Timing is Everything

Estimating strategic response using temporally aggregated data

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Social scientists often estimate how actors respond to each-other. However, data is usually aggregated temporally, into days, weeks, or years, when response occurs at higher frequency. How does temporal aggregation affect standard regression estimates? We show that studying interactions aggregated (or disaggregated) into predetermined intervals, rather than the actual response, can distort estimates, generating attenuation, amplification and even reverse signs. We provide analytic derivations, simple examples, and empirical Monte Carlo simulations. We conclude by examining how temporal aggregation can distort our understanding of the Israel-Gaza conflict from 2007 to 2017.

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And have no doubt — we will hold all those responsible to account at a time and in a manner [of] our choosing.

- Former President Biden discussing the United States' response to missile and drone attacks targeting United States military installations resulting in three casualties (2024).

1 Introduction

How can a researcher estimate a country's response to a recurring conflict (e.g. trade wars, cyber-attacks, rocket fire) using data which temporally aggregate actions into fixed time units (such as months)? Enders and Sandler (1993) provide one of the first applications using Vector Autoregressions (VAR), a flexible reduced form approach popularized within macroeconomics which relies on data recorded at fixed units (e.g., years, quarters or months). Researchers have since applied VAR to many strategic settings, including Israel/Palestinian conflicts (Jaeger and Paserman 2008; Haushofer, Biletzki, and Kanwisher 2010), politicians' campaign strategies (Box-Steffensmeier, Darmofal, and Farrell 2009), political-media interactions (Barberá et al. 2019), and most recently Myanmar state violence (Davis, Paula Lopez-Pena, and Wen 2023). In these settings players choose both *how* and *when* to respond to each other. That results in irregularly spaced responses which are oftentimes aggregated to some arbitrary time interval, like a day. Yet in a single day multiple responses and counter-responses could occur –so that aggregation might conflate them.

A useful distinction in this setting is between *calendar time* –measured in units of fixed duration (e.g., days) and *action time*, which records when actions occur, their magnitude, and duration. For example, an evening action might respond to an afternoon action, but last past midnight. We show that temporally aggregating such a sequence into daily data can cause misleading point estimates and distorted inference in VAR analyses.

We first motivate our findings using data recorded at high frequency (five-minute intervals) from the Israeli-Gaza conflict between 2007 and 2017. The conflict during this period is best described as *partial deterrence*: Israelis and Gazans launch violent attacks at one another, sometimes many times daily, but do not reach the level of war seen following October 7^{th}, 2023. We highlight three facts from the data: i) sides tend to respond to attacks within a day, ii) violence tends to last only a few days with lulls of quiet between, and iii) sides vary their response time. We then estimate reduced form VARs and impulse responses functions (IRFs) at the daily level following Jaeger and Paserman (2008). That exercise reproduces results like those of Jaeger and Passerman on the Second Intifada period: our VAR and IRF analysis suggests long, drawn out responses lasting nearly 40 days –in contrast to fact (ii).

Why does a VAR and IRF analysis produce prolonged responses when the data suggests a conflict defined by spats of violence between days of calm? We argue this can occur because of a temporally aggregated unit of observation. Strategic interactions can be thought of as sequential games, where each player performs actions (e.g. airstrikes, mortar fire) resulting in damages (e.g., casualties, property damage).[4] Studying the problem at a predefined temporal level (e.g., day or week) can cause standard VAR modeling to estimate prolonged lag lengths, misleading point estimates and empirical responses. This distortion can result from actors strategically waiting different amounts of time to respond, or from actions spanning temporal units (e.g., multiple days). For example, an airstrike is nearly instantaneous, but an incursion or barrage of rocket fire can last days.

[4] Maskin and Tirole (1988) make a similar argument when modeling pricing strategies.

Put simply, aggregating (and disaggregating) the timing of behavior can create severe misspecification in VAR analysis. We analytically show that it can cause VAR coefficients to be attenuated, inflated, or even have the wrong sign compared to an action level analysis.

We next develop a Monte Carlo simulation to examine the extent of distortion in data one might use. First, we generate data at the action level (e.g., action time), in which each side only reacts to the previous action (a Markovian game). We then aggregate that data into fixed time units observation (e.g., calendar time) and perform standard VAR and IRF analyses. VAR applied to that temporally aggregated data fails to estimate the parameters of the action-level parameters. The optimal Bayesian Information Criterion (BIC) lag length tends to equal the average response time between actions. The estimated impulse response functions also produce statistically significant effects for many periods into the future. Finally, point estimates are severely distorted compared to the action-level parameters, in magnitude and statistical significance.

We then revisit the Israeli-Gazan conflict and find signs that temporal aggregation distortions may be present and driven by multi-day actions. There is little evidence that strategic waiting is driving temporal distortions in this setting. The average time between actions is about 11 hours in our action time specification. However, there are multiple instances of Israeli and Gazan performing multi-day actions. These longer actions contribute to the prolonged impulse responses, and potential temporal aggregation distortions. Using a Markovian response function in action time, we can recreate daily Israeli impulse response functions that remain statistically significant at the 5% level over 10 days after the initial shock. These results suggest that temporally aggregated units of observation may have contributed to previously drawn conclusions about the Israel-Gaza conflict.

This paper is the first to study how strategic behavior and temporally aggregated observations may affect our understanding of the Israel-Gaza conflict. Previous work assumed a daily unit of observation (Jaeger and Paserman 2008), then proceeded to study different damage metrics (Haushofer, Biletzki, and Kanwisher 2010)[5] and functional form (Asali, Abu-Qarn, and Beenstock 2017). We can go a step further and study how the temporal unit of observation affects empirical findings, using our high frequency data on the conflict.

Our findings extend the temporal aggregation literature to the strategic setting. Previous econometric works focused on temporal aggregation distortions in nonstrategic settings, such as the association between economic indicators (Working 1960; Zellner and Montmarquette 1971; Silvestrini and Veredas 2008; Brewer 1973; Tiao and Wei 1976; Geweke 1978; Wei 1978; Freeman 1989; Marcellino 1999; Jordá 1999). We extend this work by studying how temporal aggregation affects estimates of strategic behavior with observational data. Given researchers' increasing access to high frequency data, we expect our findings to highlight an ever-growing issue in estimation.

The remainder of the paper is organized as follows: Section 2 motivates the problem. Section 3 provides the economic setting and Section 4 analytically derives potential distortions from analyzing actions in time intervals. Section 5 presents a Monte Carlo simulation. Section 6 investigates how temporal aggregation may affect understanding the Israeli-Gaza conflict. Section 7 concludes.

2 Motivating example

We first motivate our problem using data from the Israeli-Gaza conflict. Between 2007 and 2017, the United Nations recorded casualties and munitions launched across the Israeli-Gazan border at the five-minute level (Berman et al. 2024). We refer to one of the five-

[5] See Golan and Rosenblatt (2011) for a comment on this work.

minute reports as an *attack*. This includes injuries and fatalities on both sides. It also includes Israeli airstrikes, shellings, small arms fire, and incursions, and Gazan mortars, qassams, and small arms fire.

Figure 1 plots the number of Israeli and Gazan attacks per day. The magnified portion highlights December 2013, a randomly chosen month within the data.

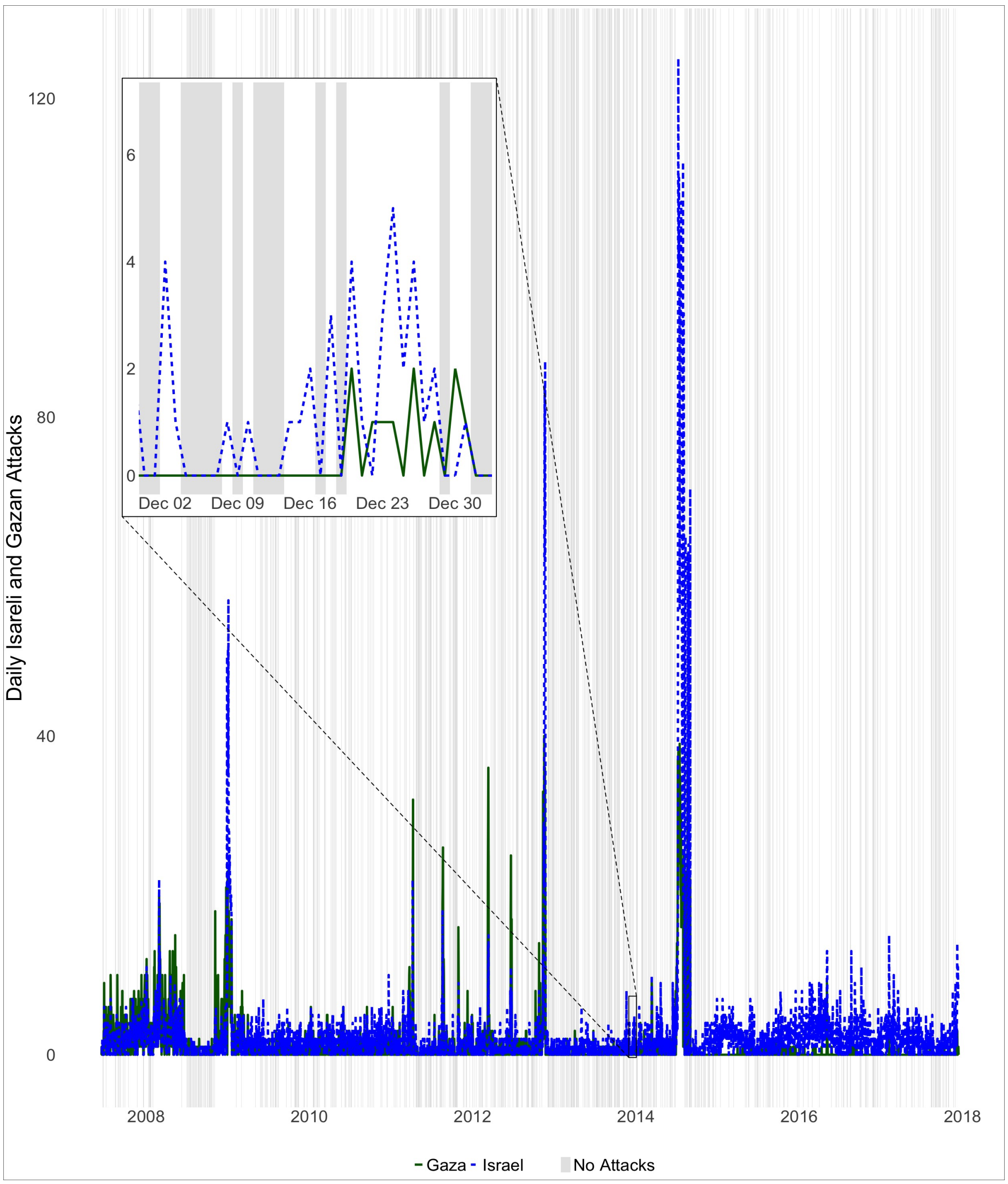

Figure 1 Israeli and Gazan attacks per day

The daily data depicts prolonged violence with interspersed lulls of quiet. The grey bars highlight days in which neither Israel nor Gaza attacked. There were no attacks during about a quarter of the days in the sample.

There are three takeaways from the graph. First, sides tend to respond to attacks within the same day. Over the full sample period, there were about 3.97 attacks per day, with no attacks on 24.5% of days and only one attack on 23.7%. There were on average 5.26 attacks per day, conditional on an attack occurring.

Second, violence is interspersed between periods of calm. When an attack occurred, the violence ended within the day about half the time, followed by at least one day of calm. The median length of time preceding at least one day of calm was 0.8 days, which typically included three attacks. The longest stretch of continuous violence lasted 78 days; the second longest was 54 days; the longest stretch of quiet was just under 12 days.

Third, the graph depicts both sides varying their reaction times. Israel reacted to the previous attack on average after 40 minutes while Gazan militants responded after about 50 minutes on average. However, response times vary considerably: the Israeli response standard deviation is 0.62 days, and the Gazan is 0.49 days. Both sides reacted within a minute about 0.2% of the time, and Israel took longer than a day to respond 8% of the time. The Gazans did so 5% of the time.

These takeaways highlight two challenges when using observational conflict data. First, daily data masks strategic behavior within days. More than one attack occurs over half the days in our sample, and nearly a quarter of the days have more than three attacks. Second, each side varies their response time. Israel and Gaza's response time both have large standard deviations, compared to their means. Statistical approaches that ignore this variation effectively average over the strategic response time.

We next estimate impulse response functions employing the methods from Jaeger and Paserman (2008) using data from Berman et al. (2024). Following their work, we first temporally aggregate to the daily level. While Jaeger and Paserman (2008) focused their analysis on fatalities, follow-up work identified strategic behavior studying fatalities and projectiles (Haushofer, Biletzki, and Kanwisher 2010). Therefore, we combine casualties and munitions into one index by regressing *fatalities* $+ 0.8 \times$ *injuries* on munitions (with no intercept) to create Israeli and Gazan expected damage metrics. Intuitively, the fitted values capture each side's intended damage, measured in casualties.

