# Cross-Country Inflation Expectations: Evidence of Heterogeneous & Synchronized 'Mistakes'

Karen Smith[1]

December 15, 2025

**Abstract**

This paper evaluates the assumption of Full-Information Rational Expectations (FIRE) in the inflation predictions of professional forecasters across forty-six countries from 1990 to 2020. I document widespread heterogeneity in the magnitude and direction of FIRE violations and provide new evidence contradicting established stylized facts about the Expectation Formation Process. In juxtaposition to this heterogeneity, I present a Bayesian Dynamic Factor Model that reveals a latent common factor in cross-country forecast errors contributing to the synchronization of forecasters' *'mistakes'*. I add another dimension to this analysis by introducing a novel real-time CPI dataset, augmenting the availability of international real-time data.

*Keywords:* Inflation expectations, monetary policy, dynamic factor analysis, real-time data
*JEL codes:* D84, E31, E32, E71.

[1]Department of Economics, Drew University, Madison NJ. Email: ksmith7@drew.edu. The quantitative part of this paper was conducted using resources from Brandeis University and Drew University. Consensus Economics Inc. owns the copyright to the Consensus Forecasts for the G7 & Western Europe, Asia Pacific, and Latin America datasets, which I used under a license agreement.

# 1 Introduction

> Another gap in our knowledge about the nature of the inflation process concerns expectations [...] *Perhaps most importantly, we need to know more about the manner in which inflation expectations are formed and how monetary policy influences them.*
>
> Janet Yellen, Chair of the Federal Reserve Board of Governors (2014 - 2018)[2]
>
> *(Emphases added)*

To understand how expectations are formed, the role they play in shaping macroeconomic outcomes, and the implications for the transmission of monetary policy, has been the goal of economists for over six decades.[3] Over this time, while our ideas on the nature of belief formation have evolved, we are yet to agree on a unified approach to modeling expectations. There is, however, a growing consensus that the longstanding assumption of Full-Information Rational Expectations (FIRE) does not sufficiently characterize the Expectations Formation Process (EFP). Findings from survey-based data – a widely used approach to gauge agents' subjective expectations – point to widespread FIRE violations with distinctions across variables, demographics, and surveys.[4] While several models have emerged to account for these violations, notably missing from this line of research is a comprehensive cross-country analysis of FIRE, including the implications for modeling the EFP and monetary policy.[5]

The main contribution of this paper is to provide empirical evidence of the cross-country landscape of FIRE in support of the view that there is a need for a more international approach to modeling the EFP. In this study, I ask to what extent cross-country forecasts depart from the assumption of FIRE and what the implications are for modeling the EFP and optimal monetary policy formulation. My analysis assesses the *consensus* inflation predictions of *professional* forecasters across 46 countries from 1990 to 2020, covering the geographical areas of North America, Latin America, Western Europe, and Asia Pacific. Using the Coibion-Gorodnichenko and Mincer-Zarnowitz tests of rational expectations, I present new evidence that FIRE is rejected across countries and document the heterogeneity in the magnitude and direction of these violations - forecaster *'mistakes'*.

In one of my most notable findings, I document the coexistence of under and over-reaction in cross-country inflation forecasts – a direct contradiction of the existing literature that regards

[2] 60th Annual Economic Conference, Federal Reserve Bank of Boston, October 2016.

[3] See Bordalo et al. (2022) for a historical perspective of research on expectations.

[4] See, Mankiw et al. (2003); Souleles (2004); Bordalo et al. (2020).

[5] These include models of: (1) Information Frictions (see Mankiw and Reis (2002); Woodford (2003); Sims (2003); Coibion and Gorodnichenko (2015); Gabaix (2014); Fuster et al. (2010)); (2) Extrapolation Bias (see Greenwood and Shleifer (2014); Barberis et al. (2018); ; (3) A mix of information frictions and extrapolation bias (see Bordalo et al. (2020); Angeletos et al. (2021); Kohlhas and Walther (2021)); (4) Adaptive Learning (see Evans and Honkapohja (2001); Eusepi and Preston (2011); Bianchi and Melosi (2016); Farmer et al. (2024)); and (5) Experience Effects (see Malmendier and Nagel (2011)Malmendier and Nagel (2016)).

under-reaction as the dominant bias in *consensus* forecasts and over-reaction in *individual*-level forecasts.[6] My results reveal that forecaster predictions can instead be ranked along a *bias continuum* of over and under-reaction. I find that in many instances, FIRE violations are driven by the presence of a dominant bias attenuated by the coexistence of weaker (*secondary*) bias in an opposing direction. Within the literature, focus has been placed on the relevance of attributes of the data-generating process, such as persistence and forecast horizon, to account for observed deviations from rational expectations.[7] In this paper, I argue that a country's unique macroeconomic dynamics determine the strength of both the dominant and secondary biases. My results imply that strict classifications of bias in FIRE violations present an incomplete picture of the dynamics of expectations. For policymakers, it becomes clear that there can be no one-size-fits-all approach to modeling belief formation; generalizations about the outcome of well-known tests of FIRE are not ubiquitous across countries. Therefore, our understanding of the EFP remains incomplete without considering the impact of country-specific macroeconomic dynamics on the evolution of beliefs.

While this paper presents an analysis of cross-country heterogeneity in forecaster *'mistakes'*, it does not discard the findings of the existing literature. Instead, it seeks to reconcile the observed heterogeneity with the existing stylized facts such as over and under-reaction in forecasts by examining the synchronization of *'mistakes'* – commonalities in cross-country departures from FIRE. Exploring the role of global linkages in domestic macroeconomic outcomes and the implications for monetary policy transmission is part of an expansive and growing literature.[8] In the context of inflation, it has long been argued that a global factor drives a country's domestic inflation and therefore should be a key consideration of optimal monetary policy.[9] The implication is that cross-country inflation can help forecast country-specific inflation. It follows that forecasters, particularly professionals, would have fully incorporated this knowledge into their forecasting models. Therefore, based on the assumption of FIRE, any synchronization in forecast errors represents an anomaly in the data.

