Providing Certainty

Andy B. Choi[1] Christoph Schlom[2] Chengyang Zhu[3]

[1]University of Michigan [2]UC Davis [3]Boston University

Motivation

Policymakers often announce or commit to relatively certain future policies

- "tax credits will last until 2030"
- "interest rate will remain low" (forward guidance by central bank)
- "will buy vaccines" (advance market commitment)

Uncertainty about future policy can cause agents to delay investment, but arrival of future information could mean the committed policy is no longer optimal. We study this tradeoff between policy certainty and loss in flexibility.

We introduce a moral hazard model, and we find that it is optimal for the principal to **provide certainty**. We also ask questions such as

1. **How** does the principal optimally provide certainty (structure)?
2. When does the **agent benefit** from certainty provision?
3. When does moral hazard **delay** investment?

Simple Example

Suppose a principal (she) wants an agent (he) to invest in a project. Investing incurs an upfront cost $I > 0$, but investing early is good: agent gets flow benefit $b > 0$ every period after investing. To encourage early investing, the principal provides a subsidy y. However, $c \in \{1, 2\}$ is an uncertain state of the world that represents the principal's cost to provide this subsidy, prior $p = \mathbb{P}(c = 1) \in (0, 1)$. The timeline of the game is

Before the game: Principal commits to a state-dependent policy rule $y(c)$

Period 0: Agent decides whether to invest.

Period 1: State $c \in \{1, 2\}$ publicly realized, $\mathbb{P}(c = 1) = p$.

If agent has not invested, he decides whether to invest.

Principal implements policy $y \geq 0$ at cost cy.

$$\text{Agent's payoff} = \begin{cases} 2b - I + y & \text{if agent invests in period 0} \\ b - I + y & \text{if agent invests in period 1} \\ 0 & \text{if agent never invests} \end{cases}$$

Assume $b < I(1 - p)/(2 - p)$, the flow benefit is not too big so the agent always wants to invest.

Suppose that the principal wants to induce the agent to invest at time 0. Her problem is to find cost-minimizing policies $y(c)$ that does this. We start by solving this cost-minimization problem under a **first-best** benchmark where the agent's investment timing is contractible. We will then consider the **second-best** case where investment is not contractible.

First-best

In this first-best case, since the principal wants to induce investing at time 0, it is clear that she can set $y = 0$ unless the agent invests in period 0. Then, the agent has no reason to wait until period 1 – he should either invest in period 0 or never invest.

$$\min_{y(1), y(2)} py(1) + 2(1 - p)y(2)$$
$$\text{subject to } 2b + py(1) + (1 - p)y(2) - I \geq 0$$

Solution is: $y^{FB}(1) = \frac{I - 2b}{p}$ and $y^{FB}(2) = 0$. There is only positive policy in the low cost state. Otherwise it is too costly for the principal to provide a positive policy.

Second-best

When investment time is not contractible, y^{FB} cannot be implemented. The agent can wait until period 1, then invest iff $c = 1$ to avoid the risk of not receiving any policy.

To prevent the agent from waiting, the principal's problem must additionally satisfy the **no-waiting** constraint:

$$\underbrace{2b - I + \mathbb{E}\left(y(c)\right)}_{\text{invest in period 0}} \geq \underbrace{p(b - I + y(1))}_{\text{wait until period 1, then invest iff } c = 1}$$

We can then compare the solutions in the first-best and in the second-best.

	$y(1)$	$y(2)$
First-best	$\frac{1-2b}{p}$	0
Second-best	$I - b$	$I - \frac{b(2-p)}{(1-p)}$

- $y^{SB}(2) > y^{FB}(2)$: To reduce A's value of waiting, P must raise y in the high-cost state
- $y^{SB}(1) < y^{FB}(1)$: Because IR has been relaxed
- $y^{FB}(2) < y^{SB}(2) < y^{SB}(1) < y^{FB}(1)$, and $\mathbb{E}\left(y^{SB}\right) = \mathbb{E}\left(y^{FB}\right)$. Moral hazard makes P provide certainty about future policy.
- Both constraints bind in equilibrium, the agent is indifferent between investing at period 0, investing at period 1, and not investing.