We then calculate the orthogonal impulse response function from a VAR with 14 lags.[6] Figure 2 plots the empirical response functions for Israel and Gaza with 95% confidence intervals. Each response maps 60 days after an initial attack. Israel appears to inflict damage above the mean for nearly 30 days following a Gazan attack. Conversely, the Gazan response is far more muted than the Israelis, with less damage and a shorter attack duration. In the online appendix, we show that damage to Israel (weakly) Granger causes damage to Gaza, but not the converse.

[6] VAR(14) has become the standard model specification for the Israel-Gaza conflict (Jaeger and Paserman 2008; Asali, Abu-Qarn, and Beenstock 2024). See the appendix for the VAR estimates, and alternative outcome and model specifications.

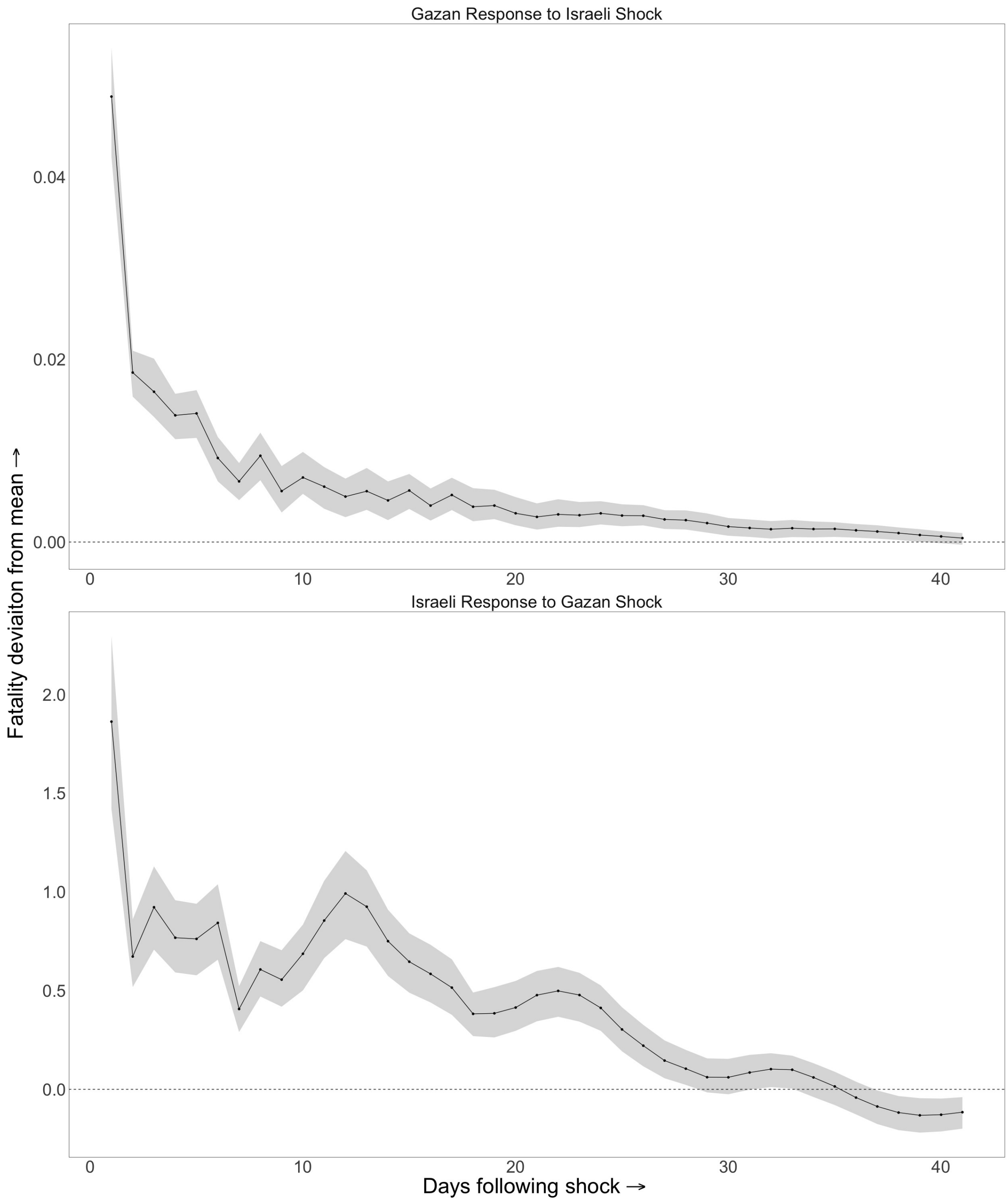

Figure 2 Daily-level impulse response functions for Expected Fatalities

Taken at face value, the empirical response functions and accompanying VAR analysis suggest a long duration of response to a single attack, 30 or 40 days. Yet, 99% of violent episodes in the dataset last less than 30 days. On average, violence lasted 0.4 days before at least one day of calm. In the remainder of the paper, we investigate how strategic response time and temporal aggregation may cause VAR and impulse response functions to produce findings that appear inconsistent with features of the underlying data.

3 Setup

Section 3.1 introduces and extends the Berman et al. (2024) economic framework to repeated conflict and then maps it to estimating equations. Section 3.2 sets up challenges of studying conflict when the data is collected in predetermined intervals.

3.1 Economic framework

Berman et al. (2024) assumes players A and B are competing in a two-sided sequential game. For every turn $i = 1, \dots, I$, players A and B alternate receiving damage, denoted as d_i^A and d_i^B respectively. We refer to data organized by action as being in *action time*.[7] This records damage inflicted per action, rather than traditional time (e.g., minutes, days, years). There can be a millisecond, day, or even a year between actions, and duration varies from action to action.

Responses are based on damage suffered: player A inflicts $d_i^B = R^A(d_{i-1}^A)$ on player B, and player B inflicts $d_i^A = R^B(d_{i-1}^B)$ damage on player A. We model both player's response curves as linear dependent only on the previous action:[8]

[7] Engle and Russell (2004) refer to this as "event-space".
[8] While we focus our attention on an AR(1) response curve, the framework can accommodate more lags, higher order polynomials, and moving averages. The AR(1) specification translates to a Markov one process mixed strategy employed in many sequential games, a common modeling choice outside conflict analysis (e.g., Noel 2007).

$$R^A\left(d_i^A\right) = d_i^B = \alpha_0^A + \alpha_1^A d_{i-1}^A + \epsilon_i \quad (1)$$

$$R^B(d_i^B) = d_i^A = \alpha_0^B + \alpha_1^B d_{i-1}^B + \epsilon_i \quad (2)$$

where ϵ_i is conditional mean zero with variance σ_i^2.

This approach to conflict provides unique escalation and deescalation estimation and hypothesis testing. A sequence of responses and counter-responses de-escalates if $R^B\left(R^A(d^A)\right) < d^A$ and $R^A\left(R^B(d^B)\right) < d^A$, at a point (d^A, d^B). Researchers can study conflict stability, multiple equilibrium, and changes in strategy with this approach.

The game's sequential nature allows us to combine the two reaction curves into one using a player indicator. Let $z_i = I(Player\ A\ moves\ )$. Then equations (1) and (2) can be equivalently written as:

$$R(d_i|d_{i-1}, z_i) = d_i = \alpha_0^B + (\alpha_0^A - \alpha_0^B)z_i + \alpha_1^B d_{i-1} + (\alpha_1^A - \alpha_1^B)d_{i-1}z_i + \epsilon_i \quad (3)$$

Leaders consider both *how* and *when* to attack. Timing therefore influences the decision process as well. Wait too long, and the antagonist may misinterpret the protagonist's response for the beginning of a new conflict. Respond too predictably, and the antagonist can parry the protagonist's response.

Based on this observation, we allow each player to also choose the duration between the end of current and previous actions, denoted $w_i \ \geq \ 0$. Let $f_w(w_i|d_i, z_i)$ be the distribution between the ending of d_{i-1} and d_i. Like before, we model Player A and Player B's duration as one function with an indicator.

The response curve and duration distribution combine to form the joint action distribution. We write the joint distribution using matrix notation, which will be used in following sections:

$$f(D, W|Z) = f_w(W|D, Z)R(D|Z)$$

$R(D|Z) = X_I\alpha + \epsilon$

$X_I = [1 \;\; L_I D \;\; Z \;\; diag(Z)L_I D]_{I \times 4}$

$\alpha' = [\alpha_0^B \;\; (\alpha_0^A - \alpha_0^B) \;\; \alpha_1^B \;\; (\alpha_1^A - \alpha_1^B)]_{1 \times 4}$

$E[X_I X_I'] < \infty$

$E[\epsilon|Z, L_I D] = 0$

$E[\epsilon^2|Z, L_I D] = \Sigma$

where *D*, *W*, and *Z* are the respective vectors, L_I is the lag matrix in action time,[9] $diag(\cdot)$ is a diagonal matrix with vector · along the diagonal, and Σ is a diagonal matrix.

This general framework follows a marked process, first popularized within financial econometrics by Engle and Russell (1998) and Engle (2000). The specification allows us to clearly highlight distortion caused from studying conflict at predetermined intervals.

3.2 Actions recorded in calendar time

Assume that a researcher observes damage to Players A and B at a specified time interval (e.g. days), which we call *calendar time*. Examples of calendar time include daily, weekly, and monthly data. Let $\mathrm{d}_\mathrm{t}^\mathrm{A}$ and $\mathrm{d}_\mathrm{t}^\mathrm{B}$ denote the damage that the researcher observes at period *t* = 1,...,*T*. As before, let D^A and D^B denote vectors with $\mathrm{d}_\mathrm{t}^\mathrm{A}$ and $\mathrm{d}_\mathrm{t}^\mathrm{B}$ the t^{th} element. Assume that the researcher observes data at the same time-unit (e.g., a day) at which the data is aggregated. action time maps to calendar time by adding all the actions within that day together (mirroring Jordá (1999) and Jordá and Marcellino (2000)):

[9] For those less familiar with the lag operator, $L_i d_i = d_{i-i}$ for any variable.

$$d_t^A = \sum_i h_i \left(\left[\sum_{j=1}^{i} w_j \right] \in [t, t+1] \right) I(z_i = 1) d_i$$

(8)

$$d_t^B = \sum_i h_i \left(\left[\sum_{j=1}^{i} w_j \right] \in [t, t+1] \right) I(z_i \neq 1) d_i$$

The h_i function captures how an action is distributed over days. If the action is instantaneous, such as an airstrike, then $h_i()$ is the indicator function, equal to one the day the action occurred and zero else. The action may be a prolong incursion, barrage or shelling, in which case the action is dispersed across days.

Equation 8 can be rewritten in matrix notation. Let P be a $\mathrm{T} \times \mathrm{I}$ aggregation matrix with $p_{t,i}$ the [*t,i*] element of *P*. $\mathrm{p}_{t,i}$ captures the percent of action *i* that occurred on day *t*. The columns of P sum to one because every action is fully mapped to the days.

Additionally, let 1 be a vector of ones, and *diag*(*Z*) is a square matrix with the Z vector along the diagonal. Then

$$\mathrm{D}^{\mathrm{A}} = Pdiag(Z)D$$

and

$$\mathrm{D}^{\mathrm{B}} = Pdiag(1 - Z)D.$$

We provide a simple example in the appendix.

4 Challenges to VAR in calendar time

A *k* order reduced-form vector autoregression is commonly applied to data in calendar time. The first equation in this example is:

$$d_t^A = \beta_0^A + \sum_{j=1}^{k}\left\{\beta_{A,j}^A d_{t-j}^A + \beta_{B,j}^A d_{t-j}^B\right\} + \eta_t \tag{9}$$

In matrix form:

$$D^A = X_T\beta^A + \eta \tag{10}$$

where X_T is the design matrix. X_T can be rewritten using action time observations and the aggregation matrix as:

$$X_T = [1\ D_{-1}^A \dots\ D_{-k}^A,\ \ D_{-1}^B \dots\ D_{-k}^B]$$

Where $D_{-k}^A = L_T^k D$, and L_T is the lag matrix in time-space. In words, the first column of X is the intercept, the next *k* columns are lags of player A's actions 1 to *k* days before, and the final *k* columns are player B's actions 1 to *k* days before.

How does β^A relate to the reaction function parameters, α? Plugging Equation (2) into the standard regression formula and taking conditional expectations yields:

$$\beta^A = E[(X_T'X_T)]^{-1}E[X_T'D^A]$$

$$= E[(X_T'X_T)]^{-1}E[X_T'Pdiag(Z)X_I]\alpha$$

Where

$$Pdiag(Z)X_I = [Pdiag(Z)1\ \ Pdiag(Z)L_ID\ \ Pdiag(Z)Z\ \ Pdiag(Z)L_ID]$$

For convenience, let $\Upsilon = E[(X_T'X_T)]^{-1}E[X_TPdiag(Z)X_I]$, and $\upsilon_{t,k}$ be the [t,k] element of υ. β^A equals α if $Pdiag(Z)X_I = X_T$. Notice that β^A equals α when X_T is a timemanipulation of X_I during A's turn. Examples of this include manipulating the data between monthly, quarterly, and yearly units of analyses (Zellner and Montmarquette 1971).