To conduct this analysis, I introduce a Bayesian Dynamic Factor Model (BDFM), which measures the degree of synchronization in cross-country forecast errors from 2001 to 2020. The results confirm the existence of a time-varying *unique* latent factor that oscillates between over and under-prediction of inflation. A variance decomposition of the factor reveals that it accounts for an average of 3% of the variability in domestic forecast errors across all 46 countries, 6% across all advanced economies, and less than 1% across emerging economies. This initial finding establishes two key points. Firstly, while the relative importance of the factor appears small, the fact that forecasters still make these synchronized mistakes strengthens the argument against FIRE. Secondly, based on these findings, I argue that the higher degree of synchronization of *'mistakes'* predominant across advanced economies may have contributed to the existing

[6]See Coibion and Gorodnichenko (2015) and Bordalo et al. (2020)

[7]See Afrouzi et al. (2020)

[8]This has been explored in the context of: (1) Business Cycle Fluctuations, see Kose et al. (2003) (2) Monetary Policy Transmission, see Bernanke (2017); Miranda-Agrippino and Rey (2020); Camara et al. (2024) (3) Globalization, see Rogoff (2003); Galí and Gertler (2010); Grossman and Helpman (2015)

[9]See Borio and Filardo (2007); Ciccarelli and Mojon (2010); Di Giovanni et al. (2023); Li et al. (2025)

generalizations about the nature of the EFP since most studies have been conducted using data from advanced economies.

Next, I evaluate the sensitivity of the variance contribution of the factor to different time intervals, including periods of macroeconomic instability. I find that during the Global Financial Crisis (GFC) from 2007 to 2009, cross-country commonalities accounted for an average of 48% of the variability in country-specific forecast errors. Across the G7 & Western European countries, the factor displayed even greater prominence with country-specific forecaster-*'mistakes'*, accounting for 62% of the variability. In the Asia Pacific and Latin American regions, these values were 40% and 37%, respectively. These findings expose important interlinkages in departures from FIRE not only across countries but also within geographical groups. In so doing, this paper adds another dimension to the literature that examines the role of international fluctuations on regional and national macroeconomic outcomes. The risk of ignoring this *cross-country bias* is that policymakers place an unduly disproportionate emphasis on the role of belief distortions in inflation predictions at the *domestic* country level. This may lead to the application of excessively restrictive or accommodative policies, particularly during times of heightened global economic uncertainty.

Within the literature, an important consideration in understanding how expectations depart from FIRE, is knowledge of the information available to forecasters at the time their predictions were made, that is, *real time data* or data that has not been revised. Among economists, there is consensus that the use of real-time data leads to substantially different and more accurate conclusions about forecaster performance compared to working with revised data (see Croushore and Stark (2001) and Croushore and Stark (2003)). Still, the availability of consistent and reliable international real-time macroeconomic data remains limited.[10] To my knowledge, there are two main sources of comprehensive international real-time data. The first is the Organization for Economic Co-Operation and Development's (OECD's) Revisions-Analysis data set – recently migrated to their Data Explorer platform – a database of 21 key economic variables with vintages of *monthly* real-time data from 1999 forward. The second, which also uses OECD data, is the Dallas Fed's real-time data compilation by Fernandez et al. (2012), which contains *quarterly* vintages of 13 macroeconomic variables from 1962 to 1998 for 26 OECD countries. This data set provides OECD real-time data for key variables such as the Money Supply, Unemployment, and GDP, that were not digitized by the OECD before 1999.

This paper, as one of its key contributions, augments the availability of international real-time data for macroeconomic analysis. I introduce a novel real-time dataset of the Consumer Price Index (CPI) variable for 18 OECD countries. Using hard copies of the OECD's historical Main Economic Indicators (MEI) publications gathered from various public sources, in an

[10] For example, Croushore and Stark's (2001) Real-Time Data Set for Macroeconomists (RTDSM) administered by the Philadelphia Fed, contains data on the US economy from 1965 onward. The Federal Reserve Bank of St. Louis also maintains ALFRED, an archive of real-time data from 1969 for the US and some advanced economies; The Bank of England publishes its Revision Triangles which contains real-time GDP data; the European Central Bank maintains a Real Time Database (context of Euro Area Business Cycle Network); The Bundesbank maintains a real-time data set for the German economy; The University of Melbourne in Australia maintains a Real-Time Macroeconomic Database and the Reserve Bank of New Zealand maintains a real-time Database for GDP.

extensive and meticulous exercise, I digitized 10 years of *monthly* real-time CPI data for each country, from 1989 to 1999, complete with the accompanying vintages.[11] This dataset differs from the Dallas Fed's real-time data compilation since it makes available *monthly* as opposed to *quarterly* data for all countries, providing researchers with the lower level of data aggregation required for some studies. A key outcome of this research direction is the expansion of the dataset made available to members of the international macroeconomic research community.

Before concluding this section, I would like to address the limitations of the analysis presented in this paper. While I conduct tests of rational expectations using the traditional approach of in-sample regressions, findings within the recent empirical literature suggest that the results of tests of FIRE may differ significantly when conducted using out-of-sample predictions (see Bianchi et al. (2022)). Farmer et al. (2024) show that many of the forecasting 'anomalies' observed in these tests, such as bias and autocorrelation, disappear in a model of slow learning when agents are endowed with 'reasonable' initial beliefs. These are undoubtedly areas for further consideration.

In summary, this paper presents an analysis of the cross-country departures from full-information rational expectations. Several tests of rational expectations reveal significant variation in the extent to which cross-country expectations depart from FIRE across forecast horizons. I find, however, that coexisting with these heterogeneous results are several commonalities. I discuss how these contrasting results can be reconciled. Through my application of dynamic factor analysis, I provide evidence of the existence of a common factor in forecast errors and argue that this factor likely accounts for commonalities observed in deviations from FIRE and the concomitant generalizations in the literature. I also show, that country-specific factors also feature prominently in the common factor. Together, these findings point to the need for a more global approach to understanding the expectation formation process. The broader implication of this study is that given the increasingly interconnected global environment, and the well-established endogeneity of inflation expectations, decision-makers must consider, more strongly, the impact of international factors on domestic monetary policy decisions.

**Roadmap** The remainder of the paper is organized as follows. The next section presents my empirical framework and estimates of cross-country departures from FIRE. Section (3) outlines the econometric framework used to model the synchronization of forecaster mistakes. Here I introduce the Bayesian Dynamic Factor Model and explain its application to cross-country forecast errors. In section (4) I estimate the Model on cross-country expectational errors and show that the latent factor explains a significant part of the common variation in the data. Section (5) concludes.

