

$$\frac{1+i_{n,t}}{1+i^{US}_{n,t}} = E_t \left[\frac{\mathcal{E}_{n,t+1}}{\mathcal{E}_{n,t}} \right] \frac{1}{1-\psi'(B^{US}_{n,t}/P^{US}_{n,t})} \qquad \text{(UIP) } n \in N-1. \quad (2)$$

The domestic bond is in net zero supply everywhere, and all countries save or dissave using U.S. bonds. In addition to the UIP condition, the rest of the arbitrage condition ensures that a country's bilateral exchange rates remain consistent with its exchange rates against the U.S. Finally, for completeness of notation, we define a country's exchange rate with itself:

$$\mathcal{E}_{n,m,t} = \frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{m,t}} \;\forall n \neq m \;\&\; m \neq US \; n, m \in N \tag{3}$$

$$\mathcal{E}_{n,n,t} = 1 \;\forall n \in N \tag{4}$$

We have $N \times N$ exchange rates, and along with the UIP condition, these two conditions uniquely determine the exchange rate.

3.2 Intratemporal problem.

We now turn to the household's intratemporal problem. The first part of the intratemporal problem is the standard labor-consumption tradeoff that determines labor supply:

$$\frac{W_{n,t}}{P^C_{n,t}} = \chi L^{\eta}_{n,t} C^{\sigma}_{n,t} \;\forall n \in N, \forall t \tag{5}$$

where $W_{n,t}$ is the wage in country n at time t.

Determining the intratemporal breakdown of consumption involves a nested CES structure. Outputs from different countries are first bundled into a country-sector consumption bundle, which is then aggregated into a country good:

$$C_{n,t} = \left[\sum_{i \in J} \Gamma_{n,i}^{\frac{1}{\theta^C_h}} C_{n,i,t}^{\frac{\theta^C_h - 1}{\theta^C_h}} \right]^{\frac{\theta^C_h}{\theta^C_h - 1}}. \tag{6}$$

Here, the index (n, i) captures the sector level (i) bundles in country n. $C_{n,i,t}$ is country n's consumption of industry bundle i and $\Gamma_{n,i}$ is the weight of the bundle i. θ^C_h is the elasticity that governs the substitution between different sectors in consumption (e.g., between automobiles and food in consumption). This bundle is then a combination of all goods of i procured by country n from countries $m \in N$ globally:

$$C_{n,i,t} = \left[\sum_{m \in N} \Gamma_{n,i,mi}^{\frac{1}{\theta^C_{l,i}}} C_{n,i,mi,t}^{\frac{\theta^C_{l,i} - 1}{\theta^C_{l,i}}} \right]^{\frac{\theta^C_{l,i}}{\theta^C_{l,i} - 1}}. \tag{7}$$

In this equation, we focus on country-sector varieties (mi) that form sectoral bundle (i) in country n, which we index with (n,i,mi). $\Gamma_{n,i,mi}$ is the weight of country m's good in this bundle (e.g., German automobiles –mi– in automobile bundle –i– for the U.S. consumers –n). $\theta^C_{l,i}$ is the elasticity of substitution between different country varieties in sector i. Prices and consumption levels of this object is indexed the same way. We can then express the relevant price levels in line with the CES structure:

$$P^C_{n,t} = \left[\sum_{i\in J} \Gamma_{n,i}(P^C_{n,i,t})^{1-\theta^C_h}\right]^{\frac{1}{1-\theta^C_h}}$$
$$P^C_{n,i,t} = \left[\sum_{m\in N} \Gamma_{n,i,mi} P^{1-\theta^C_{l,i}}_{n,mi,t}\right]^{\frac{1}{1-\theta^C_{l,i}}}$$

where $P^C_{n,i,t}$ is the local currency consumption price of the aggregated good basket i in country n at time t (We use the superscript C for denoting price bundles in the consumption side). We assume that prices are set in the producer's currency and then converted to the consumer's currency using the exchange rate under the producer currency pricing (PCP) assumption:

$$P_{n,mi,t} = \mathcal{E}_{n,m,t}(1+\tau_{n,mi,t})P_{mi,t} \tag{8}$$

where $\mathcal{E}_{n,m,t}$ is the bilateral exchange rate, $\tau_{n,mi,t}$ is the tariff imposed by country n of country-sector mi and $P_{n,mi,t}$ is the price of mi good in country n.

Remark 1. Given the prices that end users see and the aggregation of consumer prices, tariffs serve as a distortionary wedge, similar to a consumption tax or tax on labor income, in the labor-consumption tradeoff given by equation (5).

$$C_{n,i,t} = \Gamma_{n,i}\left(\frac{P^C_{n,i,t}}{P^C_{n,t}}\right)^{-\theta^C_h} C_{n,t} \tag{9}$$
$$C_{n,mi,t} = \Gamma_{n,i,mi}\left(\frac{P_{n,mi,t}}{P^C_{n,i,t}}\right)^{-\theta^C_{l,i}} C_{n,i,t} \tag{10}$$

3.3 Production

Having defined the household's side, we now turn to the production side of the economy. Output in country n, sector i, at time t follows a CES production function:

$$Y_{ni,t} = A_{ni,t}\left[\alpha_{ni}^{1/\theta^P} L_{ni,t}^{\frac{\theta^P-1}{\theta^P}} + (1-\alpha_{ni})^{1/\theta^P}(X_{ni,t})^{\frac{\theta^P-1}{\theta^P}}\right]^{\frac{\theta^P}{\theta^P-1}} \forall n \in N, \forall i \in J, \tag{11}$$

where $Y_{ni,t}$ is the output of sector i in country n, $A_{ni,t}$ is the total factor productivity, θ^P governs the elasticity between the labor and intermediate bundle $X_{ni,t}$, and α_{ni} is the labor share.

All firms within a given country-sector combination are assumed to be identical, and each firm solves the following marginal cost minimization problem:

$$MC_{ni,t} = \min_{\{X_{ni,j,t}, L_{ni,t}\}} W_t L_{ni,t} + P^X_{ni,t} X_{ni,t} \quad \text{s.t.} \quad Y_{ni,t} = 1.$$

where $P^X_{ni,t}$ is the price of the intermediate bundle for country-sector ni (We use the superscript X for denoting prices for all bundles in the production side).

As a firm faces this problem, it chooses labor and the quantities of the intermediate good specific to the producing industry in the given country. This intermediate good bundle is constructed as follows. Intermediate goods from different countries are first bundled into a country-industry-good bundle. This bundle and the relevant relative demand condition are defined below:

$$X_{ni,j,t} = \left[\sum_{m \in N} \Omega_{ni,j,mj}^{\frac{1}{\theta^P_{l,j}}} X_{ni,mj,t}^{\frac{\theta^P_{l,j}-1}{\theta^P_{l,j}}}\right]^{\frac{\theta^P_{l,j}}{\theta^P_{l,j}-1}} \tag{12}$$

$$X_{ni,mj,t} = \Omega_{ni,j,mj}\left(\frac{P_{n,mj,t}}{P^X_{ni,j,t}}\right)^{-\theta^P_{l,j}} X_{ni,j,t} \tag{13}$$

Here, we index the sector bundle j for producer sector i in country n with (ni, j, t). $P^X_{ni,j,t}$ is the price index for this bundle, and $X_{ni,j,t}$ is the quantity. This bundle is formed by country varieties mj (e.g., Chinese steel –mj– in steel bundle –j– for the U.S. automobile industry – ni), which we index for (ni, mj, t). $\Omega_{ni,j,mj}$ captures the share of industry mj in bundle j for industry ni. $\theta^P_{l,j}$ governs the elasticity of substitution among different varieties within sector j in production side. The prices and intermediate inputs follow the same subscripts.

Analogously, the intermediate bundle is constructed as follows:

$$\frac{X_{ni,j,t}}{X_{ni,t}} = \Omega_{ni,j}\left(\frac{P^X_{ni,j,t}}{P^X_{ni,t}}\right)^{-\theta^P_h} \quad \forall\, j \in J \tag{14}$$

$$X_{ni,t} = \left[\sum_{j\in J} \Omega_{ni,j}^{\frac{1}{\theta^P_h}} X_{ni,j,t}^{\frac{\theta^P_h - 1}{\theta^P_h}}\right]^{\frac{\theta^P_h}{\theta^P_h - 1}} \tag{15}$$

As we derive in detail in Appendix A.2, given the setup and definitions above, the firm's problem yields the following equilibrium conditions for the marginal cost $MC_{ni,t}$:

$$\frac{X_{ni,t}}{L_{ni,t}} = \frac{(1-\alpha_{ni})}{\alpha_{ni}}\left(\frac{W_t}{P^X_{ni,t}}\right)^{\theta^P} \tag{16}$$

$$MC_{ni,t} = \frac{1}{A_{ni,t}}\left[\alpha_{ni} W_t^{1-\theta^P} + (1-\alpha_{ni})\left(\sum_j \Omega_{ni,j}(P^X_{ni,j,t})^{1-\theta^P_h}\right)^{\frac{1-\theta^P}{1-\theta^P_h}}\right]^{\frac{1}{1-\theta^P}} \tag{17}$$

Within each country sector there is an infinite continuum of identical firms. Representative firm f in sector i of country n solves the following problem Rotemberg setup:

$$P^f_{ni,t} = \arg\max_{P^f_{ni,t}} \mathbb{E}_t\left[\sum_{T=t}^{\infty} \mathrm{SDF}_{t,T}\left[Y^f_{ni,T}(P^f_{ni,T})\left(P^f_{ni,T} - MC_{ni,T}\right)\right.\right.$$
$$\left.\left. - \frac{(1-\vartheta_{ni})\delta_{ni}}{2}\left(\frac{P^f_{ni,T}}{P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T} - \frac{\vartheta_{ni}\delta_{ni}}{2}\left(\frac{\mathcal{E}^{US}_{n,T-1}P^f_{ni,T}}{\mathcal{E}^{US}_{n,T}P^f_{ni,T-1}} - 1\right)^2 Y_{ni,T}P_{ni,T}\right]\right]$$

where a bundler puts together the sectoral output as a CES bundle such that the demand function is $Y^f_{ni,t}(P^f_{ni,t}) = \left(\frac{P^f_{ni,t}}{P_{ni,t}}\right)^{-\theta^R} Y_{ni,t}$. A given country n's exchange rate vis-a-vis the US dollar is given by $\mathcal{E}^{\$}_{n,t}$. In this Rotemberg setup, $(1-\vartheta_{ni})\delta_{ni}$ captures the real cost of changing the price in producer currency and $\vartheta_{ni}\delta_{ni}$ captures the real cost of changing the price in the dominant currency (US dollar). ϑ_{ni} captures the share of prices that are rigid in the dominant currency as opposed to the producer currency. With this formulation our model combines producer currency pricing with dominant currency pricing (DCP). In this setup $\vartheta_{ni} \to 0$ would correspond to PCP and $\vartheta_{ni} \to 1$ would correspond to DCP, so with $\vartheta_{ni} \in (0,1)$ the model combines the two pricing schemes in a manner consistent with empirical evidence. In our quantitative results, we discipline ϑ_{ni} using the share of exports invoiced in the US dollar given by the empirical DCP literature.

As we show in Appendix A.2.1, this problem yields the following equilibrium condition:

$$
\begin{aligned}
&(1-\vartheta_{ni})(\Pi_{ni,t}-1)\Pi_{ni,t}+\vartheta_{ni}\left(\frac{\Pi_{ni,t}}{D^{US}_{n,t}}-1\right)\frac{\Pi_{ni,t}}{D^{US}_{n,t}}=\frac{\theta^R}{\delta_{ni}}\left[\frac{MC_{ni,t}}{P_{ni,t}}-\frac{\theta^R-1}{\theta^R}\right]\\
&+\beta\,\mathbb{E}_t\left[(1-\vartheta_{ni})(\Pi_{ni,t+1}-1)\Pi_{ni,t+1}+\vartheta_{ni}\left(\frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}}-1\right)\frac{\Pi_{ni,t+1}}{D^{US}_{n,t+1}}\right]
\end{aligned}
\tag{18}
$$

where $\Pi_{ni,t}$ is gross inflation ($\Pi_{ni,t}=\frac{P_{ni,t}}{P_{ni,t-1}}$) and $D^{US}_{n,t}$ is gross depreciation of the producer's currency against the USD ($D^{US}_{n,t}=\frac{\mathcal{E}^{US}_{n,t}}{\mathcal{E}^{US}_{n,t-1}}$).

Equation (18) constitutes a country- and sector-specific forward-looking New Keynesian Phillips Curve, expressed in terms of nominal marginal cost deflated by the sector's producer price. As $\delta_{ni}\to 0$, prices become more flexible, leading to $\Pi_{n,t}=1$ and $\frac{MC_{ni,t}}{P_{ni,t}}=\frac{\theta^R-1}{\theta^R}$, which corresponds to the general pricing equation under monopolistic competition with steady state markups.[10]

3.4 Balance of Payments and NIIP

We track the evolution of each country's net international investment position (NIIP) as follows:

$$
\begin{aligned}
&\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}C_{n,mj,t}\right)+\sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}}\left(\frac{P_{n,mj,t}}{1+\tau_{n,mi,t}}X_{ni,mj,t}\right)+\mathcal{E}_{n,t}(1+i^{US}_{n,t-1})B^{US}_{n,t-1}\\
&+\mathcal{E}_{n,t}P^{US}_{n,t}\psi(B^{US}_{n,t}/P^{US}_{n,t})=\sum_{i\in\mathcal{J}}P_{ni,t}Y_{ni,t}+\mathcal{E}_{n,t}B^{US}_{n,t}\quad\forall n\in N-1
\end{aligned}
\tag{19}
$$

where we account for the fact that tariffs are modeled as revenue-neutral by dividing relevant prices by $(1+\tau_{n,mi,t})$, since end-user prices reflect the impact of tariffs just as they do the impact of exchange rates. The key point here is that, even tariff revenue is rebated back, both producers and consumers still see the tariff-distorted price when making their optimal consumption and production decisions.

Given market-clearing conditions and budget constraints, one country's budget constraint is redundant as an equilibrium condition. Thus, we omit that of the first country, which corresponds to the U.S. in our model. However, we still need to ensure that the market for

[10] Assuming away DCP (setting $\vartheta_{ni}=0$) would yield the standard non-linear New Keynesian Phillips Curve that is derived with Rotemberg adjustment costs: $(\Pi_{ni,t}-1)\,\Pi_{ni,t}=\frac{\theta^R}{\delta_{ni}}\left(\frac{MC_{ni,t}}{P_{ni,t}}-\frac{\theta^R-1}{\theta^R}\right)+\beta\mathbb{E}_t\left[(\Pi_{ni,t+1}-1)\,\Pi_{ni,t+1}\right]$.

USD bonds is closed:

$$B_t^{US} = \sum_{m}^{N-1} B_{m,t}^{US} \tag{20}$$

3.5 Definitions, Market Clearing, Policy and Equilibrium

We assume that all goods markets clear. Goods can be used as final (consumption) goods and as intermediate inputs in all countries. Therefore, we write the goods market-clearing condition for country-sector ni at time t as:

$$Y_{ni,t} = \sum_{n\in\mathcal{N}} C_{m,ni,t} + \sum_{m\in\mathcal{N}} \sum_{j\in\mathcal{J}} X_{mj,ni,t}, \tag{21}$$

where country m is the consuming country and n is the producing country.

To close the model, we need to specify the market-clearing condition for labor, define aggregate inflation, and specify policy. Monetary policy in each follows a generalized rule that allows for interest rate smoothing and targeting of a generic price basket.

$$L_{n,t} = \sum_{i\in J} L_{ni,t} \tag{22}$$

$$P_{n,t}^T = \left(\prod_{m\in N}\prod_{j\in J} (P_{mj,t})^{\Upsilon_{n,mj}^P}\right)\left(\prod_{m\in N\setminus\{n\}} \mathcal{E}_{n,m,t}^{\Upsilon_{n,m}^{\mathcal{E}}}\right) \tag{23}$$

$$\Pi_{n,t}^T = \frac{P_{n,t}^T}{P_{n,t-1}^T} \tag{24}$$

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho_m^n} \left(\Pi_{n,t}^T\right)^{\phi_\pi^n} e^{\hat{M}_{n,t}} \quad \forall n \in N \tag{25}$$

Here, $\rho_m^n \in [0, 1)$ denotes the degree of interest rate smoothing or inertia, $\Pi_{n,t}^T$ is the gross inflation rate of the target price basket $P_{n,t}^T$. This generic representation allows us to accommodate different alternatives including CPI targeting, specific combination of producer prices, and/or exchange rate targeting.[11]

Definition 1. A non-linear competitive equilibrium for the model is a sequence of 11 endogenous variables $\{C_{nt}, C_{ni,t}, C_{n,mj,t}, X_{ni,mj,t}, X_{ni,j,t}, X_{ni,t}, Y_{ni,t}, L_{ni,t}, L_{n,t}, MC_{ni,t}, B_{n,t}^{US}\}_{t=0}^{\infty}$ and 12 prices $\{P_{ni,t}, P_{n,mi,t}, P_{n,t}^C, P_{n,t}^T, P_{ni,t}^C, P_{ni,t}^X, P_{ni,j,t}^X, \Pi_{n,t}^T, \Pi_{ni,t}, \mathcal{E}_{n,t}, i_{n,t}, W_{n,t}\}_{t=0}^{\infty}$ given ex-

[11] This notation nests different ways of combining producer prices including PPI targeting or the divine coincidence index of Rubbo (2023), which places more weight on sectors where prices are stickier.

ogenous processes $\{\tau_t, A_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (1)-(25) hold for all countries and time periods.

3.6 Linearized Model and Analytical Solution

We linearize the 25 equations above and define an approximated equilibrium in order to use the method of undetermined coefficients and solve the model analytically.[12]

Definition 2. A linearized competitive equilibrium for the model is a sequence of 11 endogenous variables $\{\hat{C}_{n,t}, \hat{C}_{ni,t}, \hat{C}_{n,mj,t}, \hat{X}_{ni,mj,t}, \hat{X}_{ni,j,t}, \hat{X}_{ni,t}, \hat{Y}_{ni,t}, \hat{L}_{ni,t}, \hat{L}_{n,t}, \widehat{MC}_{ni,t}, \hat{B}^{US}_{n,t}\}_{t=0}^{\infty}$ and 12 prices $\{\hat{P}_{nt}, \hat{P}^{T}_{n,t}, \hat{P}_{ni,t}, \hat{P}^{C}_{ni,t}, \hat{P}^{p}_{ni,t}, \hat{P}^{p}_{ni,j,t}, \hat{P}_{n,mi,t}, \pi^{T}_{n,t}, \pi^{p}_{ni,t}, \hat{\mathcal{E}}_{n,t}, \hat{i}_{n,t}, \hat{W}_{n,t}\}_{t=0}^{\infty}$ given exogenous processes $\{\hat{\tau}_t, \hat{A}_{ni,t}, \hat{M}_{n,t}\}_{t=0}^{\infty}$ such that equations (A.1)-(A.24) hold for all countries and time periods.

It is common to linearize open economy models around a steady state with net zero debt. We take a different approach (e.g., Obstfeld and Rogoff, 1995) and allow for asymmetry of the primitive parameters (i.e., home bias and imported intermediate input dependence) across countries, which implies a certain level of debt and net exports at the steady state that has to be consistent with these parameters. This level of steady state debt is then used to parametrize the portfolio adjustment costs that discourage deviations from steady-state levels of debt. In the quantitative section, we discipline the asymmetry of parameters and the steady-state level of debt using the ICIO Table. Further details on this can be found in Appendix B.

To solve the model analytically, we make the following simplifying assumptions. The first simplifying assumption involves adopting elastic labor in the spirit of Golosov and Lucas (2007) preferences. That is we set $\chi = 1$ and $\eta = 0$, making labor infinitely elastic, which simplifies the intratemporal labor-leisure choice to $\hat{W}_{n,t} - \hat{P}_{n,t} = \sigma \hat{C}_{n,t}$. This simplification allows us to focus on consumption in our five-equation Global New Keynesian Representation and track aggregate output separately. Second simplifying assumption for analytical solution is to assume $\psi(B^{US}_{n,t}/P^{US}_{n,t}) \to 0$.[13] Third, for analytical simplicity we assume price rigidity in the producer prices, i.e., $\vartheta_{ni} = 0 \ \forall n \in N, i \in J$. Additionally, we assume that policy targets only a basket of producer prices and not the exchange rate, setting $\Upsilon^{\mathcal{E}}_{n,m} = 0 \ \forall n, m \in N$. In our analytical work, we use the weights that producer prices have in the consumption basket for the target basket such that $\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi\Gamma} \hat{\boldsymbol{P}}^{\boldsymbol{P}}_{\boldsymbol{t}}$.[14] Finally, we set shocks other than tariffs to

[12] We denote the steady-state values with the bar notation.

[13] Portfolio adjustment costs serve as a stationarity-inducing device in the model. Because, these costs are numerically small, in analytical work we simplify them away.

[14] Our results do not hinge on using the $\boldsymbol{\Gamma}$ matrix here and one target a different mix of producer prices.

zero and introduce generalized elasticities that directly link the lowest-level bundles to the highest-level aggregates, such as:[15]

$$\hat{C}_{n,t} = \sum_{m\in N}\sum_{i\in J}\Gamma_{n,mi}\hat{C}_{n,mi,t}$$
$$\hat{C}_{n,mi,t} = -\theta^{C}_{l,i}\left(\hat{P}^{p}_{mi,t} + \hat{\mathcal{E}}_{n,m,t} + \tau_{n,mi,t} - \hat{P}^{C}_{ni,t}\right)$$

3.6.1 Vector and Matrix Notation

Given the number of countries and industries involved, we can utilize the matrix form to write the equilibrium conditions. To that end, let us consider the linearized producer price inflation equation:

$$\pi^{p}_{ni,t} = \frac{\theta^{P}_{l,i}}{\delta_{ni}}\left(\underbrace{\alpha_{ni}\hat{W}_t + \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}(\hat{P}^{p}_{mj,t} + \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,mj,t})}_{\widehat{MC}_{ni,t}} - \hat{P}^{p}_{ni,t}\right) + \beta\mathbb{E}_t\pi^{p}_{ni,t+1} \tag{26}$$

This can be expressed in vector and matrix notation as follows:

$$\underbrace{\boldsymbol{\pi}^{P}_{t}}_{NJ\times 1} = \underbrace{\boldsymbol{\Lambda}}_{NJ\times NJ}\left(\underbrace{\boldsymbol{\alpha}}_{NJ\times N}\underbrace{\hat{\boldsymbol{W}}_t}_{N\times 1} + \underbrace{(\boldsymbol{\Omega}-\boldsymbol{I})}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{P}}^{P}_{t}}_{NJ\times 1} + \underbrace{\boldsymbol{L}^{P}_{\mathcal{E}}}_{NJ\times N^2}\underbrace{\hat{\boldsymbol{\mathcal{E}}}_t}_{N^2\times 1}\right.$$
$$\left. + \underbrace{\boldsymbol{L}^{P}_{\tau}}_{NJ\times N^2 J}\underbrace{\hat{\boldsymbol{\tau}}_t}_{N^2 J\times 1}\right) + \beta\mathbb{E}_t\underbrace{\boldsymbol{\pi}^{P}_{t+1}}_{NJ\times 1} \tag{27}$$

where with some slight abuse of notation, we define the $\hat{\boldsymbol{\mathcal{E}}}_t$ as the $N^2\times 1$ vector of bilateral exchange rates, the $\hat{\boldsymbol{\tau}}_t$ as the $N^2J\times 1$ vector of tariff rates. In line with these vector representations, we also use $\mathbf{L}$ to denote loadings (i.e., how the subscript variable loads onto the superscript variable).[16] These expressions compactly describe how vector variables load onto a given equation and serve as partial derivatives. The matrix notation makes our

[15]To the first order, bundles presented in Sections 3.2 and 3.3 can be directly linked to the goods that form them. We can write these relations as:

$$\Gamma_{n,mi} = \Gamma_{n,i}\ \Gamma_{n,i,mi},$$
$$\Omega_{ni,mj} = (1-\alpha_{ni})\ \Omega_{ni,j}\ \Omega_{ni,j,mj},$$

[16]In particular, $(\boldsymbol{L}^{P}_{\mathcal{E}}\hat{\boldsymbol{\mathcal{E}}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}^{P}_{\tau}\hat{\boldsymbol{\tau}}_t)_{ni} = \sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\hat{\tau}_{n,mj,t}$.

expressions compact, generalizable, and useful for computational work.

Thus, keeping in mind the labor-leisure tradeoff and using the fact that the price level at time t is the past price level plus inflation, we can express producer prices in levels as:

$$\hat{\boldsymbol{P}}_t^P = \underbrace{[(1+\beta)\boldsymbol{I} + \boldsymbol{\Lambda}(\boldsymbol{I}-\boldsymbol{\Omega})]^{-1}}_{\boldsymbol{\Psi}_\Lambda} \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\underbrace{\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right)}_{\hat{\boldsymbol{W}}_t} + \boldsymbol{L}_{\mathcal{E}}^P \cdot \hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_\tau^P \cdot \hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right]$$

where $\boldsymbol{\Psi}_\Lambda$ is a stickiness-adjusted Leontief Inverse.

We can also express the CPI using these matrices. For analytical tractability, we define the $NJ \times 1$ dimensional CPI vector $\boldsymbol{P}_t^C$ such that $\boldsymbol{P}_{mi,t}^C = P_{m,t}^C$. With this, we can write the CPI as:

$$\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma} \cdot \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \cdot \hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_\tau^C \cdot \hat{\boldsymbol{\tau}}_t,$$

where $\boldsymbol{\Gamma}$ is an $N \times NJ$ matrix.[17]

Finally, in the linearized model we define $V_{n,t} = (1 + i_{n,t}^{US})B_{n,t}^{US}$ and linearize this variable. As we do so, we stack the balance of payments equations together with the market clearing condition for U.S. bonds as we detail below.

3.6.2 Global New Keynesian Representation

With the vector and matrix notation established, the full set of linearized equilibrium conditions in Appendix A can be written in vector form as an equilibrium that satisfies the Blanchard-Kahn stability conditions. We use this representation both for interpretation and to solve the model using the method of undetermined coefficients.[18] This five-equation representation is similar in spirit to the canonical three-equation New Keynesian model, if that model were extended to a context with N open economies, including I-O linkages.

Definition 3. A linearized equilibrium comprises vector sequences $\{\hat{\boldsymbol{C}}_t, \hat{\boldsymbol{P}}_t^P, \hat{\boldsymbol{P}}_t^C, \hat{\boldsymbol{\mathcal{E}}}_t, \hat{\boldsymbol{V}}_t\}_{t_0}^\infty$ for a given sequence of $\{\hat{\boldsymbol{\tau}}_t\}_{t_0}^\infty$ and an initial condition for $\hat{\boldsymbol{V}}_0$ such that equations (28)-(32) hold:

$$\textbf{NKIS+TR:} \quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1} - \hat{\boldsymbol{C}}_t) = \boldsymbol{\Phi}\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P) - \mathbb{E}_t(\hat{\boldsymbol{P}}_{t+1}^C - \hat{\boldsymbol{P}}_t^C) \tag{28}$$

$$\textbf{CPI:} \quad \hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_\tau^C\hat{\boldsymbol{\tau}}_t \tag{29}$$

[17]Similar to the production case, $(\boldsymbol{L}_{\mathcal{E}}^C\hat{\boldsymbol{\mathcal{E}}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\mathcal{E}}_{n,m,t}$ and $(\boldsymbol{L}_\tau^C\hat{\boldsymbol{\tau}}_t)_n = \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\hat{\tau}_{n,mj,t}$.

[18]We depict prices in levels (e.g., $\hat{\boldsymbol{P}}_t^P$) rather than in first differences (e.g., $\boldsymbol{\pi}_t^P$) for two reasons in this representation. First, since prices appear both in levels and in first differences doing so allows us to write an equilibrium with 5 vector variables and 5 vector equations in a compact manner. Second, this representation is convenient for the algebra work we do with the method of undetermined coefficients.

$$\textbf{NKPC:} \qquad \hat{\boldsymbol{P}}_t^P = \boldsymbol{\Psi}_\Lambda \left[\hat{\boldsymbol{P}}_{t-1}^P + \boldsymbol{\Lambda}\left(\boldsymbol{\alpha}\left(\hat{\boldsymbol{P}}_t^C + \sigma\hat{\boldsymbol{C}}_t\right) + \boldsymbol{L}_{\mathcal{E}}^P\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{L}_\tau^P\hat{\boldsymbol{\tau}}_t\right) + \beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P\right] \tag{30}$$

$$\textbf{UIP+TR:} \qquad \tilde{\boldsymbol{\Phi}}_1\mathbb{E}_t\hat{\boldsymbol{\mathcal{E}}}_{t+1} - \tilde{\boldsymbol{\Phi}}_2\hat{\boldsymbol{\mathcal{E}}}_t = \tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P) \tag{31}$$

$$\textbf{BoP:} \qquad \beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{\Xi}_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t \tag{32}$$

where "TR" denotes that the Taylor rule has been substituted in, and $\mathbf{L}$ notation represents loadings (i.e., how the subscript variable loads onto the superscript variable as a linear combination of the entries of the vector variable, as detailed above), which also serve as partial derivatives. In the first and fourth of these equilibrium conditions, the Taylor rule is used to substitute out the nominal interest rate, where the diagonal matrix $\boldsymbol{\Phi}$ contains the Taylor rule's sensitivity to the basket of producer price inflation in the respective countries. For example, in the two-country case, we have $\boldsymbol{\Phi} = \begin{bmatrix} \phi_\pi & 0 \\ 0 & \phi_\pi^* \end{bmatrix}$. That is, we have $\hat{\boldsymbol{i}}_t = \boldsymbol{\Phi}\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ and the first $N-1$ rows of $\tilde{\boldsymbol{\Phi}}_3\boldsymbol{\Gamma}(\hat{\boldsymbol{P}}_t^P - \hat{\boldsymbol{P}}_{t-1}^P)$ load the vector form of interest rate differentials $\hat{\boldsymbol{i}}_t - \hat{i}_t^{US}$ for countries other than the first country in our system, the U.S.

The first of these equilibrium conditions is the Euler (New Keynesian IS; NKIS) equation, which is defined in terms of aggregate consumer prices. Intuitively, the impact of tariffs enters the demand side through how tariffs load onto consumer prices.

The second equation defines the consumer price index (CPI). As the CPI and the producer price index (PPI) differ, with consumer prices being a weighted average of producer prices, exchange rates, and tariffs under our producer currency pricing assumption. Here, $\boldsymbol{L}_{\mathcal{E}}^C$ captures, in matrix form, how consumer prices of various goods are exposed to the exchange rate. The scalar analogy would be $(1-\gamma_H)$, where $\gamma_H \in [0,1]$ represents the home bias parameter for consumption. Similarly, $\boldsymbol{L}_\tau^C$ captures the share of goods exposed to tariffs.

The third equation is the New Keynesian Phillips Curve for producer price inflation, defined in levels for convenience in the analytical solution. The impact of the I-O network is captured in the stickiness-adjusted Leontief inverse term $\boldsymbol{\Psi}_\Lambda$. This term multiplies the diagonal matrix of stickiness parameters $\boldsymbol{\Lambda}$ and the matrix of nominal marginal costs. Additionally, $\boldsymbol{\Psi}_\Lambda$ multiplies both the vector of lagged producer prices $\hat{\boldsymbol{P}}_{t-1}^P$ and the discounted expectation of future producer prices $\beta\mathbb{E}_t\hat{\boldsymbol{P}}_{t+1}^P$. In this setup, the exchange rate loads onto nominal marginal costs via the dependence of producers on imported intermediate inputs, which is captured by $\mathbf{L}_{\mathcal{E}}^P$. Similarly, tariffs have a direct impact, as they load onto the share of goods exposed to tariffs, captured by $\mathbf{L}_\tau^P$. If not for their additional impact on consumer prices, tariffs τ would be isomorphic to standard supply shocks in the New Keynesian context.