This equality is unlikely to hold in most strategic setting because the action-time response curve may not follow the VAR timing of lags. In our setting, α is four-dimensional while β^A is $2k + 1$. Rather than mapping directly to the reaction curve parameters, the VAR parameters are a linear combination of α scaled by Υ.

We'll focus for a moment on the intercept to solidify ideas. Let ι_I be an *I*-length column of ones, the first column in X_I. Similarly, let ι_T be a *T* length column of ones, the first column in X_T . If $\iota_T = Pdiag(Z)\iota_I$, then $(X_T' X_T)^{-1}X_T' \iota_T$ equals a *T* length vector where the first element of the vector equals one and all else are zero. Only β_0 is a function of α_0 because all the other elements are zero. Suppose instead that player A performs γ actions each period. Mathematically, $\iota_T = \gamma\iota_I$. The action-space intercept still only contributes to the time-space intercept because only the first vector's element is nonzero. However, $\beta_0^A = \gamma\alpha_0$ to account for the multiple actions per period.

Suppose instead that $\iota_T \neq \gamma\iota_I$. This occurs when side A performs a different number of actions each day. Now, $Pdiag(Z)\iota_I$ is no longer guaranteed to be a column of X_T. The first element of $(X_T' X_T)^{-1}X_T' Pdiag(Z)\iota_I$ need not equal one, nor do the other elements need not equal zero. Every element of β^A can be some linear combination of α_0 since none of the $(X_T' X_T)^{-1}X_T' Pdiag(Z)\iota_I$ elements are guaranteed to equal zero. All the time-space coefficients may be a function of the action-space intercept and number of actions per day because of Player A's strategic responses.

4.1 Illustration

In this section, we illustrate how VAR analyses using calendar time data may be misleading. We assume that each side's strategic response follows an exponential distribution, both sides react symmetrically to one another, and each action occurs instantaneously:

$$w_i | d_i, z_i \sim \exp(1)$$

$$d_i | z_i, d_{i-1} \sim N(\alpha_0 + \alpha_1 d_{i-1}, \Sigma)$$

$$h_i(*) = I(*)$$

where Σ is a diagonal matrix. In this simple example, each side has the same waiting and response function with an idiosyncratic error. We drop the α superscripts for ease of reading.

The econometrician observes data at the calendar time, as in Equation 8. They then estimate the VAR following Equation (10) with $p = 5$. This guarantees that each side's reaction is always captured in the VAR lag structure. We present VAR estimates for side A below:

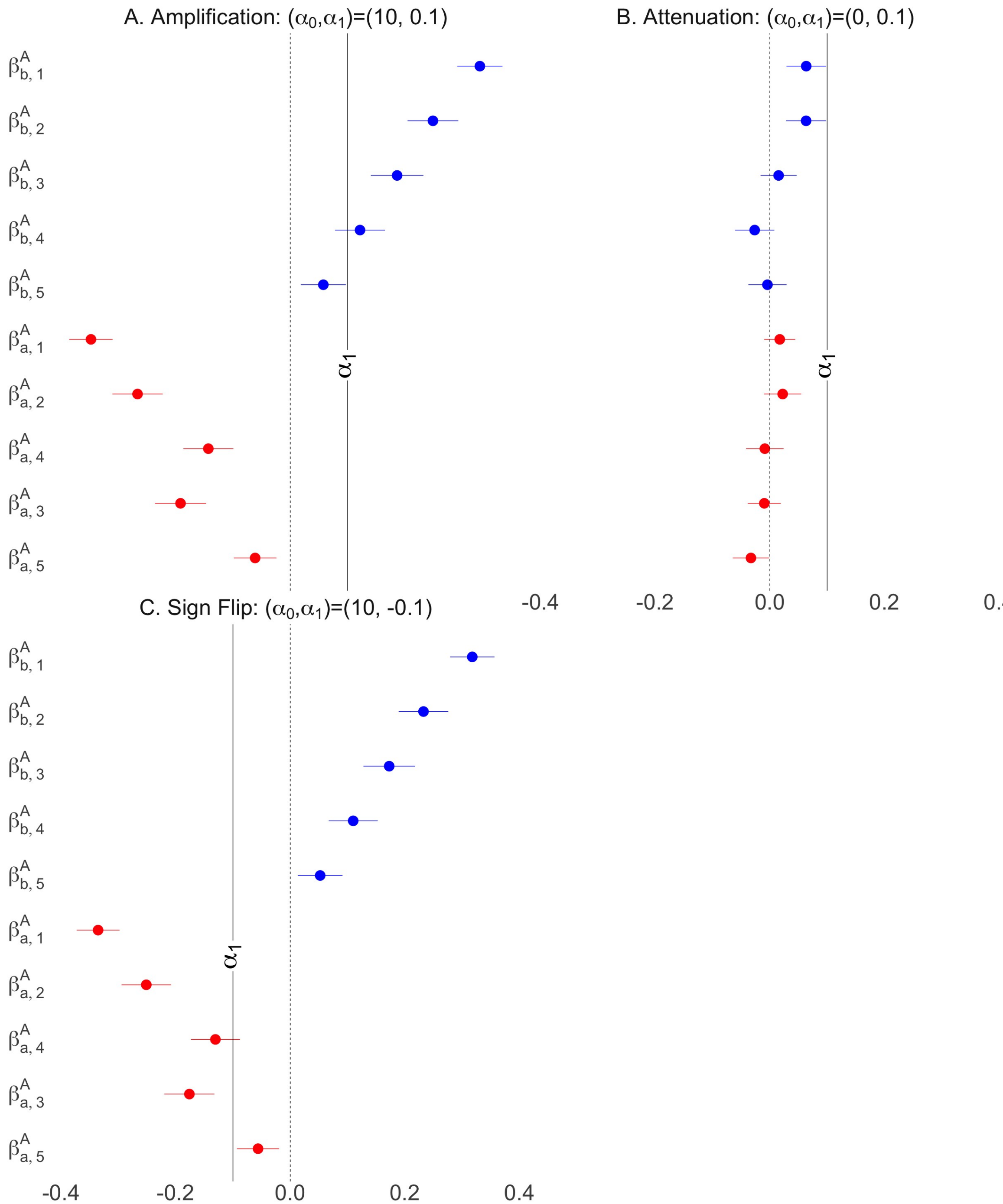

Figure 3 Player A VAR slope estimates and 95% CI under different action time response curves.

Figure 3 plots VAR estimates under three sets of parameters for the action time response function, (α_0, α_1). The three panels highlight three potentially misleading findings from a calendar time VAR analysis. In Panel A, the first three lagged estimates for side B are statistically larger than the action-level slope coefficient, $\alpha_1 = 0.1$. We refer to this as *amplification*. Panel B shows the opposite effect: the VAR coefficients are all smaller than the true reaction curve slope (again $\alpha_1 = 0.1$). We refer to this as *attenuation*. Finally; Panel C shows an example where the time-space slope coefficients for side B are all positive. This would imply that side A escalates a conflict. However, the true reaction curve slope coefficient is negative, $\alpha_1 = -0.1$, which is a de-escalatory position. We refer to this as *sign flippage*. We provide additional breakdowns and intuition in the appendix.

5 Simulations

We develop a simulation to study how calendar time may distort VAR estimates using realistic data. Section 4.1 introduces the simulation setup and Section 4.2 showcases how a VAR analysis may lead to erroneous conclusions when behavior occurs in action time.

5.1 Simulation setup

We first provide functional forms to duration and response functions. The response follows a normal distribution, and the waiting follows an exponential distribution:

$$d_i|z_i, d_{i-1} \sim N(\alpha_0^B + \alpha_1^B d_{i-1} + (\alpha_0^A - \alpha_0^B) z_{i-1} + (\alpha_1^A - \alpha_1^B) z_i d_{i-1}, \sigma^2)$$
$$w_i|d_i, z_i \sim \exp\left(\frac{1}{\lambda}\right)$$
$$h(*) = I(*)$$

where *h*() = *I*() implies that each action is instantaneous. In practice, the optimal number of lags may be more than one and can be empirically tested. We limit our simulation to one lag to clearly isolate the effects of temporal aggregation. We then estimate the simulations using the following parameters: $\{\alpha_0^B, \alpha_1^B, \alpha_0^A, \alpha_1^A, \sigma^2\} =$ $\{0.0056, 0.1354, 0.0078, -0.7404, 0.1\}$. We generate 15,000 actions per simulation.

Multiple functional forms have been proposed to model the time between financial transactions (Pacurar 2008). We opt to use a basic exponential distribution with a constant parameter λ. While an abstraction from reality, the modeling assumption allows us to concisely isolate the effect of shorter or longer wait times. We also assume that actions occur instantaneously.

Finally, the action time data is transformed to calendar time data following Equation (8). We evaluate VAR performance on three metrics:

Optimal VAR lag length: We determine optimal lag length using the Bayesian Information Criterion assuming a constant optimal lag length for both sides per simulation run. We set $\lambda \in \{.5,1,2,\ldots,7\}$. Intuitively, we allow the average response time to vary between a half day, then from a day to a week.

VAR coefficient and significance: Based on the optimal lag length, we estimate a VAR at the daily level following Equation (10). We then report the coefficients and standard errors. We set $\lambda = 7,1,\frac{1}{2}$, corresponding to sides waiting, on average, one week, one day, and half a day between actions.

Impulse Response: We plot the empirical impulse responses for each side three weeks after an initial shock with 95% confidence intervals. We set $\lambda = 7,1,\frac{1}{2}$, corresponding to sides waiting, an average, one week, one day, and half a day between actions.

We repeat this exercise 1,000 times for each λ value for 6,000 total simulation runs.

5.2 VAR findings

We first investigate how a VAR performs in this setting using Equation (10) by estimating the optimal lag length based on the BIC criterion, the coefficient estimates, and orthogonal impulse responses.

Table 1 shows the BIC-optimal lag lengths compared to varying values of λ. As the average response time increases, the optimal lag length increases, nearly one-to-one. The variance also increases as the average response time increases. This is a biproduct of the exponential variance equals the mean.

Table 1 Optimal BIC over Monte Carlo Simulations with varying lag length.

Response Time	Average BIC	Minimum BIC	Median BIC	Maximum BIC
0.5	1.0	1	1	1
1.0	1.9	1	2	2
2.0	3.1	2	3	4
3.0	4.2	3	4	6
4.0	5.2	4	5	7
5.0	6.0	5	6	8
6.0	6.8	5	7	9
7.0	7.5	6	8	10

Note: The simulation is repeated 1,000 per response time.

Even though each side responds to the previous action, each VAR coefficient estimate is a linear combination of the underlying action time parameters. Longer waiting periods cause larger scalars for each action time coefficient leading to a BIC suggesting more lags.

Figure 4 plots the percent of simulations for each lagged VAR coefficient which are statistically significant at the 5% level. The left-hand side plots the coefficients when the outcome is Player A's action, while the right-hand side plots coefficients for player B's action. Each row showcases a different simulation specification. The top row assumes each side waits, on average half a day before responding, the middle assumes a full day, and the bottom assumes a full week.

Each dot represents the percent of lagged coefficients statistically significant at the 5% level. For example, the second red triangle in the top left-hand panel indicates that Player B's action two days ago was statistically associated with Player A's action today for 100% of the simulations when the wait time was a day.