[11] The OECD's Main Economic Indicators (MEI) publication is a monthly publication of macroeconomic statistics dating back to the organization's inception in 1961. Each publication presents data with a two-month lag. There are 12 publications per year, each representing a vintage of the data which may be subject to revisions as time progresses. In addition to data revisions by the respective countries' statistical agencies, changes in the variables such as the move from CPI to HICP throughout Europe, are recorded, making these publications one of the most extensive and well-documented sources of real-time data available to the public.

# 2 The Cross-Country Landscape of FIRE

In this section I present new evidence of departures from FIRE in the survey predictions of professional forecasters across forty-six countries. The objective is to examine the variation in the direction and magnitude of these violations and to discuss the extent to which they contradict established stylized facts about the EFP. Within the existing literature of empirical studies that use survey data to assess the subjective expectations of agents, it has been widely accepted that forecasts at the mean or consensus level display under-reaction as the dominant bias (see Coibion and Gorodnichenko (2015)). In other studies examining the nature of expectations within survey predictions made at the individual level, over-reaction has been observed to be the dominant bias (see Bordalo et al. (2018) and Bordalo et al. (2020)). Relatively unexplored, however, is the extent to which these stylized facts are upheld in cross-country data. To my knowledge, this paper presents the first empirical application of cross-country tests of FIRE.

In an application of two key tests of FIRE, I confirm that consistent with the existing literature, FIRE is rejected in the inflation expectations of professional forecasters across countries. My findings reveal substantial heterogeneity in the magnitude and direction of FIRE deviations across countries and regions. Inconsistent with the literature, I find that the stylistic facts identified above are contradicted by the coexistence of over and under-reaction in the consensus expectations of forecasters.

## 2.1 The Data

To conduct this analysis I combine three main data sources. The first is the consensus survey predictions of professional forecasters across forty-six countries using data from Consensus Economics from 1990 - 2020.[12] This dataset contains monthly forecasts of average annual inflation for the current and subsequent calendar years. At the beginning of each month, forecasters predict the average annual (end-of-year) inflation rate for the current and subsequent calendar year so that each monthly forecast has a declining forecast horizon. To make use of all the available data, forecasts made in January for the current and subsequent calendar years are weighted $^{11}/_{12}$ and $^{1}/_{12}$, respectively. Similarly, forecasts made in July are weighted $^{6}/_{12}$ and $^{6}/_{12}$. In this way, this analysis maximizes the use of forecast data, utilizing all the available observations.

The second source of data is real-time data. Within the literature, the standard has been to use real-time data to evaluate the historical performance of forecasters (see Coibion and Gorodnichenko (2015)). A limitation for cross-country studies of this nature is that there is no comprehensive international database of real-time macroeconomic data, particularly, dating back to the period of analysis for this paper. The OECD, through its digitized, monthly Main Economic Indicators (MEI) publication, contains real-time data for select macroeconomic vari-

[12] Consensus Economics is a global macroeconomic survey firm. Data are purchased under a license agreement from the company.

ables for OECD countries, in many cases dating back to 1961 or from when a specific country entered the OECD. These publications, however, are available on the OECD's website from 1999.[13] Earlier publications remain in hard copy format only. One exception is the Federal Reserve Bank of Dallas digitized selection of real-time data compiled by Fernandez et al. (2012), which is sourced from the OECD's MEI. The dataset contains real-time data for 13 macroeconomic variables from 1962 to 1998 across 26 OECD countries. It is important to note that while the MEI publication is monthly, the Dallas Fed's compilation presents only quarterly data. In this paper, I augment the availability of international real-time data by sourcing and digitizing monthly real-time CPI data across 18 OECD countries from 1989 to 1999, complete with the accompanying vintages from the OECD's MEI publication (a total of 120 publications). I use these data to estimate inflation for the respective countries.

It may be argued that real-time CPI data does not differ substantially from the ex-post data or data that has been revised compared to other variables such as GDP. While it is well known that a country's referenced inflation rate reflects the central bank's preferred measure of inflation, a critical outcome of this data collection and digitization project has been the tracking and reconciliation of CPI, as its measurement has changed over time. An example of this is the United Kingdom, where the central bank's preferred measure of inflation ranged from the Retail Price Index (RPX) in 1947 to the Retail Price Index excluding Interest Payments (RPIX) in 1992. The relevant inflation measure was again changed to the Harmonized Index of Consumer Prices (HICP) in 2003 and finally to the Consumer Price Index, including owner-occupiers' Housing cost (CPIH) in 2017. This evolution of inflation measurement across some countries, casts light on what forecasters may have observed at the time of their forecasts and highlights some of the challenges of evaluating the historical performance of forecasters, particularly within an international context where the availability of real-time data is scarce.

Figure (1) presents an excerpt from the OECD's MEI for Canada and Figure (2) shows the accompanying digitized vintages.[14] Each data entry is recorded with a specific focus on documenting changes to previously recorded values of CPI. Updates to data calculations, missing data, changes in the relevant measures of CPI, and changes in indexation are all recorded and sometimes reconciled. This exercise also required the use of historical central bank records to determine exactly when changes were made to inflation measures, matching this with the dates when the OECD actually includes the change.[15]

The third source of data for this paper is the World Bank's Global Database of Inflation (Ha et al. (2021)), which provides ex-post realized inflation for all countries in the dataset.[16] Together, all three sources of data are used to evaluate the assumption of FIRE across countries.

[13] OECD Main Economic Indicators (MEI) Publications

[14] When using real-time data the question of which vintage of the real-time data should be used to make the appropriate comparison to forecasts may arise. For this empirical application, I follow Coibion and Gorodnichenko (2015) by comparing all forecasts to real-time data available one year after the period being forecasted.

[15] In some instances, it was found that the OECD's MEIs publication did not record changes until three to four months after the respective central bank changes. One example of this was the introduction of the HICP for some European countries.