The fourth equation combines the UIP condition, exchange rate arbitrage conditions, and the definition of a country's exchange rate with itself (i.e., nesting linearized versions of equations (2), (3), and (4)). Here, the $\tilde{\boldsymbol{\Phi}}$ terms ensure that the ϕ_π terms for each country, along with the arbitrage conditions, are correctly loaded in each row.

The fifth equation combines market clearing for debt with the $N-1$ equations of motion for net debt, capturing the balance of payments as a function of prices, which reflect the terms of trade for each specific country-good variety, and the aggregate consumption vector.[19] This final equation describes how a country's net external position evolves in response to changes in good-specific terms of trade, as well as fluctuations in the interest rate and the balance sheet effect of debt via exchange rates. As such, it nests all the intratemporal relative demand conditions and pricing equations. Through this equation, debt responds to automatic debt dynamics and adjustments in exports following changes in the terms of trade.

This five-equation general representation can nest a broad class of open-economy New Keynesian models. For example, models with a bundle of intermediate inputs and a final good correspond to the case where $\boldsymbol{\Omega}$ involves $J = 2$ and one of the columns of $\boldsymbol{\Omega}$ is a column of zeros. This representation is general for N-country New Keynesian models (e.g., Clarida et al., 2002). However, by collapsing the number of countries to one and making the real rate exogenous, it reduces to a small open economy model reminiscent of Galí and Monacelli (2005).

As we show in Appendix B, the system above can be solved out with the method of undetermined coefficients as follows:

$$
\begin{aligned}
\underbrace{\hat{\boldsymbol{C}}_t}_{N\times 1} &= \underbrace{\boldsymbol{c}_p}_{N\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{c}_v}_{N\times 1} \hat{V}_{t-1} + \underbrace{\boldsymbol{c}_\tau}_{N\times 1} \hat{\tau}_t \\
\underbrace{\hat{\boldsymbol{P}}^P_t}_{NJ\times 1} &= \underbrace{\boldsymbol{\Psi}^{NKOE}}_{NJ\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{p}_v}_{NJ\times 1} \hat{V}_{t-1} + \underbrace{\boldsymbol{p}_\tau}_{NJ\times 1} \hat{\tau}_t \\
\underbrace{\boldsymbol{\pi}^P_t}_{NJ\times 1} &= \underbrace{(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I})}_{NJ\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + \underbrace{\boldsymbol{p}_v}_{NJ\times 1} \hat{V}_{t-1} + \underbrace{\boldsymbol{p}_\tau}_{NJ\times 1} \hat{\tau}_t \\
\underbrace{\hat{\mathcal{E}}_t}_{1\times 1} &= \underbrace{\boldsymbol{e}_p}_{1\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + e_v \hat{V}_{t-1} + e_\tau \hat{\tau}_t \\
\underbrace{\hat{V}_t}_{1\times 1} &= \underbrace{\boldsymbol{v}_p}_{1\times NJ} \hat{\boldsymbol{P}}^P_{t-1} + v_v \hat{V}_{t-1} + v_\tau \hat{\tau}_t
\end{aligned} \tag{33}
$$

[19] The first $N-1$ rows contain linearized versions of equation (19), while the last row captures the bond market clearing condition given by equation (20). In Appendix B, we derive this equation of motion.

4 Tariffs Under Flexible Prices

The impact of tariffs on our main variables of interest, exchange rate, inflation, output, output gap, trade balance and consumption, are complex and dependent on the primitive parameters. In this section, we start with the flexible-price version of the model to establish intuition. In order to do so, we will focus on a two-country setup ($N = 2$) with an arbitrary number of industries J.

In order to understand the long-run impact of tariffs we consider a permanent increase in tariffs under flexible prices. As we detail in Appendix C.2, in our Global New Keynesian Representation this corresponds to setting $\rho \to 1$ and taking the limit of each of the diagonal entries of $\mathbf{\Lambda}$ to infinity (i.e. $\Lambda_{ni,ni} \to \infty$). Using this parametrization, with the method of undetermined coefficients we find:[20]

Proposition 1. *The first period impact of a permanent increase in tariffs under flexible prices on the endogenous variables is as follows:*

$$\frac{\partial \hat{\boldsymbol{C}}}{\partial \hat{\tau}} = \boldsymbol{c}_\tau = \left(\mathbf{\Gamma}\left(\boldsymbol{\alpha}\sigma - \Xi_4^{-1}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)\mathbf{\Xi}_2\right)\right)^{-1}\left[\Xi_4^{-1}\mathbf{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{L}_{\mathcal{E}}^P)\Xi_5 - \mathbf{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^P)\right]$$

$$\frac{\partial \boldsymbol{\pi}^P}{\partial \hat{\tau}} = \boldsymbol{p}_\tau = \mathbf{0}$$

$$\frac{\partial \hat{\mathcal{E}}}{\partial \hat{\tau}} = e_\tau = \Xi_4^{-1}\Big[-\mathbf{\Xi}_2\boldsymbol{c}_\tau - \Xi_5\Big]$$

$$\frac{\partial \hat{V}}{\partial \hat{\tau}} = v_\tau = 0$$

$$\frac{\partial \boldsymbol{\pi}^C}{\partial \hat{\tau}} = \boldsymbol{L}_{\mathcal{E}}^C e_\tau + \boldsymbol{L}_\tau^C$$

where we drop the time subscript, because under flexible prices, facing a permanent shock, the system instantly jumps to the new steady state and stays there thereafter.

Proof. See Appendix B.4. □

Corollary 1. *Under flexible prices, a permanent shock has zero impact on producer price inflation when the central bank targets a basket of producer prices. Aggregate CPI inflation*

[20] The first order approximation is around a given steady state, whereas a permanent shock will lead to the system settling at a different steady state. As a result, in general the first-order solution based on an approximation around the initial steady state may not be valid when considering a permanent change that delivers the system to a new steady state. We confirm our approach is valid in two ways. First, under flexible prices the whole system can be written with first difference variables (e.g. $\Delta\hat{\mathcal{E}}_t$) and $\Delta\hat{\tau}_t$ serves as the exogenous shock variable instead of $\hat{\tau}_t$. This resulting system satisfies Blanchard-Kahn stability conditions and a permanent shock to $\hat{\tau}_t$ is a one-time shock to $\Delta\hat{\tau}_t$, for which there is global stability. Secondly, we verify these solutions with the quantitative model. and confirm with our non-linear solution detailed in Section 7 that the first-order analytical solution here is numerically the same as the non-linear solution.

then reflects the change via the exchange rate and the direct effect of tariffs on consumer prices.

This is because prices are flexible and the policy rule targets a mix of producer price inflation.[21] As a result, in response to a permanent shock, the entire adjustment is done by variables other than producer price inflation. Notably, consumer price inflation is not zero as relative prices have to adjust. Similarly the exchange rate and consumption respond to tariffs.

Corollary 2. *Under flexible prices, a permanent shock does not change the net debt/asset position of either country denominated in the U.S. Dollar, which is the currency in which both countries save.*

This follows from the fact that $\frac{\partial \hat{V}}{\partial \hat{\tau}} = 0$. Under flexible prices, a permanent shock does not change the trade balance of either country expressed in U.S. Dollars. Note that the balance of payments can be summarized as follows from the perspective of the first country, US, whose local currency debt is used to facilitate global savings:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \beta^{-1}(1-\beta)\widehat{NX}_t + \hat{i}_t$$

When the targeted producer price basket's inflation rate is zero, this implies that the interest rate will also be zero in deviation from the steady state. Similarly, since the net debt position does not move, we have $\hat{V}_t = \hat{i}_{n,t} = 0 \ \forall n, t$. Then we necessarily have that the USD value of net exports do not change. That is $\widehat{NX}_t = 0 \ \forall t$. This is in line with the exact local neutrality result of Costinot and Werning (2025) and with the finding of Itskhoki and Mukhin (2025) that the long-run trade balance is determined by the financial position of a country.[22] Note that this does not rule out changes in quantities; trade balance in terms of quantities or expressed as a share of GDP can change, while the U.S. dollar value of net exports will remain constant. The intuition here is that in the presence of a permanent shock and flexible prices, the tariffs do not present an intertemporal tradeoff. In line with the permanent income hypothesis, the entire adjustment is done in quantities, while debt is not utilized. As a result, the USD value of net exports does not change.

[21] In Appendix C.2, we consider the case when the policy rule targets the entire CPI basket; in that case it is the level of consumer prices that remain unchanged in response to a permanent shock under flexible prices.

[22] Itskhoki and Mukhin (2025) emphasize the gross position of the tariff-imposing country. While our modeling framework allows for countries to accumulate debt or assets in more than one currency, in our analytical and quantitative work we restrict countries to net saving/dissaving in the dollar. Even if when net debt is expressed in the currency of the home country, we still have valuation effects.

Remark 2. The impact on consumption and the exchange rate is jointly determined and dependent on the sensitivity of the balance of payments to aggregate consumption vectors and the terms of trade as captured by the Ξ coefficients.

This follows from the first and third equations in Proposition 1. From the third equation we see that, from the point of view of the home country that places tariffs on the foreign country, an increase in consumption is associated with appreciation (since a negative movement in $\hat{\mathcal{E}}_t$ is defined as appreciation). The intuition here is as follows. As the optimal tariff literature has argued, when a large country places tariffs it can tilt the price vector in its favor depending on the tariff-imposing country's share in the market-clearing condition of the relevant good and elasticities. That is if the tariff imposing country is a sizable buyer of the tariffed good, it can benefit from tariffs and end up with consumption greater than steady-state levels. These effects are captured by the Ξ terms in our notation. In the first equation of Proposition 1, we see that the direct effects captured by $\boldsymbol{L}_\tau^C$ and $\boldsymbol{L}_\tau^P$) have a negative sign in front of them, when one considers the entries for the home country, which is the first country in our notation. As a tax on consumption and production, these direct effects are associated with lower consumption. However, the general equilibrium effects captured by the other terms can result in higher consumption for the tariff-imposing country especially in the absence of retaliation.

4.1 Scalar Example with One Industry ($N = 2$ & $J = 1$)

Let us now consider the scalar case for additional intuition. In order to do so, we set $J = 1$ and assume away self use by each industry. Then the matrices at hand will look as follows, when expressed in terms of the primitives:[23]

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \Omega_H \\ \Omega_F & 0 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 1-\Omega_H & 0 \\ 0 & 1-\Omega_F \end{bmatrix}, \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1-\gamma_H & \gamma_H \\ \gamma_F & 1-\gamma_F \end{bmatrix}$$

$$\boldsymbol{\Psi} = (\mathbf{I}-\boldsymbol{\Omega})^{-1}, \quad \boldsymbol{L}_{\mathcal{E}}^C = \begin{bmatrix} \gamma_H \\ -\gamma_F \end{bmatrix}, \quad \boldsymbol{L}_\tau^C = \begin{bmatrix} \gamma_H L_\tau^C \\ \gamma_F L_\tau^C \end{bmatrix}, \quad \boldsymbol{L}_{\mathcal{E}}^P = \begin{bmatrix} \Omega_H \\ -\Omega_F \end{bmatrix}, \quad \boldsymbol{L}_\tau^P = \begin{bmatrix} \Omega_H L_\tau^P \\ \Omega_F L_\tau^P \end{bmatrix}$$

where L_τ^C and L_τ^P are dummy variables that take on the value 0 or 1, indicating whether a given country imposes tariffs on the other one. We use subscripts H and F to refer countries in the two country case.

The first case to consider involves symmetry in parameters and symmetric retaliatory

[23] To make the notation easier to follow in the scalar case we simplify subscripts such that $\gamma_{H,F}$ becomes γ_H and $\Omega_{H,F}$ becomes Ω_H.

tariffs by both sides. Where we employ symmetry, we drop subscripts such that $\Omega_H = \Omega_F = \Omega$ and $\gamma_H = \gamma_F = \gamma$. We assume $0 \leq \gamma < \frac{1}{2}$ and $0 \leq \Omega < 1$.

Corollary 3. *Under symmetric parameters and retaliation, the impact of tariffs on consumption and the exchange rate is:*

$$\frac{\partial C_H}{\partial \tau} = \frac{\partial C_F}{\partial \tau} = -\frac{1}{\sigma}\left[\gamma + \frac{\Omega}{1-\Omega}\right] < 0$$
$$\frac{\partial \hat{\mathcal{E}}}{\partial \hat{\tau}} = \hat{\mathcal{E}}_\tau = 0$$

When parameters are symmetric and tariffs involve symmetric retaliation, the exchange rate response is zero. This, in turn, implies that a contraction in consumption by both countries is guaranteed. Import dependence both on the consumption side and production side sharpen this decline in consumption.

Next, we consider the case where parameters are asymmetric across the two countries but there is no retaliaton; tariffs are only placed by H on F.

Corollary 4. *Under asymmetric parameters and no retaliation, the impact of tariffs on consumption is:*

$$\frac{\partial \hat{C}_{H,t}}{\partial \tau_t} = -\underbrace{\frac{(\Omega_H + \gamma_H(1-\Omega_H))}{\sigma\,(1-\Omega_H)}}_{+}(e_\tau + 1)$$
$$= -\frac{(\Omega_H + \gamma_H(1-\Omega_H))}{\sigma(1-\Omega_H)}\underbrace{\frac{\Xi_{22}\big[\Omega_F + \gamma_F(1-\Omega_F)\big] + \Xi_4\sigma(1-\Omega_F) + \Xi_5\sigma(\Omega_F - 1)}{\Xi_{22}\big[\Omega_F + \gamma_F(1-\Omega_F)\big] - \Xi_{21}\big[\Omega_F + \gamma_H(1-\Omega_F)\big] + \Xi_4\sigma(1-\Omega_F)}}_{e_\tau}$$

With the home bias assumption under which γ_H and γ_F are less than 1/2 and given boundary $\Omega < 1$ we can sign this expression. For tariffs to expand tariff-imposing home country's consumption a sufficiently large appreciation of the home country's currency is needed. In this version of our model this corresponds to a more than one for one appreciation (i.e. for $\frac{\partial \hat{C}_{H,t}}{\partial \hat{\tau}_t} > 0$ it must be that $-e_\tau > 1$).

Two observations are noteworthy here. The first is that the rest of the world's parameters matter beyond picking export and import elasticities, when considering tariffs by the home country on the foreign country. This is in contrast with the small open economy approach. The rest of the world's parameters are loaded onto consumption via the impact on the exchange rate. The Ξ coefficients capture the sensitivity of the balance of payments to changes in aggregate demand, and good-specific terms of trade (i.e. change in quantities induced by granular price changes, exchange rate and tariffs). As we show in Appendix

B, these coefficients also contain the elasticities of substitution and the weights associated with the respective goods. Secondly, the solution for the exchange rate turns into a complex object as soon as one leaves the case of symmetry combined with symmetric retaliation and one expands the Ξ terms to express them in terms of the γ, Ω and θ terms. In Online Appendix B.4, we show the complexity of this term under asymmetry. For intuition let us turn momentarily to the solution for the exchange rate under the symmetry assumption:

$$e_\tau = -\frac{\overbrace{\Omega + (1-\Omega)\gamma + \sigma\theta\big[1-(1-\Omega)\gamma\big]}^{+}}{\underbrace{2\Big(\Omega + (1-\Omega)\gamma + \underbrace{\sigma\theta\big[1-(1-\Omega)\gamma\big]}_{+}\Big)}_{+} - \underbrace{\sigma(1+\Omega)}_{+}} \tag{34}$$

Since the numerator is strictly positive under our parameter restrictions, the sign of e_τ is determined entirely by the denominator. In particular, the leading minus sign implies $e_\tau > 0$ (a depreciation) if and only if the denominator is negative.

The sign of the expression above depends on the subtraction in the denominator. While tariffs are generally assumed to appreciationary, this implies that there is a range for the parameters θ, Ω and γ that result in depreciation. Setting $\sigma = 1$, we can see this analytically:

$$\theta \ < \frac{(1-\Omega)\big(\frac{1}{2}-\gamma\big)}{1-(1-\Omega)\gamma} \tag{35}$$

When θ is low, relative-price changes generate little expenditure switching: quantities are weakly responsive, so the tariff delivers limited terms-of-trade and volume effects. If, in addition, trade exposure is limited (small Ω and γ), the adjustment needed to satisfy the balance-of-payments condition operates primarily through valuation rather than quantities, making a depreciation consistent with external balance even after a tariff shock. To understand the intuition behind this result, let us consider the fifth equation in our model, the balance of payments equation, expressed in this context under *symmetric* parameters:[24]

$$\begin{aligned} \beta\hat{V}_t = \hat{V}_{t-1} + \underbrace{\frac{\mathcal{A}}{1+\Omega}}_{\Xi_2}\big(\hat{C}_{H,t} - \hat{C}_{F,t}\big) + \underbrace{\frac{\mathcal{A}(\theta-1)}{1-\Omega}}_{\Xi_3}\big(\hat{p}_{H,t} - \hat{p}_{F,t}\big) \\ + \underbrace{\frac{\mathcal{A}(1+\Omega-2\theta)}{(1-\Omega)(1+\Omega)}}_{\Xi_4}\hat{\mathcal{E}}_t - \underbrace{\frac{\mathcal{A}\theta}{(1-\Omega)(1+\Omega)}}_{\Xi_5}\hat{\tau}_t \end{aligned} \tag{36}$$

[24]Under symmetry, we have symmetric coefficients such that for example $\boldsymbol{\Xi}_2 = [\Xi_{21}\Xi_{22}]$, where $\Xi_{21} = -\Xi_{22} = \Xi_2$.

where $\mathcal{A} \equiv \gamma+(1-\gamma)\Omega > 0$. This expression demonstrates that the size of θ determines the sign of Ξ_4, which in turn determines whether the Marshall Lerner condition holds and a depreciation improves the trade balance. For intuition let us consider $\theta \to 0$: when goods are impossible to substitute and quantities remain unaffected, when the exchange rate depreciates imports (exports) become more expensive in domestic currency (less valuable in foreign currency), while export revenues in domestic currency (the import bill in foreign currency) remain the same. Via this mechanism depreciation can worsen the trade balance and increase the net debt of the home country, which corresponds to the case when $\Xi_4 > 0$ as $\theta \to 0$. In this regime, a depreciation raises the domestic-currency value of the import bill with little offsetting quantity adjustment, so it can worsen the trade balance; therefore, when a tariff mechanically pushes the external balance toward surplus, equilibrium may require a depreciation to offset that surplus. Thus equation (35) is based on the case in which θ is sufficiently low and the Marshall Lerner condition does not hold.[25] Therefore, after a tariff mechanically pushes the external balance toward surplus, equilibrium can require a depreciation because it is the way to undo that surplus when quantities are too inelastic.

4.2 Risk Sharing Wedge Under Incomplete Markets

The discussion above about the exchange rate highlights the role that incomplete markets play in this context. To explore this, let us begin by establishing some notation.

The perfect risk-sharing (complete-markets) benchmark implies

$$\sigma\big(\hat{C}_{H,t} - \hat{C}_{F,t}\big) = \hat{Q}_t, \qquad \hat{Q}_t \equiv \hat{P}^C_{F,t} + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t},$$

where $\hat{Q}_t$ is the real exchange rate. and the risk-sharing arrangement. Intuitively, with ex-ante full insurance the allocation is arranged via state-contingent transfers, not by trade in a single nominal bond, so that in each state the planner reallocates resources toward the location where a unit of consumption delivers higher marginal utility once its local price is accounted for. In particular, when $\hat{Q}_t$ is high (i.e. the home consumption basket is cheaper relative to the foreign one when converted into the same currency) efficiency requires relatively higher home consumption, because the same numeraire resources purchase more consumption at home and hence raise utility more there at the margin.

The perfect risk-sharing (complete markets) benchmark is given by $\sigma(\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{Q}_t$, where $\hat{Q}_t$ is the real exchange rate: $\hat{Q}_t = \hat{P}^C_{F,t} + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}$.[26] Under prefect risk sharing,

[25] Note that (35) is more restrictive than $\Xi_4 > 0$: any θ satisfying (35) also implies $\Xi_4 > 0$, but there exist parameter values for which $\Xi_4 > 0$ holds while (35) fails. In that region, $\Xi_4 > 0$ is only necessary—not sufficient for tariffs to be depreciationary in general equilibrium.

[26] This condition follows from the complete-markets optimality condition that equates state-contingent

which one can derive by including Arrow Debreu securities in the model instead of nominal bonds as a saving instrument, there is ex-ante full insurance. Intuitively, with ex-ante full insurance the allocation is arranged via state-contingent transfers, not by trade in a single nominal bond, so that in each state resources are allocated toward the location where a unit of consumption delivers higher marginal utility once its local price is accounted for. That is ex-ante consumption plans are made such that in states of the world where the consumption basket is relatively cheaper in the home country (indicated by a higher level of $\hat{Q}_t$), capital flows in the direction of the home country to ensure relative consumption is higher in the home country relative to the steady state.

Under incomplete markets, this perfect risk-sharing condition is violated, because there is not ex-ante full insurance. To see this we can combine the Euler equation with the UIP condition, which yields the Backus-Smith condition in expectation:

$$\sigma(\mathbb{E}_t \Delta \hat{C}_{H,t+1} - \Delta \hat{C}_{F,t+1}) = \mathbb{E}_t \Delta \hat{Q}_{t+1}$$

Rearranging this as $\hat{w}_t = \mathbb{E}_t \hat{w}_{t+1}$, where $\hat{w}_t \equiv \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$, we can see that the risk sharing wedge given by $\hat{w}_t$ is a martingale. For a given shock, $\hat{\tau}_t$ that reveals itself at $t = 0$ (and then decays thereafter with some persistence ρ), $\hat{w}_t$ will be some linear function of the initial shock: $\hat{w}_t = f(\hat{\tau}_0)\ \forall t$. That is in response to the tariff shock a wedge opens up on impact and then that wedge remains constant $\forall t > 0$. This wedge determines whether there is a positive or negative deviation from perfect risk sharing from the perspective of the home country. That is the tariff shock, as it is unexpected and agents do not have perfect ex-ante insurance for it, serves as a wealth transfer when the shock reveals itself. If the valuation effects and terms of trade gains from tariffs favor the home country (e.g. in the case of a unilateral tariff), $\hat{w}_t$ will be negative for reasonable values of θ, the home country's exchange rate will face appreciationary pressure, while its aggregate consumption will be pushed upwards relative to the perfect risk sharing benchmark.

The wedge representation is useful because instead of tracking debt as a time-varing state variable, the equilibrium can be written with the tariff shock and the risk-sharing wedge as a state variable. To demonstrate this consider the flexible-price version of our model with one good as we did above under symmtery. Let us make the additional assumption that monetary policy perfectly stabilizes aggregate prices in both countries instead of following a Taylor rule; under flexible prices, this does not matter for real variables and serves to

marginal utilities, when valued in the same currency: $\frac{U_C(C_{H,t})}{P^C_{H,t}} = \lambda \frac{U_C(C_{F,t})}{\mathcal{E}_t P^C_{F,t}}$ for a constant λ pinned down by initial wealth. Linearizing this condition around a steady-state under CRRA utility yields $\sigma(\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{Q}_t$.

simplify the notation. The equilibrium can then be defined as follows:

Definition 4. An approximated equilibrium comprises 7 sequences $\{\hat{P}^P_{H,t}, \hat{P}^P_{F,t}, \hat{\mathcal{E}}_t, \hat{\imath}_{H,t}, \hat{\imath}_{F,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \hat{w}_t\}_{t=0}^{\infty}$, the following equations hold:

1. Euler equations hold:

$$(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{\imath}_{H,t}$$
$$(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{\imath}_{F,t}$$

2. The price of the consumption basket is defined as follows:

$$0 = (1-\gamma)\,\hat{P}^P_{H,t} + \gamma\,(\hat{\mathcal{E}}_t + \hat{P}^P_{F,t} + \hat{\tau}_t)$$
$$0 = (1-\gamma)\,\hat{P}^P_{F,t} + \gamma(\hat{P}^P_{H,t} - \hat{\mathcal{E}}_t)$$

3. Price equals marginal cost:

$$\hat{P}^P_{H,t} = (1-\Omega)\hat{C}_{H,t} + \Omega(\hat{P}^P_{F,t} + \hat{\tau}_t + \hat{\mathcal{E}}_t)$$
$$\hat{P}^P_{F,t} = (1-\Omega)\hat{C}_{F,t} + \Omega(\hat{P}^P_{H,t} - \hat{\mathcal{E}}_t)$$

4. The two countries share risk imperfectly, with wedge $\hat{w}_t$:

$$\hat{\mathcal{E}}_t - (\hat{C}_{H,t} - \hat{C}_{F,t}) = \hat{w}_t$$

In this representation, the risk-sharing condition replaces both the UIP equation and the balance of payments equation from the five equation representation, and in the solution every variable can be expressed purely as a function of tariffs and the risk-sharing wedge. Following, once again, the method of undetermined coefficients we have:

$$\hat{C}_{H,t} = -\underbrace{\frac{\Omega(1-\gamma)+\gamma}{1+\Omega}}_{>0}\left(\frac{1}{1-\Omega}\hat{\tau}_t + \hat{w}_t\right)$$
$$\hat{\mathcal{E}}_t = -\underbrace{\frac{(\Omega(1-\gamma)+\gamma)}{1+\Omega}}_{>0}\hat{\tau}_t + \underbrace{\frac{(1-\Omega)(1-2\gamma)}{1+\Omega}}_{>0}\hat{w}_t$$

Under perfect risk sharing, when $\hat{w}_t = 0$, a unilateral tariff increase by the home country leads to appreciation ($\frac{\partial \hat{E}_t}{\partial \hat{\tau}_t} < 0$). The intuition behind this real appreciation has to do with the fact that tariffs shift demands towards domestic goods and under home bias the home country is the larger consumer of those goods; in that sense its consumption basket becomes more expensive. With the home country's consumption basket being more expensive, under prefect risk sharing resources (e.g. Arrow Debreu transfers) are allocated towards the country where consumption is cheaper; home country's consumption declines ($\frac{\partial \hat{C}_{H,t}}{\partial \hat{\tau}_t} < 0$).

For these unambiguous signs to change (e.g. for tariffs to improve home country's consumption or for there to be depreciation), tariffs must also lead to a deviation from perfect risk sharing (i.e. $\hat{w}_t \neq 0$) thereby constituting a wealth transfer that occurs when the shock is revealed. This is what happens, when we add back the UIP condition and the balance of payments equation in (36), thereby including in the model both net debt $\hat{V}_t$ and the risk sharing wedge $\hat{w}_t$ as endogenous variables. When we do so and solve the model again, the risk-sharing wedge that opens up and remains constant when tariffs are imposed can be written as follows:

$$\frac{\partial \hat{w}_{t+j}}{\partial \hat{\tau}_t} = -\frac{(1-\beta)}{1-\beta\rho^{\tau}} \frac{\gamma(1-\Omega)(\theta-1)^2-\theta}{(1+\Omega-2\theta)\big(\Omega(2(1-\gamma)+1)-(1-2\gamma)\big)}(1+\Omega) \quad \forall j > 0$$

The three dimensional grid with γ, Ω and θ contains more than one region where the sign flips. However, when Ω and γ are relatively small (e.g. between 0 and 0.2), which is reasonable given real-life data, this expression is positive for values of θ below a threshold and then it turns negative for values greater than the threshold. A negative $\hat{w}_t$ implies a wealth transfer that favors the tariff-imposing home country. When goods are easier to substitute, this favors the tariff imposing home country as terms of trade gains improve the net external position of the country. A final point to note is that the persistence of the tariff makes the risk-sharing wedge larger. More persistent tariffs lead to terms of trade gains for longer and that indicates a larger wealth transfer.

This discussion of the risk-sharing wedge that constitutes a wealth transfer is relevant for two purposes. First, the fact that tariffs open up a risk-sharing wedge is one of the three main aspects of tariff shocks as we explore in Section 5 with the other two components resembling an Euler equation shock and a cost-push shock. In other words, as we build up to the New Keynesian parts of the model, the intuition developed here regarding the open-economy core of the model with which tariffs open up a risk-sharing wedge and constitute a wealth transfer will continue to hold.

Second, this risk-sharing wedge representation is empirically relevant as it allows us to shed light on why the US dollar depreciated in 2025 as tariffs were introduced. As explored above, under perfect-risk sharing with $\hat{w}_t = 0$, tariffs are expected to be appreciationary and thus, a type of shock that renders $\hat{w}$ positive is needed to offset the appreciationary impact of tariffs. As explored in greater detail in Kalemli-Özcan et al. (2026), volatility and increased uncertainty can turn $\hat{w}_t > 0$ and assert pressures that simultaneously depress consumption and create depreciationary pressure on the exchange rate. Kalemli-Özcan et al. (2026) extends the model in this paper to a setting with CARA utility and risk-sensitive financial intermediaries. The former leads to variance of consumption serving effectively as an Euler equation shock that leads to precautionary saving, while the latter leads to variance of the exchange rate leading to endogenous UIP violations. Since the variance of consumption and the exchange rate are driven in turn by the variance of tariffs, an increase in the volatility of tariffs can lead to depreciation since both Euler equation shocks and a widening of the UIP wedge can lead to depreciation.

In this paper, in our quantitative analysis in Section 7, we will assume that the introduction of tariffs occur in a way that exogenously widens the UIP wedge in line with Kalemli-Özcan et al. (2026), which finds empirically that the UIP wedge increased by 2.98 percentage points in 1Q2025 relative to 4Q2024. Thereafter it remained elevated at 1.62 and 0.52 percentage points respectively over the next two quarters.[27]

This works in line with the mechanism described above to offset the appreciationary pressure from $\hat{w}_t < 0$. Let us suppose now the UIP condition has an exogenous wedge, κ_t: $\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t] + \kappa_t$. Using the Euler equations of the two countries we can substitute out the interest rates in the UIP condition, and we arrive at the Backus Smith condition in expectation, which now includes the exogenous wedge: $\sigma(\mathbb{E}_t \Delta \hat{C}_{H,t+1} - \Delta \hat{C}_{F,t+1}) = \mathbb{E}_t \Delta \hat{Q}_{t+1} + \kappa_t$. This in turn yields a wedge between the real exchange rate and relative consumption that depends on both the constant risk component explored above, $\hat{w}_t$ and the UIP premium: $\hat{w}_t + \kappa_t = \hat{Q}_t - \sigma(\hat{C}_{H,t} - \hat{C}_{F,t})$. Even as tariffs lead to $\hat{w}_t < 0$, sufficiently large κ can overcome the appreciationary pressure, and yield depreciation and a decrease in relative consumption.

4.3 Contributions of Primitives

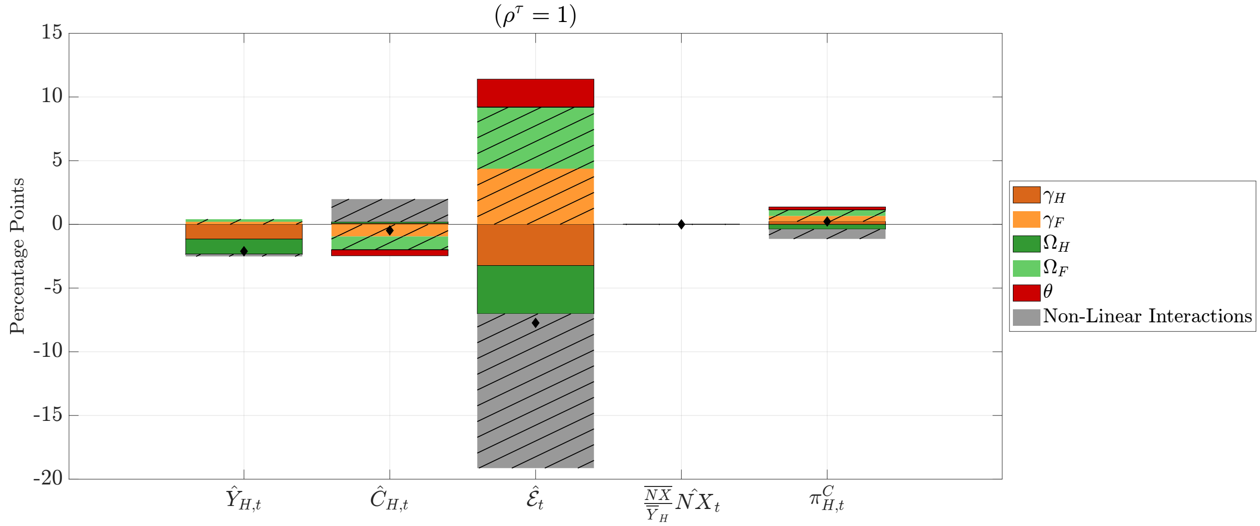
As is evident in the expressions above, while the solution is linear in the state variables, it is not linear in the parameters. Since the solved out terms can involve mathematically long expressions, below we visualize how the solution changes in response to changes in the primitives at hand as we re-introduce asymmetry of parameters: θ, Ω_H, Ω_F, γ_H and γ_F. That is we initialize these parameters respectively at $\theta = 4$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$ and look at changes in home country's macroeconomic variables of interest in the period of impact for a 10% tariff imposed by the home country on the foreign country, as one varies one parameter at a time. Each primitive's contribution comes from comparing the baseline results to the case when that primitives is set to 0.