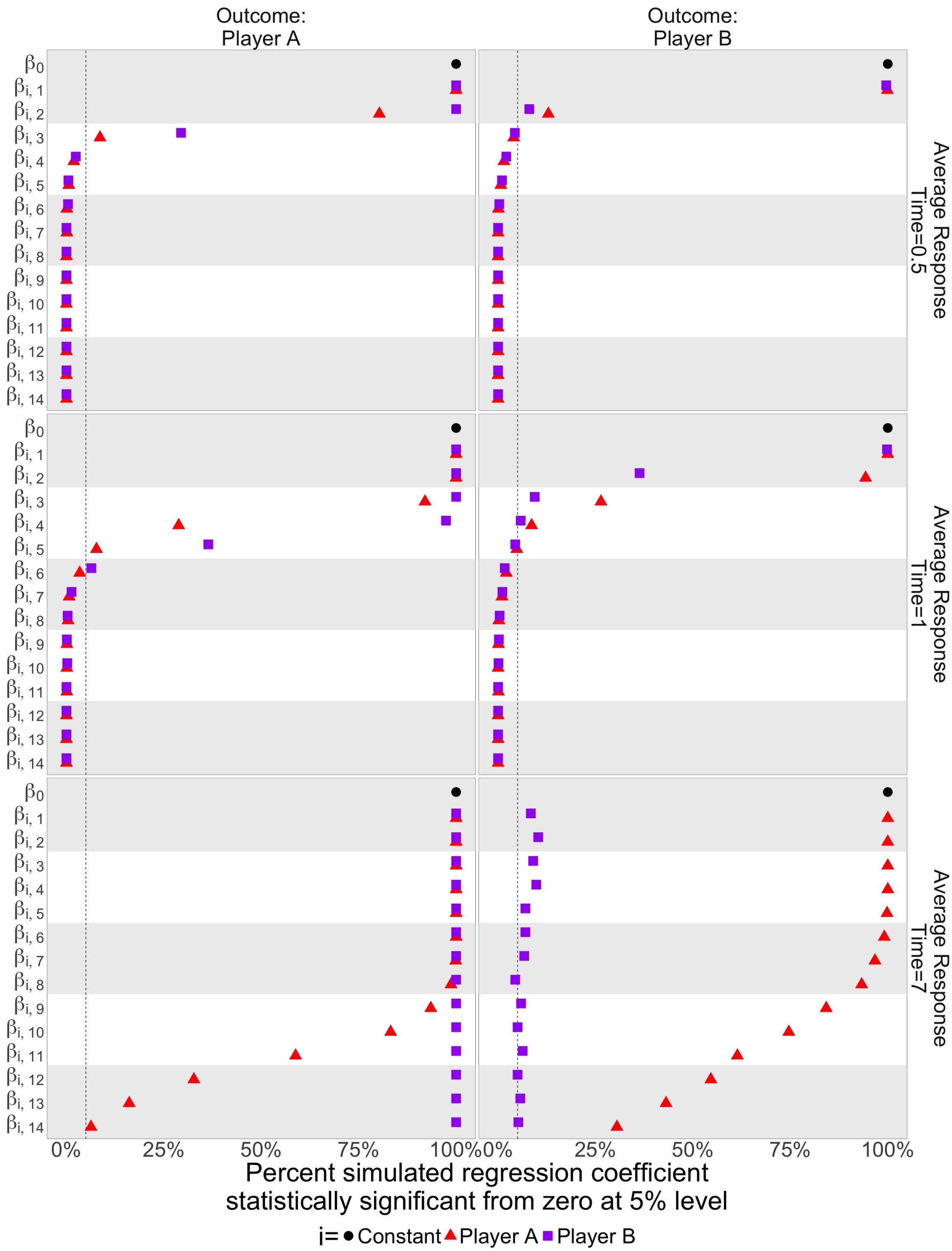

Figure 4 VAR coefficients over 1,000 Monte Carlo Simulations for players A and B.

The wait time, *W*, affects the percent of statistically significant lags. When the wait time is less than a day, only the first few lags are statistically significant for most of the simulations. As the wait time increases, the number of statistically significant lags increase. When the wait time is on average seven days, coefficients on the opposite side's previous days are statistically significant over the past two weeks. Moreover, the own-lagged values are also statistically significant more than 5% of the simulations eight lags back. This highlights how calendar time VAR coefficients are a linear combination of the action time parameters.

Finally, Figure 5 plots orthogonal impulse response functions over 1,000 Monte Carlo simulations. The top left-hand panel maps Player A's response to a shock to Player B's action 21 days after the shock when the average response time is $\frac{1}{2}$, one, five, and seven. The bottom three panels present the same for A's response to a shock to B.

The first period following a shock is statistically different from zero for all waiting periods for both A and B's empirical responses using 95% confidence intervals. The empirical response effects quickly die out when the response time is one day or less on average. Less than 5% of the simulations identified a statistically significant effect at the 5% level four periods after the shock or after. Assuming an average wait time of five days, the A empirical response is statistically significant at the 5% level for 16 of the time periods after the shock. The B player empirical response is statistically significant at the 5% level more for 18 periods after the shock. The impulse response statistically differs from zero more than 5% of the time for every lag studied when the wait time is increased to seven days on average.

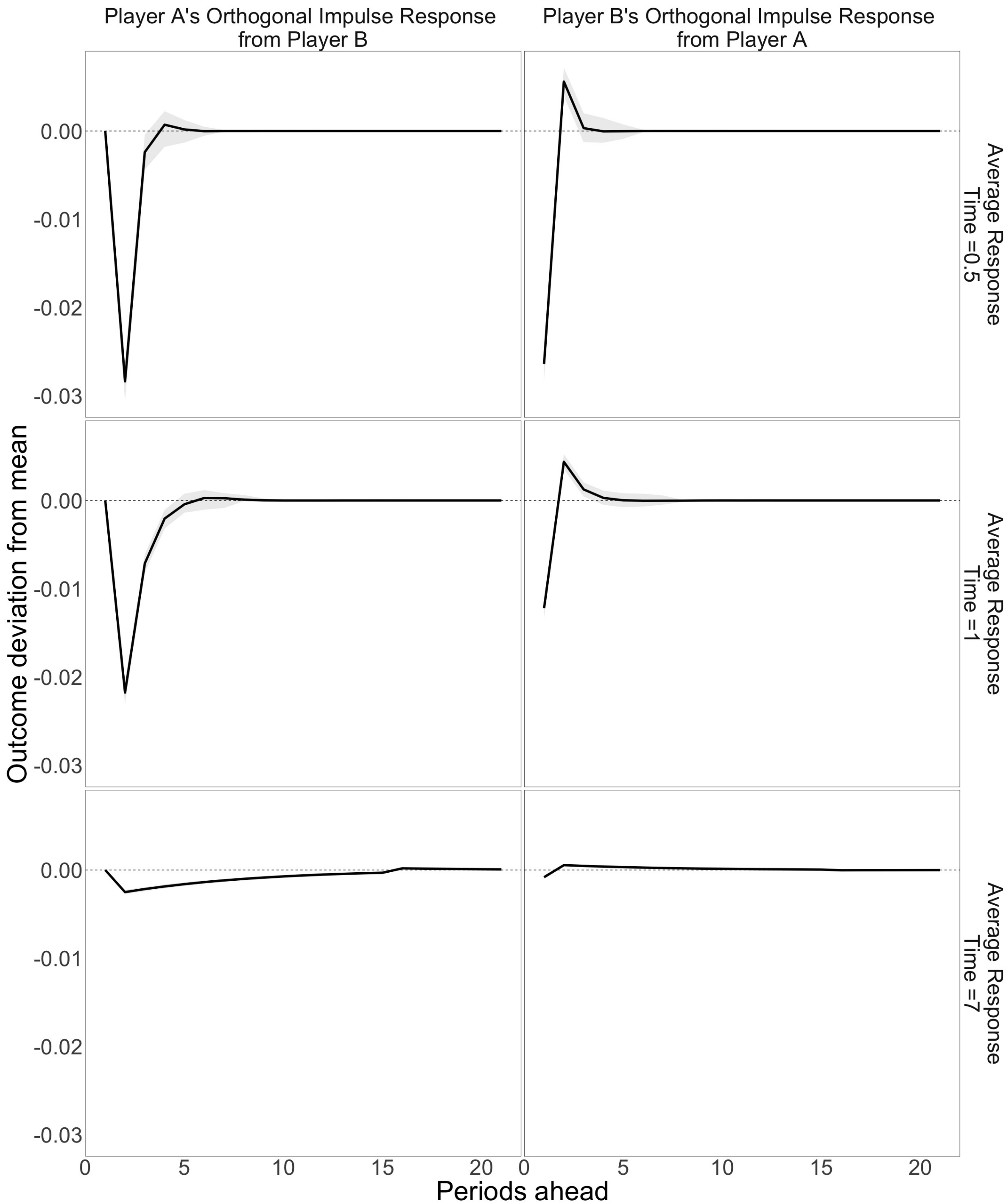

Figure 5 Simulated Orthogonal Impulse Response Function.

The simulations highlight potential challenges to analyzing calendar time data. Standard regression tools can suggest prolonged responses even when the underlying action time responses resemble a tit-for-tat strategy, with only immediate response.

6 Empirical example: Temporal aggregation and the Israeli-Gaza Conflict from 2007-2017

How much of the long impulse response functions presented in Section 2 can be explained exclusively by temporal aggregation? Using the five-minute report data from the Israeli-Gaza conflict, we first create action time data. We then estimate R following a Markov one process and calculate the P matrix from the data. Daily level data is simulated 1,000 times from empirically estimated reaction curve R and P, and VAR/IRF analysis is performed on each dataset.

We vary P to include all actions, only actions that occurred within one day, and only actions that occur over multiple days. Simulating over all actions allows us to investigate how data aggregation contributes to the estimated prolonged responses. Simulating only daily actions with varying wait times and multi-day actions with a daily response helps us identify which temporal aggregate contributes more to prolonged impulse responses in this setting.

6.1 Setup

We convert five-minute attack reports into actions using a simplified version of the Berman et al. (2024) approach. The authors use action time data to estimate Israeli and Gazan response curves from 2007 to 2017, omitting the Cast Lead, Pillar of Defense, and

Protective Edge major operations. They transform the UN-reported five-minute attack data into a sequence of actions based on time between attacks, munitions used, and location.

Attacks are organized in chronological order, ignoring location. An *action* is a sequence of attacks by one side i) uninterrupted by the other side's attack and ii) with less than 48 hours between attacks. These coding rules assume that both sides perform coordinated attacks over time and across munitions. They also assume that both sides have the capabilities to respond within two days to the other side's action.[10] Finally, we add two actions where no damage occurs, which we refer to as *zero damage action*, for every 48 hours of calm. This captures each side's choice *not* to respond.

These transformations create a dataset where the unit of observation is an action. The dataset includes an indicator if the Israelis executed the action, the expected damage as described in Section 2, the days an action occurred, and what percent of the action occurred each day.

Table 2 presents action-level statistics for Israel and Gaza. Because of the sequential setup, Israel performs 4,451 actions while Gaza performs 4,450 for a total of 8,901 actions. Both sides perform zero damage actions between a fifth to a fourth of the total actions. Both sides take just under half a day to respond on average and responded after at least a day under 10% of the time. Israeli actions lasted just over half a day on average, while Gazan actions tend to last just under a day. Finally, Israeli actions tended to cause more damage. The average Israeli action caused 0.521 expected damage (measured in *fatalities* + $.8 \times$ *casualties*), while the Gazans cause 0.0216 in expected damage.

[10] See Berman et al. (2024) for more discussion.

Table 2 Action-level summary statistics

Variable	Gaza	Israel
Number of Actions	2,952	2,953
Proportion Zero Damage Actions	0.36	0.30
Average Days Between Actions	0.65	0.65
Proportion Days Between Actions Longer than One Day	0.10	0.10
Average Length of Actions (days)	0.53	0.88
Average Number of Attacks per Action	1.30	2.35
Proportion Actions Longer than One Day	0.11	0.16
Predicted Damage	0.03	0.21

Next, we approximate the expected damage conditional expectation function following Equation 3:

$$R(d_i|d_{i-1}, z_i) = d_i = \alpha_0^G + (\alpha_0^I - \alpha_0^G)z_i + \alpha_1^G d_{i-1} + (\alpha_1^I - \alpha_1^G)d_{i-1}z_i + \epsilon_i$$

where $z_i = I(Israel\ moves\ on\ turn\ i)$. We limit our estimation to a single lag linear regression to isolate the effects of potential time aggregation.

Table 3 presents the results. Both Israel and Gaza have positive intercept and positive slopes. The Israelis appear to respond to Gazan attacks, while we fail to find evidence the Gazans respond linearly to the Israelis.

We next calculate the P matrix directly from the data. We identify how many days occur before each action, and how many days an action span. We calculate $p_{i,t}$ as the percent of action *i*'s damage on day *t* divided by the total damage. For example, if action i occurred over days $j, j + 1$ and $j + 2$ causing 0.1, 0.2, and 0.3 expected damage, then $\{p_{i,j}, p_{i,j+1}, p_{i,j+2}\} = \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}\right\}$.

Together, we can calculate the daily level time series as:

$$D^I = P diag(Z) D$$

$$D^G = P(I - diag(Z))D$$

where D^I is a vector of daily level predict damage to Gaza (caused by Israel) and D^G is daily level predicted damage to Israel (caused by Gaza).

Table 3 Estimated Israeli-Gaza Response Curves

	(1)	(2)	(3)
α_0^G	0.034*** (0.001)	0.031*** (0.001)	0.034*** (0.001)
α_1^G		0.014*** (0.004)	0.003 (0.002)
α_0^I	0.214*** (0.008)	0.213*** (0.008)	0.191*** (0.009)
α_1^I		0.014*** (0.004)	0.634*** (0.148)
Observations	5905	5901	5901
Standard Errors	HAC	HAC	HAC

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

Notes: outcome is expected damage. Fitted values from regressing deaths + .8 x fatalities on munitions type. We drop the first action in the dataset and the first action following each major operation.

6.2 Simulation

We simulate 3,000 daily level datasets using the model described above. Algorithm 1 summarizes the steps.

Algorithm 1: Simulating Israeli-Gazan violence.

Result: VAR and IRF simulation estimates Using action level data; **for** *i in (all actions, daily actions, and multi-day actions with daily waits)* **do**

for $s = 1,2, \ldots ,1000$ **do**

Simulate D^s using column 3 in Table 3 for 8,901 actions. ϵ_i is drawn from the empirical residuals;

Simulate P^s by drawing 8,901 actions with replacement following *i*;

Generate $D^{I,s} = Psdiag(Z)D^s \, and \, D^{G,s} = P^s(I - diag(Z))D^s$;

Estimate optimal lag length VAR (up to 14 lags) and IRF 25 periods after the initial shock;

Save results

end

end

The data is first generated at the action level. Damage is generated using Table 3, column 3. We then draw strategic behavior, which includes the wait time before an action begins, how many days it occurs and the distribution of the days. We first draw from all possible actions, then limit the simulations to daily actions and multi-day actions. Finally, the simulated action level data is transformed to daily level data, then standard regression analysis is performed. We repeat this process 3,000 times.