[16] The World Bank's Global Inflation Database

**Figure 1.** Historical CPI - Canada

**CANADA**

| 1990 JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC | 12-month rate of change / Variation sur 12 mois | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | **PRIX** 1985 = 100 |
| | | | | | | | | | | | | | **Prix à la production** (ind. manufacturières) |
| 109.6 | 110.4 | 110.2 | 110.0 | 110.5 | 110.3 | 110.0 | 110.1 | 111.1 | 111.5 | 111.6 | 111.4 | 1.8 | Total |
| 114.8 | 115.1 | 115.2 | 115.8 | 116.6 | 116.8 | 116.6 | 116.4 | 116.4 | 116.5 | 116.3 | 116.1 | 1.6 | Produits alimentaires et boissons |
| 127.6 | 129.9 | 128.1 | 126.9 | 127.5 | 127.8 | 126.5 | 125.5 | 126.4 | 125.0 | 124.8 | 124.1 | -2.6 | Papier et industries connexes |
| 113.6 | 114.1 | 113.9 | 114.0 | 114.2 | 114.4 | 114.1 | 114.2 | 115.5 | 118.1 | 119.9 | 120.0 | 6.0 | Produits chimiques |
| 111.9 | 111.5 | 113.5 | 113.7 | 115.2 | 113.9 | 114.7 | 117.8 | 119.8 | 117.2 | 114.4 | 111.3 | -3.4 | Métaux de base |
| 114.6 | 114.9 | 114.7 | 114.4 | 114.4 | 114.3 | 114.2 | 114.5 | 114.6 | 114.7 | 114.7 | 114.7 | 0.5 | Ouvrages en métaux |
| 112.5 | 112.5 | 111.7 | 111.8 | 112.4 | 111.9 | 111.2 | 111.4 | 112.4 | 111.7 | 111.4 | 111.4 | -1.0 | Machines électriques |
| 77.4 | 78.0 | 77.7 | 77.0 | 76.5 | 76.4 | 75.5 | 76.9 | 81.0 | 87.1 | 90.8 | 92.1 | 23.3 | Produits du pétrole et du charbon |
| | | | | | | | | | | | | | **Prix à la consommation** |
| 121.8 | 122.6 | 123.0 | 123.0 | 123.6 | 124.1 | 124.7 | 124.9 | 125.2 | 126.2 | 126.9 | 126.8 | 5.0 | Total |

*Notes:* The figure shows an excerpt from the OECD's Main Economic Indicators publication from its February 1991 report. These publications are released with a two-month delay, therefore, the February 1991 report contains data up to and including December 1990. Here, the term *Prix à la consommation* translates to Consumer Prices and measures the level of the respective consumer price indices. There are twelve monthly issues with each subsequent issue releasing one new data point as well as updating data points from the previous months if changes are released by the respective countries. It is in this way that vintages of countries' price indices are formed. In the context of this paper, these documents were sourced for 18 countries, for each month, from 1989 to 1999, digitized, and then merged with the OECD's digitized data from 2000 onward.

**Figure 2.** Digitized Real-Time Dataset - Canada

| Publication Reporting Period | | Measure: CPI- All Items Canada (Indexation: 1985 = 100) Publication Month/Vintage (Columns) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Mar-90 | Apr-90 | May-90 | Jun-90 | Jul-90 | Aug-90 | Sep-90 | Oct-90 | Nov-90 | Dec-90 | Jan-91 | Feb-91 | Mar-91 | Apr-91 |
| January | 1990 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 | 121.8 |
| February | 1990 | | 122.5 | 122.5 | 122.5 | 122.5 | 122.6 | 122.6 | 122.6 | 122.6 | 122.6 | 122.6 | 122.6 | 122.6 | 122.6 |
| March | 1990 | | | 122.9 | 122.9 | 122.9 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 |
| April | 1990 | | | | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 | 123.0 |
| May | 1990 | | | | | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 | 123.6 |
| June | 1990 | | | | | | 124.1 | 124.1 | 124.1 | 124.1 | 124.1 | 124.1 | 124.1 | 124.1 | 124.1 |
| July | 1990 | | | | | | | 124.7 | 124.7 | 124.7 | 124.7 | 124.7 | 124.7 | 124.7 | 124.7 |
| August | 1990 | | | | | | | | 124.7 | 124.9 | 124.9 | 124.9 | 124.9 | 124.9 | 124.9 |
| September | 1990 | | | | | | | | | 125.2 | 125.2 | 125.2 | 125.2 | 125.2 | 125.2 |
| October | 1990 | | | | | | | | | | 126.2 | 126.2 | 126.2 | 126.2 | 126.2 |
| November | 1990 | | | | | | | | | | | 126.9 | 126.9 | 126.9 | 126.9 |
| December | 1990 | | | | | | | | | | | | 126.8 | 126.8 | 126.8 |
| January | 1991 | | | | | | | | | | | | | 130.2 | 130.2 |
| February | 1991 | | | | | | | | | | | | | | 130.2 |

Data Revision

Methodology | USA | Japan | Germany | France | Italy | **Canada** | UK | Belgium | Greece | Austria | Denmark | Finland | Ireland | Neth ...

*Notes:* The figure shows the digitized vintages of real-time data for Canada organized by publication month. Datapoints shaded in yellow represent data revisions. Note that throughout the dataset, revisions not only resulted from updates reflecting more accurate calculation of the respective variables but also included updates resulting from changes in indexation, missing data, and changes in the respective central banks' preferred inflation measure. These are captured and documented throughout the entire dataset.

## 2.2 Evidence of Heterogeneous 'Mistakes'

Several econometric tests of Full-Information Rational Expectations (FIRE) have emerged over the years. This paper focuses on specifications that evaluate the consistency of *consensus* survey expectations with 'Muthian' rationality. In his ground-breaking 1961 Econometrica article, *"Rational Expectations and the Theory of Price Movement"*, Muth outlines the key conjecture of the rational expectations hypothesis.

> "Expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory [...] we call such expectations 'rational.' "

The underlying principle of Muth's rational expectations hypothesis is that on average, for a given information set, an agent's subjective expectations are consistent with the predictions of macroeconomic models – that is, their expectations are model-consistent. A further implication is that economic agents form expectations consistent with the actual stochastic behavior of the economy. For this to hold, Muth assumes that the agent's subjective expectations share the same mathematical properties as probabilistic conditional expectations. Sheffrin (1996) takes this a step further to show that all tests of rational expectations are essentially tests of the underlying properties of conditional expectations and can be classified accordingly.[17] Two well-known properties of conditional expectations are 'Unbiasedness' and 'Forecast Error Predictability'. In the following sections, I discuss how these properties relate to the specified tests, display the associated regression results, and discuss the implications of the findings in the cross-section of the data.