[27] To align major tariff announcements in the first quarter of the presidency, Kalemli-Özcan et al. (2026) shifts quarters by 20 days such that the first quarter of 2025 starts with January 20, Inauguration Day.

Figure 1 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Specifically, to calculate contributions, we set each primitive of interest to 0, except for θ whose lower bound is set at 1.01, and recompute the outcome variables in that case.[28] Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic, except for the sign change in the exchange rate dependent on θ, which is delineated above and controlled for. That is why we can interpret these as contributions.

Figure 1. Contribution of Primitives to Macro Aggregates Under Flexible Prices

($\rho^\tau = 1$)



NOTE: Figure 1 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic and that is why we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 1.5$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 1$. This figure is consistent with our analytical work and simulations in Dynare.

In Figure 1, we see that consumption is declining in both γ_H and Ω_H, while they are increasing in the foreign country's parameters. The exchange rate appreciates in response

[28] Since θ can lead to a reversal of the exchange rate's sign, as explored above we set its lower bound at 1.01, so the contribution of θ can be interpreted as deviation from the Cobb-Douglas baseline.

to tariffs. This appreciation is stronger as one lowers the home bias in consumption and production for the home country. The intuition here is that as once Ω_H and γ_H increase, H becomes a larger buyer of goods produced by F and thus one has a larger change in the relative demand for F's goods, which in turn leads to a larger appreciation. This appreciation is not large enough to flip the sign of consumption into positive territory. Output is mostly responsive to the elasticity of substitution, which allows both production and consumption to respond to prices in both countries.[29] Output is declining in γ_H and γ_H, while it is not significantly responsive to foreign country parameters.

5 Tariffs Under Sticky Prices

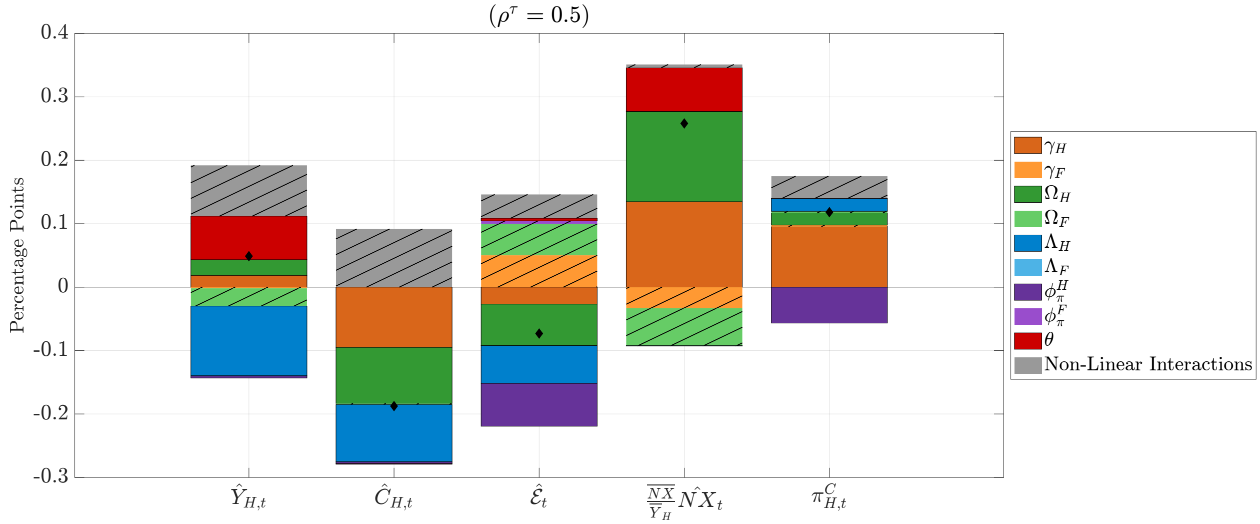
Having reviewed the impact of the first three of the five primitive factors, we now turn to the impact of the remaining two. That is, in this section, we add nominal rigidity in the form of sticky prices and policy. These additions change the impact of the first three primitives as well. To provide intuition, in the $N = 2$ & $J = 1$ case the primitives we are adding correspond to the following matrices and scalar objects:

$$\mathbf{\Lambda} = \begin{bmatrix} \Lambda_H & 0 \\ 0 & \Lambda_F \end{bmatrix}, \mathbf{\Phi} = \begin{bmatrix} \phi_\pi^H & 0 \\ 0 & \phi_\pi^F \end{bmatrix} \tag{37}$$

With these, to capture the core intuition of our model, we can build on the example in Figure 1 by constructing a similar figure for the sticky price version of the model. Figure 2 depicts the linear contributions of primitives to macro aggregates under sticky prices. Comparisons with Figure 1 are illustrative. In the presence of a transitory shock (AR(1) persistence $\rho^\tau = 0.5$), the trade balance temporarily improves and the exchange rate appreciation is smaller. Increases in elasticities of substitution incentivize production and net exports. Of the two newly added primitives, policy tends to put downward pressure on inflation and create additional appreciationary impact, because policy reacts to inflation by raising interest rates. Of the two primitives, Λ, on the other hand, adds a positive impulse to inflation. As the home country's sectoral NKPC grows steeper, inflationary impact of tariffs increase and there is an added downward pressure on output and consumption.

[29] Additionally, while output is solved out from the five-equation representation, we can compute it based on the solution of other variables.

Figure 2. Contribution of Primitives to Macro Aggregates Under Sticky Prices



Note: Figure 2 visualizes how the first period impact of 10% tariffs by the home country changes as the primitive parameters are changed. Each primitive's contribution is calculated by re-running the model with that primitive set to 0 one at a time and comparing the results to the baseline case. Throughout the paper we plot contribution figures like this one; these can be thought of as numerical second derivatives, capturing what happens to the impact of tariffs on variables of interest (the first derivative) as one varies the primitive parameters. In Section D, we plot bivariate plots that show these impacts are monotonic and that is why we interpret these as contributions. Hatching emphasizes the foreign country's parameters and the non-linear interaction terms that involve the foreign country's parameters. Net exports are measured as a share of steady-state Nominal GDP to make its interpretation more intuitive. We initialize these parameters respectively at $\theta = 1.5$ and $\Omega_H = \Omega_F = \gamma_H = \gamma_F = 0.1$. The AR(1) persistence of the tariff shock is set at $\rho^\tau = 0.5$. This figure is consistent with our analytical work and simulations in Dynare.

5.1 Tariffs as Three Separate Shocks

To understand the intuition behind Figure 2 and what stickiness adds to our model, we can think of the model as having two blocks: (i) an open economy block, which comprises imperfect risk sharing, and (ii) a New Keynesian block.

In this context, tariffs serve as three separate shocks. (i) First, on the open economy side, in the presence of incomplete markets, tariffs act as a shock that opens up a wedge in the Backus Smith risk sharing condition as detailed in Section 4.2. That is, tariffs lead to a deviation from the complete markets benchmark via their impact on the the balance of payments equation and thereby on the net debt position of the country. This third shock depends on all the primitives, but is primarily driven by $\boldsymbol{\Omega}$, $\boldsymbol{\Gamma}$ and $\boldsymbol{\theta}$.[30] (ii) Second, on the

[30] Once the wedge is opened up in the period when the shock is introduced, it remains constant thereafter unless portfolio adjustment costs eventually return the system to the initial steady state in terms of the real variables.

New Keynesian side, non-transitory tariffs act similar to an Euler equation shock (e.g. a patience shock or a consumption tax) and we denote this impact with $\boldsymbol{L}_{\tau}^{C}$. This is because the direct effects of tariffs can make it expensive to consume in a given period compared to other periods and as such the intertemporal tradeoff can be affected depending on monetary policy. Additionally, like other Euler equation shocks, this shock distorts the household's labor supply condition. This depends on the consumption shares primitive, given by $\boldsymbol{\Gamma}$. (ii) Third, again on the New Keynesian side, when tariffs are placed on intermediate inputs they can additionally act as a cost push shock. We denote this with $\boldsymbol{L}_{\tau}^{P}$. Similar to other supply shocks, this makes it expensive to produce and is dependent on the primitive $\boldsymbol{\Omega}$.

In our quantitative work we find that the risk-sharing wedge, as it takes into account the infinite horizon, is not very sensitive to price stickiness. For that reason we do not re-derive an expression for it under price stickiness. The latter two aspects can lead to an inflation-unemployment tradeoff and thus they interact with each other and the stickiness primitive $\boldsymbol{\Lambda}$ and policy primitive $\boldsymbol{\Phi}$. The presence of the network can make inflation persist for longer and/or output losses deeper. Stickiness determines how fast prices will adjust and policy determines the inflation output tradeoff.

With the risk sharing wedge representation introduced in Section 4.2, the model can be summarized with the following three equations along with definitions of inflation:

$$\textbf{NKIS+TR:}\quad \sigma(\mathbb{E}_t\hat{\boldsymbol{C}}_{t+1}-\hat{\boldsymbol{C}}_t)=\underbrace{\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\pi}_t^P}_{\hat{\boldsymbol{i}}_t}-\underbrace{\left(\boldsymbol{\Gamma}(\mathbb{E}_t\boldsymbol{\pi}_{t+1}^P)+\boldsymbol{L}_{\mathcal{E}}^C(\mathbb{E}_t\hat{\mathcal{E}}_{t+1}-\hat{\mathcal{E}}_t)+(\rho-1)\boldsymbol{L}_{\tau}^C\hat{\tau}_t\right)}_{\mathbb{E}_t\boldsymbol{\pi}_{t+1}^C}$$

$$\textbf{NKPC:}\quad \boldsymbol{\pi}_t^P=\boldsymbol{\Lambda}\underbrace{\left[(\boldsymbol{\Omega}-\boldsymbol{I})\,\hat{\boldsymbol{P}}_t^P+\boldsymbol{\alpha}\underbrace{\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^C\hat{\tau}_t+\sigma\hat{\boldsymbol{C}}_t\right)}_{\text{Nominal Wage}}+\boldsymbol{L}_{\mathcal{E}}^P\,\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^P\,\hat{\tau}_t\right]}_{\text{Real Marginal Cost}}+\beta\mathbb{E}_t\,\boldsymbol{\pi}_{t+1}^P$$

$$\textbf{Risk Sharing}\quad -\boldsymbol{Z}(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^P+\boldsymbol{L}_{\mathcal{E}}^C\hat{\mathcal{E}}_t+\boldsymbol{L}_{\tau}^C\hat{\tau}_t)+\hat{\mathcal{E}}_t=\sigma\boldsymbol{Z}\hat{\boldsymbol{C}}_t+\hat{w}_t$$

With this representation we see that the Euler equation shock aspect, captured by $\boldsymbol{L}_{\tau}^{C}$ acts both similar to a preference shock in the Euler equation and as a shock that distorts the nominal wage. If we turn off other two aspects (i.e. have no tariff on intermediate inputs and have complete markets), then the impact of only a tariff on final goods, acts as a preference shock. It reduces consumption, while the nominal exchange rate and inflation is unaffected. Of the three aspects of the tariff shock, this one is not directly related to the presence of a production network as the aggregate share of the consumption basket that is exposed to tariffs is what matters.

If we next, turn to the cost push shock aspect, captured by $\boldsymbol{L}_{\tau}^{P}$, we see that it will act as a supply shock. If we turn off the other two shocks (i.e. have complete markets and have no tax on final goods), this new distortion in production leads to higher producer price inflation, lower output and lower consumption on impact. The exchange rate appreciates as the home country's goods become more scarce. The presence of a production network with IO linkages can make the effects of this shock more persistent. Additionally, by way of parameter heterogeneity, a sector that is small in share can become more important depending on how its price stickiness parameter differs from other sectors and or how much weight policy places on that sector. Our NKOE Leontief Inverse captures persistence across time and how network granularity can amplify or deamplify importance of sectors. This cost push aspect is similar to Werning et al. (2025) and speaks to the broader production network literature (e.g., Rubbo, 2023; Pasten et al., 2020).

In the first two cases, above there is perfect insurance under complete markets. However, in the presence of incomplete markets, for reasonable parameters $\hat{w}_t < 0$. That is the tariff shock opens up a risk sharing wedge that favors and transfers wealth towards the home country, appreciating the home country's exchange rate while also possibly allowing it to increase its consumption. This wedge captures the terms of trade gains and the valuation effects emphasized by Itskhoki and Mukhin (2025). This occurs because of terms of trade gains and valuation effects that can render the tariff shock a positive shock for the net asset position of the tariff imposing country via the balance of payments equation. Under asymmetrty, if the home country is a large buyer of the foreign country's goods and if goods are highly substitutable the absolute value of home country's gains captured by $\hat{w}_t$ will be larger. The network structure makes this a more complicated term. The gains from the risk sharing wedge depend not only on aggregate shares but granular quantities. A sector that is small can become more important based on its position in the network. This is because elasticities of substitution differ in the two layers of CES. When comparing Japanese steel to Chinese steel, goods are substitutes, but when comparing steel to plastic these are complements. As such when aggregating granular quantities depending on the position in the network, can have a larger impact on $\hat{w}_t$ and thus become more important, if a higher elasticity gets applied to it to greater degree. The analogy here is to how the network can amplify or deamplify importance of sectors that are stickier or less sticky than other sectors.

5.2 Impact of Tariffs on Macroeconomic Aggregates

We next turn to the analytical solution for our model under sticky prices.

Proposition 2. *The first period impact of a transitory increase in tariffs ($0 < \rho < 1$) under* *sticky prices on the endogenous variables is as follows:*

$$\frac{\partial \hat{\boldsymbol{C}}_t}{\partial \hat{\tau}_t} = \boldsymbol{c}_\tau = \frac{1}{\sigma\,(1-\rho)}\Bigg[\Big[\sigma\,\boldsymbol{c}_p - (\boldsymbol{I} - \boldsymbol{L}^C_{\mathcal{E}}\boldsymbol{Z})\,\boldsymbol{\Phi\Gamma} + \boldsymbol{\Gamma}(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I}) + \rho\,\boldsymbol{\Gamma}\Big]\boldsymbol{p}_\tau + (\sigma\,\boldsymbol{C}_2 + \boldsymbol{\Gamma}\,\boldsymbol{p}_v)v_\tau - (1-\rho)\,\boldsymbol{L}^C_\tau\Bigg]$$

$$\frac{\partial \boldsymbol{\pi}^P_t}{\partial \hat{\tau}_t} = \boldsymbol{p}_\tau = \Big[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\,\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\Big]^{-1} \Big(\boldsymbol{\Lambda}\big(\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}})e_\tau + (\boldsymbol{\alpha}\boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau)\big) + \beta\,\boldsymbol{p}_v v_\tau\Big).$$

$$\frac{\partial \hat{\mathcal{E}}_t}{\partial \hat{\tau}_t} = e_\tau = \frac{1}{(1-\rho)}\left[(\boldsymbol{e}_p - \boldsymbol{Z\Phi\Gamma})\boldsymbol{p}_\tau + e_v v_\tau\right]$$

$$\frac{\partial \hat{V}_t}{\partial \hat{\tau}_t} = v_\tau = \beta^{-1}\Big[\boldsymbol{\Xi}_2\boldsymbol{c}_\tau + (\boldsymbol{\Xi}_3 + \boldsymbol{\Xi}_6\boldsymbol{\Phi\Gamma})\boldsymbol{p}_\tau + \Xi_4 e_\tau + \Xi_5\Big]$$

$$\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = \boldsymbol{\Gamma}\boldsymbol{p}_\tau + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau$$

where $\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda = \boldsymbol{\Psi}^{-1}_\Lambda - \boldsymbol{\Lambda\alpha\Gamma}$. *In these expressions, expectations are solved out; however, for notational compactness we maintain the undetermined coefficients on the right-handside.*[31]

Proof. See Appendix B.6. □

The representation of the system in Proposition 2 captures key intuitions of our model. First, the impact on consumption and the exchange rate depends on tariffs' impact on the balance of payments equation v_τ. This is sensible, because this equation captures the intertemporal budget constraint.[32]

Remark 3. If tariff shocks are not persistent and/or they are not perceived to be persistent, the on-impact exchange rate response will be smaller. Relatedly, the ability of the tariff-imposing country to tilt the price vector in its favor and have a welfare-improving increase in consumption will be more limited.

When tariffs are placed, if terms of trade gains are large enough to overcome the inefficiencies in production, prices can move in a manner that favors the tariff imposing country. That is, in line with the flexible-price case above, simultaneously one can see a sufficiently

[31] In Appendix B.6, we show these coefficients in greater detail.

[32] To see this we can forward the relevant equation:

$$\hat{V}_t = E_t\left[\overline{V} + \sum_{k=1}^{\infty}\beta^{k-1}\Big((1-\beta)\widehat{NX}_{t+k} - \beta\hat{i}_{t+k}\Big)\right]$$

where $\overline{V} = \lim_{k\to\infty}\hat{V}_{t+k}$.

large appreciation and consumption improve. This is dependent on the persistence of tariffs as evidenced by $(1-\rho)$ term in the denominator for the e_τ expression, as well as the market power that the tariff-imposing country has as a large buyer and how easy it is to substitute, both of which are captured by the Ξ terms in the balance of payments equation and in the expression for v_τ. In our model, the standard intuition regarding the signs of e_v and v_τ holds. Having a larger net debt position (having more net foreign assets) is depreciationary (appreciationary), so $e_v > 0$. Tariffs improve the trade balance and reduce net debt in the short run so $v_\tau < 0$. The signs of these two terms explain why tariffs are generally appreciationary on impact given that $e_\tau \approx \frac{e_v v_\tau}{(1-\rho)}$ significantly contributes to e_τ. In this expression we additionally see the role that policy plays through the $(-\boldsymbol{Z}\boldsymbol{\Phi}\boldsymbol{\Gamma})\boldsymbol{p}_\tau)$ term; in a country that suffers positive (negative) tariff-related inflation, if policy is more reactive to there will be an additional appreciationary (depreciationary) impact from policy.

The second key aspect that Proposition 2 highlights is the role played by $\boldsymbol{\Psi}^{NKOE}$, which is the coefficient matrix multiplying the lagged price vector in the solution for producer prices, since producer prices feed into all other equations. We call this term the New Keynesian Open Economy Leontief Inverse, because it is a dynamic general equilibrium Leontief Inverse that is closely linked with the stickiness-adjusted Leontief Inverse. $\boldsymbol{\Gamma}(\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I})$ appears in the first equation in Proposition 2, because expected consumer price inflation involves producer price inflation as it loads onto consumption prices. $\boldsymbol{\Psi}^{NKOE}$ additionally appears in the second equation in the propagation term alongside the stickiness-adjusted Leontief Inverse: $\left[\tilde{\boldsymbol{\Psi}}_\Lambda^{-1} - \beta\big(\rho\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1}$. Notably, how $\boldsymbol{\Psi}^{\text{NKOE}}$ compares to $\tilde{\boldsymbol{\Psi}}_\Lambda^{-1}$ makes a difference and can amplify or mute entries in conjunction with the $\boldsymbol{\Lambda}$ matrix, containing price stickiness parameters.

Whereas a regular Leontief Inverse and the stickiness-adjusted Leontief Inverse captures propagation of pricess across sectors at a given point in time, $\boldsymbol{\Psi}^{NKOE}$ is a DGE object, and it can amplify or mute the importance of certain prices in the network as prices propagate across time, since this matrix is the matrix that multiplies the lagged price vector in the solution for producer prices. $\boldsymbol{\Psi}^{NKOE}$ is closely related to the stickiness-adjusted Leontief Inverse, $\tilde{\boldsymbol{\Psi}}_\Lambda$. This is because $\boldsymbol{\Psi}^{NKOE}$, solves the following quadratic equation.

$$\left[\big(\boldsymbol{\Psi}_\Lambda^{-1} - \beta\boldsymbol{\Psi}^{NKOE}\big)\boldsymbol{\Psi}^{NKOE} - \underbrace{\boldsymbol{\Lambda}\big(\boldsymbol{\alpha} + \mathbf{L}_{\mathcal{E}}^{P}\mathbf{Z}\big)\boldsymbol{\Phi}\boldsymbol{\Gamma}}_{\substack{\text{Policy}\\\text{Impact}}} - \mathbf{I}\right](\boldsymbol{\Psi}^{NKOE} - \mathbf{I}) = 0$$

As we detail in Section 6, when the number of sectors J=1 the first term in brackets on the left is invertible, which results in $\boldsymbol{\Psi}^{NKOE} = \boldsymbol{I}$. When that is not the case we end up a with a more complex matrix that takes into account, both the input-output structure as it is captured by $\boldsymbol{\Psi}_\Lambda$, but also the impact of policy. Policy is featured here in two ways: i) as it impacts consumption and thereby wages, ii) via the exchange rate. The intuition has to do with the following. The lagged price vector serves two purposes: 1) the NKPC depends on it for propagation and 2) monetary policy depends on it as it targets inflation. How prices persist into the future will balance these two aspects. That is $\boldsymbol{\Psi}^{NKOE}$ tells us dependencies across sectors and how shocks spread through the network in DGE over time with the twist that it depends on both input-output structure and monetary policy.

Via the terms that it involves, the NKOE Leontief inverse, which relates the tariff-related distortions on both consumption (demand) and production (supply) to the dynamics of the system inclusive of the impacts of policies. Intuitively, if a given sector is central to production- either because it is widely used across industries (e.g., steel and aluminum) or due to its downstream importance (e.g., semiconductor chips)- it will carry significant weight in the standard Leontief inverse. If this sector also exhibits highly flexible (or rigid) prices- corresponding to a vertical (or horizontal) supply curve with fixed quantity (or highly elastic supply)- the inflationary or deflationary impact of a tariff on that sector will be amplified (or muted) by the network captured in the NKOE Leontief inverse. In these cases this object resembles the stickiness-adjusted Leontief Inverse; however, in our context it additionally takes into account the impact of policy. In that sense like the regular stickiness-adjusted Leontief Inverse can amplify or mute sectors based on $\boldsymbol{\Lambda}$, for the NKOE Leontief inverse it will also matter if a given sector is located in a country with relatively loose (or tight) monetary policy (e.g. via the impact on consumption and the exchange rate).

5.3 Decomposing Inflationary Impacts of Tariffs

To see the role that $\boldsymbol{\Psi}^{NKOE}$ plays, let us decompose the impact of tariffs on inflation. Combining the second and last equations of Proposition 2 yields the following decomposition of the effect of tariffs on consumer prices as we detail in C:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \hat{\tau}_t} = \underbrace{\boldsymbol{L}_\tau^C}_{\text{Direct CPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{L}_\tau^P}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\boldsymbol{L}_\tau^C}_{\text{Direct Effect on Real Wages}} + \underbrace{\boldsymbol{\Gamma}\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau}_{\text{Demand channel}}$$
$$+ \underbrace{(\boldsymbol{\Gamma}(\boldsymbol{\alpha}\boldsymbol{L}_\mathcal{E}^C + \boldsymbol{L}_\mathcal{E}^P) + \boldsymbol{L}_\mathcal{E}^C)e_\tau}_{\text{ER channel}} + \underbrace{\beta\boldsymbol{\Gamma}\boldsymbol{\Lambda}^{-1}\boldsymbol{p}_v v_\tau}_{\text{Debt channel}} + \underbrace{\boldsymbol{\Gamma}\Big(\Big[\tilde{\boldsymbol{\Psi}}_\Lambda^{-1} - \beta\big(\rho\boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\Big]^{-1}\boldsymbol{\Lambda} - \boldsymbol{I}\Big)\boldsymbol{H}_2}_{\text{Propagation with Stickiness}} \tag{38}$$

where $\boldsymbol{H}_2 = \big(\boldsymbol{\Lambda}\boldsymbol{\alpha}\sigma\boldsymbol{c}_\tau + \boldsymbol{\Lambda}(\boldsymbol{\alpha}\boldsymbol{L}_\mathcal{E}^C + \boldsymbol{L}_\mathcal{E}^P)e_\tau + \boldsymbol{\Lambda}(\boldsymbol{\alpha}\boldsymbol{L}_\tau^C + \boldsymbol{L}_\tau^P) + \beta\,\boldsymbol{p}_v v_\tau\big)$.

In this decomposition the first term captures the direct effect on CPI. The second term captures the direct effect on PPI as it loads onto consumer prices, accounting for stickiness. The third term captures the direct effect on real wages; when tariffs are placed, they change the real wages perceived by the household and thereby distort the labor supply mechanism. The fourth term, captures the impact of tariffs on producer prices via their impact on demand. The fifth term captures the impact of tariffs via the impact on the exchange rate, which takes into account how the exchange rate impacts both producer prices and consumer prices. The sixth term captures the impact of tariffs on producer prices via the impact on the net debt position. The final term captures propagation under price stickiness.

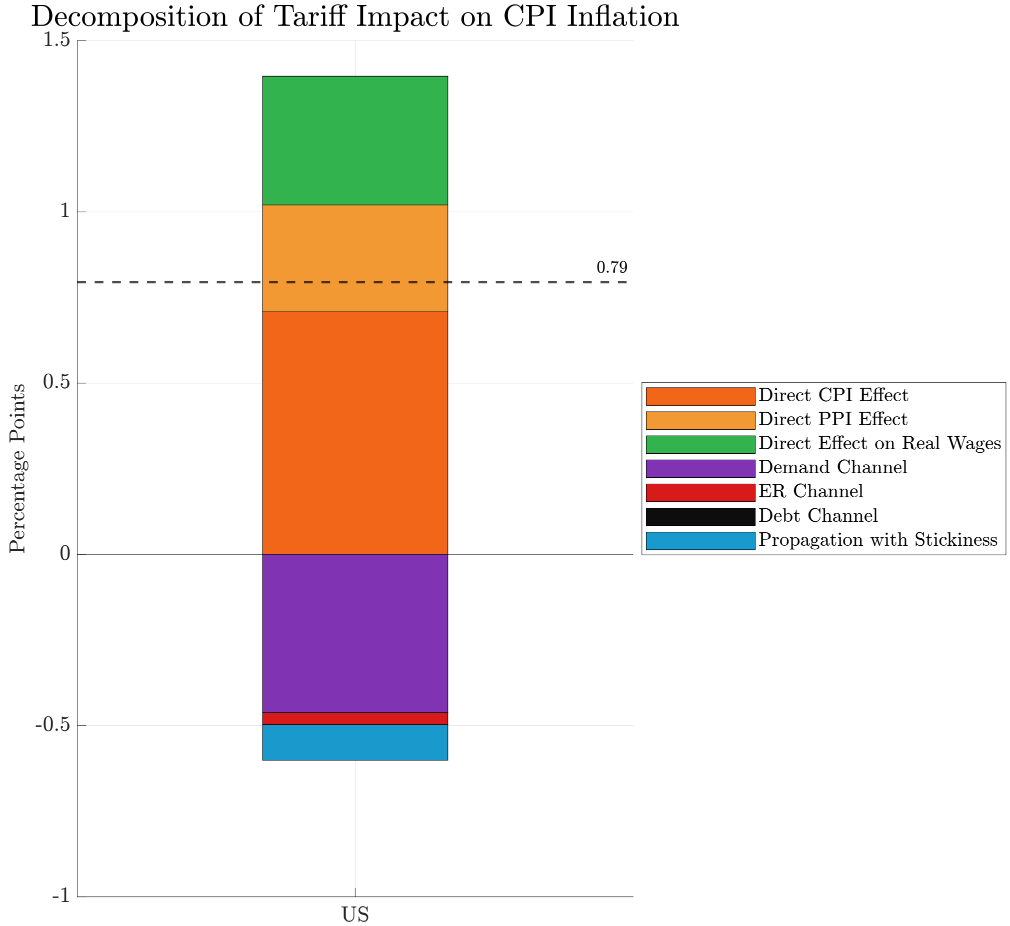
To illustrate how these channels operate and to build intuition around the model, let us consider an example based purely on the analytical solution above. Our objective here is not to conduct a full quantitative exercise- that is reserved for Section 7. Imagine dividing the world into two regions: the United States and the rest of the world. Suppose the United States imposes a 10% tariff on all goods and industries imported from the rest of the world with persistence $\rho^\tau = 0.5$. Agents in both regions anticipate that these tariffs will decay across time in line with ρ^τ. We use the parameter values described in greater detail in Section 7 and Table 2, except where simplifications of the analytical model apply (e.g., $\sigma = 1$, $\eta = 0$). The impact of this theoretical tariff shock is illustrated in Figure 3 below.

When this transitory tariff shock occurs, the direct impact on CPI and PPI generates an inflationary impulse of approximately 1 percentage point in the tariff-imposing country. The magnitude of these direct effects is related to the trade openness of the United States. We further observe direct effects on households' real wage, which distorts their labor supply decision. Beyond these direct effects, we also observe indirect effects.

In this example, demand declines in response to tariffs. This is for two reasons. First, policy responds to a mix of producer prices, which are increasing. As a result, with the central bank raising real interest rates to stabilize producer prices, demand will contract. Second reason behind the decline in demand is that the tariff shock is not persistent enough to generate a sufficiently large terms of trade gains to turn consumption positive. Exchange rate appreciation in this example adds a modest downward pressure on inflation as well. Finally, the propagation term, which combines the Leontief Inverses and the price stickiness matrix Λ adds a downward pressure on inflation on impact.

A core takeaway from Figure 3 is that for a reasonable parametrization, the direct impacts of tariffs drive a significant portion of the initial inflationary impact of tariffs. This is the context in which tariffs are often described as a "one time increase in prices." In our work below, we show that in the presence of production networks inflation can be more persistent precisely via the $\mathbf{\Psi}^{NKOE}$ term when more sectors are included in the model and the production network is thus modeled in a more granular manner.

Figure 3. Theoretical Example: Tariffs Without Retaliation



Note: Figure 3 visualizes the decomposition of CPI inflation in a two-country case, namely the U.S. and the rest of the world (RoW). We assume the US imposes an additional 10% tariff on RoW. Using Equation 38, we break down the different contributing effects. The dashed line represents the total effect, showing an inflation increase of 0.79% in the U.S.

5.4 Alternative Policy Formulations

In our Online Appendix, we solve the model under different policy regimes and show that propagation of inflation is different under different regimes. In Section F, we start with the special case when there is a real rate rule that fixes consumption in all countries of interest. Next we develop the case when policy fixes nominal demand, and the pressure from tariffs is shared equally by the the aggregate price level and aggregate consumption within each country.

In line with our two main research questions, these cases serve two purposes. First, we consider these cases to see what happens to macroeconomic aggregates in response to tariffs if monetary policy targets quantities (e.g., consumption), or prices (e.g., inflation targeting), or a mix of both (e.g., fixing nominal demand). Second, we explore how network propagation changes under different policy regimes. To capture propagation, we develop New Keynesian Open Economy Leontief Inverse matrices for these cases.