6.3 Results

Figure 6 presents the rejection frequency over 1,000 simulations. The column headers identify the outcome, while the row labels indicate whether all the actions, daily actions, or multi-day actions were used to simulate the data. The green squares represent coefficients for lagged damage to Israel (Gazan actions) while the blue value represent the lagged damage to Gaza (Israeli actions).[11]

[11] As an example, focus as the top left panel. It shows the percent of simulations the coefficients are statistically significant when regressing Gazan action on lagged actions using all actions. The first green square in the top left panel shows that the previous Gazan action is statistically significant nearly all the simulations. The first blue triangle in the top left-hand panel shows that the previous Israeli action is statistically significant in nearly three quarters of the simulations.

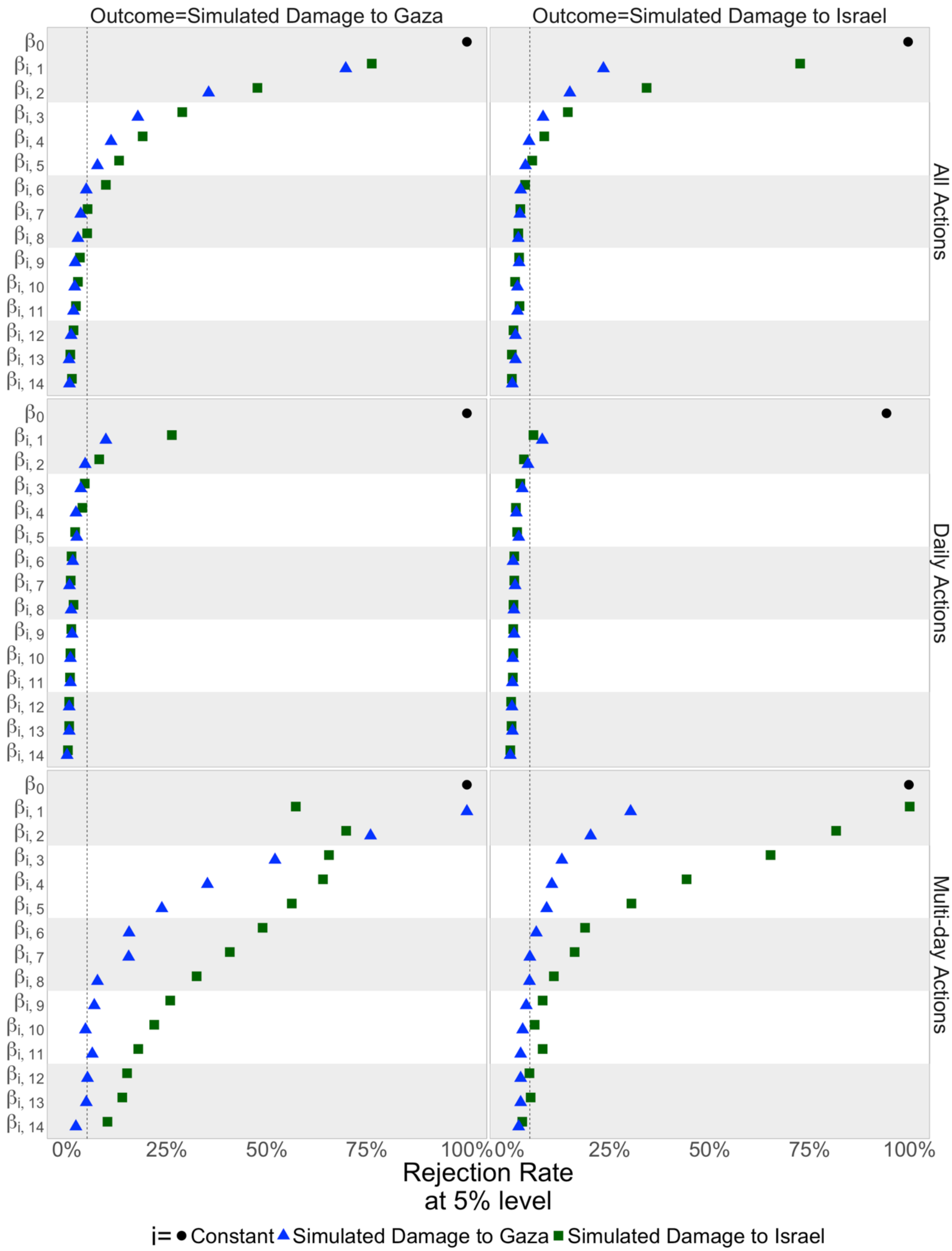

Figure 6 Percent coefficients significant from 1,000 Monte Carlo simulations. Coefficients calculated using VAR at daily level.

Focusing first on the top row, lagged coefficients tend to be statistically significant at least the first five lags. Lagged damage to Israel actions are statistically significant for both Israeli and Gazan actions. The lags coefficients in the Israel regression (top-right hand panel) tend to remain statistically significant for the first seven lags, while previous damage to Gaza remains significant the first four.

The prolonged significant lags are mostly driven by the multi-day actions. The bottom row simulates the data only using multi-day actions. The lagged coefficients remain statistically significant at a higher rate compared to using all actions. In comparison, daily actions with strategic timing does not lead to many significant lagged coefficients. This suggest that temporal aggregation may bias point estimates. Furthermore, this bias is driven by multi-day actions, not strategic waiting, in this context.

We next turn our attention to the simulated impulse response functions. Figure 7 plots the average Gazan (left hand side) and Israeli (right hand side) impulse response functions simulating from all the actions, only daily actions, and only multi-day actions. As before, the top row plots the percent of impulse responses significant at the 5% level using all actions, the middle using only daily actions, and the bottom using multi-day actions. We show the IRF magnitudes in Figure A7.

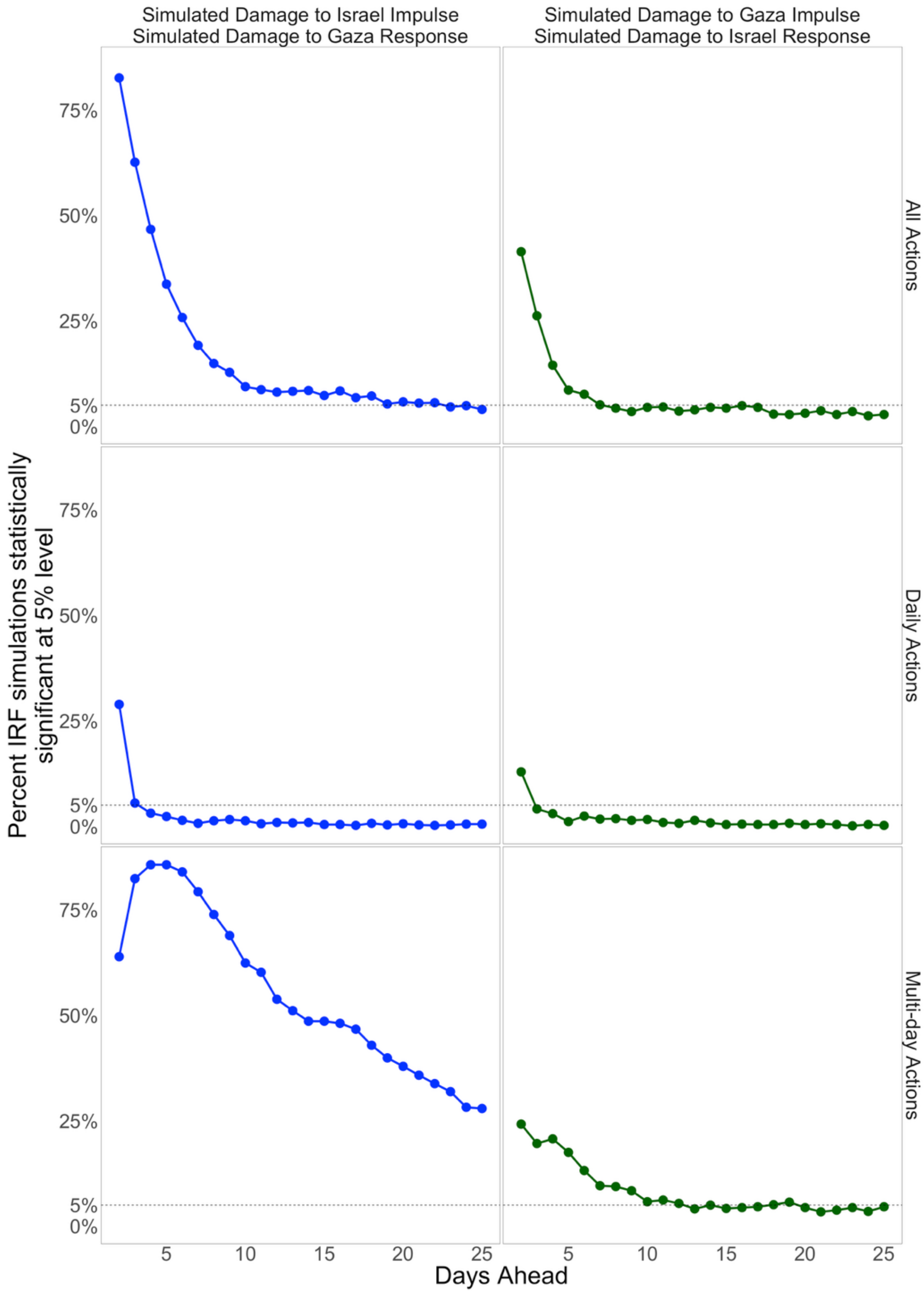

Figure 7 Percent IRFs significant at 95% level from 1,000 Monte Carlo simulations. Initial shock based on estimated standard deviation from each simulation.

The model recreates prolonged impulse responses for Israel using all actions, but not Gaza. Averaging over all action simulations, the Israeli impulse response remains statistically significant over 5% of the time for all 25 periods. The Gazan impulse response shows signs of statistical significance within the first week, then dies out. The effect again is driven by the multi-day actions. Impulse responses generated by daily data exhibit strong reactions in the first one to two days but die out quickly.

Temporal aggregation can cause statistically significant lagged coefficients and prolonged impulse responses through strategic waiting and multi-day actions. Our findings suggest that temporal distortion in the Israeli-Gazan context is driven primarily from multi-day actions.

7 Conclusion

Estimating a player's reaction curve during strategic interactions is important for both academics and policymakers. We show that if the underlying nature of a conflict is not considered, then standard econometric practices can lead to misleading results. In our examples, we assume that two players engage in a sequential game with varying response times, and responses that can last days. Applying standard regressions techniques to temporally aggregated data, e.g., aggregated in days, rather than to the sequence of actions, can generate misleading coefficient estimates and impulse response analyses.

When we take these insights to the Israeli-Gaza conflict between 2007 and 2017, we find evidence that response times are especially influenced by multi-day actions. Researchers studying the conflict using data aggregated to days may well infer that responses are long and drawn out, when they may be better described as responses to the previous action.

Our findings can be summarized as follows: study strategic action at the unit of behavior, not predetermined intervals. If a researcher has data in action time, they should seriously consider performing analysis with it. If they have data in calendar time, then they should try to convert it to action time before modeling response curves. Studying strategic behavior in calendar time, instead of action time, can meaningfully distort our understanding of actors' strategies.

Work Cited

Asali, Muhammad, Aamer S. Abu-Qarn, and Michael Beenstock. 2017. "The Cycle of Violence in the Second Intifada: Causality in Nonlinear Vector Autoregressive Models." *Journal of Applied Econometrics* 32 (6): 1197–1205. https://doi.org/10.1002/jae.2563.

Asali, Muhammad, Aamer Abu-Qarn, and Michael Beenstock. 2024. "Violence and Cooperation in Geopolitical Conflicts: Evidence from the Second Intifada." *Journal of Economic Behavior & Organization* 217 (January): 261–86. https://doi.org/10.101 6/j.jebo.2023.11.012.

Barberá, Pablo, Andreu Casa, Jonathan Nagler, Patrick J. Egan, Richard Bonneau, John T. Jost, and Joshua A. Tucker. 2019. "Who Leads? Who Follows? Measuring Issue Attention and Agenda Setting by Legislators and the Mass Public Using Social Media Data." *American Political Science Review* 113 (4): 883–901. https: //doi.org/10.1017/s0003055419000352.

Berman, Eli, Prabin B. Khadka, Danny Klinenberg, and Esteban Klor. 2024. "Deterrence Through Response Curves: An Empirical Analysis of the Gaza-Israel Conflict." Working {Paper}. Working Paper Series. National Bureau of Economic Research. https://doi.org/10.3386/w33273.