## 2.3 Unbiasedness

Unbiasedness tests evaluate whether the forecast is an unbiased predictor of the realized variable. One such test is the Mincer and Zarnowitz (1969) (MZ) Test which is specified as,

$$\pi^i_{t+h} = \alpha^i + \gamma^i F_t \pi^i_{t+h} + u^i_{t+h}. \tag{1}$$

Here, the left-hand-side variable is the realized inflation of country $i$ at time $t{+}h$, $F_t\pi^i_{t+h}$ is the $h$-period-ahead inflation forecast made at time $t$, and $u^i_{t+h}$ is the rational expectations error term. FIRE implies that the joint null hypothesis of $(\alpha, \gamma) = (0,1)$ will not be rejected at standard confidence levels, indicating that forecasts are *on average* unbiased and that realized values move one-for-one with the forecasted variable. Table (1) shows the estimated p-values from the joint test for each country. Consistent with the existing literature, the results confirm

[17] Sheffrin (1996) notes that in keeping with the properties of conditional expectations, tests of rational expectations can be classified into four main categories: (1) Unbiasedness, (2) Efficiency, (3) Forecast Error Predictability, and (4) Consistency.

that FIRE is rejected in the inflation predictions of professional forecasters. In the context of this paper, one can observe that the null hypothesis is rejected across thirty-three of the forty-six countries at the 10% significance level. Further, we can observe that while rejections of the test are not isolated to any specific region, they appear to be most prevalent in countries within the Asia Pacific region. While these results provide new evidence of cross-country violations of FIRE – a previously undocumented finding – the deeper contribution of this analysis emerges from examining the heterogeneity in the estimated gamma coefficients and what they reveal about the nature of the EFP across countries.

**Table 1.** Mincer-Zarnowitz Test: *p-values*

| **G7 and Western Europe** | $(\alpha,\gamma)=(0,1)$ | **Asia Pacific** | $(\alpha,\gamma)=(0,1)$ | **Latin America** | $(\alpha,\gamma)=(0,1)$ |
|---|---|---|---|---|---|
| USA | 0.04 | Australia | 0.00 | Argentina | 0.03 |
| Japan | 0.08 | Bangladesh | 0.01 | Bolivia | 0.11 |
| Germany | 0.45 | China | 0.00 | Brazil | 0.12 |
| France | 0.05 | Hong Kong | 0.00 | Chile | 0.93 |
| UK | 0.94 | India | 0.06 | Colombia | 0.55 |
| Italy | 0.88 | Indonesia | 0.12 | Costa Rica | 0.00 |
| Canada | 0.01 | Malaysia | 0.00 | Dom. Rep. | 0.26 |
| Austria | 0.32 | New Zealand | 0.05 | Ecuador | 0.00 |
| Belgium | 0.00 | Pakistan | 0.21 | Mexico | 0.01 |
| Denmark | 0.00 | Philippines | 0.00 | Panama | 0.11 |
| Finland | 0.00 | Singapore | 0.03 | Paraguay | 0.00 |
| Greece | 0.16 | South Korea | 0.02 | Peru | 0.00 |
| Ireland | 0.20 | Sri Lanka | 0.04 | Uruguay | 0.00 |
| Netherlands | 0.64 | Thailand | 0.00 | | |
| Norway | 0.00 | Vietnam | 0.14 | | |
| Portugal | 0.79 | | | | |
| Spain | 0.98 | | | | |
| Sweden | 0.00 | | | | |
| Switzerland | 0.00 | | | | |

*Notes:* The table shows the estimated p-values under $H_0 : \alpha, \gamma = 0, 1$. Estimates are computed using Newey-West standard errors with lag length $L = [0.75 \times T^{1/3}]$. Average number of observations per country: G7 and Western Europe: 371; Asia Pacific: 340; Latin America: 283. *p $<$ 0.1, **p$<$ 0.05, ***p$<$ 0.01.

### 2.3.1 A Note on Under and Over-Reaction in the Forecasts

The magnitude of this coefficient reveals the extent to which the forecast moves one-for-one with the realized variable. In the most straightforward case, a gamma of one indicates that forecasters are on average accurate in their predictions, that is, forecasted and realized inflation coincide. A positive gamma coefficient that is less than one indicates that a one percent increase (decrease) in expected inflation correlates with a less than one percent increase (decrease) in realized inflation. More generally, it implies that throughout the analysis, given new information about the state of the economy, a key assertion of Muthian rationality, forecasters were biased in the direction of overreaction. When negative news is revealed about the economy, inflation is predicted to be higher than realized, conversely, when positive news comes to light, inflation is predicted to be lower than realized.

In instances when the estimated gamma coefficient is positive and greater than one, this suggests that forecasters are biased in the direction of under-reaction. In this case, a one percent increase (decrease) in expected inflation correlates with a greater than one percentage point increase (decrease) in expected realized inflation. As forecasters receive negative data about the state of the economy, realized inflation is higher than forecasted and given positive news, realized inflation is lower than forecasted. Table (2) summarizes the implication of the estimated gamma coefficients from the MZ Test. Applying these evaluation criteria to the results reveals new facts about the EFP across countries and regions with varying macroeconomic settings.