Intuitively, we find that network propagation is different under different policy regimes. Under a real rate rule that stabilizes consumption, tariffs lead to depreciation via expenditure switching and home and foreign monetary policies in the UIP equation. This is in marked contrast with the case when policy fixes nominal demand; this renders inflation in each sector and each country weakly positive, as tariffs act as a marginal cost shock and a marginal cost shock in one part of the network propagates as a marginal cost shock in all parts of the network. Below we highlight the latter result.

Consider a version of the model whereby policy follows a nominal demand rule. That is we replace the Taylor rule with the equation: $\hat{P}_t + \hat{C}_t = \hat{M}_t$ which fixes nominal domestic demand. Additionally we set $\sigma = 1$ and we obtain $\hat{W}_{n,t} = \hat{M}_{n,t} = \hat{P}_{n,t} + \hat{C}_{n,t}$. This approach is similar to menu cost models such as Golosov and Lucas (2007); Caratelli and Halperin (2023) and can be microfounded using a cash-in-advance constraint.[33] The economic interpretation is that with an exogenous $\hat{M}_{n,t}$, policy sets the overall aggregate domestic demand stance, similar to earlier generations of models such as Salter-Swan (Swan, 1963; Salter, 1959). In a closed-economy setting, the policy rule would be analogous to nominal GDP targeting.

Using the method of undetermined coefficients yields the following solution for CPI inflation with a different NKOE Leontief Inverse, $\mathbf{\Psi}_{\Lambda}^{NKOE}$, that is particular to this context.

Proposition 3. *With future shocks set to zero such that (i.e., $\tau_{t+j} = \hat{M}_{t+j} = \hat{M}^*_{t+j} = 0 \; \forall j > 0$) the solution for consumer price inflation is:*

$$\boldsymbol{\pi}_t^C = \left(\underbrace{\mathbf{\Gamma}\,\mathbf{\Psi}_{\Lambda}^{NKOE}\mathbf{\Lambda}}_{\substack{\textit{NKPC}\\ \textit{propagation}}} \quad \underbrace{(\boldsymbol{I} - \mathbf{\Omega})}_{\substack{\textit{via Wages and}\\ \textit{via ER for producers}}} + \underbrace{(\boldsymbol{I} - \mathbf{\Gamma})}_{\textit{via ER for consumers}} \right) \hat{\boldsymbol{M}}_t + \left(\underbrace{\mathbf{\Gamma}\,\mathbf{\Psi}_{\Lambda}^{NKOE}\mathbf{\Lambda}}_{\substack{\textit{NKPC}\\ \textit{propagation}}} \quad \underbrace{\boldsymbol{L}_{\tau}^{P}}_{\substack{\textit{Tariff incidence}\\ \textit{for Producers}}} + \underbrace{\boldsymbol{L}_{\tau}^{C}}_{\substack{\textit{Tariff incidence}\\ \textit{for consumers}}} \right) \hat{\boldsymbol{\tau}}_t + \underbrace{\mathbf{\Gamma}\left(\mathbf{\Psi}_{\Lambda}^{NKOE} - \boldsymbol{I}\right)\hat{\boldsymbol{P}}_{t-1}^{P}}_{\textit{Impact of lagged prices}} \tag{39}$$

Proof. See Online Appendix □

[33] This approach can also be microfounded by incorporating money in the utility function.

As seen above in Equation (39), policy and tariffs affect consumer price inflation through two channels: first, via producer prices, and second, through the exchange rate and tariffs that convert a producer price into a consumer price. Tariffs load onto producer prices and propagate with $\boldsymbol{\Psi}_{\Lambda}^{\text{NKOE}}\boldsymbol{\Lambda}$ and they also directly load onto prices. Since policy fixes nominal demand, tariffs do not impact the exchange rate in this context. That is why, as we highlight below, fixing nominal demand as a policy rule produces a very particular propagation for inflation: a tariff shock in one place is inflationary everywhere.

Proposition 4. *The impact of a one-time tariff ($\tau_t \geq 0$) on consumer price inflation is always weakly positive under fixed nominal demand. That is, let $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t}$ be an $NJ \times 1$ vector such that $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$.*

Proof. We can derive the necessary derivative from (39) as follows:

$$\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} = \boldsymbol{L}_{\tau}^C + \boldsymbol{\Gamma}\boldsymbol{\Psi}_{\Lambda}^{\text{NKOE}}\boldsymbol{\Lambda}\boldsymbol{L}_{\tau}^P \tag{40}$$

In this context, $\boldsymbol{L}_{\tau}^P = \tilde{\boldsymbol{\Omega}}^F$ and $\boldsymbol{L}_{\tau}^C = \tilde{\boldsymbol{\Gamma}}^F$ correspond to the row sums of the foreign elements in intermediate inputs and final consumption, respectively. All matrices on the right-hand side of Equation (40) contain weakly positive entries. As a result, $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t} \geq \mathbf{0}$.

This holds because $\boldsymbol{\Lambda}$ has weakly positive entries by construction. The matrix $\boldsymbol{\Psi}^{\text{NKOE}}\Lambda$ is a sign-preserving transformation of the stickiness-adjusted Leontief inverse $\boldsymbol{\Psi}\Lambda$. Like the standard Leontief inverse, $\boldsymbol{\Psi}_{\Lambda}$ has weakly positive entries. This is because it can be expressed as a Neumann series, namely an infinite sum of matrices with nonnegative entries.

By definition, $\tilde{\boldsymbol{\Omega}}^F$ also retains nonnegative entries. The product of a matrix and a vector with non-negative entries is another vector with nonnegative entries. Thus, every entry of $\frac{\partial \boldsymbol{\pi}_t^C}{\partial \tau_t}$ is weakly positive. □

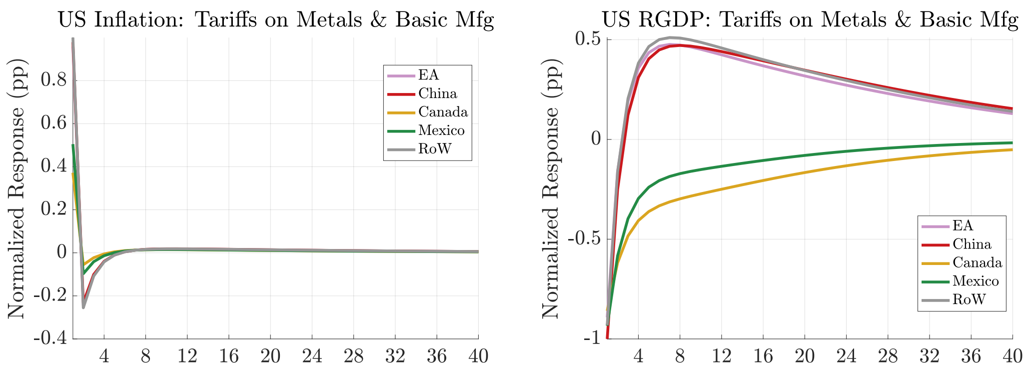
6 Why Network Matters? The Case for $N > 2, J > 1$

Since we aim to understand how tariffs affect macroeconomic aggregates and how this answer changes in the presence of a production network, a natural question to ask is when having a more granular network (i.e. with more than one sector) matters?

In network models, having more than one sector ($J > 1$) can produce different results for macro aggregates in two related and established cases. First, when parameters are heterogenous at the granular level, aggregating the parameters and multiplying them will produce a different result compared to multiplying them at the granular level and then aggregating. Pasten et al. (2020) and Rubbo (2023) show this in the case of the New

Keynesian Phillips Curve and in our notation this can be seen with Equation (26). where the granular price updating frequency multiplied with the granular weight will produce a different result if the frequencies are aggregated and then multiplied with an aggregated weight. Secondly, having more than one sector in the model can produce different results if one is studying shocks at the sectoral level and sectors different along various parameters (e.g. shocking a sector with a more vs. less vertical sectoral New Keynesian Phillips Curve). The current tariff context fits both of these conditions, which is why we find it appropriate the study this subject with a multi-sector model. Figure 4 constitutes an example of US tariffs on even the same sector across different countries can be different. This is because parameters are heterogenous at the country-sector level and shocks are sector-specific.

Figure 4. Impact of Sectoral Tariffs



NOTE: Figure 4 visualizes the impact of 10% ad valorem tariff applied to metals and basic manufacturing sector from different countries on inflation (left panel) and output (right panel).

These two cases apply to the two aspects of how tariff shocks work in our model. Having more than one sector will impact the risk sharing wedge, because sectors that are small in size can produce differing results if their elasticities of substitution (θ) are different. Similarly sectors that are small in size can impact macro aggregates differently depending on how their price stickiness parameter (Λ) compares to that of other sectors.

Beyond these cases, our model yields a third way in which having $J > 1$ produces different results for macro aggregates: specifically, we find that lagged prices can matter for future inflation in the presence of a production network. This has to do with how propagation across time changes based on the number of sectors. To develop this point, consider Corollary 5:

Corollary 5. *When the number of sectors, $J = 1$, we have:*

$$\boxed{\mathbf{\Psi}^{NKOE} = \mathbf{I}, \quad \boldsymbol{p}_v = \mathbf{0}, \quad \boldsymbol{v}_p = \mathbf{0}, \quad v_v = 1}.$$

Proof. This follows from Proposition 2 and the fact that when $J = 1$, $N \times NJ$ matrices like $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}$ are invertible. For details, see Appendix B.5. □

In order to understand the implications of Corollary 5, let us note that the solution to our Five-Equation Global NK Representation in (33), has a VAR(1) representation of the following type: $\boldsymbol{x}_t = \boldsymbol{A}\boldsymbol{x}_{t-1} + \boldsymbol{B}\,\epsilon_t$ where

$$\boldsymbol{x}_t \equiv \begin{bmatrix} \boldsymbol{\pi}_t^P \\ \hat{\boldsymbol{P}}_t^P \\ \hat{V}_t \\ \hat{\tau}_t \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{\Psi}^{NKOE} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{v}_p & v_v & v_\tau\,\rho \\ \mathbf{0} & \mathbf{0} & 0 & \rho \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{p}_\tau \\ \boldsymbol{p}_\tau \\ v_\tau \\ 1 \end{bmatrix}.$$

With the VAR(1) representation, we can derive analytical impulse response functions. For example, producer price inflation, j periods after tariffs with persistence ρ are imposed can be written as:

$$\frac{\partial\,\boldsymbol{\pi}_{t+j}^P}{\partial \epsilon_t} = \boldsymbol{S}_\pi\,\boldsymbol{A}^j\,\boldsymbol{B} \text{ where } \boldsymbol{S}_\pi \equiv \begin{bmatrix} \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Remark 4. It follows from Corollary 5 that the propagation matrix $\boldsymbol{A}$ looks different when the number of sectors, $J = 1$, versus $J > 1$.

$$\boldsymbol{A}_{J=1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \mathbf{0} & 1 & v_\tau\,\rho \\ \mathbf{0} & \mathbf{0} & 0 & \rho \end{bmatrix}, \quad \boldsymbol{A}_{J>1} = \begin{bmatrix} \mathbf{0} & \boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{\Psi}^{NKOE} & \boldsymbol{p}_v & \boldsymbol{p}_\tau\,\rho \\ \mathbf{0} & \boldsymbol{v}_p & v_v & v_\tau\,\rho \\ \mathbf{0} & \mathbf{0} & 0 & \rho \end{bmatrix}$$

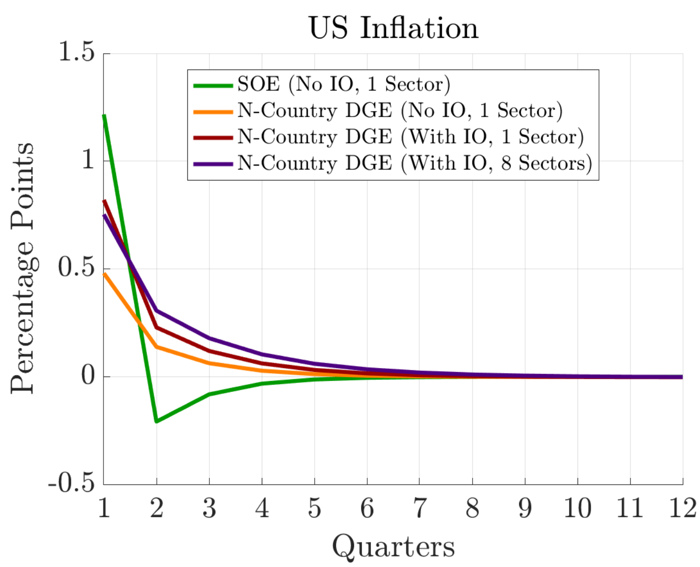
That is the number of sectors changes how endogenous lagged variables help propagate shocks. Most importantly, the shape of $\boldsymbol{\Psi}^{NKOE}$ can lead to persistence. When $j = 1$, $\boldsymbol{\Psi}^{NKOE}$ collapses to the identity matrix and the impact of lagged prices on contemporaneous inflation $\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} = \mathbf{0}$. On the other hand, when $J > 1$, $\boldsymbol{\Psi}^{NKOE} - \boldsymbol{I} \neq \mathbf{0}$ and the lagged price vector can lead to persistence in inflation.

The degree of inflation persistence hinges on both the network setup and the price stickiness matrix $\boldsymbol{\Lambda}$. In Figure 3, we show that the on-impact contribution of the propagation term, which is part of the $\boldsymbol{B}$ vector in our notation above, can be negative as the Leontief Inverses get multiplied with the stickiness matrix. Beyond the period of impact, however, in some network setups $\boldsymbol{\Psi}^{NKOE}$ can lead to persistent inflation.

One key example of $\boldsymbol{\Psi}^{NKOE}$ leading to persistence in inflation is the case of price rigidity at the buyer-seller level. The trade literature has extensively studied how international trade at the granular level is dependent on matching of buyers and sellers; for example, Bocola and Bornstein (2023) demonstrates the importance of trade credit in this context. We do not model these matches with a matching function, but our framework is flexible enough to incorporate the case whereby contracts are signed between buyers and sellers such that the price ridigity cost (and prices) are at the bilateral level. That is instead of having producer prices $\hat{\boldsymbol{P}}_t^P$ be an $NJ \times 1$ matrix, the relevant matrices can be adjusted to render it an $NNJ \times 1$ matrix, such that consumers in a given country might face a different price for the price of a good made in country n industry j compared to the price that buyers of the same good might face in a different country.

With this extension, our model can capture the fact that it takes time to adjust trade at the bilateral buyer-seller level. The intuition in this context is that, one buyer-seller pair updates a contract at a given point, it then prompts other buyer-seller pairs to update. This takes longer than each producing sector updating its price at once for all its buyers. Algebraically this corresponds to more granular $\boldsymbol{\Omega}$ and $\boldsymbol{\Gamma}$, which in turn changes $\boldsymbol{\Psi}^{NKOE}$.

Figure 5. Model Comparison for Inflation



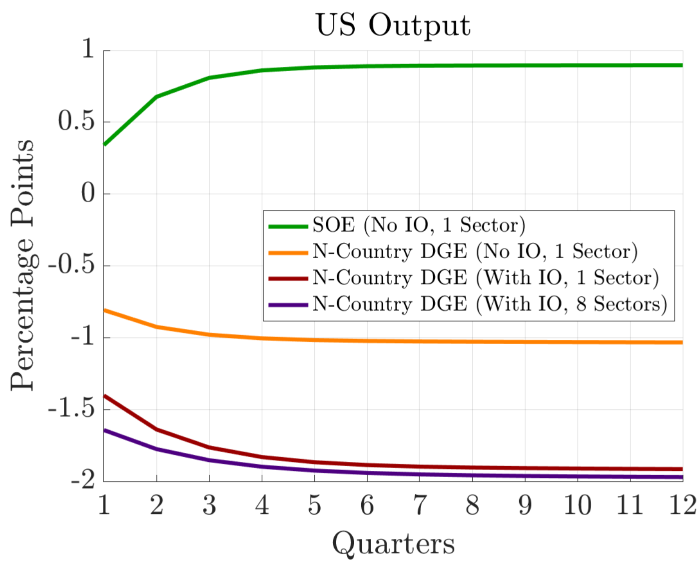
In Figure 5, in light of the theory, we show how the propagation of inflation changes across time in different network setups using the example of US tariffs placed on the rest of the world as another. The blue line shows the propagation of inflation in the absence of intermediate inputs (i.e. $\boldsymbol{\Omega} = \boldsymbol{0}$); inflation in this case is lower. The brown line depicts the case when $J = 1$ (i.e. $\boldsymbol{\Omega}$ is a, while the purple line depicts the case when $J = 8$ and prices are rigid at the buyer-seller level. Comparing the latter two cases, inflation on impact is slightly larger when $J = 1$; however, inflation persists for longer and at a higher level when $J > 1$ in line with Remark 4. The green line depicts the case when one takes the version of the model with one sector and without IO and converts it into an SOE model, by making the rest of the world arbitrarily larger; this case does not have inflation persistence.

A related point can be made about output in our model. While the Five-Equation Global New Keynesian Representation does not directly track output as an endogenous variable, in Appendix B, we show that output can be expressed in terms of the endogenous variables of the five-equation system, which are solved in Proposition 2:

$$\hat{\boldsymbol{Y}}_{ni,t} = \boldsymbol{\Psi}_T \Bigg[\Big(\overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{\alpha} \sigma \Big) \hat{\boldsymbol{C}}_t + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_P^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_P^P \Big) \hat{\boldsymbol{P}}_t^P + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_{\mathcal{E}}^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_{\mathcal{E}}^P \Big) \hat{\mathcal{E}}_t + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_\tau^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_\tau^P \Big) \hat{\tau}_t \Bigg] \quad (41)$$

where overline notation denotes steady state values, while θ^C and θ^P are generalized elasticities of substitution respectively on the consumption and production side. $\boldsymbol{T}$ matrices capture how each specific price compares to the average price basket of a user. That is for households each granular good price is compared to the average consumer price basket, and for producers each intermediate input price is compared to the price of the overall intermediate input basket, and then with some elasticity (θ^C or θ^P) quantities respond.

Figure 6. Model Comparison for Output



This expression illustrates why the elasticity of substitution contributes largely to output in Figure 2 and why under complementarities, in the presence of a network the decline in output resulting from tariffs can be larger. Figure 6 makes this point in a manner similar to Figure 5. An additional point to highlight here is that, in the SOE case, when the rest of the world is made arbitrarily larger, the same set of parameters that otherwise yield a recession in the N-country DGE case can yield an expansion. This is because the rest of the world is able to easily increase their demand for the home country's goods since the tariffs imposed by the small open economy do not pose a negative demand shock for the rest of the world.

7 Quantitative Analysis

7.1 Data on Input - Output Network

As the basis for consumption shares and intermediate input shares, we use the OECD Inter-Country Input-Output (ICIO) tables (Yamano and et al., 2023) for the year 2019.[34] We aggregate the ICI-O data to align with the country and industry groupings used in our analysis. we include the United States, Euro Area, China, Canada, and Mexico- reflecting the countries most affected by the tariff announcements as of April 2025- along with an aggregate entity representing the Rest of the World (RoW). On the industry side, we aggregate sectors into eight broad categories: agriculture, energy, mining, food, basic manufacturing, advanced manufacturing, residential services, and other services to match with sectoral rigidity data of Nakamura and Steinsson (2008) (see below).

We visualize the I-O network in Figure 7. The thickness of the edges in this network captures the input shares. The layout of the network was generated automatically using the edge-weighted spring embedded layout feature of Cytoscape. Global shocks could be carried over the links shown on this network. Strikingly, many Canadian and Mexican sectors are naturally grouped together with American industries. In contrast, the Chinese sectors are not very well integrated. This might be due to the fact that many Chinese goods imported by the U.S. could be for final consumption.

In Table 1, we show the basic stats for the U.S. industries. The U.S. economy heavily relies on services, with more than 75% GDP attributed to this sector. Most of the U.S. output is consumed domestically, with shares ranging from 80 to 99 %. The home share in consumption and intermediate inputs exhibit the lowest rates in manufacturing sectors. Interestingly, close to one third of consumer goods and intermediate inputs are sourced from foreign countries in advanced manufacturing. The energy sector's intermediate products are sourced at a higher level internationally. In Table D.2 of the Appendix, we provide a more detailed breakdown of the final and intermediate input shares at country-sector level.

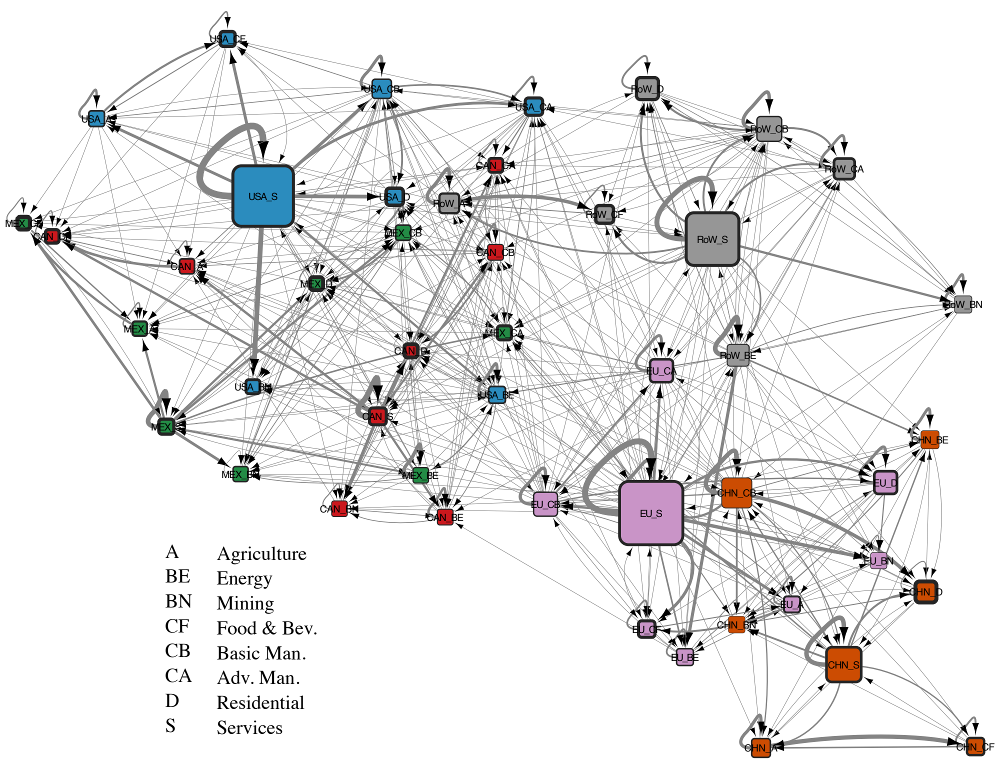
[34] Although the latest available data at the time of writing was for 2020, we use 2019 data to avoid distortions arising from the COVID-19 pandemic.

Table 1. Sector Statistics for USA (%)

Industry	Output Share	VA Share	Consumption Share	Output Home Share	Consumption Home Share	Intermediate Home Share
Agriculture	1.3	0.9	0.6	87.2	88.5	89.3
Energy	3.0	2.0	1.5	85.7	89.4	75.0
Mining	0.5	0.5	0.5	91.2	98.5	89.9
Food and Beverages	2.6	1.2	3.1	94.0	91.2	91.7
Basic Manufacturing	6.6	4.7	4.1	87.6	66.0	82.5
Advanced Manufacturing	6.2	5.1	8.2	81.7	67.0	66.9
Residential Services	6.4	6.1	7.7	99.9	99.9	99.5
Services	73.4	79.4	74.3	95.3	96.7	96.2

Notes: The values are calculated from OECD ICI-O for year 2019 Yamano and et al. (2023). Output Share is the share of the sector in total U.S. output. VA share is the share of the sector in total U.S. GDP. Consumption share is calculated as the sector's weight in the household expenditure. Output Home Share represents the share of the output of the sector sold domestically. Consumption Home Share captures the share of domestic production in consumption and Intermediate Home Share captures the share of intermediate goods supplied domestically.

Figure 7. Visualizing the Input-Output Network



Note: Figure 7 visualizes the inter-country inter-industry I-O network. The color of the node represents the country. Size of the node represents the total output. The thickness of the edges show the share of inputs of target node coming from the source node (we do not show the edges smaller than 1%). The thickness of the borders of nodes represents the share of final goods in the output of the sector. The layout was generated automatically using the edge-weighted spring embedded layout using the openly available Cytoscape software.

7.2 Parameters

The quantitative model incorporates a permanent real capital account wedge in each country to treat the year 2018 as the steady state to which the economy eventually returns. These wedges are added for the following reason. If a country has a trade deficit at the steady state, this requires that the country have positive net foreign assets that pay interest to finance this deficit (e.g., past trade surpluses finance the steady-state deficit). However, in practice, the United States has persistently maintained trade deficits and negative net foreign assets. In order to treat consumption and I-O tables for a given year (e.g., 2018) as the steady state and at the same time embed a realistic net foreign asset (NFA) position for all relevant country blocks, one needs to reconcile steady state algebra with real-life data. The real permanent capital account wedges help with reconciling the two. These wedges can be interpreted as a persistent difference in the patience of nations or alternatively can be thought of as a persistent exogenous difference in the interest paid on assets versus liabilities that render having trade deficits and net debt at the steady state possible.

The calibration parameters are summarized in Table 2. The model employs sector-specific Calvo parameters based on the empirical estimates in Nakamura and Steinsson (2008), adjusted to a quarterly frequency. The production and intratemporal consumption structures are similar to those in Çakmaklı et al. (2025) and di Giovanni et al. (2023). On the production side, firms combine labor and intermediate input bundles to produce goods. Based on Atalay (2017), we set the elasticity of substitution between labor and intermediates $\theta^P = 0.6$. Intermediate input bundles are composed of sectoral bundles, which are assumed to be complements. Following Boehm et al. (2019) and Baqaee and Farhi (2024), we set this elasticity to $\theta^P_h = 0.2$. Each sectoral bundle consists of varieties sourced from different countries. In our baseline specification, we set the Armington elasticity across countries at the sectoral level to $\theta^P_{li} = 0.6$. On the intratemporal consumption side, we follow Baqaee and Farhi (2024) and assume Cobb–Douglas preferences across sectors, setting the sectoral elasticity to $\theta^C_h = 1$. Boehm et al. (2023) estimate short-run trade elasticities of approximately 0.76 and long-run elasticities around 2. In line with this and with di Giovanni et al. (2023), for the aggregation of varieties within sectoral consumption and production bundles (e.g. for the substitutability between cars from country A and country B), we allow for higher substitutability and conduct sensitivity analyses with θ^C_{li} and θ^P_{li} in the range between 0.6 and 2. In the baseline calibration, both parameters are set to 1.5.

Additionally, for realism, we incorporate monetary policy inertia by modifying the baseline Taylor rule. Specifically, Equation (25) is replaced with the following specification:

$$1 + i_{n,t} = (1 + i_{n,t-1})^{\rho^n_m} (\Pi_{n,t})^{\phi^n_\pi} \quad \forall n \in N$$

Here, ρ_m^n captures the degree of interest rate smoothing (or policy inertia), ϕ_π^n is the weight place on inflation in the Taylor rule. This specification is applied to all countries $n \in N$ in the model.

For the United States, we set $\rho_m^{\text{US}} = 0.82$ and $\phi_\pi^{\text{US}} = 1.29$, based on the estimates provided by Carvalho et al. (2021a). Following Clarida et al. (2000), we use $\rho_m^n = 0.95$ and $\phi_\pi^{EA} = 1$ for the rest of the world and the Euro Area, respectively. For other countries in the rest of the world, we assume $\phi_\pi^n = 0.2$, except for Mexico, where we use a slightly higher value of $\phi_\pi^{\text{MX}} = 0.3$. These ϕ_π values are calibrated using a model-consistent interpretation of the long-run average of quarterly inflation rates. Specifically, following the logic in Clarida et al. (2000), we set $\phi_\pi^n = \frac{1-\rho_m^n}{\overline{\pi}_n^C}$, where $\overline{\pi}_n^C$ denotes the long-run average of quarterly CPI inflation in country n. Using quarterly data from 2002Q2 to 2024Q4 and setting $\rho_m^n = 0.95$, we calibrate the inflation response coefficients accordingly. This calibration captures the empirical observation that central banks in many countries outside the United States are less responsive to inflation fluctuations and are therefore less likely to adhere strictly to a Taylor rule.

Table 2. Parameter values

Parameter	Explanation	Value	Source
σ	Intertemporal EoS	2	e.g., Itskhoki and Mukhin (2021)
η	Elasticity of Labor	1	e.g., Itskhoki and Mukhin (2021)
ψ	Reactivity of UIP to Debt	0.001	Standard
ρ_m^n	Inertia in Taylor Rule for $n \neq US$	0.95	Clarida et al. (2000)
ρ_m^{US}	Inertia in Taylor Rule for U.S.	0.82	Carvalho et al. (2021a)
ϕ_π^{US}	Weight on inflation in Taylor Rule for U.S.	1.29	Carvalho et al. (2021a)
λ_n	Sector specific price rigidities		Nakamura and Steinsson (2008)
θ^P	EoS between intermediates and VA	0.6	Atalay (2017)
θ_h^C	Intratemporal EoS of consumption among sectors	1	di Giovanni et al. (2023)
θ_h^P	EoS among intermediate inputs	0.2	Baqaee and Farhi (2019); Boehm et al. (2019)
θ_{li}^C	Sector level consumption bundle EoS	0.6-2	di Giovanni et al. (2023)
θ_{li}^P	Sector level input bundle EoS	0.6-2	di Giovanni et al. (2023)

Notes: "EoS" is the elasticity of substitution.

Finally, as the existing policy environment has started to feature permanently higher tariffs (e.g. as evidenced by the agreement between the US and EU), we diverge from the the analytical model's specification of tariffs following an AR(1) process (i.e., $\tau_t = \rho^\tau \tau_{t-1} + \epsilon_t^\tau$). In the cases below, we instead feed into the model a tariff shock that raises tariffs to a higher level for 100 periods, which correspond to 25 years in our calibration. Quantitatively, for our horizon of interest this produces results that are similar to an AR(1) tariff shock with near-permanent persistence, $\rho^{\tau} = 0.999$. We find that this approach produces impulse responses that are more realistic since they do not involve a decaying process in our horizon of interest.