Box-Steffensmeier, Janet M., David Darmofal, and Christian A. Farrell. 2009. "The Aggregate Dynamics of Campaigns." *The Journal of Politics* 71 (1): 309–23. https: //doi.org/10.1017/s0022381608090208.

Brewer, K. R. W. 1973. "Some Consequences of Temporal Aggregation and Systematic Sampling for ARMA and ARMAX Models." *Journal of Econometrics* 1 (2): 133–54. https://doi.org/10.1016/0304-4076(73)90015-8.

Davis, C. Austin, Paula Lopez-Pena A. Mushfiq Mobarak, and Jaya Wen. 2023. "Causes and Consequences of State Violence Against Civilians: The Rohingya of Myanmar Working Paper

- Faculty & Research - Harvard Business School.”
https://www.hbs.edu/faculty/Pages/item.aspx?num=63388.

Enders, Walter, and Todd Sandler. 1993. “The Effectiveness of Antiterrorism Policies: A Vector-Autoregression-Intervention Analysis.” *American Political Science Review* 87 (4): 829–44. https://doi.org/10.2307/2938817.

Engle, Robert F. 2000. “The Econometrics of Ultra-High-Frequency Data.” *Econometrica* 68 (1): 1–22. https://doi.org/10.1111/1468-0262.00091.

Engle, Robert F., and Jeffrey R. Russell. 1998. “Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data.” *Econometrica* 66 (5): 1127. https://doi.org/10.2307/2999632.

———. 2004. “Analysis of High Frequency Data.” *Selfpublished Notes*.

Freeman, John R. 1989. “Systematic Sampling, Temporal Aggregation, and the Study of Political Relationships.” *Political Analysis* 1 (January): 61–98. https://doi.org/10.1093/pan/1.1.61.

Freeman, John R., John T. Williams, and Tse-min Lin. 1989. “Vector Autoregression and the Study of Politics.” *American Journal of Political Science* 33 (4): 842. https://doi.org/10.2307/2111112.

Geweke, John. 1978. “Temporal Aggregation in the Multiple Regression Model.” *Econometrica* 46 (3): 643. https://doi.org/10.2307/1914238.

Golan, David, and Jonathan D. Rosenblatt. 2011. “Revisiting the Statistical Analysis of the Israeli–Palestinian Conflict.” *Proceedings of the National Academy of Sciences* 108 (15). https://doi.org/10.1073/pnas.1016378108.

Haushofer, Johannes, Anat Biletzki, and Nancy Kanwisher. 2010. “Both Sides Retaliate in the Israeli–Palestinian Conflict.” *Proceedings of the National Academy of Sciences* 107 (42): 17927–32. https://doi.org/10.1073/pnas.1012115107.

Jaeger, David A, and M. Daniele Paserman. 2008. “The Cycle of Violence? An Empirical Analysis of Fatalities in the Palestinian-Israeli Conflict.” *American Economic Review* 98 (4): 1591–1604. https://doi.org/10.1257/aer.98.4.1591.

Jordá, Oscar. (1999). “Random-Time Aggregation in Partial Adjustment Models”. Journal of Business & Economic Statistics, 17:3, 382-395, DOI: 10.1080/07350015.1999.10524827

Jordá, Oscar, and Massimiliano Giuseppe Marcellino. 2000. “Stochastic Processes Subject to Time-Scale Transformations: An Application to High-Frequency Fx Data.” https://doi.org/10.2139/ssrn.224788.

Marcellino, Massimiliano. 1999. “Some Consequences of Temporal Aggregation in Empirical Analysis.” *Journal of Business & Economic Statistics* 17 (1): 129–36. https://doi.org/10.1080/07350015.1999.10524802.

Maskin, Eric, and Jean Tirole. 1988. “A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles.” *Econometrica* 56 (3): 571. https://doi.org/10.2307/1911701.

Noel, Michael D. 2007. “Edgeworth Price Cycles: Evidence from the Toronto Retail Gasoline Market.” *The Journal of Industrial Economics* 55 (1): 69–92. http://www.jstor.org/stable/4622374.

Pacurar, Maria. 2008. “AUTOREGRESSIVE CONDITIONAL DURATION MODELS IN FINANCE: A SURVEY OF THE THEORETICAL AND EMPIRICAL LITERATURE.” *Journal of Economic Surveys* 22 (4): 711–51. https://doi.org/10.1111/j.14676419.2007.00547.x.

Silvestrini, Andrea, and David Veredas. 2008. “TEMPORAL AGGREGATION OF UNIVARIATE AND MULTIVARIATE TIME SERIES MODELS: A SURVEY.” *Journal of Economic Surveys* 22 (3): 458–97. https://doi.org/10.1111/j.14676419.2007.00538.x.

Sims, Christopher A. 1980. “Macroeconomics and Reality.” *Econometrica* 48 (1): 1. https://doi.org/10.2307/1912017.

Tiao, G. C., and W. S. Wei. 1976. “Effect of Temporal Aggregation on the Dynamic Relationship of Two Time Series Variables.” *Biometrika* 63 (3): 513–23. https://doi.org/10.1093/biomet/63.3.513.

Wei, William W. S. 1978. “The Effect of Temporal Aggregation on Parameter Estimation in Distributed Lag Model.” *Journal of Econometrics* 8 (2): 237–46. https://doi.org/10.1016/0304-4076(78)90032-5.

Working, Holbrook. 1960. “Note on the Correlation of First Differences of Averages in a Random Chain.” *Econometrica* 28 (4): 916. https://doi.org/10.2307/1907574.

Zellner, Arnold, and Claude Montmarquette. 1971. “A Study of Some Aspects of Temporal Aggregation Problems in Econometric Analyses.” *The Review of Economics and Statistics* 53 (4): 335–42. https://doi.org/10.2307/1928734.

A Appendix

A.1 An example for Section 3.2

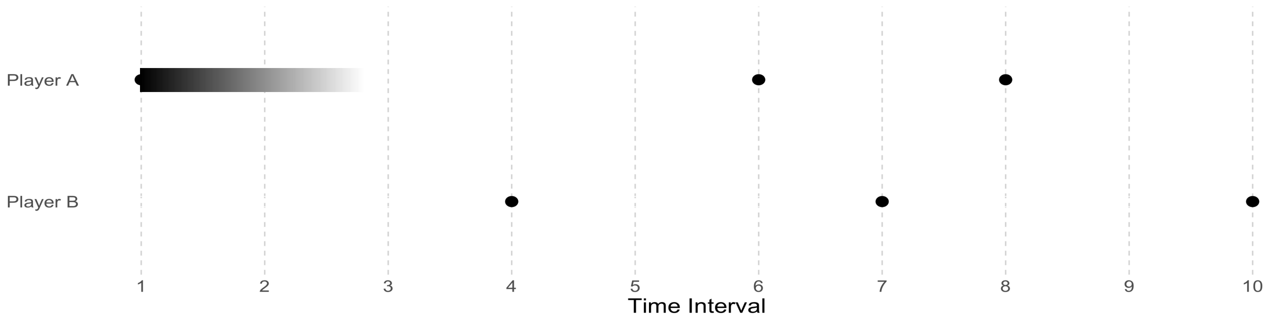

Figure A1: Graphical example of actions recorded at pre-specified time intervals.

Figure A1 provides a graphical illustration of six instantaneous actions over ten days when the time interval is always shorter than the response time.[12] Suppose Player A and Player B are observed over 10 time-units (e.g. days). Within each unit, a player may perform an action causing positive damage or wait to perform an action. The black circle represents an action, and empty slots represent waiting. In this example, Player A first inflicts positive damage to Player B over two days (t = 1 and t = 2). Player B then waits two time periods and attacks Player A (t = 4). Player A then attacks inflicting positive damage at t = 6, leading to a retaliation from Player B followed by further attacks from Player A.

The example highlights that studying the sequential game in arbitrary time intervals can cause the lag structure to differ across actions. Even though Player B's first and second actions (e.g. t = 4 and t = 7) are reacting to Player A's previous action, Player B is reacting to the third lagged time in their first action (t = 1) and first lagged time period in the second

[12] Mathematically, $w_i \geq 1 \; \forall i$.

action (t = 6). We refer to the mapping between recording actions at the action-level and time-interval level as the *data aggregation process*.

We present the example mappings in matrix form of Player A:

$$Pdiag(Z) = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 1-\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where the ones are bolded for easier reading. The waiting periods lead to many rows of only zeroes. This is one of three data distortion from aggregating to arbitrary time intervals. The second is many actions occurring in one day. This is represented by rows having multiple actions.

Finally, an action may be spread over multiple days. This is represented by a column having two nonzero entries, but that add to unity. For example, side A's i^{th} action (like a shelling campaign) may begin on day t and conclude on t + 1. If they dropped a third of their bombs on t and the rest on t+1, then the [t,i] entry for *PZ* is $\frac{1}{3}$ and $\frac{2}{3}$ for [t+1,i].

A.2 Additional parameter decomposition for Section 3.3.1

Equation (12) shows that each β is a linear combination of α, where each scalar term is unbounded. Formally and without loss of generality, $\beta^{A}_{b,j} = \sum_{j=1}^{4} v_{(1+k+j),\mathrm{p}}\alpha_p$.

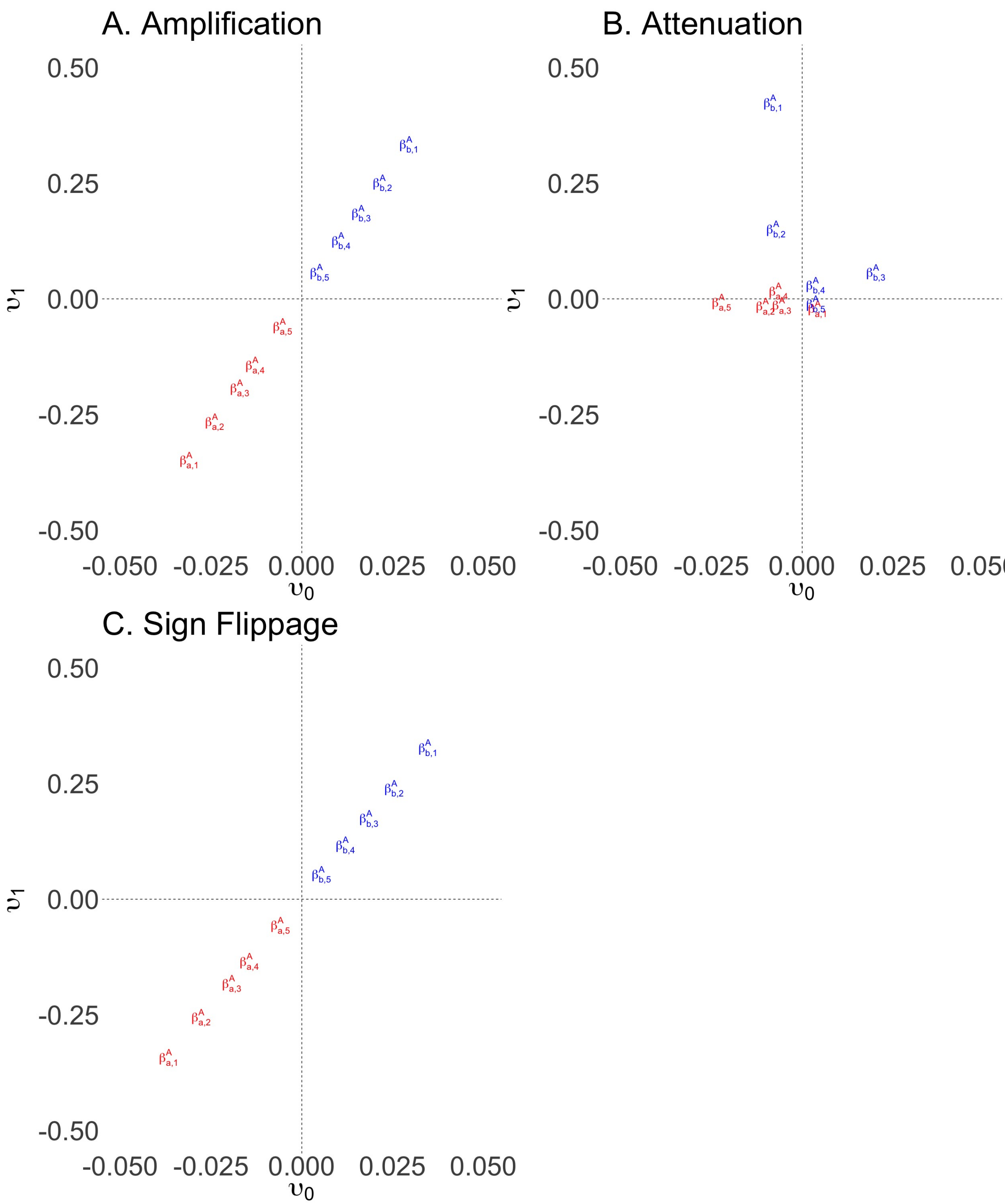

Figure A2: VAR decomposition under different action-level response curves

Figure A2 decomposes each β slope coefficient between the Υ values. The x-axis plots the υ value multiplied by α0 for all the β coefficients, while the y-axis does so for α_1. The panels correspond accordingly to the panels in Figure 3. Most slope coefficients are a combination of both the slope and the intercept. Previous B action scalars (depicted in blue) are positive in Panels A and C, but less than one. The scalars become smaller with more lags, highlighting the diminishing effects of previous lags. Conversely, previous A actions tend to receive negative weights on both the slope and intercept in Panels A and C. The coefficients in Panel B tend to have larger slope scalars than intercept scalars, in line with the intercept being null.