**Table 2.** Under and Over-Reaction in the (MZ) Test

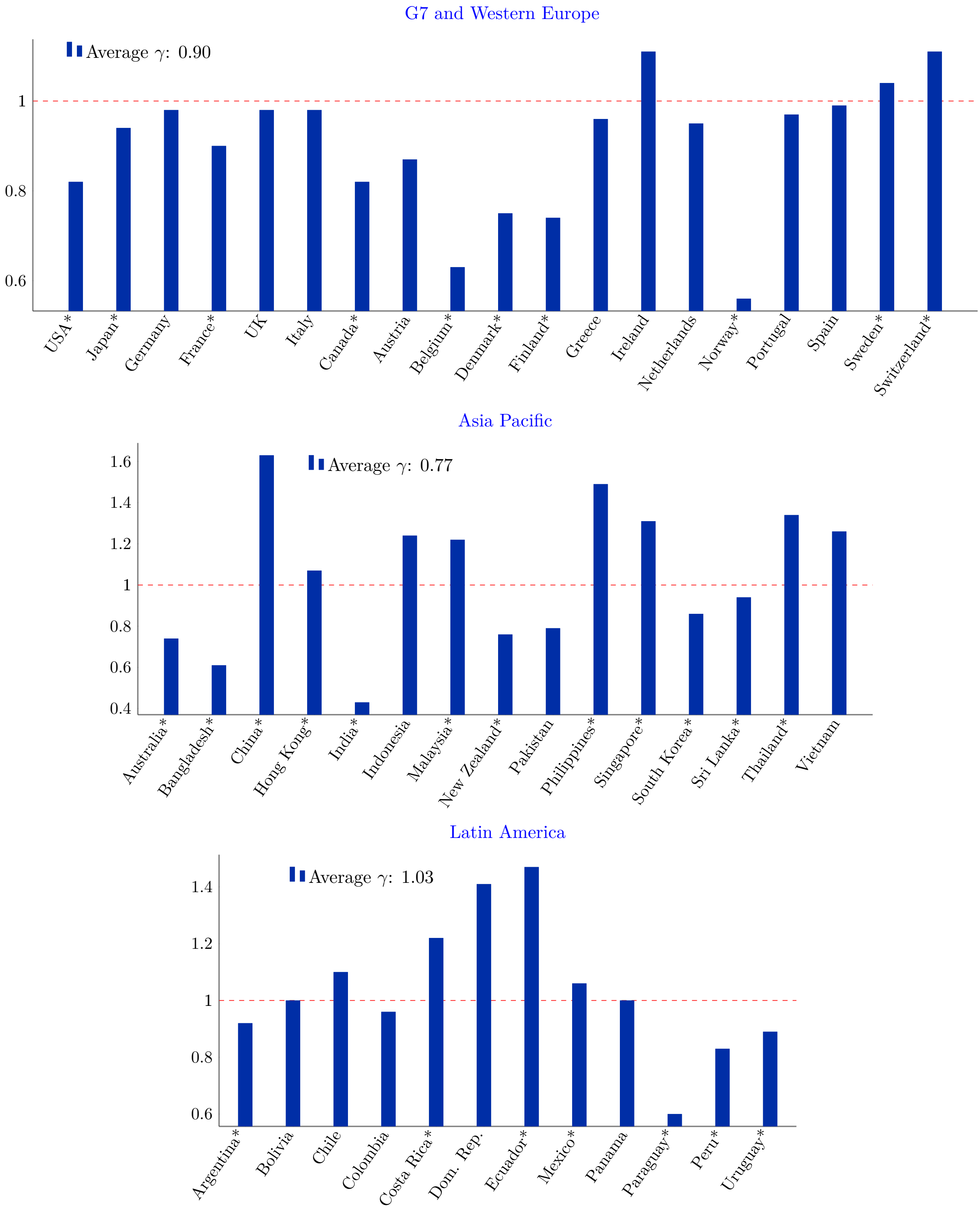
| **Estimated Gamma** | **Direction of Bias** |
|---|---|
| $\gamma > 1$ | Underreaction |
| $\gamma < 1$ | Over-reaction |
| $\gamma = 1$ | Perfect forecast |

#### 2.3.2 The Estimated Coefficients

Figure (3) plots the estimated gamma coefficients of the MZ Test by region with the countries for which the null hypothesis of FIRE is rejected being marked with an asterisk. While the average gamma across all countries is 0.89, indicating that over-reaction is the predominant bias, the results reveal distinct cross-country and regional heterogeneity. Across the G7 and Western Europe region, over-reaction is common among forecasts in most countries except Sweden and Switzerland. Notably, Belgium and Norway have the lowest gamma coefficients of 0.63 and 0.56 respectively, indicating that over-reaction is strongest in these countries.

Across countries in the Asia Pacific region, a similar conclusion holds. Forecasts, on average, appear to be biased in the direction of over-reaction, given the average gamma of 0.77. Upon closer examination, this may be somewhat misleading given the presence of statistically significant outliers across countries such as China, Hong Kong, Indonesia, Malaysia, Philippines, Singapore, and Thailand where under-reaction is predominant. In fact, within this region, the number of statistically significant countries displaying under-reaction outnumber those displaying over-reaction. It is also important to note that across all three regions, forecasts within the Asia Pacific countries exhibit the strongest variation in the magnitude of the estimated coefficients, indicating greater divergence of bias likely reflecting stronger heterogeneity in macroeconomic settings across these countries.

**Figure 3.** Mincer-Zarnowitz Test - Estimated $\gamma$ Coefficients




*Notes:* The figure shows the $\gamma$ coefficient for each country estimated from the Mincer-Zarnowitz specification in equation (1) which tests the null hypothesis $(\alpha, \gamma = (0, 1)$. The red dashed lines represent the evaluation line for $\gamma = 1$. Countries for which the test results are statistically significant are marked with *. The average gamma across all countries is 0.89. All standard errors are Newey-West with lag length $L = [0.75 \times T^{1/3}]$. Average number of observations per country: G7 and Western Europe: 371; Asia Pacific: 340; Latin America: 283.

Within the Latin American region while the average gamma appears to be almost indistinguishable from one, a similar trend exists. For countries such as Costa Rica, Ecuador, and

Mexico forecasts are biased in the direction of under-reaction whereas over-reaction is the dominant bias in forecasts from Argentina, Paraguay, Peru, and Uruguay. In terms of the magnitude of the estimated coefficients, Latin America, despite its history with volatile inflation displays less variation in results than Asia and excluding the results for Costa Rica and Ecuador, appears quite similar to the results for the G7 & Western Europe region.

Together these results reveal the extent of heterogeneity in violations of FIRE but more so, the coexistence of bias in divergent directions, a finding that stands in direct contradiction to the existing literature. To my knowledge, this paper is the first to document the existence of bias in the direction of both under and over-reaction in cross-country consensus inflation predictions of professional forecasters. The implication is that variation in macroeconomic settings plays a more pivotal role in the formation of expectations than previously acknowledged and that models of expectation formation may inadequately account for such differences. To continue this analysis, I turn to another test of FIRE.