7.3 Validating the Model: 2018 Trade War

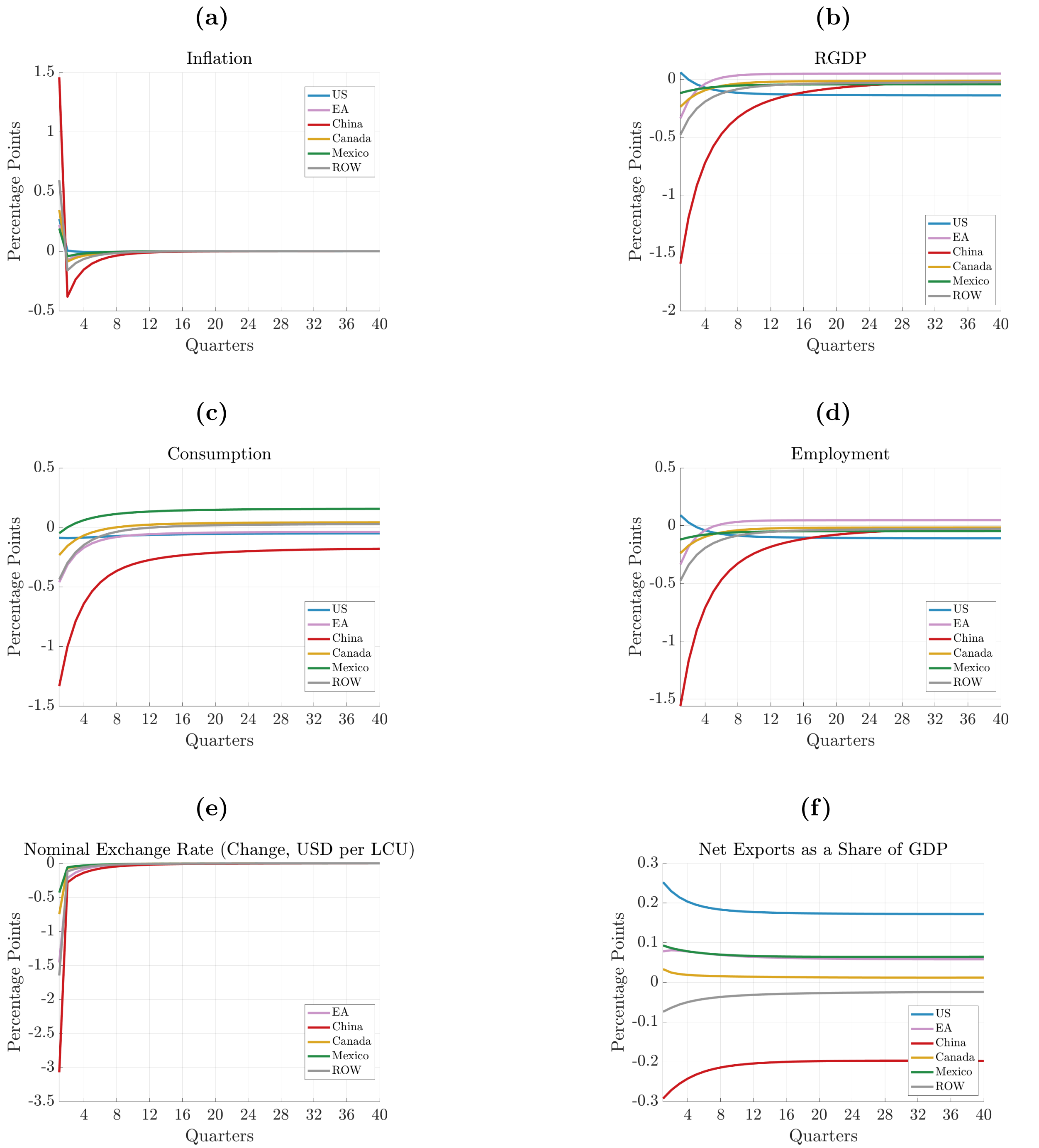
As a validation exercise, we study the trade war between United States on China and other countries with tariffs imposed from February 2018 to September 2018. In this period, the U.S. implemented tariffs ranging from 10% to 25% to China, 10% tariff to aluminum, 25% to iron and steel, 30% to solar and 20 to 50 % tariffs to washers with some exceptions at country levels. In return, Canada, China, European Union, Mexico, Russia and Turkey retaliated with tariffs ranging from 5 to 20%. We obtained the detailed tariff data for this episode from Fajgelbaum et al. (2020) and trade values to calculate the weighted tariff rates from USITC website.[35]

As the model is non-linear, we solve it with Dynare (Adjemian et al., 2011) under three alternative solution methods: first-order approximation, second-order approximation, and MIT shocks under perfect foresight. For small shocks, these methods yield nearly identical impulse response functions. However, our preferred solution approach, which we report below, employs MIT shocks under perfect foresight, because of the sizeable nature of the trade shocks at hand and given the non-linearities in the model, especially with regards to complementarities on the production side.

Tariffs imposed by the United States on China and other countries between February 2018 and September 2018 (See Section 7.4 for details of the data). As shown in Figure 8, the model predicts a 3% nominal appreciation of the U.S. dollar (USD) against the Chinese yuan on impact, eventually reaching 4.2% nominal appreciation in the long run. This aligns with the observed 5.6% appreciation of the USD between June 2018 and December 2018. Real GDP loss reaches 0.1 percentage points. This is in the range of the estimate of Fajgelbaum et al. (2020), which found an aggregate real income loss of 0.04% of GDP. Finally, the model predicts an inflation impact of 0.27 percentage points, which is close to the 0.1-0.2 percentage point estimate of Barbiero and Stein (2025).

[35] Exports: https://dataweb.usitc.gov/trade/search/TotExp/HTS, Imports: https://dataweb.usitc.gov/trade/search/GenImp/HTS.

Figure 8. Case 1: Impact of 2018's Trade War



Note: Figure 8 visualizes simulated responses to the 2018 U.S. tariff package targeting China. Impulse responses are computed with MIT shocks.

Table 3. On-Impact Response of Variables in Case 1: 2018's Trade War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.06%	-0.34%	-1.59%	-0.24%	-0.12%	-0.48%
C_n	-0.09%	-0.46%	-1.33%	-0.23%	-0.05%	-0.44%
π_n	0.27%	0.22%	1.46%	0.35%	0.19%	0.60%
i_n	0.00%	0.22%	0.29%	0.07%	0.06%	0.12%
$\Delta\mathcal{E}_n$	0.00%	-1.46%	-3.07%	-0.75%	-0.43%	-1.65%
RER_n	0.00%	-1.51%	-1.92%	-0.67%	-0.51%	-1.33%
L_n	0.09%	-0.33%	-1.56%	-0.24%	-0.12%	-0.48%
$\frac{W_n}{P_n}$	-0.09%	-1.25%	-4.17%	-0.70%	-0.22%	-1.34%
$\frac{NX_n}{NGDP_n^{ss}}$	0.25%	0.07%	-0.29%	0.03%	0.09%	-0.07%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.06%	-0.02%	-0.31%	-0.01%	0.00%	-0.13%

NOTE: First-period impact of the U.S. tariffs in 2018. Effects are reported in deviation from the pre-tariff steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

7.4 2025 Tariffs

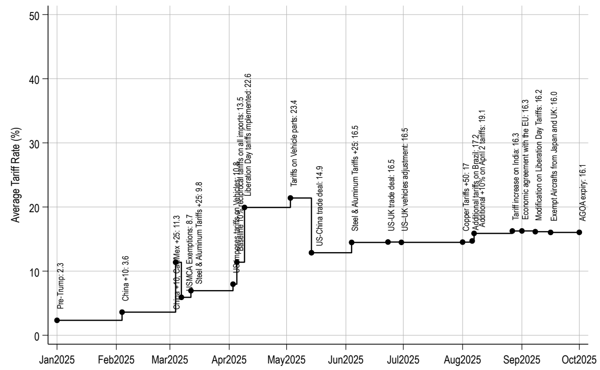
In the quantitative exercises that follow, we are motivated by the renewed interest among policymakers in using tariffs as a tool to manage external imbalances and exert geopolitical influence. This interest predates the second Trump presidency and reflects a broader global re-evaluation of trade policy not only for the standard terms of trade manipulation but also both for strategic and retaliatory purposes. In the quantitative section of our paper we solely focus on the tariffs announced in the early months of the second Trump administration.

As shown in Figure 9a, the tariffs proposed on April 2- referred to as "Liberation Day" by the administration- are projected to raise the effective U.S. tariff rate to 22.4%, the highest level in over a century. We obtain the country - sector levels tariffs from the WTO – IMF Tariff Tracker (WTO and IMF, 2025) at Harmonized System 6-digit level. We aggregate these tariff rates to ICI-O sectoral level by weighing them with the imports of the countries, provided in the same dataset. Figure 9b shows the implemented tariff rates since January 1, 2025 until June 20, 2025. The "liberation day tariffs," were announced on April 2, 2025 but with most tariffs going into effect on April 9th. Between these two dates, there was also a steep escalation between the U.S. and China tariffs to each other, resulting in tariff rates exceeding 125% for Chinese goods in the U.S.

Figure 9. Effective Tariff Rates

(a) Historic and Estimated, (%)

(b) Since January 1, 2025, (%)

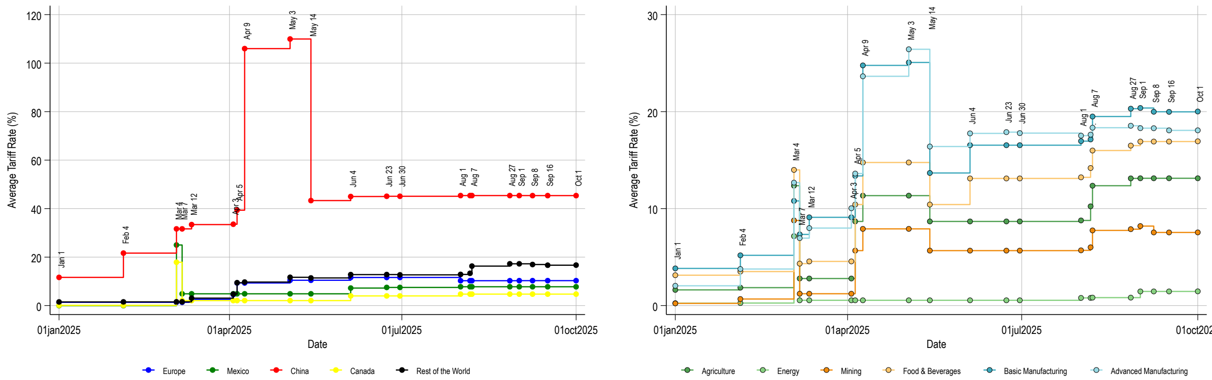


NOTE: Figure 9a visualizes effective tariff rate stands for customs duty revenue as a proportion of goods imports. Data from *Historical Statistics of the United States* Ea424-434, *Monthly Treasury Statement*, Bureau of Economic Analysis. Estimated effective tariff rates of for 2025 provided by Yale Budget Lab using the GTAP Model v7 (Corong et al., 2017). Figure 9b visualizes estimated effective tariff rates based on WTO - IMF Tariff Tracker (WTO and IMF, 2025). The dates here correspond to the actual implementation change of the tariffs. The data was accessed on October 20, 2025.

In Table D.1, we document the episodes of implemented tariff changes for the U.S. reported by the WTO-IMF Tariff Tracker (WTO and IMF, 2025). We summarize the tariff rates at the country and sector level in Figure 10. The largest swings are observed for China with escalating tariff announcements with a moratorium on May 14, 2025 (Figure 10a). At the sectoral level, the tariffs are the highest for basic and advanced manufacturing goods. Figure D.2 in the Appendix shows the size of country-sector-level tariffs implemented in 2025 until the time of our writing in panel (a). Panel (b) focuses on the "Liberation Day" tariffs. Figure D.2a shows that the highest tariff rates are applied to the Chinese goods. Among Chinese sectors, basic manufacturing (e.g., textiles), food and beverages, and agriculture have the highest values with tariffs ranging from 45-50%. For most other countries, the tariffs started from very low levels but increased around 10-20% for many goods.We will use the most recent data (June 4, 2025) levels for our quantitative analysis. In Table D.2 of the Appendix, we provide detailed breakdown of the tariff rates as of October 1, 2025 and maximum tariff rate observed between January 1, 2025 and October 1, 2025.

Figure 10. Effective Country and Sector Level Tariff Rates

(a) Country Level Tariffs (%) **(b)** Sector Level Tariffs (%)



NOTE: Figure 10 visualizes estimated effective tariff rates at the country level and at the sectoral level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) between January 1, 2025 and October 1, 2025 (last available data as the manuscript was prepared). Both country level and sectoral level tariff rates are calculated as the weighted average of the 6-digit tariff rates by using the latest available import values reported in the dataset as weights.

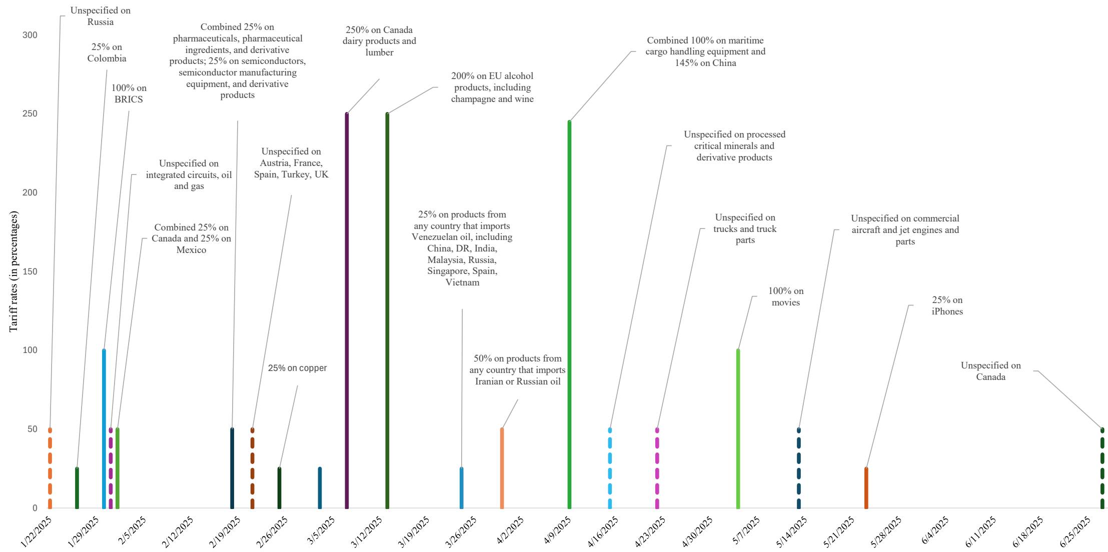
According to both the St Louis Fed[36] and the Tax Foundation[37], the 2018 tariffs affected \$376 billion of goods from China, which is around 1.66% of the 2018 U.S. GDP. As of June 2025, most of the tariffs enacted on the "Liberation Day" have been halted via an injunction by the U.S. Court of International Trade. Those not affected still represent \$500 billion worth of U.S. imports, or 1.68% of the 2024 U.S. GDP. If all of the "Liberation Day" tariffs were to come into effect again, they would represent \$2.3 trillion worth of U.S. imports, which is 7.7% of 2024 U.S. GDP.

The tariff rates changed considerably with very frequent announcements, repeals, threats, deals, and various negotiations. In Figure 11, we show some of the tariff threats, which include non-implemented tariffs and some announcements with uncertain future implementation. In Appendix D, we also show tariffs announcements (Appendix Figure D.1a) and impementations (Appendix Figure D.1) by date. This also leads to a great deal of uncertainty surrounding which tariffs will be implemented in the end. That is why we also model the tariff threats in our quantitative section.

[36] https://www.stlouisfed.org/on-the-economy/2025/may/what-have-we-learned-us-tariff-increases-2018

[37] https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/

Figure 11. Tariff Threats - not implemented and future implementation uncertain



Note: Figure 11 visualizes tariff threats between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far). The tariffs that are classified as "threats" are those that –as of June 30th - had not been implemented and were unlikely to be implemented based on available information. These threats were identified by an extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

7.5 Reversed Tariff Threats

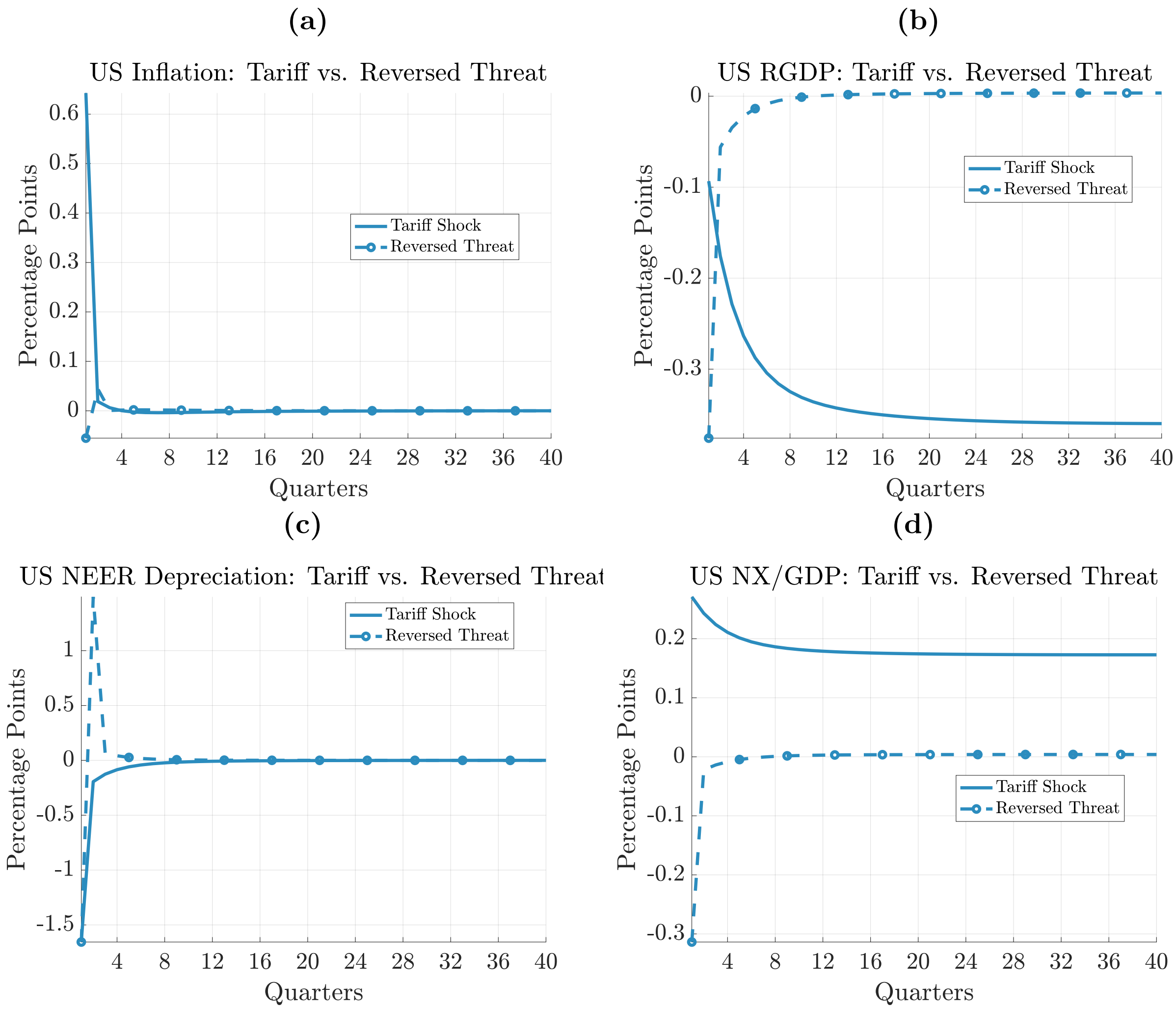
As seen in Figure 11, there have been many tariff threats that are not implemented or uncertain to be implemented. In this section, we apply our model to the case of reversed tariff threats- scenarios in which a country announces future tariffs but subsequently reverses the decision before implementation. This case also incorporates retaliation: specifically, the United States announces in period 1 that tariffs will be imposed in period 2, prompting other countries to announce retaliatory measures for the same period. However, when period 2 arrives, it is announced that no tariffs will be levied by either side.

This scenario not only mimics the reality of how tariffs were introduced in 2025 but also it allows us to isolate the role of the expectations channel. In standard scenarios, when a tariff is introduced, as is the case in Cases 1, 3,4 and 5, under price stickiness it is the case that inflation shows up first and the decline in GDP takes some time. This is because of the theoretical mechanisms we explored in Section 5. A significant portion of the inflationary impacts of tariffs come from direct effects, which are present when tariffs are first placed, while the indirect effects especially those involving output take longer when prices take time to adjust. That is why, with a standard model, monetary policymakers might expect after tariffs are placed that the worst is over on inflation after a one-time increase in prices and that there is more to worry with regards to output. A world with production networks and reversed tariff threats differs from this expectation in two ways. First, if actual tariffs are preceded by reversed tariff threats, there could be some deflation initially as we show below and output might begin to contract before inflation starts ticking up. Second, in setups with production networks, inflation can be more persistent even if reversed tariff threats delay direct effects.

To analyze the effects of reversed tariff threats, we construct two impulse responses under perfect foresight. First, we simulate the all-out tariff war shock examined in Case 5, assuming it is both announced and implemented in the first period of the simulation. Second, we simulate the same shock- identical in magnitude- but announced to take effect in the second period, only to be withdrawn before implementation. The impulse response to the reversed tariff threat is then obtained by shifting the first (implemented) impulse response forward by one period and subtracting it from the second (announced-but-not-implemented) response. This approach isolates the effect of the anticipatory behavior triggered by the announcement, net of the effects of actual implementation. Importantly, we observe that from the second period onward, the quantity variables in both simulations converge and remain nearly identical. This reflects the fact that agents discount the future and adjust quantities in response to the announcement, but not to the same extent as they would if the shock were immediate and fully realized. Our approach here is inspired by the *fake news* algorithm of Auclert et al. (2021), in which agents receive information about a future increase in income and optimize accordingly, only to later discover that the anticipated change does not materialize. While Auclert et al. (2021) employ this construct as a computational device for solving models in sequence space, we interpret and apply it literally to study the macroeconomic implications of trade policy reversals.

Figure 12 compares the impact on inflation, real GDP, and U.S. NEER appreciation in Case 5 (Tariff Shock) to the reversed tariff threat scenario. First striking observation is that reversed tariff threats are deflationary, whereas actual tariffs are inflationary. The intuition here is closely linked to the decomposition of inflation in Figure 3. When the direct inflationary effects of tariffs are absent, indirect effects that are deflationary can overcome inflationary indirect effects as we see here.

Figure 12. Case 2: Impact of Reversed Tariff Threats



Note: Figure 12 visualizes simulated response to reversed tariff announcements. Tariffs are announced in the first period, with retaliation expected, and later canceled in the second period.

Second, a future in which the United States demands fewer goods from abroad prompts an immediate appreciation of the USD, as the intertemporal budget constraint adjusts and agents incorporate expectated future income streams into current allocations and pricing. In this scenario, the U.S. trade-weighted nominal effective exchange rate appreciates by 1.7% on impact. In contrast, quantity variables respond more gradually. When agents realize in the second period that the shock will not materialize, they reoptimize, resulting in a partial recovery. In line with this, although tariffs are never actually implemented, we see that real GDP declines by 0.4 percentage points. Notably this decline is larger than the decline in the event of actual tariffs, because in the case of the latter direct effects on prices start encouraging demand for domestic goods, whereas in the reversed tariff threat case anticipated supply distortions discourage production. The period before tariffs actually take place is a period in which it is cheaper to import than it will be in the future. As a result, one can simultaneously see appreciation and frontloaded net imports leading to a worsening of the trade balance as is the case here.

It is notable that, once tariffs are reversed, the U.S. dollar depreciates: agents had previously priced in a future in which the U.S. would reduce demand for foreign goods, but upon receiving new information in the second period that this scenario would not materialize, the exchange rate response is reversed. Expectations-linked overshooting is interesting since this does not happen with regular tariffs. A more realistic interpretation of the observed and somewhat sustained U.S. dollar depreciation in response to tariffs requires accounting for a large uncertainty (VIX) shock and policy volatility more than our simple one period on-off tariff threat exercise, or other shocks such as fiscal uncertainty, that are outside the scope of our paper.

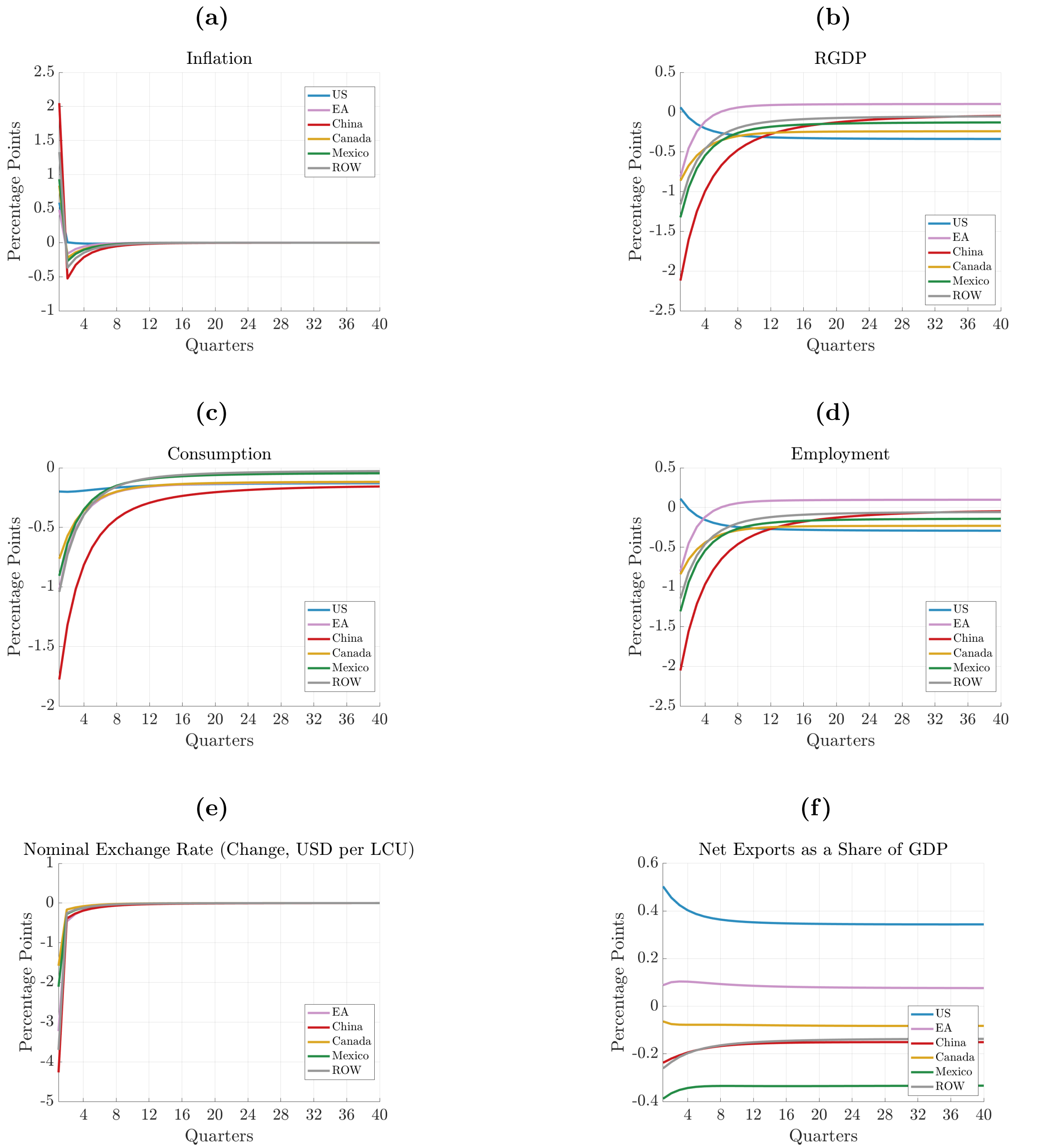
Overall, this exercise demonstrates that the expectations channel, emphasized in our theoretical analysis, plays a central role. Reversed tariff announcements operate similarly to demand shocks, particularly when announcements are perceived as credible. Importantly, the macroeconomic distortion introduced through the expectations channel does not dissipate immediately with the reversal announcement. Variables exhibit persistence, and the economy does not return to steady state instantaneously.

7.6 Baseline: 2025's Trade War

In 2025, the United States announced several rounds of tariffs targeting Mexico, Canada, Europe, China, and many other countries. In Section 7.4, We explained in detail the tariffs announced, implemented, changed and limited retaliation from others happened so far as of the time of our writing.

As shown in Figure 13 and Table 4, predicts a contraction in U.S. real GDP reaching 0.4 percent in the long-run. This is accompanied by almost 0.2% decrease in consumption, a 0.5 percentage point increase in net exports as a share of steady-state GDP (i.e. an improvement in trade deficit), and a 0.3% decline in real wages. Inflation rises by 0.6 percentage points. Additionally, the U.S. trade-weighted nominal effective exchange rate (NEER) appreciates by 3.3%.

Figure 13. Case 3: Impact of 2025 Tariffs



Note: Figure 8 visualizes simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, Europe and the RoW. Impulse responses are computed with MIT shocks.

The effects are pronounced for China, Mexico, and Canada. China's real GDP contracts by 2.12%, while Mexico's declines by 1.32% and Canada's by 0.86%. Labor market impacts are also substantial, with employment falling by 2.05% in China, 1.31% in Mexico, and

0.84% in Canada. Net exports decline by 0.24%, 0.38%, and 0.06% of steady-state GDP, respectively. Inflation rises by 2.05 percentage points in China, 0.93 percentage points in Mexico, and 0.84 percentage points in Canada. The Euro Area (EA) experiences a decline in real GDP of 0.81%, while the rest of the world (RoW) contracts by 1.16%. Consumption falls by 1.02% in the EA and 1.04% in the RoW. Inflation rises by 0.48 percentage points in the EA and 1.33 percentage points in the RoW. Employment declines by 0.80% and 1.15%, respectively. Real wages fall by 2.81% in the EA and 3.20% in the RoW. Net exports as a share of steady-state GDP increase slightly in the EA (0.07%) but decline in the RoW (–0.26%).

Table 4. On-Impact Response of Variables in Case 3: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.06%	-0.81%	-2.12%	-0.86%	-1.32%	-1.16%
C_n	-0.20%	-1.02%	-1.78%	-0.76%	-0.91%	-1.04%
π_n	0.59%	0.48%	2.05%	0.84%	0.93%	1.33%
i_n	0.00%	0.49%	0.41%	0.17%	0.28%	0.27%
$\Delta\mathcal{E}_n$	0.00%	-3.23%	-4.27%	-1.58%	-2.11%	-3.70%
RER_n	0.00%	-3.33%	-2.88%	-1.34%	-1.77%	-2.99%
L_n	0.11%	-0.80%	-2.05%	-0.84%	-1.31%	-1.15%
$\frac{W_n}{P_n}$	-0.29%	-2.81%	-5.50%	-2.35%	-3.09%	-3.20%
$\frac{NX_n}{NGDP_n^{ss}}$	0.50%	0.07%	-0.24%	-0.06%	-0.38%	-0.26%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.11%	-0.01%	-0.47%	-0.01%	0.02%	-0.27%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package. Tariff rates vary by country-sector; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

In this scenario, the tariff shock is inflationary on impact for all countries. Then in non-US countries there is a period of deflation. This occurs because US tariffs constitute a negative demand shock for the rest of the world and monetary policy reacts with a lag. In our model, DCP contributes to this dynamic as well. Other countries have a meaningful share of their exports invoiced in USD and since the model expects USD appreciation, the on-impact inflation seen in non-US countries is driven not only by supply distortions but also by exchange rate passthrough. It is notable that this tariff shock is a negative shock for the real GDP of all countries except for the Euro Area, which tends to benefit from trade diversion in the long run.

7.7 Non-Reactive Monetary Policy

A natural question to ask is how much of the observed results in Case 3 are driven by the specific monetary policy rule that the model assigns to various countries. After all, macroeconomic outcomes like inflation are ultimately policy driven. To that end we consider a second version of Case 4 in which, nominal interest rates are held constant across the globe. One interpretation of this exercise is that this is an attempt to understand the impact of tariffs before policy weighs in.