The decomposition highlights two points. First, the time-space β slope parameters are a linear combination of all the action-space α parameters. The VAR estimates capture a mix of the slope and intercept for both sides A and B. Second, the scaling terms are, without further assumptions, unbounded. Therefore, the β slope parameters may be amplified, attenuated or even switch signs. Sign flippage can even occur if all the action-space α coefficients are positive because the scaling terms need not be positive.

A.3 Additional Estimation for Section 4.2

Figure A3 plots the average VAR coefficients over the simulations. The optimal lag length is identified using BIC for each simulation, then estimated. The points show the average coefficient estimate per lagged value. The 95% confidence intervals use the average standard error over the average response time simulations.

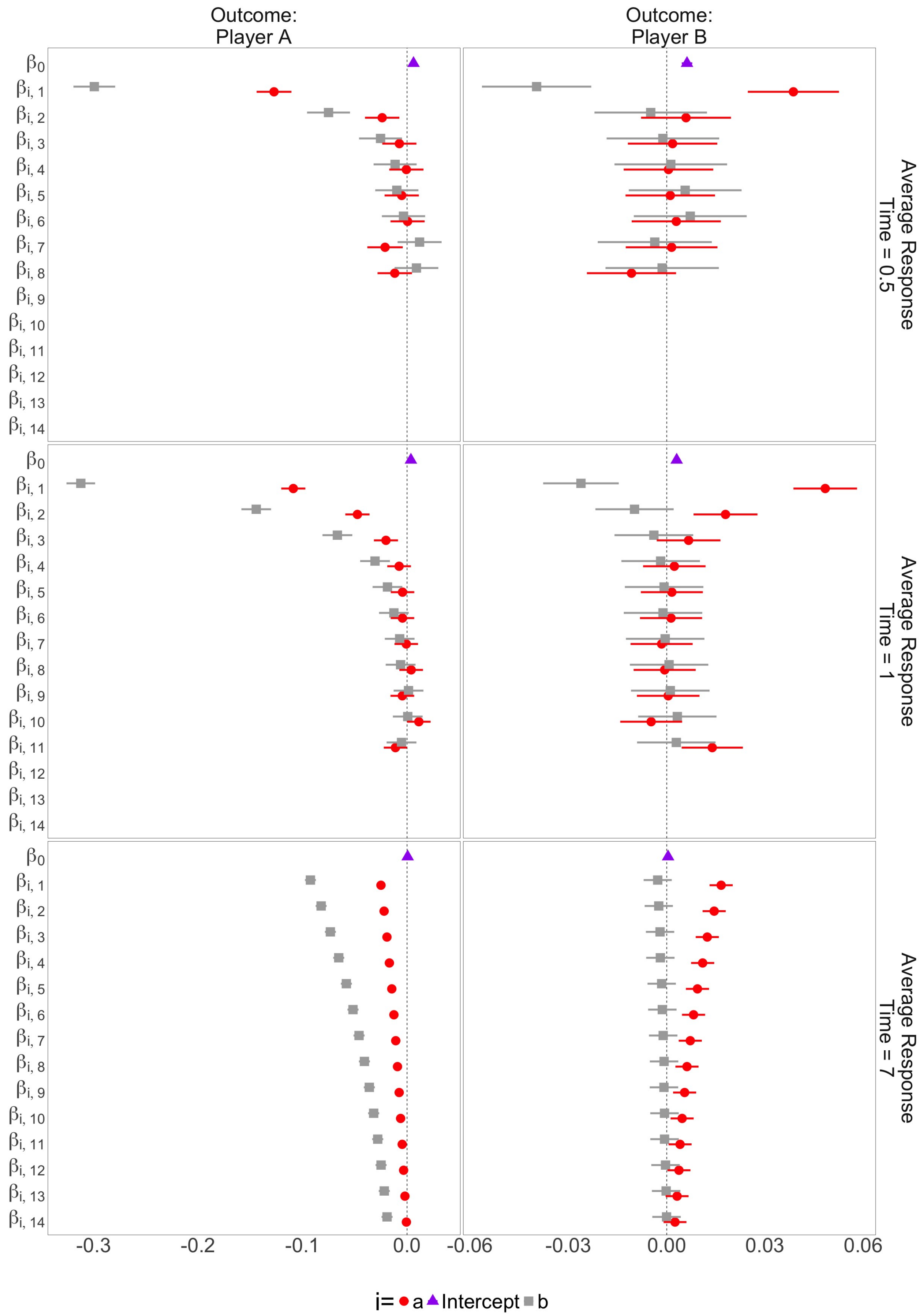

Figure A3: VAR coefficients over 1,000 Monte Carlo Simulations for players A and B.

Recall Player A's action-level response curve is $0.01 + -0.74\ d_{i-1}$, implying a negative slope. In all three iterations, the coefficients on Player B's past actions are nonpositive. When the response time is shorter (i.e. 1 day), the lagged coefficients beyond four periods tend to be economically and statistically insignificant. Additionally, the magnitudes tend to be much smaller than the action-level slope. Player B's lagged actions also remain statistically significant well beyond the average response times. The time distortion may lead a researcher to erroneously attribute Player A's previous actions in determining future reactions.

Conversely, Player B's action-level response curve had a positive slope. In all three simulation iterations, Player A's lagged daily actions are nonnegative. Player B's own lagged daily actions tend to be negative, but statistically insignificant. Together, these simulations highlight how longer response times create a facade of prolong reactions.

A.4 Additional Facts and Figures for Section 5

Table A1: VAR results Comparing Data to Simulations

	Outcome: Expected Israeli				Outcome: Expected Gazan			
	Data	Simulated All Actions	Simulated Daily actions	Simulated Multi actions	Data	Simulated All Actions	Simulated Daily Actions	Simulated Multi Actions
Expected Israeli Action, lag 1	0.345***	0.0311	-0.031	0.2937	0.00302***	-0.0169	0.0097+	-0.0172
	(0.017)	(0.0092)	(0.031)	(0.0054)	(0.00047)	(0.0065)	(0.0201)	(0.0039)
Expected Israeli Action, lag 2	0.326***	0.0079	-0.011	-0.0059	-0.00207***	-0.0032	0.0068*	0.0049
	(0.018)	(0.0092)	(0.030)	(0.0056)	(0.00049)	(0.0065)	(0.0204)	(0.0041)
Expected Israeli Action, lag 3	0.107***	0.0019	0.0044+	0.0016	0.00094+	-0.00047	0.010*	-0.00023
	(0.019)	(0.0091)	(0.0306)	(0.0056)	(0.00051)	(0.00648)	(0.021)	(0.00411)
Expected Israeli Action, lag 4	0.052**	0.0016	0.011*	-0.00073	-0.00011	-0.00062+	0.011*	-0.00069
	(0.019)	(0.0091)	(0.031)	(0.00562)	(0.00051)	(0.00648)	(0.021)	(0.00410)
Expected Israeli Action, lag 5	0.092***	0.00065	0.020*	0.0013	0.00198***	0.00018+	0.0066*	-0.00014
	(0.019)	(0.00903)	(0.031)	(0.0056)	(0.00051)	(0.00650)	(0.0207)	(0.00408)
Expected Israeli Action, lag 6	-0.170***	-0.0011	0.025*	-0.0011	-0.00051	-0.00039+	0.015*	-0.00031+

Continued next page

Table A1: VAR results Comparing Data to Simulations (Continued)

	(0.018)	(0.0089)	(0.031)	(0.0056)	(0.00051)	(0.00650)	(0.021)	(0.00407)
Expected Israeli Action, lag 7	0.027	-0.0018	0.016**	0.0026	-0.00176***	0.00032*	0.0072*	-0.00013+
	(0.019)	(0.0089)	(0.031)	(0.0055)	(0.00051)	(0.00652)	(0.0203)	(0.00404)
Expected Israeli Action, lag 8	0.098***	-0.0024+	0.019*	-0.0024	-0.00151**	0.00031*	0.0076**	-0.00013+
	(0.019)	(0.0089)	(0.031)	(0.0055)	(0.00051)	(0.00647)	(0.0206)	(0.00403)
Expected Israeli Action, lag 9	0.112***	-0.00021+	0.016*	0.00058	0.00201***	-0.00018*	0.003**	-0.00027*
	(0.018)	(0.00888)	(0.031)	(0.00549)	(0.00051)	(0.00646)	(0.020)	(0.00402)
Expected Israeli Action, lag 10	0.113***	-0.00057+	0.042*	-8.9e-05	0.00105*	-0.00026*	0.025**	0.00011*
	(0.018)	(0.00881)	(0.031)	(5.5e-03)	(0.00051)	(0.00644)	(0.021)	(0.00403)
Expected Israeli Action, lag 11	0.094***	0.00012*	0.035**	0.0026	0.00131*	0.0006*	0.0071**	-0.00018*

Table A1: VAR results Comparing Data to Simulations (

	(0.019)	(0.00875)	(0.032)	(0.0054)	(0.00051)	(0.0065)	(0.0213)	(0.00402)
Expected Israeli Action, lag 12	-0.090***	-0.0058*	0.036**	-0.0012	-0.00163**	-0.0012*	0.027**	0.00025*
	(0.019)	(0.0088)	(0.032)	(0.0054)	(0.00051)	(0.0066)	(0.022)	(0.00402)
	-0.176***	-0.0053*	0.036**	0.0026	-0.00203***	-0.0023**	0.025**	-8.6e-05*

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Expected Israeli Action, lag 13								
	(0.017)	(0.0088)	(0.031)	(0.0053)	(0.00048)	(0.0068)	(0.021)	(4.0e-03)
Expected Israeli Action, lag 14	-0.097***	0.0038*	0.021**	0.0037+	0.00041	0.0050*	0.026**	5.1e-05*
	(0.017)	(0.0087)	(0.031)	(0.0051)	(0.00046)	(0.0071)	(0.022)	(3.8e-03)
Constant	-0.0051	0.0673	0.692	0.0178	0.00449***	0.0206	0.186	0.00559
	(0.0335)	(0.0046)	(0.057)	(0.0012)	(0.00092)	(0.0032)	(0.037)	(0.00084)
Expected Gazan Action, lag 1	1.95**	0.240	0.172	0.0905	0.380***	0.1031	-0.0048*	0.3490
	(0.62)	(0.014)	(0.051)	(0.0084)	(0.017)	(0.0093)	(0.0312)	(0.0054)

Expected Gazan Action, lag 2	3.79***	0.104	0.103	0.106	0.187***	0.0327	-0.0042*	0.0203
	(0.66)	(0.015)	(0.049)	(0.009)	(0.018)	(0.0095)	(0.0312)	(0.0058)
Expected Gazan Action, lag 3	-1.50*	0.067	0.054+	0.0688	0.073***	0.0166	-0.0018*	0.0238
	(0.67)	(0.015)	(0.049)	(0.0091)	(0.018)	(0.0095)	(0.0315)	(0.0059)

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Expected Gazan Action, lag 4	0.51	0.046	0.041+	0.0711	0.089***	0.0109	-0.0025*	0.0026
	(0.67)	(0.014)	(0.050)	(0.0092)	(0.018)	(0.0095)	(0.0318)	(0.0059)
Expected Gazan Action, lag 5	-1.26+	0.045	0.026*	0.0524	-0.046*	0.0102	0.0041*	0.014
	(0.67)	(0.014)	(0.050)	(0.0092)	(0.018)	(0.0095)	(0.0319)	(0.006)
Expected Gazan Action, lag 6	-2.01**	0.038	0.012*	0.0345	-0.030	0.0084	-0.0047*	0.0042
	(0.67)	(0.014)	(0.050)	(0.0093)	(0.018)	(0.0094)	(0.0319)	(0.0060)

Expected Gazan Action, lag 7	2.05**	0.042	0.038*	0.0348	0.074***	0.0075+	0.006*	0.0082
	(0.67)	(0.014)	(0.051)	(0.0093)	(0.018)	(0.0094)	(0.032)	(0.0060)
Expected Gazan Action, lag 8	-3.76***	0.033	-0.005*	0.0285	-0.040*	0.0029*	-0.007**	0.00059
	(0.67)	(0.014)	(0.051)	(0.0092)	(0.018)	(0.0094)	(0.032)	(0.00601)
Expected Gazan Action, lag 9	0.62	0.029	0.057*	0.0227	0.065***	0.0081+	0.010**	0.0058
	(0.67)	(0.014)	(0.051)	(0.0092)	(0.018)	(0.0094)	(0.032)	(0.0060)