## 2.4 Forecast Error Unpredictability

Tests of forecast error predictability evaluate the agent's ability to incorporate all relevant information into the forecast. These tests generally regress the forecast error on relevant variables within the forecaster's information set such as the forecast revision or the value of the realized variable in the current period. By construction, these tests follow from the orthogonality property of forecast errors – that errors should be uncorrelated with any information available to forecasters at time $t$ or earlier. One well-known test within this category is the Coibion-Gorodnichenko (2015) Test – CG Test. The general specification of the test is:

$$\pi^i_{t+h} - F_t\pi^i_{t+h} = \alpha^i + \beta^i(F_t\pi^i_{t+h} - F_{t-1}\pi^i_{t+h}) + u^i_{t+h}, \tag{2}$$

where, the left hand side variable, $(\pi^i_{t+h} - F_t\pi^i_{t+h})$, represents the one-period ahead forecast error. This term is regressed against the time $t$ forecast revision or one-period change in the forecast, $(F_t\pi^i_{t+h} - F_{t-1}\pi^i_{t+h})$, for each country. From a conceptual standpoint, the forecast revision measures the forecasters' reaction to new news on the state of the economy that has been incorporated into their information set. The final term $u^i_{t+h}$ is the rational expectations error which should be uncorrelated with all information received at time $t$ or earlier. Equation (2) tests $H_0 : \beta^i = 0$.

The novelty of this test is that it also allows us to quantitatively and qualitatively assess the implication of a departure from FIRE on the agent's use of the available information set. For example, a positive beta coefficient, $\beta^i > 0$ implies that given the arrival of *negative* new information about the state of the economy, the forecaster's prediction of next period's inflation was revised higher, but insufficiently so. Consequently, the time $t$ forecast revision is positive and the time $t+1$ forecast error is also positive. In a similar line of reasoning, given *optimistic* news about the state of the economy a positive beta implies that in this case the forecaster's inflation prediction was revised lower, but not low enough. This gives the general idea of some level of inertia or *under-reaction* to new information amongst forecasters. Coibion and Gorodnichenko (2015) argue that this *under-reaction* is a feature of consensus forecasts.

In their expanded study of the evolution of expectations across various macroeconomic and financial variables, Bordalo et al. (2020) show that a negative beta coefficient, $\beta^i < 0$, is indicative of *over-reaction*, a feature that characterizes individual-level forecasts. That is, given new information about the state of the economy, forecasters' predictions are revised too high or too low resulting in a negative relationship between the forecast error and the forecast revision. The authors argue that this is largely a result of the extrapolative tendencies of decision makers which are rooted in psychologically founded biases such as overconfidence. Table (3) summarizes the implication of the estimated beta coefficients from the CG Test. Applying these evaluation criteria to the results reveals new facts about the EFP across countries and regions with varying macroeconomic settings.

**Table 3.** Under and Over-Reaction in the CG Test

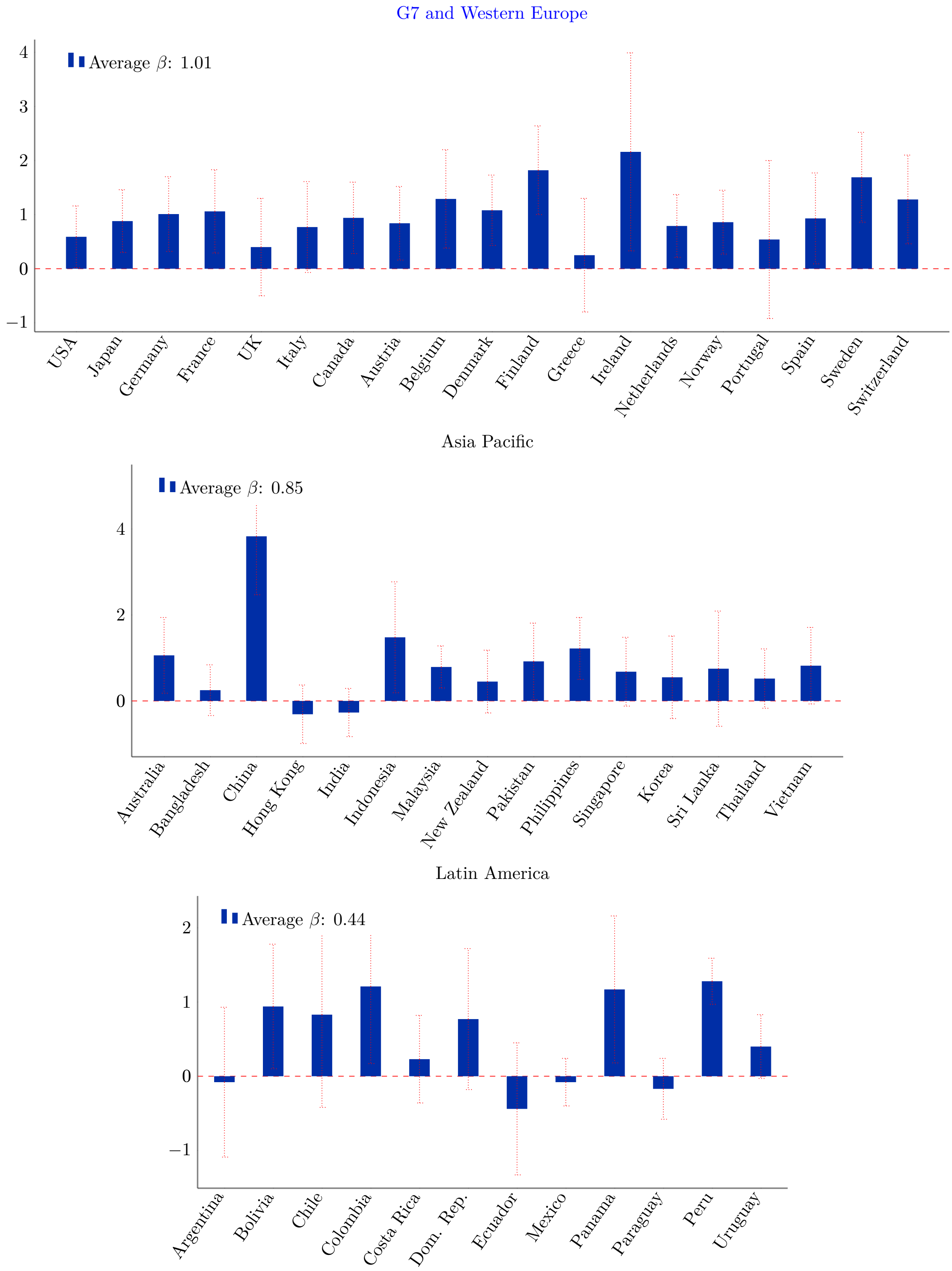
| | **Favorable News** | **Unfavorable News** |
|---|---|---|
| Under-reaction $\beta > 0$ | Forecast revision insufficiently low | Forecast revision insufficiently high |
| Over-reaction $\beta < 0$ | Forecast revision too low | Forecast revision too high |

### 2.4.1 The Estimated Coefficients

Figure (4) presents the results of the Coibion-Gorodnichenko Test for all forty-six countries organized by region. Consistent with the existing literature, FIRE is rejected in the inflation predictions of professional forecasters. The test was rejected by 28 of the 46 countries spanning all three regions with the average beta coefficient at 0.83, confirming that under-reaction is the dominant bias. A closer look at the findings by region reveals that the test appears to have greater statistical significance and the strongest degree of under-reaction among countries within the G7 & Western European countries compared to countries within the other two regions.