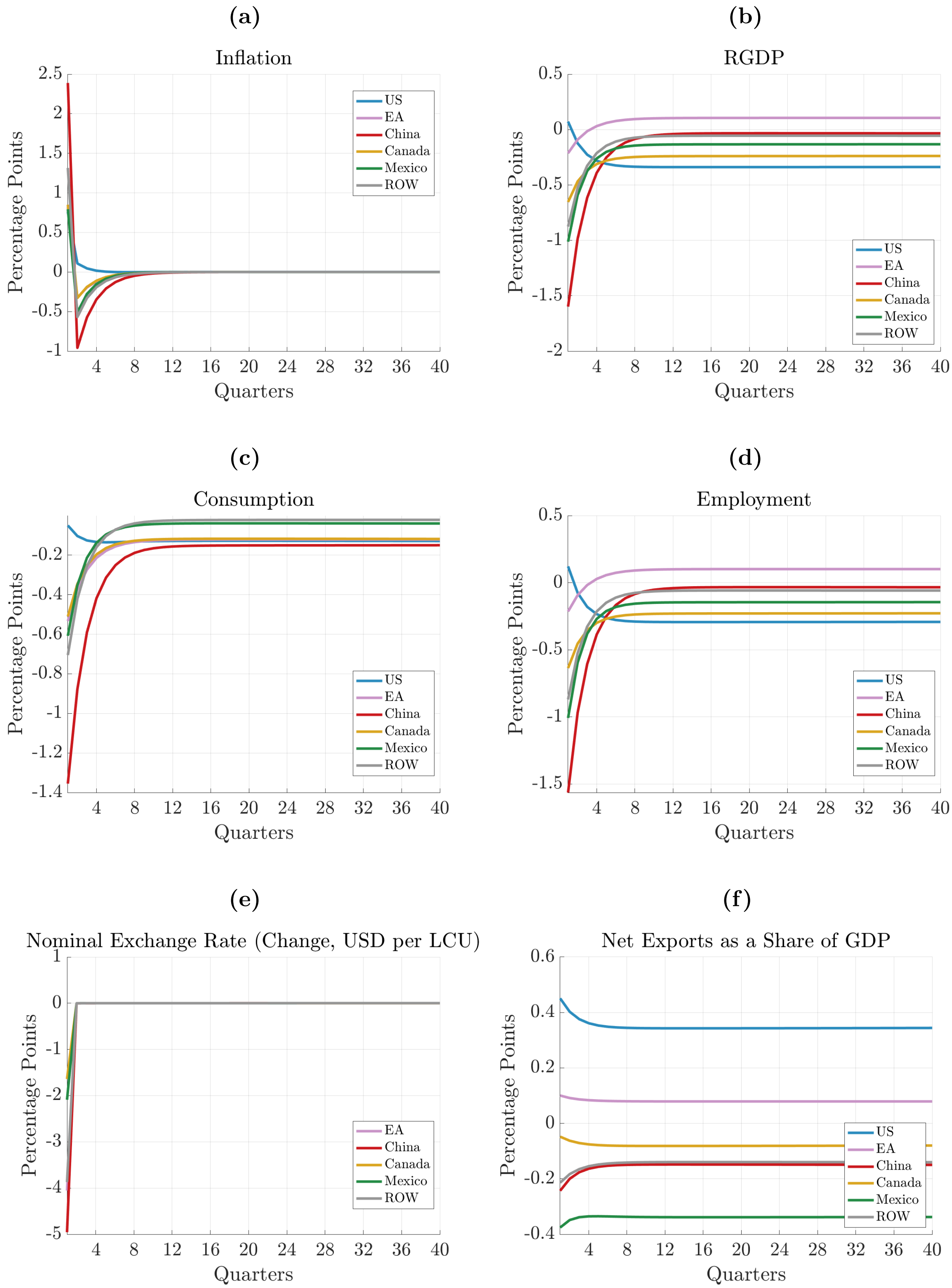
Table 5. On-Impact Response of Variables in Case 4: 2025's Tariffs

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	0.07%	-0.21%	-1.60%	-0.65%	-1.01%	-0.88%
C_n	-0.05%	-0.53%	-1.35%	-0.51%	-0.61%	-0.71%
π_n	0.76%	0.82%	2.39%	0.85%	0.79%	1.32%
i_n	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$\Delta\mathcal{E}_n$	0.00%	-4.05%	-4.96%	-1.64%	-2.09%	-3.87%
RER_n	0.00%	-4.00%	-3.42%	-1.55%	-2.05%	-3.34%
L_n	0.12%	-0.21%	-1.56%	-0.64%	-1.01%	-0.87%
$\frac{W_n}{P_n}$	0.02%	-1.28%	-4.21%	-1.65%	-2.21%	-2.26%
$\frac{NX_n}{NGDP_n^{ss}}$	0.44%	0.11%	-0.24%	-0.05%	-0.37%	-0.22%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.10%	-0.02%	-0.55%	-0.01%	0.02%	-0.30%

NOTE: First-period outcomes of the 2025 unilateral U.S. tariff package. Tariff rates vary by country-sector; effects are reported in deviation from the steady state. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

In this case, inflation is higher on impact globally and inflation's persistence is higher in the US since in this case policy does not respond. Since policy is non-reactive the deflation in non-US countries in the second period onwards is larger as well. Beyond the changes in inflation, the results for other variables are largely similar to Case 3. This is not surprising since the baseline Taylor rule calibration already features a large degree of policy inertia.

Figure 14. Case 4: Impact of 2025 Tariffs With Non-Reactive Monetary Policy



Note: Figure 14 visualizes simulated responses to the 2025 U.S. tariff package, targeting China, Canada, Mexico, Europe and the RoW in the case whereby monetary policy is non-reactive. Impulse responses are computed with MIT shocks.

Naturally this can raise questions about determinacy. In general, New Keynesian models suffer from indeterminacy if the nominal interest rate is held exogenous and does not respond to endogenous outcomes like inflation. To overcome this problem, we adopt the following exercise influenced by the HANK literature's use of real rate rules. When linearized, the policy rule used in our quantitative work is $i_{n,t} = \rho_m^n i_{n,t-1} + \phi_\pi^n \pi_{n,t}^C$. Setting $\rho_m^n = 1$. We can take the limit of $\phi_\pi^n \to 0$ and as long as $\phi_\pi > 0$, the system stays in the region of determinacy. With this approach, in Case 4 we depict how the results in Case 3 would change if the interest rate was numerically held constant.

7.8 Inflation Persistence and Depreciation

We now add two additional features to the baseline model to match real-life data better. The first of these features is the gradual nature of pass-through from tariffs onto prices in the presence of retail importers and the second has to do with the U.S. dollar depreciating in the presence of a widening UIP premium.

As described in Section 2, the empirical literature on tariff pass-through documents substantial heterogeneity, with estimates ranging from significantly below to more than full pass-through, reflecting adjustments in markups, exchange rates, and pricing strategies. Evidence from the 2018 tariffs on Chinese imports shows heterogeneity in pass-through across products and firms (Amiti et al., 2019; Cavallo et al., 2021), while recent work on tariffs in the European wine industry finds pass-through that can exceed the statutory tariff due to markup responses along the distribution chain (Flaaen et al., 2025). This literature motivates two central modeling choices in open-economy settings with tariffs: the currency in which prices are nominally rigid—producer currency pricing, dominant currency pricing, or local currency pricing with pricing-to-market—and whether nominal rigidity applies to ex-post tariff-ed producer price or the price faced by importers or consumers after endogenous responses. In the baseline model, pricing rigidity was on the pre-tariff price with end-users directly paying tariffs.

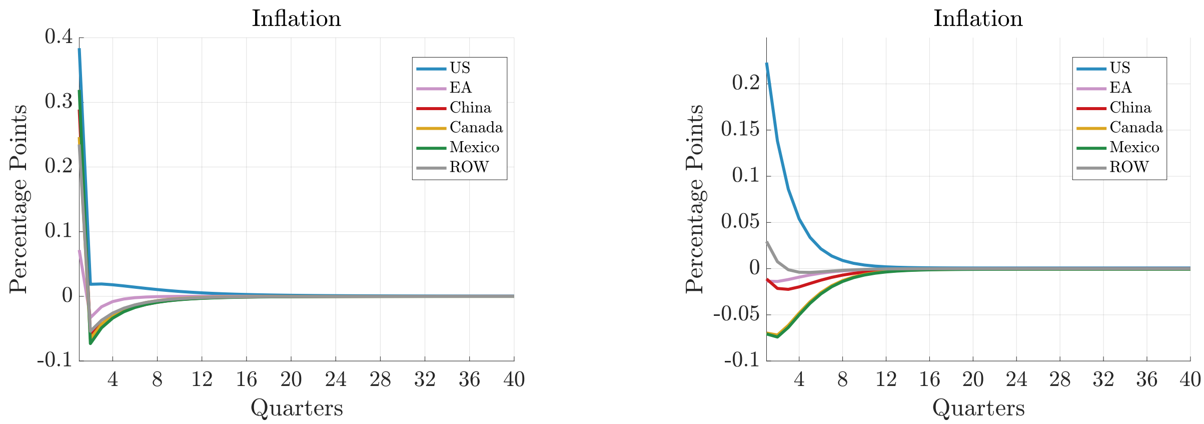
To align the model more closely with empirical work we introduce a sector of retailers. This change corresponds to a change in our Ω and Γ matrices. Instead of households and firms directly importing from foreign counterparts, we create domestically owned distribution sectors. Households and firms, in turn, import from this domestically owned sector. This ensures that there is rigidity on the post-tariff price.[38] Figure 15, shows that adding real importers slow down passthrough from tariffs onto prices and generates more inflation

[38] Our approach here differs from LCP where losses would be incurred by foreign firms. In this context, if the US is imposing tariffs and the post-tariff price for these goods in the US is rigid, then American importing firms experience profit losses.

persistence. This is in line with the intuition established in Section 6. Because retailers mainly bundle goods in a sector sourced from the rest of the world, their labor share is low and the NKPC is flat for these sectors. That is why adding retailers to the network structure and making imports go through that sector in the network slows the on-impact inflation and prolongs the time it takes for the shock to work through the system.

Figure 15. Impact of Adding Retail Importers

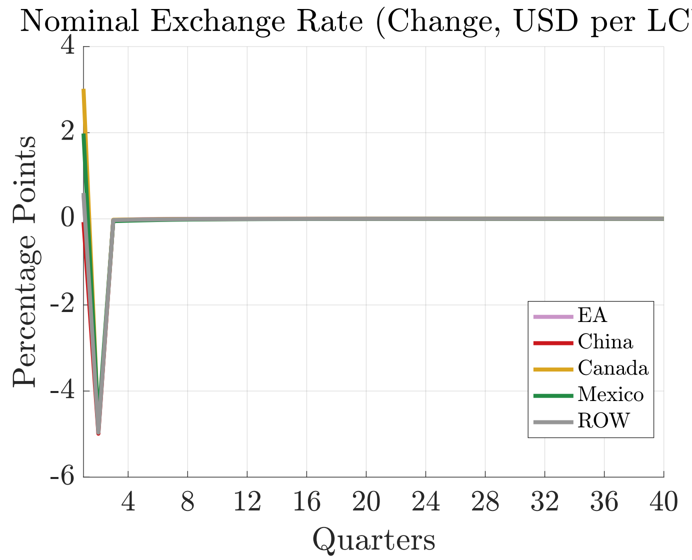
(a) Without Retail Importers **(b)** With Retail Importers



The second feature at hand is relevant for the fact that the US dollar depreciated in the first quarter of 2025 while tariffs began to be introduced. As noted in Section 4.2 and in Kalemli-Özcan et al. (2026), while this runs counter to the standard intuition regarding tariffs being appreciationary, increases in tariff volatility as it widens the UIP premium can lead to depreciation. In Figure 16, we incorporate this idea to our model. That is in the period when tariffs are initially introduced, an exogenous UIP wedge shock, following the quantitative work in Kalemli-Özcan et al. (2026), is fed in at the same time as the tariff shock. This generates depreciation of the US dollar on impact.[39] Two observations are of note here. First, here we model a one-time in crease in the UIP premium. A longer shock series capturing uncertainty beyond the initial quarter when tariffs were introduced, can help match the path of the exchange rate. Secondly, as noted in Section 4.2, the size of the risk-sharing wedge, which in this context transfers wealth to the United States, is scaled by $\frac{1}{1-\beta rho^{\tau}}$; persistence (and perceived persistence) of the tariff shock matters. If agents do not believe tariffs will be persistent or if they do not find their size to be credible that can mute the appreciationary forces of tariffs and help match the scale of depreciation in real-life in greater detail.

[39]Because our model has endogenous production, tariffs are more appreciationary than they are in the quantitative analysis of Kalemli-Özcan et al. (2026). This is the case because tariffs reduce production and a negative supply shock that increases the scarcity of the output of one country relative to others, all else being equal, leads to appreciationary pressure for that country's currency.

Figure 16. Tariff Shocks, Uncertainty and Depreciation



7.9 All-Out Trade War

We now turn to a counterfactual quantitative exercise exploring what would happen if non-US countries retaliated symmetrically to US tariffs. Case 3 was based on actual tariffs that have been placed and these involve a small degree of retaliation, but by and large these have been small. As a counterfactual we explore how the results would look under an an all-out symmetric tariff war. In this case, the United States imposes tariffs on all major trade partners at the same rates as specified in Case 3 and trade partners retaliate by imposing symmetric tariffs on U.S. exports.

China experiences a sizable contraction in GDP, declining by 1.4%, while consumption drops by 1.2%. The real exchange rate depreciates by 2.0%. Inflation rises by 1.3 percentage points, and employment declines by 1.3%. Real wages fall by 3.6%. The Euro Area experiences a moderate contraction: real GDP declines by 0.5%, consumption falls by 0.6%, and real wages decrease by 1.7%. Inflation rises by 0.3 percentage points. The euro depreciates by 1.6% against the U.S. dollar, while the real exchange rate declines by 2.0%. These exchange rate adjustments help absorb part of the external shock, limiting further output losses.

The United States experiences a mild contraction, with real GDP declining by 0.1% and consumption falling by 0.3%. Inflation rises by 0.6 percentage points, and employment decreases slightly by 0.0%. Real wages fall by 0.6%. Net exports as a share of steady-state GDP increase by 0.3 percentage points.

Canada and Mexico both experience moderate contractions. Real GDP declines by 0.6%

in Canada and 0.8% in Mexico. Consumption falls by 0.6% and 0.5%, respectively. Employment decreases by 0.6% in Canada and 0.7% in Mexico. Inflation rises by 0.5 percentage points in Canada and 0.4 percentage points in Mexico. Nominal exchange rates depreciate by 0.8% for Canada and 0.5% for Mexico relative to the U.S. dollar, while real exchange rates fall by 1.0% and 0.7%, respectively. Real wages decline by 1.7% in Canada and 1.8% in Mexico.

The rest of the world (RoW) also experiences a contraction, with real GDP declining by 0.7%, consumption by 0.6%, and employment by 0.6%. Inflation rises by 0.7 percentage points. The nominal exchange rate depreciates by 1.8%, while the real exchange rate declines by 1.8%. Real wages fall by 1.8%. Net exports as a share of steady-state GDP decline by 0.1%.

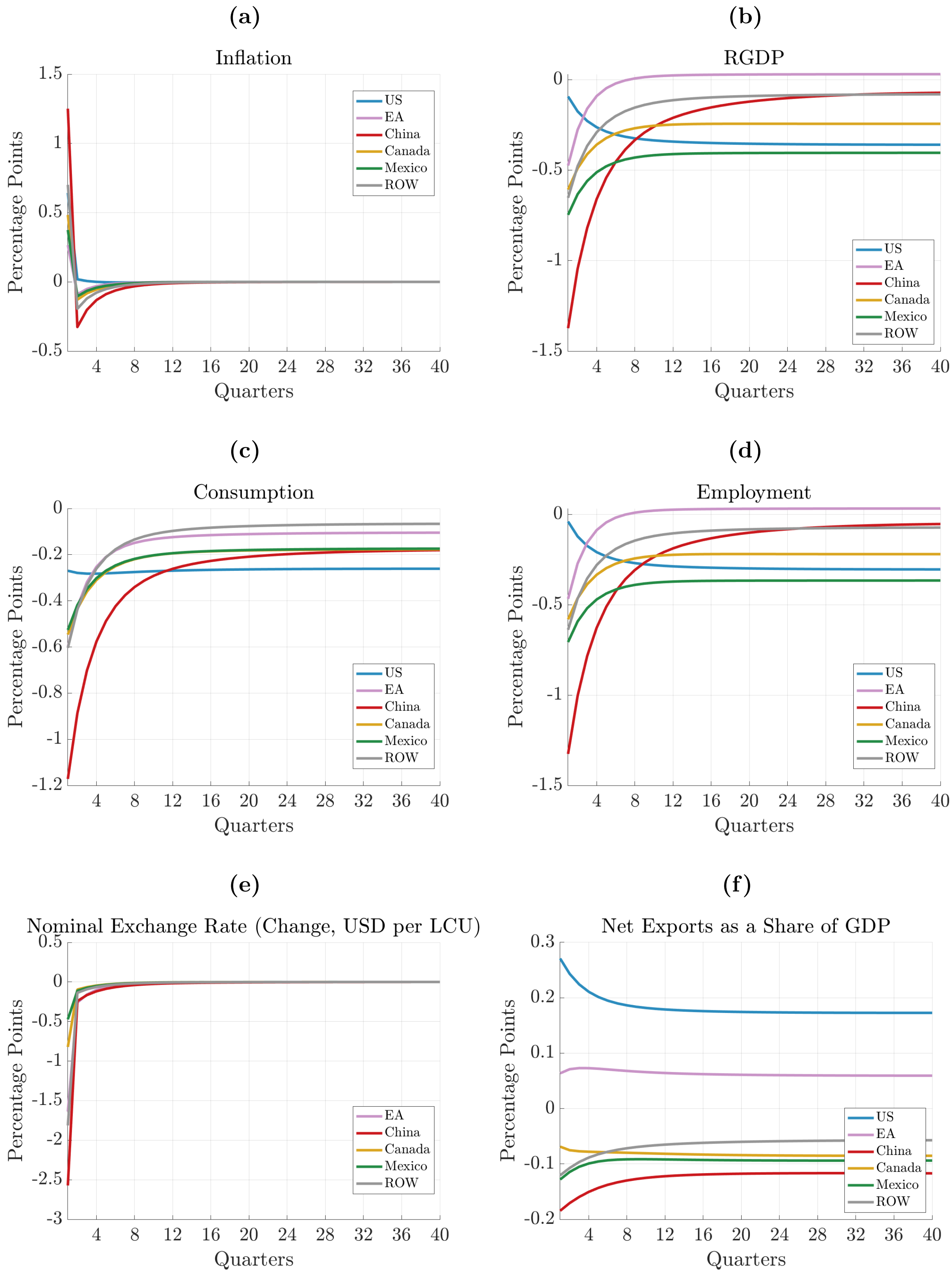
Overall, the all-out tariff war scenario leads to a synchronized global slowdown. Output and employment contract across all regions, while inflation rises moderately. The largest adverse effects occur in China, followed by Mexico, the RoW, and the Euro Area, reflecting their higher trade exposure to the United States and each other.

Table 6. On-Impact Response of Variables in Case 5: All-Out Tariff War

	US	EA	China	Canada	Mexico	RoW
$RGDP_n$	-0.09%	-0.47%	-1.37%	-0.61%	-0.75%	-0.65%
C_n	-0.27%	-0.60%	-1.17%	-0.55%	-0.53%	-0.60%
π_n	0.64%	0.27%	1.25%	0.48%	0.37%	0.70%
i_n	0.00%	0.27%	0.25%	0.10%	0.11%	0.14%
$\Delta\mathcal{E}_n$	0.00%	-1.64%	-2.57%	-0.82%	-0.47%	-1.81%
RER_n	0.00%	-2.01%	-1.98%	-0.98%	-0.74%	-1.76%
L_n	-0.04%	-0.47%	-1.33%	-0.58%	-0.71%	-0.64%
$\frac{W_n}{P_n}$	-0.58%	-1.66%	-3.62%	-1.66%	-1.75%	-1.84%
$\frac{NX_n}{NGDP_n^{ss}}$	0.27%	0.05%	-0.19%	-0.06%	-0.11%	-0.12%
$\frac{Debt_n}{NGDP_n^{ss}}$	-0.06%	-0.01%	-0.27%	-0.01%	0.01%	-0.13%

Note: First-period outcomes from a global tariff war scenario with full retaliation. Tariff magnitudes and persistence match Case 2. Variables listed here comprise real GDP ($RGDP_n$), real consumption (C_n), consumer price inflation (π_n), interest rate (i_n), depreciation of U.S. nominal exchange rate vis-a-vis country in the column ($\Delta\mathcal{E}_n$), depreciation of the U.S. real exchange rate vis-a-vis country in the column (ΔRER_n), employment (L_n), real wages ($\frac{W_n}{P_n}$), net exports as a share of steady-state GDP ($\frac{NX_n}{NGDP_n^{ss}}$) and debt as a share of steady-state GDP ($\frac{Debt_n}{NGDP_n^{ss}}$).

Figure 17. Case 5: Impact of All-Out Tariff War



NOTE: Figure 17 visualizes an all-out tariff war scenario in which trade partners retaliate symmetrically. Impulse responses are calculated with MIT shocks.

7.10 Discussion

Our analytical and quantitative analyses allow us to engage with several central questions. Under what conditions are tariffs appreciationary or depreciationary for the nominal exchange rate? Under what conditions are tariffs inflationary or deflationary? And under what conditions tariffs can be contractionary? We know these answers from the model but here in the light of the quantitative results that takes into account non-linearities, we provide further discussion.

7.10.1 Trade Deficits and the Dollar

In our quantitative framework, we find that tariffs can lead to an appreciation of the currency of the tariff-imposing country on impact. However, once retaliation is introduced, the exchange rate response becomes sensitive to the relative hawkishness of central banks. For instance, in a scenario where the U.S. imposes tariffs and the rest of the world responds, the U.S. dollar (USD) may depreciate on impact if the rest of the world has higher ϕ_π parameters-leading to greater interest rate differentials in favor of non-USD currencies.

Other work, such as Jiang et al. (2025) and Itskhoki and Mukhin (2025), explain the observed depreciation of the dollar, since the beginning of April 2025, with the loss of safe heaven status or convenience yield, where the two are related as shown before (e.g., Kekre and Lenel, 2024). Pinter et al. (2025) highlight the importance of non-trade related orthogonal shocks that coincide with a deterioration in Treasury market liquidity.

Another alternative for the observed dollar depreciation could be due to tariff uncertainty. Kalemli-Özcan et al. (2026) uses a simplified version of the model presented here to show that the impact of policy uncertainty embedded in tariff threats can create a depreciationary pressure that might dwarf the standard appreciationary effect of a regular tariff shock.

In our model,the trade balance can move during the transition with transitory tariffs but not in the long run as long as portfolio adjustment costs ensure steady state stability. In the absence of portfolio adjustment costs, on-impact valuation effects can lead to the model settling at a different steady-state level of debt. Under permanent tariffs and flexible prices, the USD value of the trade balance does not change regardless of the presence of portfolio adjustment costs. This case is in line with the argument put forth by Obstfeld (2025).

7.10.2 Inflation-Output Trade-Off and Employment

Our analytical work and calibrations show that tariffs can be inflationary or deflationary for the country on which they are imposed. A more subtle question is whether tariffs can be deflationary for the tariff-imposing country itself, such as the United States. Within our modeling framework, and barring extreme parameterizations, the direct effect of tariffs, which mechanically exerts upward pressure on prices, dominates the deflationary forces from other channels. If inflation were to turn negative, monetary policy would reverse direction and cut interest rates, thereby supporting prices. Consequently, in both our analytical solution and baseline simulations, tariffs are inflationary for the imposing country and output declines in the short-run and also in the long-run with retaliation. The key exception is Case 2 with reversed tariff threats, in which tariffs threats lead to deflation due to expectation channels.

Overall tariffs can create a stagflationary outcome with increasing inflation and declining employment and output. The response of monetary policy is critical here and our work shows that the circumstances faced by monetary policymakers might be different from some standard models.

On-impact inflation is driven by direct effects. That is why in standard models, inflation tends to precede unemployment, since it takes time for output to decline under nominal rigidity. We qualify this mechanism in two ways. First, if reversed tariff threats precede the actual imposition of tariffs there could be deflation and a decline in output at the outset, then stagflation would follow. Second, whereas in a standard model tariffs mostly result in one-time price increases, tariff-related inflation may take longer to work through in a production network. In line with our theoretical results, inflation can be more persistent in production networks.

8 Conclusion

We develop a new global general equilibrium framework to study the macroeconomic impact of tariffs under trade imbalances. Our N-country-J-sector NKOE model incorporates full global I-O linkages, heterogeneity in sectoral price rigidities and in monetary policy responses across countries involved in a trade war. We formulate the model around five primitives composed of structural parameters (consumption shares, production shares, elasticities of substitution), frictions (nominal rigidities), and endogenous monetary policy response.

Our core contribution is to delineate how the economic impact of tariffs can differ by adding dynamics, monetary policy, international borrowing/lending, and unbalanced trade into a general trade and production network with nominal rigidities. In our environment, monetary policy changes the transmission of the tariff shock, both within a given economy and across different but connected economies through trade and debt. To analyze this transmission, we derive the NKOE Leontief inverse and decompose the effects of tariffs into direct and indirect channels- each of which maps directly onto structural components of the model. Our results highlight the inflationary and contractionary effects of tariff shocks in an environment with forward-looking agents, where these effects are further amplified through the expectations channel. Although our focus is on the effects of short-run tariff shocks, we show that the effects of permanent tariff shocks can also be contractionary: the inability to substitute between domestic and foreign inputs makes goods more expensive leading to a decline in output.

Our work yields two key implications, relevant both for scholars and policy makers. First, models that omit a multi-sector structure may miss key aspects of the inflation output tradeoff. Namely inflation can be more persistent, even if it is lower on impact in production network models and the drop in output can be larger. Second, tariff threats carry real macroeconomic consequences- even when they are subsequently reversed. When agents expect future price increases, they begin to adjust consumption and production decisions in anticipation, leading to output declines and deflation on impact. Because the exchange rate is forward-looking, it appreciates immediately in response to expectated tariffs, but then reverses itself and depreciates when the threat turns empty. When tariff announcements are accompanied by heightened policy uncertainty that widens the UIP premium, this forward-looking appreciation can be muted or even reversed on impact, generating short-run exchange rate depreciation despite unchanged trade fundamentals. A deeper understanding of both production network structures and expectation-driven dynamics- such as those modeled here- can help central banks navigate a policy environment in which tariffs, retaliation, and related threats are becoming increasingly common. As Federal Reserve Chair Jerome Powell recently emphasized: "We may find ourselves in the challenging scenario in which our dual-mandate goals are in tension....There aren't historical experiences we can consult here. So it may turn out that the tariff pass-through is less or more than we think. We are perfectly open to the idea that the pass-through will be less than we think, and, if so, that will matter for our policy.[40]" Our analysis can shed light on these pressing policy questions.

By theoretically unifying long- and short-run perspectives on the impact of trade barriers, our framework echoes foundational insights from classical economic literature, dating back to Hume (1758), which emphasized the price–specie flow mechanism. This mechanism illustrates how price levels adjust endogenously through trade flows, ultimately rendering trade restrictions self-defeating. Restrictions on exports and imports induce exchange rate movements that offset perceived gains. For countries imposing import restrictions, rising labor and input costs typically follow, forcing firms to reduce employment and scale back production—ultimately undermining domestic economic performance. This core insight traces back even further to Gervaise (1720), underscoring the long-standing understanding that trade

[40]Semiannual Monetary Policy Report to the Congress, June 24, 2025.

barriers distort price signals, resource allocation and economic growth.

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Appendix

A Approximated Linear Equilibrium Conditions

Before simplifications are introduced, linearized equilibrium conditions are as follows:[41]

$$E_t\hat{C}_{n,t+1} - \hat{C}_{n,t} = \frac{1}{\sigma}\left(\hat{i}_t - E_t\pi_{n,t+1}\right) \tag{A.1}$$

$$\hat{i}_{n,t} - \hat{i}_{US,t} = E_t\hat{\mathcal{E}}_{n,t+1} - \hat{\mathcal{E}}_{n,t} + \hat{\psi} \tag{A.2}$$

$$\hat{\mathcal{E}}_{n,m,t} = \hat{\mathcal{E}}^{US}_{n,t} - \hat{\mathcal{E}}^{US}_{m,t} \tag{A.3}$$

$$\hat{\mathcal{E}}_{n,n,t} = 0 \tag{A.4}$$

$$\hat{W}_{n,t} - \hat{P}^C_{n,t} = \eta\hat{L}_{n,t} + \sigma\hat{C}_{n,t} \tag{A.5}$$

$$\hat{C}_{nt} = \sum_{j\in J}\Gamma_{n,j}\hat{C}_{n,j,t} \tag{A.6}$$

$$\hat{C}_{n,j,t} = \sum_{m\in N}\Gamma_{n,j,mj}\hat{C}_{n,mj,t} \tag{A.7}$$

$$\hat{P}_{n,mj,t} = \hat{\mathcal{E}}_{n,m,t} + \hat{\tau}_{n,m,t} + \hat{P}_{mj,t} \tag{A.8}$$

$$\hat{C}_{n,j,t} = \hat{C}_{n,t} - \theta^C_h\left(\hat{P}^C_{n,j,t} - \hat{P}^C_{n,t}\right) \tag{A.9}$$

$$\hat{C}_{n,mj,t} = \hat{C}_{n,j,t} - \theta^C_{l,j}\left(\hat{P}^C_{n,mj,t} - \hat{P}^C_{n,j,t}\right) \tag{A.10}$$

$$\hat{X}_{ni,j,t} = \sum_{m\in N}\Omega_{ni,j,mj}\hat{X}_{ni,mj,t} \tag{A.11}$$

$$\hat{X}_{ni,mj,t} = \hat{X}_{ni,j,t} - \theta^P_{l,j}\left(\hat{P}_{n,mj,t} - \hat{P}^X_{ni,j,t}\right) \tag{A.12}$$

$$\hat{X}_{ni,t} = \sum_{j\in J}\Omega_{ni,j}\hat{X}_{ni,j,t} \tag{A.13}$$

$$\hat{X}_{ni,j,t} = \hat{X}_{ni,t} - \theta^P_h\left(\hat{P}^X_{ni,j,t} - \hat{P}^X_{ni,t}\right) \tag{A.14}$$

$$\hat{Y}_{ni,t} = \hat{A}_{ni,t} + \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t} \tag{A.15}$$

$$\widehat{MC}_{ni,t} = -\hat{A}_{ni,t} + \alpha_{ni}\hat{W}_{n,t} + (1-\alpha_{ni})\hat{P}^X_{ni,t} \tag{A.16}$$

$$\hat{X}_{ni,t} - \hat{L}_{ni,t} = \theta^P\hat{W}_{n,t} - \theta^P\hat{P}^X_{ni,t} \tag{A.17}$$

$$\pi_{ni,t} = \frac{\theta^R}{\delta_{ni}}\left(\widehat{MC}_{ni,t} - \hat{P}_{ni,t}\right) + \beta\mathbb{E}_t\pi_{ni,t+1} \tag{A.18}$$

$$\bar{B}^{US}\hat{B}^{US}_t = \sum_{m}^{N-1}\bar{B}^{US}_m\hat{B}^{US}_{m,t} \tag{A.19}$$

[41] Please note in this set of equilibrium conditions the highest layer of the intermediate input bundle is simplified away.

$$\bar{Y}_{ni}\hat{Y}_{ni,t} = \sum_{n\in\mathcal{N}} \bar{C}_{m,ni}\hat{C}_{m,ni,t} + \sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{X}_{mj,ni}\hat{X}_{mj,ni,t}, \tag{A.20}$$

$$\bar{L}_n\hat{L}_{n,t} = \sum_{i\in J} \bar{L}_{ni}\hat{L}_{ni,t} \tag{A.21}$$

$$\pi_{n,t} = \hat{P}^C_{n,t} - \hat{P}^C_{n,t-1} \tag{A.22}$$

$$\hat{i}_{n,t} = \phi_\pi \pi_{n,t} + \hat{M}_{n,t} \tag{A.23}$$

and:

$$\begin{aligned} &\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\ &+ \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) \\ =&\sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}), \end{aligned} \tag{A.24}$$

where we denote the steady state (and limit) values with the bar notation.