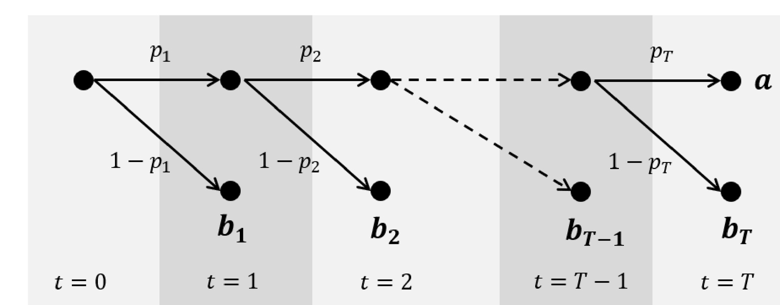
Model

The main results from the simple example are:

1. Providing certainty is distortionary: it is costly for the principal, and the agent has no value for certainty (conditional on investing early). So y^{SB} is Pareto dominated by y^{FB}.
2. Agent receives zero rent in equilibrium.
3. Moral hazard weakly delays investment.

We now consider a general model with a richer payoff structure and multiple periods. This generalizes the results from the example and obtains new insights. Consider the following:

- Time is discrete: $0, \ldots, T$. Unknown binary state $\theta \in \{0, 1\}$
- In each period t, a public signal x_t arrives via a breakdown process, survival probability p_t.



- A (terminal) **history** $\mathbf{h} \in \mathcal{H}$ is a sequence of signals, x_t, of length T. Principal's policy $y : \mathcal{H} \to \mathbb{R}_+$

$$\mathbf{a} := (\overbrace{1, \ldots, 1}^{T \text{ entries}}) \qquad \mathbf{b}_t := (\overbrace{1, \ldots, 1}^{t-1 \text{ entries}}, \overbrace{0, \ldots, 0}^{T-t+1 \text{ entries}}).$$

If the realized state is θ with history $\mathbf{h}$, agent invested at time t, and policy y,

- agent's payoff: $u(\theta, y(\mathbf{h}), t)$. We assume that it is of the form $\beta(t)w(\theta, y(\mathbf{h})) + g(\theta, t)$
  - $\beta(t)$ is weakly decreasing: complementarity between early investing and policy
  - Increasing and concave in $y(\mathbf{h})$
  - $u(1, y(\mathbf{h}), t) \geq u(0, y(\mathbf{h}), t)$ and $u_y(1, y(\mathbf{h}), t) \geq u_y(0, y(\mathbf{h}), t)$, for all $y(\mathbf{h}), t$.
  - $u(\theta, y(\mathbf{h}), \infty) = 0$, for all $\theta, y(\mathbf{h})$.
  - $u(0, 0, t) < 0$ for all $t \leq T$.
- principal's payoff: $v(\theta, y(\mathbf{h}), t)$.
  - Decreasing and concave in $y(\mathbf{h})$
  - $v(1, y(\mathbf{h}), t) \geq v(0, y(\mathbf{h}), t)$ and $v_y(1, y(\mathbf{h}), t) \geq v_y(0, y(\mathbf{h}), t)$, for all $y(\mathbf{h}), t$.
  - $v(\theta, 0, t)$ is weakly decreasing in t, for all θ.
  - $v(0, 0, \infty) = 0$.