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Expected Gazan Action, lag 10	0.45	0.025	0.00056*	0.0326	0.0013	0.0057*	-0.012**	0.0021
	(0.67)	(0.014)	(0.05143)	(0.0091)	(0.0185)	(0.0093)	(0.032)	(0.0060)
Expected Gazan Action, lag 11	2.30***	0.023+	-0.0059**	0.019	-0.018	0.0061*	0.0051**	0.0062

Table A1: VAR results Comparing Data to Simulations (

	(0.67)	(0.014)	(0.0513)	(0.009)	(0.018)	(0.0093)	(0.0325)	(0.0060)
Expected Gazan Action, lag 12	-0.97	0.043+	0.074*	0.0253	0.0093	0.0087*	0.031**	-0.00061
	(0.67)	(0.014)	(0.050)	(0.0089)	(0.0184)	(0.0093)	(0.032)	(0.00598)
Expected Gazan Action, lag 13	3.51***	0.037+	0.122**	0.0286	-0.0033	0.0067*	-0.018**	0.0041
	(0.66)	(0.013)	(0.052)	(0.0087)	(0.0181)	(0.0093)	(0.032)	(0.0059)
Expected Gazan Action, lag 14	-1.17+	0.047+	0.046**	0.0323	0.017	0.0096*	0.049**	0.0032
	(0.62)	(0.013)	(0.054)	(0.0084)	(0.017)	(0.0092)	(0.033)	(0.0057)

+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001

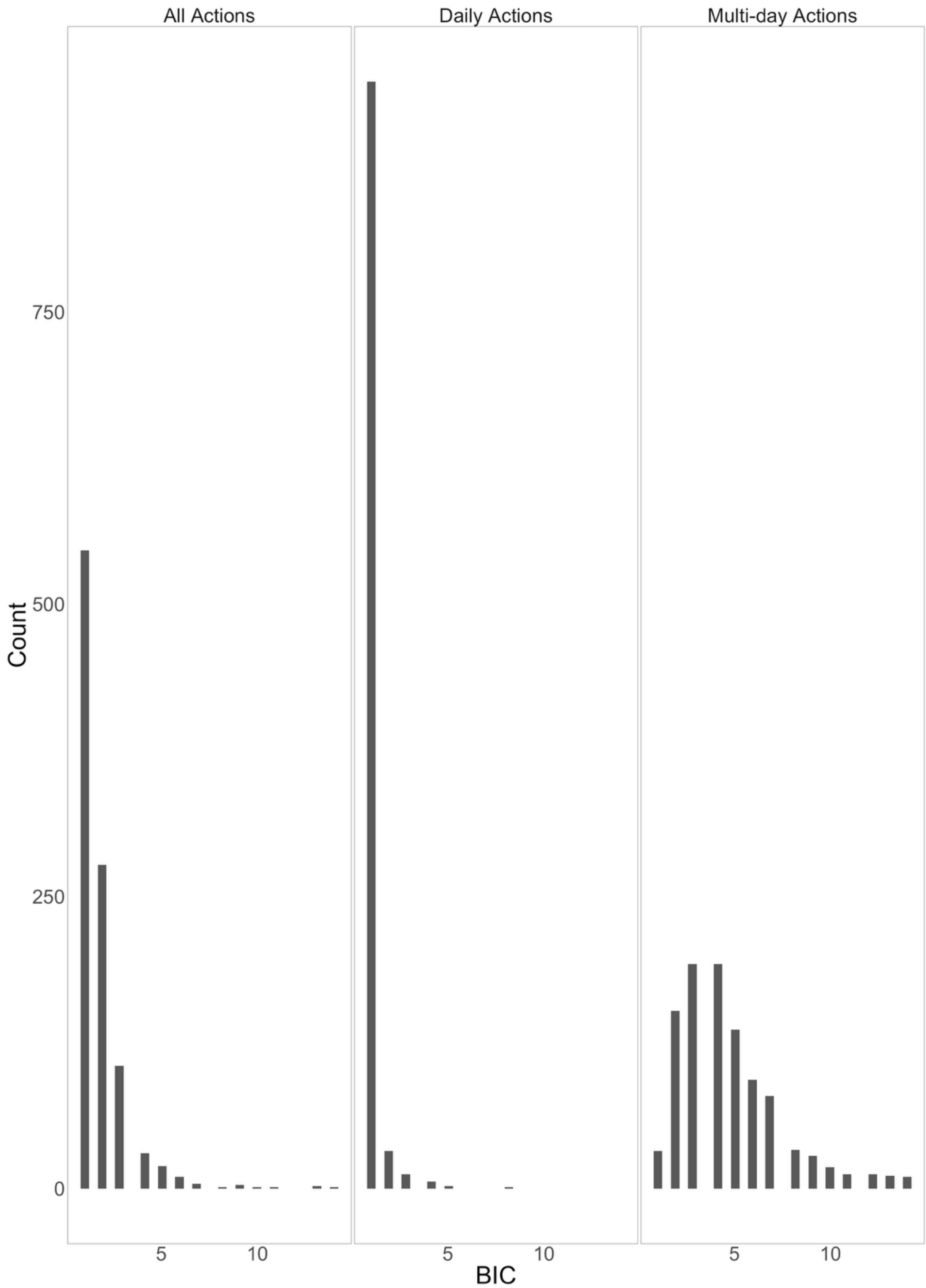

Figure A5: Optimal VAR lag length over 1,000 Monte Carlo simulations using Bayesian Information Criterion.

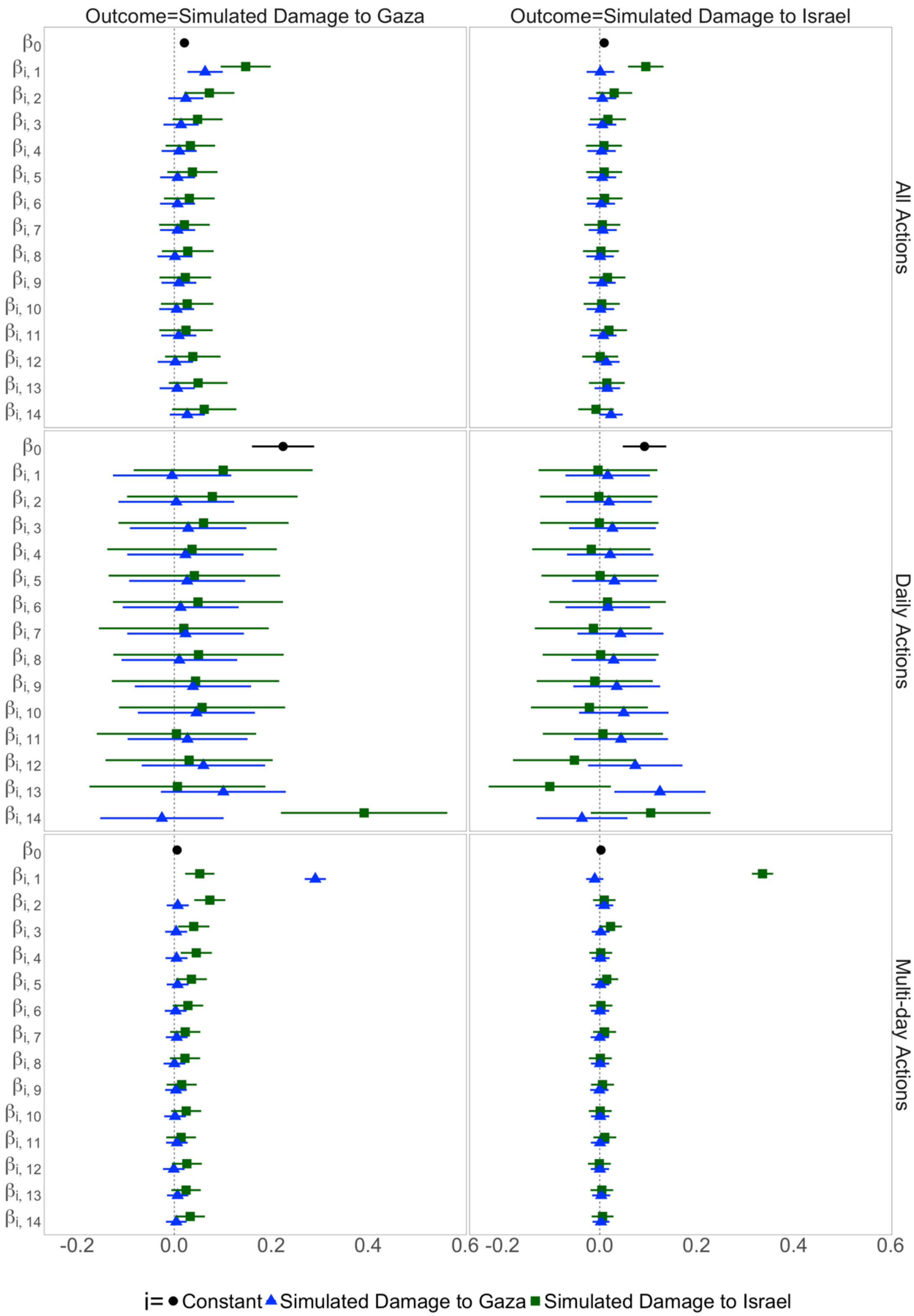

Figure A6: Coefficients from 1,000 Monte Carlo simulations.

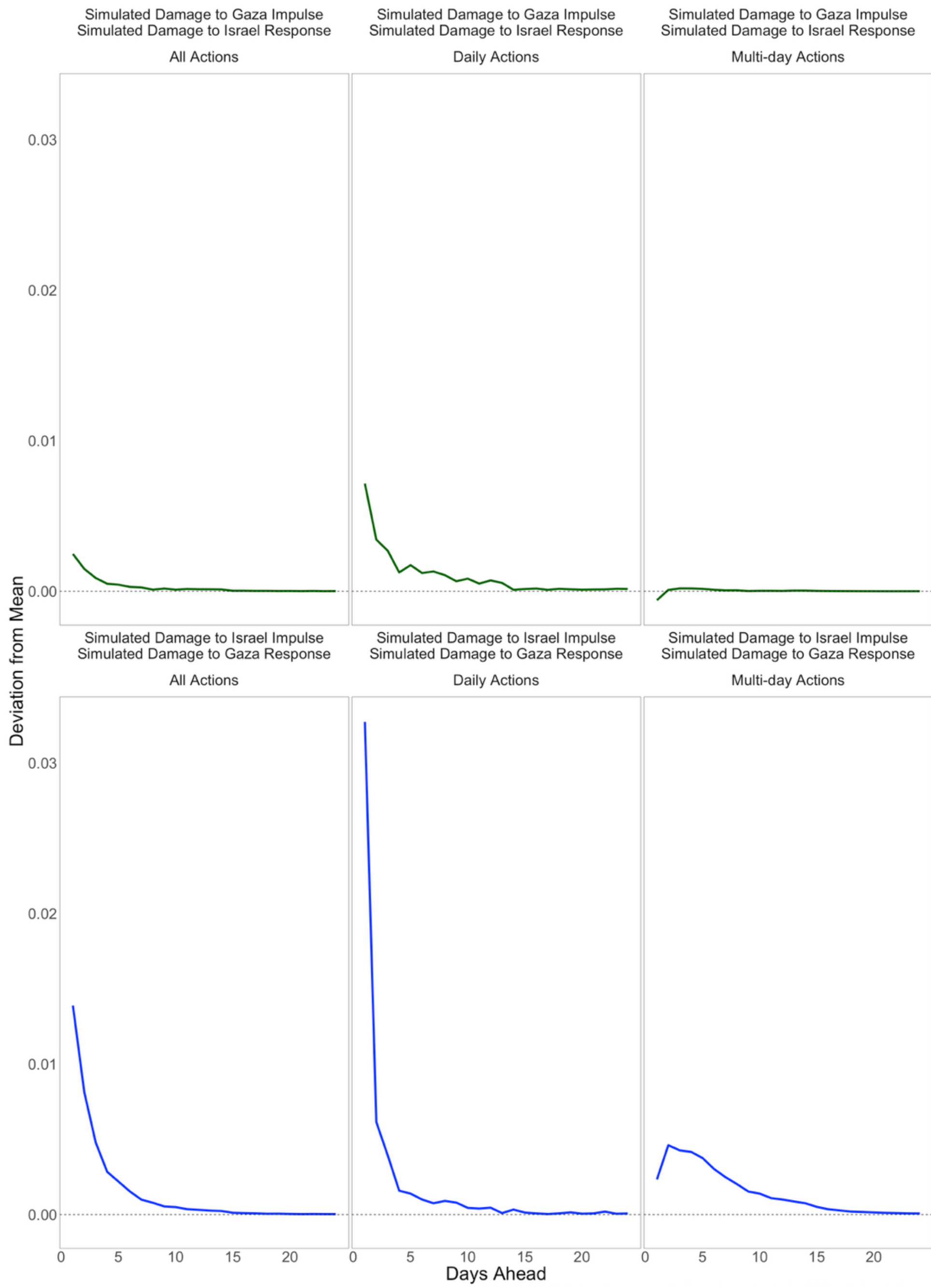

Figure A7: Average IRFs from 1,000 Monte Carlo Simulations. Initial shock removed from graph. Initial one standard deviation shock varies based on estimated standard error.