B Relating the Balance of Payments to Prices

In this section, we show that the balance of payments can be captured by the following equation:

$$\beta\hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1\hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2\hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3\hat{\boldsymbol{P}}^P_t + \Xi_4\hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5\hat{\boldsymbol{\tau}}_t$$

To start with, we can rewrite the BoP as follows:

$$\begin{aligned} &\sum_{m\in\mathcal{N}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{C}_{n,mj}(\hat{P}_{n,mj,t} + \hat{C}_{n,mj,t}) + \sum_{m\in\mathcal{N}}\sum_{i\in\mathcal{J}}\sum_{j\in\mathcal{J}} \bar{P}_{n,mj}\bar{X}_{ni,mj}(\hat{P}_{n,mj,t} + \hat{X}_{ni,mj,t}) \\ &+ \bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) = \sum_i \bar{P}_{ni}\bar{Y}_{ni}(\hat{P}_{ni,t} + \hat{Y}_{ni,t}) + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}) \\ &\bar{\mathcal{E}}_n(1+\bar{i}^{US}_n)\bar{B}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{i}^{US}_{n,t-1} + \hat{B}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \bar{\mathcal{E}}_n\bar{B}^{US}_n(\hat{\mathcal{E}}_{n,t} + \hat{B}^{US}_{n,t}) \end{aligned}$$

Defining $\hat{V}_t$ as dollar-denominated debt inclusive of interest payments: $\hat{V}_t = B^{US}_{n,t}(1+i_t)$:

$$\bar{\mathcal{E}}_n\bar{V}^{US}_n\left(\hat{\mathcal{E}}_{n,t} + \hat{V}^{US}_{n,t-1}\right) = \overline{NX}_n\widehat{NX}_{n,t} + \frac{\bar{\mathcal{E}}_n\bar{V}^{US}_n}{1+\bar{i}^{US}}(\hat{\mathcal{E}}_{n,t} + \hat{V}^{US}_{n,t} - \hat{i}^{US}_t)$$

WLOG $\bar{\mathcal{E}}_n = 1$. Also noting $(1 + \bar{i}_n^{US}) = \beta^{-1}$ and $\overline{NX} = (1-\beta)\bar{V}_n^{US}$:

$$\begin{aligned}
\bar{V}_n^{US}\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\bar{V}_n^{US}\widehat{NX}_{n,t} + \beta\bar{V}_n^{US}(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
\left(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US}\right) &= (1-\beta)\widehat{NX}_{n,t} + \beta(\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t}^{US} - \hat{i}_t^{US}) \\
(1-\beta)\hat{\mathcal{E}}_{n,t} + \hat{V}_{n,t-1}^{US} &= (1-\beta)\widehat{NX}_{n,t} + \beta\hat{V}_{n,t}^{US} - \beta\hat{i}_t^{US} \\
\beta\hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} &= (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta\hat{i}_t^{US}
\end{aligned}$$

Our five-equation representation will stack these equations of motion for net debt for each country along with the market clearing condition for US bonds. To that end let us define net exports as follows:

$$\begin{aligned}
\underbrace{\overline{\boldsymbol{NX}}_n}_{N\times N}\underbrace{\hat{\boldsymbol{NX}}_{n,t}}_{N\times 1} &= \underbrace{\overline{\boldsymbol{Y}}^{NNJ}}_{N\times NJ}(\hat{\boldsymbol{P}}^P + \hat{\boldsymbol{Y}}_{ni,t}) - \underbrace{\overline{\boldsymbol{C}}}_{N\times N}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \underbrace{\overline{\boldsymbol{X}}^{ni}}_{N\times NJ}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \hat{\boldsymbol{X}}_{ni,t}) \\
\overline{\boldsymbol{NX}}_n\hat{\boldsymbol{NX}}_{n,t} &= \overline{\boldsymbol{Y}}^{NNJ}\left[(\hat{\boldsymbol{P}}_t^P + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}_t^{C,\tau} + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} + \hat{\boldsymbol{X}}_{ni,t})\right] \qquad \text{(B.1)}
\end{aligned}$$

where the overline matrices contain appropriately mapped steady-state values. A key thing to note is that the price vectors here are ex-tariff indices as noted by the superscript. The second line follows from the fact that we can assume α share of output goes to labor income at the steady state, which is then spent on consumption. That is we have:

$$\underbrace{\overline{\boldsymbol{C}}}_{N\times N} = \underbrace{\overline{\boldsymbol{Y}}^{NNJ}}_{N\times NJ}\underbrace{\boldsymbol{\alpha}}_{NJ\times N}$$

B.1 Market-Clearing Condition

Consider the following scalar market-clearing condition for a generic good before linearization:

$$Y_{ni,t} = \sum_{m\in N} C_{m,ni,t} + \sum_{m\in N}\sum_{j\in J} X_{mj,ni,t}$$

This yields the following when linearized:

$$\overline{Y}_{ni}(\hat{Y}_{ni,t}) - \sum_{m\in N}\overline{C}_m\Gamma_{m,ni,t}(\hat{C}_{m,ni,t}) - \sum_{m\in N}\sum_{j\in J}\overline{Y}_{mj}\Omega_{mj,ni}(\hat{X}_{mj,ni,t}) = 0 \;\forall\; n\in N, i\in J$$

where we add the steady state values because Γ terms are reported as a share of the aggregate consumption basket at the steady state in our model and similarly Ω terms are as a share of production.

In vector form we can write the market-clearing condition for goods as follows:

$$\underbrace{\overline{\boldsymbol{Y}}^{ni}}_{NJ\times NJ}\underbrace{\hat{\boldsymbol{Y}}_{ni,t}}_{NJ\times 1}=\underbrace{\overline{\boldsymbol{C}}_{n}}_{NJ\times NJ}\underbrace{\boldsymbol{\Gamma}^{M}}_{NJ\times NNJ}\underbrace{\hat{\boldsymbol{C}}_{t}^{nmj}}_{NNJ\times 1}+\overline{\boldsymbol{Y}}^{ni}\underbrace{\boldsymbol{\Omega}^{M}}_{NJ\times NJNJ}\underbrace{\hat{\boldsymbol{X}}_{t}^{nimj}}_{NJNJ\times 1}$$

where overline notation indicates appropriately scaled diagonal matrices thatContain steady-state values.

Aggregate consumption and the intermediate input bundles are CES bundles; as such they have appropriately defined price indices and relative demand conditions. However, in our context there is no such object for output. In essence relating granular quantities, $\hat{\boldsymbol{C}}_t^{nmj}$ and $\hat{\boldsymbol{X}}_t^{nimj}$ respectively to aggregates $\hat{\boldsymbol{C}}_t^{n}$ and $\hat{\boldsymbol{X}}_t^{ni}$ via CES structure and the relative demand conditions we want to be able to substitute $\hat{\boldsymbol{Y}}_{ni,t}$ in the BoP equation, using only prices, exchange rate, aggregate consumption and tariffs.

Note key identities:

$$\sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\,\hat{C}_{n,mj,t}=\hat{C}_{n,t},\qquad \sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\big(\hat{P}^{P}_{mj,t}+\hat{\mathcal{E}}_{n,m,t}\big)=\hat{P}^{C,\tau}_{n,t},$$

$$\sum_{m\in N}\sum_{j\in J}\Gamma_{n,mj}\big(\hat{P}^{P}_{mj,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mj,t}\big)=\hat{P}^{C}_{n,t}.$$

$$\sum_{m\in N}\sum_{j\in J}\Omega_{ni,mj}\,\hat{X}_{ni,mj,t}=\frac{\overline{X}_{ni}}{\overline{Y}_{ni}}\,\hat{X}_{ni,t},\qquad \sum_{m\in N}\sum_{j\in J}\frac{\overline{X}_{ni,mj}}{\overline{X}_{ni}}\big(\hat{P}^{P}_{mj,t}+\hat{\mathcal{E}}_{n,m,t}\big)=\hat{P}^{X,\tau}_{ni,t},$$

$$\sum_{m\in N}\sum_{j\in J}\frac{\overline{X}_{ni,mj}}{\overline{X}_{ni}}\big(\hat{P}^{P}_{mj,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mj,t}\big)=\hat{P}^{X}_{ni,t}.$$

$$\Omega_{ni,mj}=\frac{\overline{X}_{ni,mj}}{\overline{Y}_{ni}},\qquad \overline{X}_{ni}=(1-\alpha_{ni})\,\overline{Y}_{ni}.$$

By the relative demand conditions we have:

$$\begin{aligned}\hat{C}_{n,mi,t}&=\hat{C}_{n,t}+\theta^{C}(\hat{P}^{C}_{n,t}-(\hat{P}^{P}_{mi,t}+\hat{\mathcal{E}}_{n,m,t}+\hat{\tau}_{n,mi,t}))\\ \hat{X}_{mj,ni,t}&=\hat{X}_{mj,t}+\theta^{P}(\hat{P}^{X}_{mj,t}-(\hat{P}^{P}_{ni,t}+\hat{\mathcal{E}}_{m,n,t}+\hat{\tau}_{m,ni,t}))\end{aligned}$$

Vectorizing:

$$\hat{\boldsymbol{C}}_t^{nmj} = \underbrace{\boldsymbol{S}_1}_{NNJ\times N} \hat{\boldsymbol{C}}_{n,t} + \theta^C(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmi})$$
$$\hat{\boldsymbol{X}}_t^{nimj} = \underbrace{\boldsymbol{S}_2}_{NJNJ\times NJ} \hat{\boldsymbol{X}}_{ni,t} + \theta^P(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \hat{\boldsymbol{P}}_{ni,mj,t}^X)$$

Plugging these into the market-clearing condition:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} = \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M \left(\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \hat{\boldsymbol{P}}_t^{nmi}\right)\right) \\ + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\left(\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \hat{\boldsymbol{P}}_{ni,mj,t}^X\right)\right)\end{aligned}$$

Defining these granular price variables:

$$\underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1} = \underbrace{\boldsymbol{S}_3}_{NNJ\times NJ}\underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_4}_{NNJ\times 1}\hat{\mathcal{E}}_t + \underbrace{\boldsymbol{S}_5}_{NNJ\times 1}\hat{\tau}_t$$
$$\underbrace{\hat{\boldsymbol{P}}_{ni,mj,t}^X}_{NJNJ\times 1} = \underbrace{\boldsymbol{S}_6}_{NJNJ\times NNJ}\underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1}$$

where $\boldsymbol{S}$s are just selector matrices that map these variables to appropriate scale by "selecting" the right value from the vector on the right.

Inserting granular price variables into the last expression:

$$\begin{aligned}\overline{\boldsymbol{Y}}^{ni}\hat{\boldsymbol{Y}}_{ni,t} &= \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\left(\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\left(\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\right) \\ &\quad + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\left(\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\left(\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \boldsymbol{S}_6\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right)\right) \\ &= \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_1\hat{\boldsymbol{C}}_{n,t} + \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_1\hat{\boldsymbol{P}}_t^C - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^C\overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^M\boldsymbol{S}_5\hat{\tau}_t \\ &\quad + \overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_2\hat{\boldsymbol{X}}_{ni,t} + \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_2\hat{\boldsymbol{P}}_{ni,t}^X - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^P \\ &\quad - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_4\hat{\mathcal{E}}_t - \theta^P\overline{\boldsymbol{Y}}^{ni}\boldsymbol{\Omega}^M\boldsymbol{S}_6\boldsymbol{S}_5\hat{\tau}_t \end{aligned} \tag{B.2}$$

Note that the $\boldsymbol{\Gamma}$ matrix contains steady-state values divided by aggregate consumption such that:

$$\underbrace{\hat{\boldsymbol{C}}_{n,t}}_{N\times 1} = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ}\underbrace{\hat{\boldsymbol{C}}_t^{nmj}}_{NNJ\times 1}$$

$$\hat{\boldsymbol{P}}_t^C = \underbrace{\boldsymbol{\Gamma}}_{N\times NJ} \underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^C \hat{\tau}_t$$

$$\hat{\boldsymbol{P}}_t^C = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ} \underbrace{\hat{\boldsymbol{P}}_t^{nmi}}_{NNJ\times 1} = \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ} \left(\underbrace{\boldsymbol{S}_3}_{NNJ\times NJ} \underbrace{\hat{\boldsymbol{P}}_t^P}_{NJ\times 1} + \underbrace{\boldsymbol{S}_4}_{NNJ\times 1} \hat{\mathcal{E}}_t + \underbrace{\boldsymbol{S}_5}_{NNJ\times 1} \hat{\tau}_t \right)$$

That is we can define our loading notation in the main 5 equation representation:

$$\begin{aligned} \underbrace{\boldsymbol{\Gamma}}_{N\times NJ} &= \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ} \underbrace{\boldsymbol{S}_3}_{NNJ\times NJ} \\ \boldsymbol{L}_{\mathcal{E}}^C &= \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ} \underbrace{\boldsymbol{S}_4}_{NNJ\times 1} \\ \boldsymbol{L}_\tau^C &= \underbrace{\boldsymbol{\Gamma}^{nmj}}_{N\times NNJ} \underbrace{\boldsymbol{S}_5}_{NNJ\times 1} \end{aligned}$$

Remark 5.

$$\boxed{\overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_1 = \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}}} \qquad \text{and} \qquad \boxed{\boldsymbol{\Omega}^M \boldsymbol{S}_2 = \boldsymbol{\Omega}^\top}$$

where $\overline{\boldsymbol{C}}$ is $N \times 1$ and $\overline{\boldsymbol{C}}_n$ is that matrix mapped to the $NJ \times 1$ context.

Continuing from (B.2) and using Remark 5

$$\begin{aligned} \overline{\boldsymbol{Y}}^{ni} \hat{\boldsymbol{Y}}_{ni,t} = \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_{n,t} + \theta^C \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \theta^C \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P - \theta^C \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_4 \hat{\mathcal{E}}_t - \theta^C \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_5 \hat{\tau}_t \\ + \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} + \theta^P \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \theta^P \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P - \theta^P \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_4 \hat{\mathcal{E}}_t - \theta^P \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_5 \hat{\tau}_t \end{aligned}$$

Now grouping variables:

$$\begin{aligned} \overline{\boldsymbol{Y}}^{ni} \hat{\boldsymbol{Y}}_{ni,t} &= \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_{n,t} + \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} \\ &\quad + \theta^C \left(\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_4 \hat{\mathcal{E}}_t - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_5 \hat{\tau}_t \right) \\ &\quad + \theta^P \left(\overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_4 \hat{\mathcal{E}}_t - \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_5 \hat{\tau}_t \right) \\ &= \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_{n,t} + \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} \\ &\quad + \theta^C \left(\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \left(\boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4 \hat{\mathcal{E}}_t + \boldsymbol{S}_5 \hat{\tau}_t \right) \right) \\ &\quad + \theta^P \left(\overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^M \boldsymbol{S}_6 \left(\boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4 \hat{\mathcal{E}}_t + \boldsymbol{S}_5 \hat{\tau}_t \right) \right) \end{aligned}$$

Multiplying by $\overline{\boldsymbol{Y}}^{ni^{-1}}$ on the left:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = &\left(\overline{\boldsymbol{Y}}^{ni}\right)^{-1} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_{n,t} + \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} \\
&+ \theta^C \left(\overline{\boldsymbol{Y}}^{ni}\right)^{-1} \left(\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \left(\boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4 \hat{\mathcal{E}}_t + \boldsymbol{S}_5 \hat{\tau}_t\right)\right) \\
&+ \theta^P \left(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \boldsymbol{\Omega}^M \boldsymbol{S}_6 \left(\boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4 \hat{\mathcal{E}}_t + \boldsymbol{S}_5 \hat{\tau}_t\right)\right) \qquad \text{(B.3)}
\end{aligned}$$

This expression is intuitive in that, for each entry, it compares the average price index relevant to a user—whether on the consumption or production side—with the bilateral price that user actually faces for a specific good. The resulting deviation induces an adjustment in quantities, governed by the corresponding demand elasticity.

Using the mapping identities

$$\boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_3 = \boldsymbol{\Omega}^\top, \qquad \boldsymbol{L}_{\mathcal{E}}^X \equiv \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_4, \qquad \boldsymbol{L}_\tau^X \equiv \boldsymbol{\Omega}^M \boldsymbol{S}_6 \boldsymbol{S}_5,$$

the θ^P-block can be written compactly as

$$\theta^P \overline{\boldsymbol{Y}}^{ni} \Big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^X \hat{\tau}_t\big)\Big),$$

Then:

$$\begin{aligned}
\overline{\boldsymbol{Y}}^{ni} \hat{\boldsymbol{Y}}_{ni,t} = &\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_t + \overline{\boldsymbol{Y}}^{ni} \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} \\
&+ \theta^C \left(\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_4 \hat{\mathcal{E}}_t - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_5 \hat{\tau}_t\right) \\
&+ \theta^P \overline{\boldsymbol{Y}}^{ni} \Big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^X \hat{\tau}_t\big)\Big)
\end{aligned}$$

We can also write:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = &\overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^\top \hat{\boldsymbol{X}}_{ni,t} \\
&+ \theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \left(\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \hat{\boldsymbol{P}}_t^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M (\boldsymbol{S}_3 \hat{\boldsymbol{P}}_t^P + \boldsymbol{S}_4 \hat{\mathcal{E}}_t + \boldsymbol{S}_5 \hat{\tau}_t)\right) \\
&+ \theta^P \Big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_{ni,t}^X - \big(\boldsymbol{\Omega}^\top \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^X \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^X \hat{\tau}_t\big)\Big) \qquad \text{(B.4)}
\end{aligned}$$

Substituting in $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^C \hat{\tau}_t$, $\hat{\boldsymbol{P}}_t^{C,\tau} = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t$,$\hat{\boldsymbol{P}}_{ni,t}^X = \left(\boldsymbol{\Omega} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t + \boldsymbol{L}_\tau^P \hat{\tau}_t\right)$ and $\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} = \left(\boldsymbol{\Omega} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t\right)$:

$$
\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = {} & \overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^{\top}\hat{\boldsymbol{X}}_{ni,t} \\
& + \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\left(\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{C}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{C}\hat{\tau}_t\right) - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\left(\boldsymbol{S}_3\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{S}_4\hat{\mathcal{E}}_t + \boldsymbol{S}_5\hat{\tau}_t\right)\right) \\
& + \theta^{P}\Big(\boldsymbol{\Omega}^{\top}\left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{P}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{P}\hat{\tau}_t\right) - \left(\boldsymbol{\Omega}^{\top}\hat{\boldsymbol{P}}_t^{P} + \boldsymbol{L}_{\mathcal{E}}^{X}\hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^{X}\hat{\tau}_t\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = {} & \overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t + \boldsymbol{\Omega}^{\top}\hat{\boldsymbol{X}}_{ni,t} \\
& + \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\Big(\left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_3\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_4\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\tau}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_5\right]\hat{\tau}_t\Big) \\
& + \theta^{P}\Big(\left[\boldsymbol{\Omega}^{\top}\boldsymbol{\Omega} - \boldsymbol{\Omega}^{\top}\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Omega}^{\top}\boldsymbol{L}_{\mathcal{E}}^{P} - \boldsymbol{L}_{\mathcal{E}}^{X}\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Omega}^{\top}\boldsymbol{L}_{\tau}^{P} - \boldsymbol{L}_{\tau}^{X}\right]\hat{\tau}_t\Big) \qquad \text{(B.5)}
\end{aligned}
$$

B.2 Substituting out $\hat{X}_{ni,t}$

We have the following expressions from relative demand on the construction of the production bundle and production function:

$$
\begin{aligned}
\hat{X}_{ni,t} &= \hat{L}_{ni,t} + \theta(\hat{W}_t - \hat{P}_{ni,t}^{X}) \\
\hat{Y}_{ni,t} &= \alpha_{ni}\hat{L}_{ni,t} + (1-\alpha_{ni})\hat{X}_{ni,t}
\end{aligned}
$$

Solving for labor in the first equation and plugging it into the second:

$$
\hat{X}_{ni,t} = \hat{Y}_{ni,t} + \theta^{P}\alpha_{ni}(\hat{W}_t - \hat{P}_{ni,t}^{X})
$$

Vectorizing we have:

$$
\hat{\boldsymbol{X}}_{ni,t} = \hat{\boldsymbol{Y}}_{ni,t} + \theta^{P}(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}_{ni,t}^{X})
$$

where the $\tilde{\boldsymbol{\alpha}}$ matrix, which is $NJ \times NJ$ is an appropriately scaled version of the $\boldsymbol{\alpha}$, which is $NJ \times N$. Putting this expression together with (B.4) and solving for $\hat{\boldsymbol{Y}}_{ni,t}$:

$$
\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = {} & \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega}^{\top})^{-1}}_{\boldsymbol{\Psi}_T}\Big[\overline{\boldsymbol{Y}}^{ni-1}\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\hat{\boldsymbol{C}}_t \\
& + \theta^{C}\overline{\boldsymbol{Y}}^{ni-1}\Big(\left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_3\right]\hat{\boldsymbol{P}}_t^{P} + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\mathcal{E}}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_4\right]\hat{\mathcal{E}}_t + \left[\boldsymbol{\Gamma}^{\top}\overline{\boldsymbol{C}}\boldsymbol{L}_{\tau}^{C} - \overline{\boldsymbol{C}}_n\boldsymbol{\Gamma}^{M}\boldsymbol{S}_5\right]\hat{\tau}_t\Big)
\end{aligned}
$$

$$+ \theta^P \Big(\left[\boldsymbol{\Omega}^\top \boldsymbol{\Omega} - \boldsymbol{\Omega}^\top\right] \hat{\boldsymbol{P}}_t^P + \left[\boldsymbol{\Omega}^\top \boldsymbol{L}_{\mathcal{E}}^P - \boldsymbol{L}_{\mathcal{E}}^X\right] \hat{\mathcal{E}}_t + \left[\boldsymbol{\Omega}^\top \boldsymbol{L}_{\tau}^P - \boldsymbol{L}_{\tau}^X\right] \hat{\tau}_t + \boldsymbol{\Omega}^\top (\boldsymbol{\alpha} \hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}} \hat{\boldsymbol{P}}_{ni,t}^X) \Big) \Bigg]$$

We can further substitute in $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}_t^C + \sigma \hat{\boldsymbol{C}}_t = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C \hat{\tau}_t + \sigma \hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}_t^C = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^C \hat{\tau}_t$, $\hat{\boldsymbol{P}}_t^{C,\tau} = \boldsymbol{\Gamma} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^C \hat{\mathcal{E}}_t$,$\hat{\boldsymbol{P}}_{ni,t}^X = \Big(\boldsymbol{\Omega} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t + \boldsymbol{L}_{\tau}^P \hat{\tau}_t\Big)$ and $\hat{\boldsymbol{P}}_{ni,t}^{X,\tau} = \Big(\boldsymbol{\Omega} \hat{\boldsymbol{P}}_t^P + \boldsymbol{L}_{\mathcal{E}}^P \hat{\mathcal{E}}_t\Big)$:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1}}_{\boldsymbol{\Psi}_T} \Bigg[& (\overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{\alpha} \sigma) \hat{\boldsymbol{C}}_t \\
& + \theta^C \overline{\boldsymbol{Y}}^{ni-1} \Big(\underbrace{\left[\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \boldsymbol{\Gamma} - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_3\right]}_{\boldsymbol{T}_P^C} \hat{\boldsymbol{P}}_t^P + \underbrace{\left[\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \boldsymbol{L}_{\mathcal{E}}^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_4\right]}_{\boldsymbol{T}_{\hat{\mathcal{E}}}^C} \hat{\mathcal{E}}_t + \underbrace{\left[\boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} \boldsymbol{L}_{\tau}^C - \overline{\boldsymbol{C}}_n \boldsymbol{\Gamma}^M \boldsymbol{S}_5\right]}_{\boldsymbol{T}_\tau^C} \hat{\tau}_t \Big) \\
& + \theta^P \boldsymbol{\Omega}^\top \Big(\underbrace{\left[\boldsymbol{I} - (\boldsymbol{\Omega}^\top)^{-1} \boldsymbol{\Omega} + \boldsymbol{\alpha} \boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}} \boldsymbol{\Omega}\right]}_{\boldsymbol{T}_P^P} \hat{\boldsymbol{P}}_t^P \\
& \quad + \underbrace{\left[\boldsymbol{L}_{\mathcal{E}}^P - (\boldsymbol{\Omega}^\top)^{-1} \boldsymbol{L}_{\mathcal{E}}^X + \boldsymbol{\alpha} \boldsymbol{L}_{\mathcal{E}}^C - \tilde{\boldsymbol{\alpha}} \boldsymbol{L}_{\mathcal{E}}^P\right]}_{\boldsymbol{T}_{\hat{\mathcal{E}}}^P} \hat{\mathcal{E}}_t \\
& \quad + \underbrace{\left[\boldsymbol{L}_{\tau}^P - (\boldsymbol{\Omega}^\top)^{-1} \boldsymbol{L}_{\tau}^X + \boldsymbol{\alpha} \boldsymbol{L}_{\tau}^C - \tilde{\boldsymbol{\alpha}} \boldsymbol{L}_{\tau}^P\right]}_{\boldsymbol{T}_\tau^P} \hat{\tau}_t \Big) \Big) \Bigg] \\
= \boldsymbol{\Psi}_T \Bigg[& (\overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{\alpha} \sigma) \hat{\boldsymbol{C}}_t \\
& + \theta^C \overline{\boldsymbol{Y}}^{ni-1} \Big(\boldsymbol{T}_P^C \hat{\boldsymbol{P}}_t^P + \boldsymbol{T}_{\hat{\mathcal{E}}}^C \hat{\mathcal{E}}_t + \boldsymbol{T}_\tau^C \hat{\tau}_t \Big) \\
& + \theta^P \boldsymbol{\Omega}^\top \Big(\boldsymbol{T}_P^P \hat{\boldsymbol{P}}_t^P + \boldsymbol{T}_{\hat{\mathcal{E}}}^P \hat{\mathcal{E}}_t + \boldsymbol{T}_\tau^P \hat{\tau}_t \Big) \Bigg]
\end{aligned}$$

Rearranging:

$$\begin{aligned}
\hat{\boldsymbol{Y}}_{ni,t} = \boldsymbol{\Psi}_T \Bigg[& \Big(\overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{\alpha} \sigma \Big) \hat{\boldsymbol{C}}_t \\
& + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_P^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_P^P \Big) \hat{\boldsymbol{P}}_t^P \\
& + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_{\mathcal{E}}^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_{\mathcal{E}}^P \Big) \hat{\mathcal{E}}_t \\
& + \Big(\theta^C \overline{\boldsymbol{Y}}^{ni-1} \boldsymbol{T}_\tau^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_\tau^P \Big) \hat{\tau}_t \Bigg]
\end{aligned} \tag{B.6}$$

This expression tells us, in terms of market clearing that, output gets shifted by aggregate demand (directly and indirectly via the impact of wages and thereby the substitution between goods and intermediate inputs) and by goods-specific terms of trade captured by the $\boldsymbol{T}$ matrices.

Returning to the net exports equation:

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \overline{\boldsymbol{Y}}^{NNJ} \left[(\hat{\boldsymbol{P}}^P_t + \hat{\boldsymbol{Y}}_{ni,t}) - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}^{C,\tau}_t + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}^{X,\tau}_{ni,t} + \hat{\boldsymbol{X}}_{ni,t}) \right]$$

Substituting out $\hat{\boldsymbol{X}}_{ni,t}$:

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \overline{\boldsymbol{Y}}^{NNJ} \left[(\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} + \hat{\boldsymbol{P}}^P_t - \boldsymbol{\alpha}(\hat{\boldsymbol{P}}^{C,\tau}_t + \hat{\boldsymbol{C}}_t) - \boldsymbol{\Omega}(\hat{\boldsymbol{P}}^{X,\tau}_{ni,t} + \theta^P(\boldsymbol{\alpha}\hat{\boldsymbol{W}}_t - \tilde{\boldsymbol{\alpha}}\hat{\boldsymbol{P}}^X_{ni,t})) \right]$$

We can, once again, substitute in $\hat{\boldsymbol{W}}_t = \hat{\boldsymbol{P}}^C_t + \sigma\hat{\boldsymbol{C}}_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\tau}\hat{\tau}_t + \sigma\hat{\boldsymbol{C}}_t$, $\hat{\boldsymbol{P}}^C_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\tau}\hat{\tau}_t$, $\hat{\boldsymbol{P}}^{C,\tau}_t = \boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\mathcal{E}}_t$, $\hat{\boldsymbol{P}}^X_{ni,t} = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^P_{\tau}\hat{\tau}_t\right)$ and $\hat{\boldsymbol{P}}^{X,\tau}_{ni,t} = \left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t\right)$:

$$\begin{aligned}
\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} &= \overline{\boldsymbol{Y}}^{NNJ} \Big[(\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} + \hat{\boldsymbol{P}}^P_t - \boldsymbol{\alpha}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \hat{\boldsymbol{C}}_t\right) \\
&\quad - \boldsymbol{\Omega}\Big(\boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \theta^P \left[\boldsymbol{\alpha}\left(\boldsymbol{\Gamma}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^C_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^C_{\tau}\hat{\tau}_t + \sigma\hat{\boldsymbol{C}}_t\right) - \tilde{\boldsymbol{\alpha}}\left(\boldsymbol{\Omega}\hat{\boldsymbol{P}}^P_t + \boldsymbol{L}^P_{\mathcal{E}}\hat{\mathcal{E}}_t + \boldsymbol{L}^P_{\tau}\hat{\tau}_t\right)\right]\Big) \\
&= \overline{\boldsymbol{Y}}^{NNJ} \Bigg[(\boldsymbol{I} - \boldsymbol{\Omega})\hat{\boldsymbol{Y}}_{ni,t} \\
&\quad + \left[\boldsymbol{I} - \boldsymbol{\alpha}\boldsymbol{\Gamma} - \boldsymbol{\Omega}^2 - \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}}\boldsymbol{\Omega}\right)\right]\hat{\boldsymbol{P}}^P_t \\
&\quad - \left[\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{\Omega}\boldsymbol{L}^P_{\mathcal{E}} + \theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}^C_{\mathcal{E}} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\mathcal{E}}\right)\right]\hat{\mathcal{E}}_t \\
&\quad - \left[\theta^P\boldsymbol{\Omega}\left(\boldsymbol{\alpha}\boldsymbol{L}^C_{\tau} - \tilde{\boldsymbol{\alpha}}\boldsymbol{L}^P_{\tau}\right)\right]\hat{\tau}_t \\
&\quad - \left[\boldsymbol{I} + \theta^P\sigma\boldsymbol{\Omega}\right]\boldsymbol{\alpha}\hat{\boldsymbol{C}}_t \Bigg]
\end{aligned}$$

We can plug in $\hat{\boldsymbol{Y}}_{ni,t}$ from (B.6):

$$\overline{\boldsymbol{NX}}_n \hat{\boldsymbol{NX}}_{n,t} = \Xi_P\,\hat{\boldsymbol{P}}^P_t + \Xi_{\mathcal{E}}\,\hat{\mathcal{E}}_t + \Xi_{\tau}\,\hat{\tau}_t + \Xi_C\,\hat{\boldsymbol{C}}_t, \tag{B.7}$$

$$\Xi_P = \overline{\boldsymbol{Y}}^{NNJ} \Bigg\{ \underbrace{\boldsymbol{\Psi}_\Delta \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_P^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_P^P \Big) - \theta^P \boldsymbol{\Omega} \left(\boldsymbol{\alpha} \boldsymbol{\Gamma} - \tilde{\boldsymbol{\alpha}} \boldsymbol{\Omega} \right)}_{\text{Change in Quantities (ToT)}} + \underbrace{\Big[\boldsymbol{I} - \boldsymbol{\alpha} \boldsymbol{\Gamma} - \boldsymbol{\Omega}^2 \Big]}_{\text{Change in Prices (Valuation)}} \Bigg\}, \tag{B.8}$$

$$\Xi_{\mathcal{E}} = \overline{\boldsymbol{Y}}^{NNJ} \Bigg\{ \underbrace{\boldsymbol{\Psi}_\Delta \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_{\mathcal{E}}^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_{\mathcal{E}}^P \Big) - \theta^P \boldsymbol{\Omega} \left(\boldsymbol{\alpha} \boldsymbol{L}_{\mathcal{E}}^C - \tilde{\boldsymbol{\alpha}} \boldsymbol{L}_{\mathcal{E}}^P \right)}_{\text{Change in Quantities (ToT)}} - \underbrace{\Big[\boldsymbol{\alpha} \boldsymbol{L}_{\mathcal{E}}^C + \boldsymbol{\Omega} \boldsymbol{L}_{\mathcal{E}}^P \Big]}_{\text{Change in Prices (Valuation)}} \Bigg\}, \tag{B.9}$$

$$\Xi_\tau = \overline{\boldsymbol{Y}}^{NNJ} \Bigg\{ \underbrace{\boldsymbol{\Psi}_\Delta \Big(\theta^C \overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{T}_\tau^C + \theta^P \boldsymbol{\Omega}^\top \boldsymbol{T}_\tau^P \Big) - \Big[\theta^P \boldsymbol{\Omega} \left(\boldsymbol{\alpha} \boldsymbol{L}_\tau^C - \tilde{\boldsymbol{\alpha}} \boldsymbol{L}_\tau^P \right) \Big]}_{\text{Change in Quantities (ToT)}} \Bigg\}, \tag{B.10}$$

$$\Xi_C = \overline{\boldsymbol{Y}}^{NNJ} \Bigg\{ \underbrace{\boldsymbol{\Psi}_\Delta \Big(\overline{\boldsymbol{Y}}^{ni^{-1}} \boldsymbol{\Gamma}^\top \overline{\boldsymbol{C}} + \theta^P \sigma \boldsymbol{\Omega}^\top \boldsymbol{\alpha} \Big) - \Big[\boldsymbol{I} + \theta^P \sigma \boldsymbol{\Omega} \Big] \boldsymbol{\alpha}}_{\text{Aggregate Demand Effect}} \Bigg\}. \tag{B.11}$$

where

$$\boldsymbol{\Psi}_\Delta = (\boldsymbol{I} - \boldsymbol{\Omega})(\boldsymbol{I} - \boldsymbol{\Omega}^\top)^{-1}.$$

Recall that the equation of motion for net debt for a given country is $\beta \hat{V}_{n,t}^{US} - \hat{V}_{n,t-1}^{US} = (1-\beta)\hat{\mathcal{E}}_{n,t} - (1-\beta)\widehat{NX}_{n,t} + \beta \hat{i}_t^{US}$. To avoid a redundant equation, one of the equations of motion for net debt is replaced with a market clearing condition for US bonds. Then stacking these expressions we arrive at the fifth equation capturing the balance of payments:

$$\beta \hat{\boldsymbol{V}}_t = \boldsymbol{\Xi}_1 \hat{\boldsymbol{V}}_{t-1} + \boldsymbol{\Xi}_2 \hat{\boldsymbol{C}}_t + \boldsymbol{\Xi}_3 \hat{\boldsymbol{P}}_t^P + \Xi_4 \hat{\boldsymbol{\mathcal{E}}}_t + \boldsymbol{\Xi}_5 \hat{\boldsymbol{\tau}}_t$$

where $\Xi_1 = 1$ in the case of the two-country model; aggregating this yields the fifth equation in the five-equation representation.