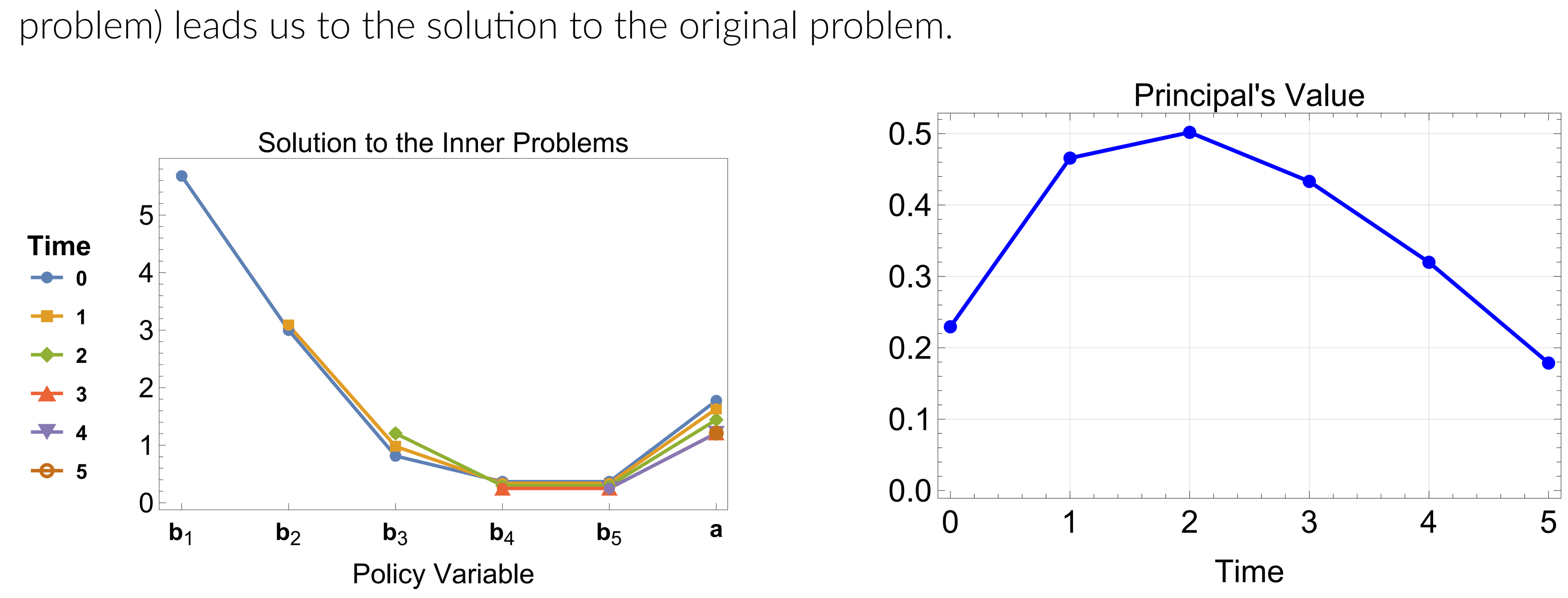
Given principal's policy, the agent faces an optimal stopping problem. Let $\tau \in \mathcal{T}$ be a strategy (a stopping time) for the agent. The principal's problem is

$$\max_{y(\cdot), \tau} \mathbb{E}[v(\theta, y(\mathbf{h}), \tau)], \quad \text{subject to } \mathbb{E}[u(\theta, y(\mathbf{h}), \tau)] \geq \mathbb{E}[u(\theta, y(\mathbf{h}), \tau')] \quad \forall \tau' \in \mathcal{T}.$$

Analysis

In general, the set of strategies that can be adopted by the agent is complicated. However, we can show that it is without loss to focus on the "simple" class of stopping times where the agent chooses some time t, does nothing until time t, and stops iff a breakdown has not happened yet at time t. Let $U_t(y)$ be agent's expected utility from adopting this τ_t strategy, given policy y.

We can further show that this problem can be solved by a classic two-step approach: first solve for a policy rule that induces each simple stopping (we call this the inner problem), then verify that maximizing the principal's payoff across simple stopping times (we call this the outer problem) leads us to the solution to the original problem.



(a) Solutions to inner problems (b) Values of inner problems

Figure 1. The two-step procedure. Panel (a) shows the solution $y^{(t)}$ to the inner problem, for each t. Panel (b) shows the value of each inner problem.

Main results

1. For any $s \leq t$, $y(\mathbf{b}_s) = 0$
2. $y(\mathbf{b}_{t+1}) \geq y(\mathbf{b}_{t+2}) \geq \ldots \geq y(\mathbf{b}_T)$
3. $y(\mathbf{b}_{t+s}) = y(\mathbf{b}_{t+s+1})$ if and only if the constraint $U_t \geq U_{t+s}$ is slack
4. $u(1, y(\mathbf{a}), t) \geq u(0, y(\mathbf{b}_{t+1}), t)$
5. $\dfrac{v_y(0, y(\mathbf{b}_T), t)}{u_y(0, y(\mathbf{b}_T), t)} \leq \dfrac{v_y(1, y(\mathbf{a}), t)}{u_y(1, y(\mathbf{a}), t)}$, with equality if and only if the constraint $U_t \geq U_T$ is slack
6. If $\beta \equiv 1$, the agent receives zero rent in equilibrium. If $\beta(\cdot)$ is strictly decreasing, the agent receives positive rent.