From the expression above and from intuition, we can see that a higher elasticity of substitution makes the balance of payments more reactive to changes in prices. More broadly, we see net exports react to the aggregate demand stance of countries and the terms of trade in each sector.

Stacking the final expression above for different countries n, alongside a market-clearing condition for U.S. bonds, yields the fifth equation in the five-equation Global New Keynesian Representation.

B.3 Scalar Example ($N = 2, J = 1$)

Let us consider a scalar example ($N = 2, J = 1$), whereby we simplify away the home country's use of its own goods as intermediate inputs. With steady-state consumption normalized to 1, we can express steady-state values for variables like $C_{H,H,t}$ and $X_{F,H,t}$ in terms of home bias in consumption $(1-\gamma_H)$ and imported input dependence Ω_H, which is transformed into $\Psi_H = \frac{1}{1-\Omega_H}$.[42] Thus, when linearized, we have the following equations:

$$
\begin{aligned}
\hat{Y}_{H,t} &= (1-\Omega_H)(1-\gamma_H)\hat{C}_{H,H,t} + (1-\Omega_F)\gamma_F\hat{C}_{F,H,t} + \Omega_F\hat{X}_{F,H,t} \\
\hat{Y}_{F,t} &= (1-\Omega_F)(1-\gamma_F)\hat{C}_{F,F,t} + (1-\Omega_H)\gamma_H\hat{C}_{H,F,t} + \Omega_H\hat{X}_{H,F,t} \\
\hat{X}_{H,F,t} &= \theta^P(1-\Omega_H)(\hat{P}^C_{H,t} + \sigma\hat{C}_{H,t} - (\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t)) + \hat{Y}_{H,t} \\
\hat{X}_{F,H,t} &= \theta^P(1-\Omega_F)(\hat{P}^C_{F,t} + \sigma\hat{C}_{F,t} - (\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t)) + \hat{Y}_{F,t} \\
\hat{C}_{H,H,t} &= -\theta^C\left(\hat{P}^P_{H,t} - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{H,F,t} &= -\theta^C\left(\hat{P}^P_{F,t} + \hat{\tau}^H + \hat{\mathcal{E}}_t - \hat{P}^C_{H,t}\right) + \hat{C}_{H,t} \\
\hat{C}_{F,F,t} &= -\theta^C\left(\hat{P}^P_{F,t} - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{C}_{F,H,t} &= -\theta^C\left(\hat{P}^P_{H,t} + \hat{\tau}^F - \hat{\mathcal{E}}_t - \hat{P}^C_{F,t}\right) + \hat{C}_{F,t} \\
\hat{P}^C_{H,t} &= (1-\gamma_H)\hat{P}^P_{H,t} + \gamma_H(\hat{P}^P_{F,t} + \mathcal{E}_t + \hat{\tau}^H_t) \\
\hat{P}^C_{F,t} &= (1-\gamma_F)\hat{P}^P_{F,t} + \gamma_F(\hat{P}^P_{H,t} - \mathcal{E}_t + \hat{\tau}^F_t) \\
\overline{NX}\widehat{NX}_t &= \left[1-(1-\Omega_H)(1-\theta^P)\right]\hat{P}^P_{H,t} + \left[-1+(1-\Omega_F)(1-\theta^P)\right]\hat{P}^P_{F,t} \\
&\quad + (\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\Omega_F)(\hat{C}_{F,t} + \theta^P\hat{P}^C_{F,t}) - (1-\Omega_H)(\hat{C}_{H,t} + \theta^P\hat{P}^C_{H,t}) - \Omega_F\hat{\mathcal{E}}_t
\end{aligned}
$$

These equations can express net exports as a share of prices, which can then be plugged into the following balance of payments equation:

$$\hat{V}_t = \beta^{-1}\hat{V}_{t-1} - \frac{(1-\beta)}{\beta}\widehat{NX}_t + \hat{i}_t$$

The rest of the model, as covered by the other four equations of the Global NK representation would be as follows:

$$
\begin{aligned}
\sigma(E_t\hat{C}_{H,t+1} - E_t\hat{C}_{H,t}) &= \hat{i}_t - E_t\pi^C_{H,t+1} \\
\sigma(E_t\hat{C}_{F,t+1} - E_t\hat{C}_{F,t}) &= \hat{i}_t - E_t\pi^C_{F,t+1}
\end{aligned}
$$

[42] Note $\overline{NX} = (1-\beta)\overline{V} = -(1-\Omega_H)(1-\gamma_H) + (1-\Omega_F)(1-\gamma_F)$.

$$
\begin{aligned}
\pi^P_{H,t} &= \Lambda_H \left(\alpha_H(\hat{P}^C_{H,t} + \sigma C_{H,t}) + \Omega_H \left(\hat{P}^P_{F,t} + \hat{\mathcal{E}}_t + \hat{\tau}^H_t\right) - \hat{P}^P_{H,t}\right) + \beta \mathbb{E}_t \pi^P_{H,t+1} \\
\pi^P_{F,t} &= \Lambda_F \left(\alpha_F(\hat{P}^C_{F,t} + \sigma C_{F,t}) + \Omega_F \left(\hat{P}^P_{H,t} + \hat{\tau}^F_t - \hat{\mathcal{E}}_t\right) - \hat{P}^P_{F,t}\right) + \beta \mathbb{E}_t \pi^P_{F,t+1} \\
\pi^C_{H,t} &= \hat{P}^C_{H,t} - \hat{P}^C_{H,t-1} \\
\pi^C_{F,t} &= \hat{P}^C_{F,t} - \hat{P}^C_{F,t-1} \\
\pi^P_{H,t} &= \hat{P}^P_{H,t} - \hat{P}^P_{H,t-1} \\
\pi^C_{P,t} &= \hat{P}^P_{F,t} - \hat{P}^P_{F,t-1} \\
\hat{i}_{H,t} - \hat{i}_{F,t} &= E_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t \\
\hat{i}_{H,t} &= \phi^H_\pi \pi^P_{H,t} \\
\hat{i}_{F,t} &= \phi^F_\pi \pi^F_{H,t}
\end{aligned}
$$

C Decomposing the Impact on Inflation

Starting with the second equation of Proposition 2 and combining with the CPI definition, we can write:

$$
\begin{aligned}
\frac{\partial \boldsymbol{\pi}^P_t}{\partial \hat{\tau}_t} &= \boldsymbol{p}_\tau = \left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1} \\
&\quad \Big(\boldsymbol{\Lambda}\big(\boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau + (\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) e_\tau + (\boldsymbol{\alpha} \boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau)\big) + \beta\, \boldsymbol{p}_v v_\tau\Big) \\
\to \frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} &= \boldsymbol{\Gamma} \underbrace{\left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1} \boldsymbol{\Lambda}}_{\boldsymbol{H}_1} \\
&\quad \underbrace{\big(\boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau + (\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) e_\tau + (\boldsymbol{\alpha} \boldsymbol{L}^C_\tau + \boldsymbol{L}^P_\tau) + \beta \boldsymbol{\Lambda}^{-1} \boldsymbol{p}_v v_\tau\big)}_{\boldsymbol{H}_2} + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau \\
\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} &= \boldsymbol{\Gamma}\Bigg(\boldsymbol{H}_2 + (\boldsymbol{H}_1 - \boldsymbol{I})\boldsymbol{H}_2\Bigg) + \boldsymbol{L}^C_{\mathcal{E}} e_\tau + \boldsymbol{L}^C_\tau
\end{aligned}
$$

This is the desired decomposition:

$$
\begin{aligned}
\frac{\partial \boldsymbol{\pi}^C_t}{\partial \hat{\tau}_t} = &\underbrace{\boldsymbol{L}^C_\tau}_{\text{Direct CPI effect}} + \underbrace{\boldsymbol{\Gamma} \boldsymbol{L}^P_\tau}_{\text{Direct PPI effect}} + \underbrace{\boldsymbol{\Gamma} \boldsymbol{\alpha} \boldsymbol{L}^C_\tau}_{\text{Direct Effect on Real Wages}} + \underbrace{\boldsymbol{\Gamma} \boldsymbol{\alpha}\sigma \boldsymbol{c}_\tau}_{\text{Demand channel}} \\
&+ \underbrace{(\boldsymbol{\Gamma}(\boldsymbol{\alpha} \boldsymbol{L}^C_{\mathcal{E}} + \boldsymbol{L}^P_{\mathcal{E}}) + \boldsymbol{L}^C_{\mathcal{E}}) e_\tau}_{\text{ER channel}} + \underbrace{\beta \boldsymbol{\Gamma} \boldsymbol{\Lambda}^{-1} \boldsymbol{p}_v v_\tau}_{\text{Debt channel}} + \underbrace{\boldsymbol{\Gamma}(\left[\tilde{\boldsymbol{\Psi}}^{-1}_\Lambda - \beta\big(\rho\, \boldsymbol{I} + \boldsymbol{\Psi}^{\text{NKOE}}\big)\right]^{-1} \boldsymbol{\Lambda} - \boldsymbol{I}) \boldsymbol{H}_2}_{\text{Propagation with Stickiness}}
\end{aligned}
$$

D Additional Results

Table D.1. U.S. Tariff events from the WTO-IMF Tariff Tracker.

Event Date	Average Tariff (%)	Event Label	Event Description
1/1/2025	2.3	Pre-Trump	The baseline tariff rates for U.S. imports from China have been updated to reflect actual tariff rates applied per tariff line, based on data from the U.S. Census for 2024. These were then compared with the Most Favored Nation (MFN) tariff rates for 2024 to identify pre-existing tariff hikes before the start of 2025. The resulting tariff rates were rounded to the nearest 0.5%. For other exporters, the baseline tariff rates are a combination of MFN and preferential rates for 2024.
2/4/2025	3.6	China +10	On February 4, 2025, the United States imposed an additional 10% tariff on all imports from China.
3/4/2025	11.3	China +10	On March 3, 2025, the United States further increased tariffs from 10% to 20% on all imports from China.
3/4/2025	11.3	Can/Mex +25	On March 4, 2025, the United States implemented an additional 25% tariff on imports from Canada and Mexico. Energy resources from Canada will have a lower 10% tariff.
3/7/2025	8.7	USMCA Exemptions	Effective on 7 March 2025 the United States announced an exemption for all imports complying with the United States-Mexico-Canada Agreement (USMCA). Compliance rate has been estimated using 2023 imports notification submitted by the U.S. to WTO's IDB. Additionally, tariffs on potash imports have been reduced from 25% to 10%.
3/12/2025	9.7	Steel & Alum. Tariffs +25	On March 12, 2025, the United States imposed additional duties on steel and aluminum imports. Specifically, a 25% tariff was applied to steel and aluminum imports, with the exception of Russian Federation, which faced a 200% tariff on aluminum.
4/3/2025	10.7	U.S. tariffs on Vehicles	Effective April 3, 2025, the United States imposed new tariffs on vehicle imports. Additional 25% tariff was applied to vehicles from all countries.
4/5/2025	13.4	Baseline 10% reciprocal tariffs	On April 05, 2025, the United States imposed a baseline additional 10% tariff on imports (there are exemptions) from all countries, except for Canada, Mexico, and countries subject to rates set forth in Column 2 of the HTSUS (Russian Federation, Cuba and Belarus, which is a WTO Observer).
4/9/2025	22.6	Liberation Day tariffs implemented	On April 9, 2025, the United States imposed additional tariffs of 34% on imports from China. On April 9, 2025, the United States increased the additional tariffs from 34% to 84% on imports from China. On April 10, 2025, the United States increased the additional tariffs from 84% to 125% on imports from China. The increased tariffs on imports from the other 55 countries with implementation date on April 9, 2025, were suspended effective April 10, 2025, for 90-days until July 9, 2025.
5/3/2025	23.3	Tariffs on Vehicle parts	Effective May 3, 2025, new tariffs were imposed on vehicle part imports. A 25% tariff was applied to vehicles' parts from all countries.
5/14/2025	14.9	U.S.-China trade deal	U.S. and China agreed to a trade deal that reduces 125% tariffs to 10%.
6/4/2025	16.5	Steel & Alum. Tariffs +25	U.S. doubled tariffs on foreign steel and aluminum imports to 50%. This applies to all trading partners except the UK.
6/23/2025	14.54	US-UK trade deal	The United States and the United Kingdom have reached a trade agreement that imposes a 10% import tariff on vehicle parts from the UK. For other countries, US imposed additional tariffs on specific consumer and household items made with steel, depending on how much steel they contain.
6/30/2025	14.48	US–UK vehicles adjustment	The United States and the United Kingdom have reached a trade agreement that establishes a 10% US import tariff on vehicles from the UK, with a quota limit of 100,000 vehicles.
8/1/2025	14.52	Copper Tariffs +50%	The United States implemented an extra 50% tariff on semi-finished copper products and related items under Section 232, citing reasons related to national security. US also imposed additional tariffs to Canada.

8/6/2025	14.70	Additional tariffs on Brazil	The United States introduced an extra 40% tariff on the majority of imports from Brazil, to take effect seven days later, while allowing exemptions and special provisions for certain products.
8/7/2025	15.88	Additional +10% on April 2 tariffs	The United States issued a new Executive Order that changes the "liberation day" tariffs for specific trading partners, and imposes an extra 10% tariff on partners not included on the list. US also implements agreements with South Korea and Japan.
8/27/2025	16.26	Tariff increase on India	The United States increased tariffs on exports from India to 50%, doubling the earlier rate of 25% and affecting a wide variety of products.
9/1/2025	16.27	Economic agreement with the EU	A new tariff schedule for the US-EU agreement is being applied retroactively from August 1, 2025.
9/8/2025	16.15	Modifications on Liberation Day Tariffs	The United States updated the liberation day tariffs.
9/16/2025	16.04	Exempt Aircrafts from Japan and UK	The United States granted an exemption from tariffs for the "Aircraft" product category on imports coming from the UK and Japan.
10/1/2025	16.05	AGOA expiry	The African Growth and Opportunity Act (AGOA) has officially ended, concluding 25 years during which goods imported into the U.S. from sub-Saharan African countries received tariff-free treatment.

Note: The tariff events are described by WTO - IMF Tariff Tracker (WTO and IMF, 2025). Note that this table only includes the actual implemented tariffs but do not include the tariffs to be implemented until October 20, 2025.

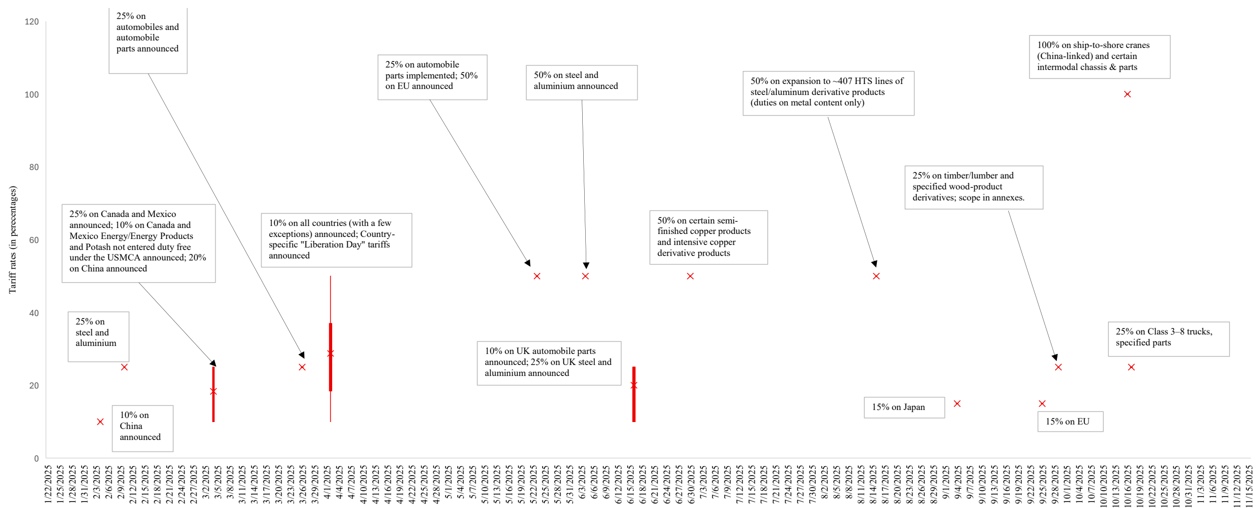
Table D.2. Sectoral Shares and Tariffs for the U.S.

Country	Industry	World Share (%)	U.S. Share (%)	U.S. Import Share (%)	U.S. Final Share (%)	U.S. Int Share (%)	U.S. Curr. Tariff (%)	U.S. Max. Tariff (%)	Ret. Curr. Tariff (%)	Ret. Max. Tariff (%)
USA	Agriculture	7.0	89.1	0.0	88.5	89.3	0.0	0.0	0.0	0.0
USA	Energy	15.0	79.0	0.0	89.4	75.0	0.0	0.0	0.0	0.0
USA	Mining	11.1	94.8	0.0	98.5	89.9	0.0	0.0	0.0	0.0
USA	Food & Bev.	13.1	91.3	0.0	91.2	91.7	0.0	0.0	0.0	0.0
USA	Basic Man.	11.2	77.4	0.0	66.0	82.5	0.0	0.0	0.0	0.0
USA	Adv. Man.	13.1	67.0	0.0	67.0	66.9	0.0	0.0	0.0	0.0
USA	Resid. Serv.	13.1	99.7	0.0	99.9	99.5	0.0	0.0	0.0	0.0
USA	Services	29.1	96.5	0.0	96.7	96.2	0.0	0.0	0.0	0.0
EUU	Agriculture	13.3	2.1	19.6	2.3	2.1	9.5	9.5	0.0	0.0
EUU	Energy	14.0	1.7	8.2	2.0	1.6	13.2	13.2	0.0	0.0
EUU	Mining	13.9	0.7	14.0	0.4	1.2	13.8	13.8	0.0	0.0
EUU	Food & Bev.	23.9	2.6	29.8	2.8	2.1	13.2	13.2	0.0	0.0
EUU	Basic Man.	23.2	7.6	33.7	12.4	5.5	6.5	6.5	0.0	0.0
EUU	Adv. Man.	29.2	8.8	26.7	8.7	9.0	10.4	15.2	0.0	0.0
EUU	Resid. Serv.	28.5	0.1	35.7	0.1	0.2	0.0	0.0	0.0	0.0
EUU	Services	30.7	1.5	42.2	1.4	1.7	0.0	0.0	0.0	0.0
CHN	Agriculture	31.7	0.4	3.9	0.4	0.4	26.2	126.6	14.0	94.6
CHN	Energy	17.6	0.1	0.4	0.1	0.1	17.6	17.6	12.7	106.7
CHN	Mining	21.4	0.1	1.4	0.1	0.1	22.1	70.8	8.5	106.1
CHN	Food & Bev.	24.1	0.8	9.3	0.8	0.8	25.7	124.1	11.8	113.3
CHN	Basic Man.	38.0	5.3	23.4	8.6	3.8	29.3	98.8	9.1	107.8
CHN	Adv. Man.	32.1	9.0	27.4	8.9	9.2	30.8	82.3	9.4	113.6
CHN	Resid. Serv.	27.2	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0
CHN	Services	12.9	0.3	9.1	0.3	0.3	0.0	0.0	0.0	0.0
CAN	Agriculture	1.1	1.7	15.3	1.8	1.6	2.2	25.0	4.0	4.0
CAN	Energy	2.3	5.9	28.2	2.0	7.4	1.0	10.0	0.0	0.0
CAN	Mining	3.1	1.4	26.9	0.5	2.6	0.5	25.0	2.3	2.3
CAN	Food & Bev.	1.4	1.2	13.9	1.1	1.4	1.9	24.5	5.8	5.8
CAN	Basic Man.	1.2	2.3	10.3	1.5	2.7	12.9	25.0	9.0	9.0
CAN	Adv. Man.	1.1	2.3	6.8	2.3	2.2	5.9	25.0	6.7	6.7
CAN	Resid. Serv.	1.9	0.1	44.5	0.1	0.2	0.0	0.0	0.0	0.0
CAN	Services	2.1	0.4	11.1	0.3	0.5	0.0	0.0	0.0	0.0
MEX	Agriculture	1.0	1.7	15.3	1.8	1.6	1.8	25.0	0.0	0.0
MEX	Energy	1.4	1.5	7.1	0.5	1.8	0.3	25.0	0.0	0.0
MEX	Mining	1.7	0.1	1.7	0.0	0.2	0.2	25.0	0.0	0.0
MEX	Food & Bev.	2.0	0.8	9.6	0.9	0.8	14.0	25.0	0.0	0.0
MEX	Basic Man.	1.0	1.3	5.7	1.2	1.3	12.4	25.0	0.0	0.0
MEX	Adv. Man.	2.1	6.3	19.1	6.3	6.3	7.7	25.0	0.0	0.0
MEX	Resid. Serv.	1.1	0.0	7.6	0.0	0.0	0.0	0.0	0.0	0.0
MEX	Services	1.1	0.3	8.6	0.3	0.3	0.0	0.0	0.0	0.0
ROW	Agriculture	45.9	5.0	46.0	5.2	5.0	18.3	18.3	0.0	0.0
ROW	Energy	49.8	11.8	56.1	5.9	14.0	0.1	0.1	0.0	0.0
ROW	Mining	48.8	2.9	56.0	0.5	6.1	9.0	10.0	0.0	0.0
ROW	Food & Bev.	35.5	3.2	37.4	3.2	3.2	18.8	18.8	0.0	0.0
ROW	Basic Man.	25.5	6.1	26.9	10.3	4.2	17.7	18.5	0.0	0.0
ROW	Adv. Man.	22.4	6.6	20.0	6.7	6.4	16.7	17.3	0.0	0.0
ROW	Resid. Serv.	28.0	0.0	12.0	0.0	0.1	0.0	0.0	0.0	0.0
ROW	Services	24.2	1.0	29.0	1.0	1.0	0.0	0.0	0.0	0.0

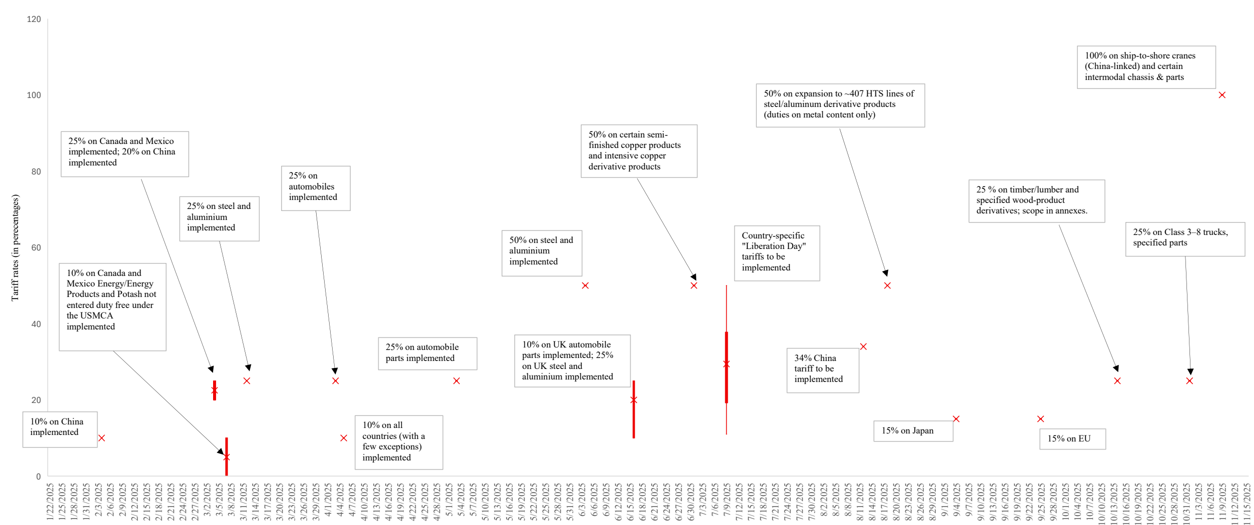
NOTE: Share data is obtained from OECD ICI-O Tables (Yamano and et al., 2023) and tariff data is obtained from WTO - IMF Tariff Tracker database (WTO and IMF, 2025). World Share is the share of the industry in the world in that industry, U.S. Share is the share of the industry in both U.S. final goods and intermediate goods, U.S. Import Share is the share of the industry in the U.S. imports in that industry, U.S. Final Share is the share in the final good consumption in that industry, U.S. Int. Share is the intermediate use share in that industry, U.S. Curr. Tariff is the tariff as of June 30, 2025, U.S. Max Tariff is the maximum tariff observed since January 1, 2025. Ret. Curr. Tariff and Ret. Max. Tariff are the retaliatory tariff levels that countries adapted against the U.S. industries.

Figure D.1. Tariff Announcements and Implementations

(a) Tariff Announcements



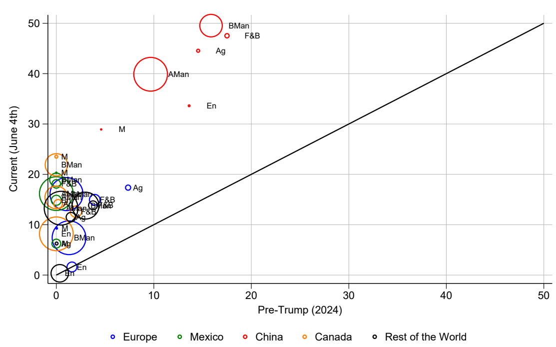
(b) Tariffs - Implemented (and to be Implemented)



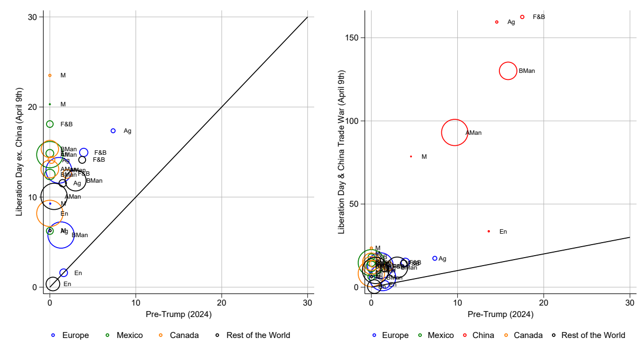
Note: Figure D.1 visualizes tariff announcements and implementations between January 20, 2025 and June 30, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs as of June 27th. This list is cross-referenced with Tax Foundation's Trump Trade War timeline as of June 17th (https://taxfoundation.org/research/all/federal/trump-tariffs-trade-war/), and a corresponding list from the PBS news article detailing a timeline of Trump's tariff actions as of May 26th (https://www.pbs.org/newshour/economy/a-timeline-of-trumps-tariff-actions-so-far). The tariffs that are classified as "threats" are those that –as of June 30th —had not been implemented and were unlikely to be implemented based on available information. These threats were identified by an extensive look into past and latest news, as well as the use of large language models. We created the data as of June 27, 2025. This website curates the all the tariff announcements by the U.S.

Figure D.2. Effective Country-Sector Level Tariff Rates

(a) As of June 4, 2025 (%)

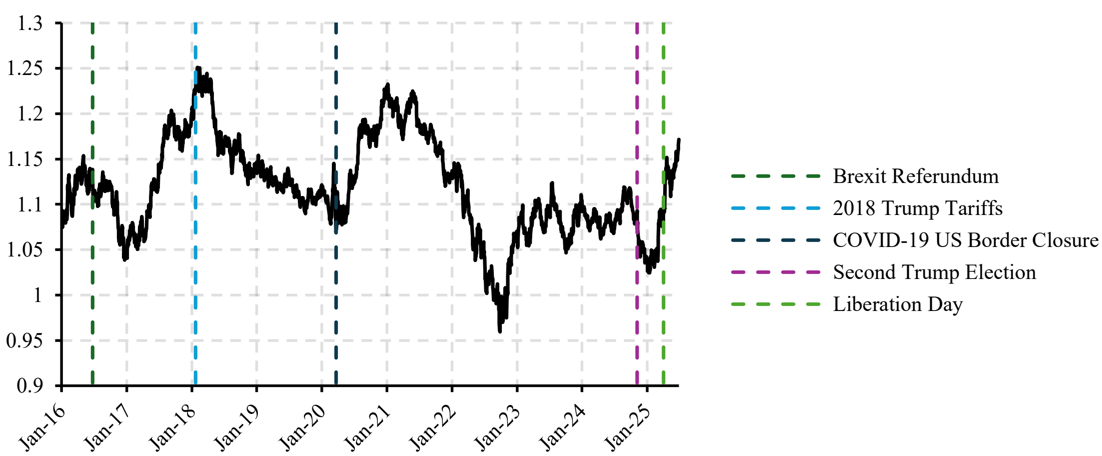


(b) As of the "Liberation Day", (%)



Note: Figure D.2a visualizes estimated effective tariff rates at the country sector level based on WTO - IMF Tariff Tracker (WTO and IMF, 2025) as of the last available day (June 4, 2025) when we accessed the data on June 20, 2025. Figure D.2b visualizes estimated effective tariff rates at the country sector level when the tariffs announced on the "liberation day" and extra tariffs on China went into effect. In the left panel, we remove the Chinese sectors. In the right panel, we show all country-sector combinations. Bubble size corresponds to the U.S. imports from that country-sector pair for the last available data at WTO. The colors code for countries: Canada, China, euro area, Mexico and the Rest of the World. Sectoral Acronyms are Ag: Agriculture, En: Energy, M: Mining, F&B: Food & Beverages, BMan: Basic Maufacturing, AMan: Advanced Manufacturing.

Figure D.3. USD - Euro Exchange Rate 2016-2025



Note: Figure D.3 visualizes the USD Euro Exchange Rate from 2015 to 2025. The vertical lines indicate different events. Source: Bloomberg.