Alternatively, this result implies that given new information about the state of the economy, forecasters in the Asia Pacific and Latin American regions are subject to less information processing constraints than forecasters within the G7 & Western European regions. In other words, these forecasters adjust their forecasts with less bias suggesting that they incorporate new information about the state of the economy more quickly and possibly with greater accuracy than their counterparts in the G7 & Western Europe regions – a key underlying assumption of Muthian rationality. To support this assertion, I draw on two key findings within the literature,

**Figure 4.** The Coibion-Gorodnichenko Test: Estimated $\beta$ Coefficients




*Notes:* The figure shows the results of the Coibion-Gorodnichenko Test from equation (2) testing the null hypothesis, $\beta = 0$ across countries within each of the three regions. The blue bars represent the estimated coefficients and dashed red lines represent the respective 95% confidence intervals. The average beta across all countries is 0.83. All standard errors are Newey-West with lag length $L = [0.75 \times T^{1/3}]$. Average number of observations per country: G7 and Western Europe: 338; Asia Pacific: 312; Latin America: 255.

Coibion and Gorodnichenko (2015) and Weber et al. (2025).

In Coibion and Gorodnichenko (2015), the authors argue that the degree of under-reaction in forecasts is related to the presence of information frictions which prevent forecasters from readily or accurately updating their forecasts. This draws from the literature on sticky information models where agents' limited attention leads to the gradual update of their forecasts, as in Mankiw and Reis (2002). This is also closely related to the literature on noisy-information or rational inattention models that account for agents' limited ability to readily observe the true state of the economy due to information processing constraints.

These constraints may be reflected in their inability to distinguish between the noisy data and the data that truly reflects changes to economic conditions as in Woodford (2003). Alternatively, under these models, the agent may selectively choose the information they pay attention to when developing forecasts. This information processing constraint leads to the formation of expectations anchored to the past forecast and only partially updated to reflect new information about the state of the economy as in Sims (2003) and Maćkowiak and Wiederholt (2009).

Using these models, Coibion and Gorodnichenko (2015) map the degree of information frictions to the estimated beta coefficient from the CG Test, assessing information friction as both the frequency with which agents update their information set under the sticky information model and as the weight forecasters place on new information in their forecasts in the noisy information model. In the context of this paper, given the cross-country estimated beta of 0.83, this implies that on average, across all countries, forecasters update their information set every five to six months under the sticky information model and place a weight of around 55% on new information (or 45% on the existing forecast) when updating their forecasts under the noisy information model.[18] Table (4) presents measures of information frictions under both models across all three regions based on the estimated beta coefficient from the CG Test.

**Table 4.** Estimated Measures of Information Friction

| **Region** | **CG Test Coeff.** $\hat{\beta}$ | **Sticky-Information** $\hat{\lambda} = \hat{\beta}/(1+\hat{\beta})$ | **Noisy Information** $\hat{\lambda} = 1/(1+\hat{\beta})$ |
|---|---|---|---|
| G7 & Western Europe | 1.01 | $0.50 \approx 6$ mths. | $0.50 \approx 50\%$ |
| Asia Pacific | 0.85 | $0.46 \approx 5.5$ mths. | $0.54 \approx 54\%$ |
| Latin America | 0.44 | $0.31 \approx 3.7$ mths. | $0.69 \approx 69\%$ |

These calculations reveal that forecasters in the Latin American region update their information sets more frequently than forecasters in the other two regions – approximately every four months versus every six months in the Asia Pacific and G7 & Western Europe regions. It also provides evidence that on average, forecast revisions are weighted as much as sixty-nine percent on new economic information across Latin American countries, compared to around fifty percent in the remaining two regions. Together, these results suggest that forecasters in Latin America were more attentive to changes in economic conditions that may inform their inflation predictions than their counterparts in the Asia Pacific and G7 & Western European regions. Alternatively, the evidence from the CG Test suggests that the lower magnitude of

[18]The authors show that the sticky information model maps beta to the degree of information rigidity, $\hat{\lambda}$, using the formula $\hat{\lambda} = \hat{\beta}/(1+\hat{\beta})$. Under the noisy information model, the mapping is similar, $\hat{\lambda} = 1/(1+\hat{\beta})$, however, here $\hat{\lambda}$ proxies the degree of information rigidity for the weight the agent places on new information.

underreaction observed in Latin American forecasts may be driven in part by lower information processing constraints compared to the other two regions.

This finding is consistent with Weber et al. (2025). The authors conduct randomized control trials (RCTs) across countries where inflation has been historically high and low.[19] Their empirical analysis reveals that agents' inflation expectations vary with their attention to information and that attention is a function of the unique macroeconomic setting, specifically, the prevailing inflation regime. The implication is that in countries where inflation has been historically high, agents are likely more attentive to recent news and the strength of their reaction to new information on inflation is abated compared to countries where inflation has been lower. Consequently, the results presented in Figure (4) *on average* reflect the broad variation in information attention across regions and its linkage to levels of historical inflation.

## 2.5 A Summary of Heterogeneous 'Mistakes'

The findings presented in this section reveal new evidence of the heterogeneity in the magnitude and direction of departures from FIRE across countries. One of the most striking results is evidence of the simultaneous occurrence of bias in the direction of under and over-reaction in consensus forecasts – a violation of the widely accepted finding that aggregate forecasts are subject to under-reaction. Figure (5) maps the degree of under or over-reaction from the CG Test on the x-axis, to the estimated coefficients from the MZ Test on the y-axis. The results are laid out in quadrants I through IV with quadrants II including countries where forecasts were biased in the direction of *strictly* under-reaction, quadrant III with countries displaying *strictly* over-reaction, and quadrants I and IV including countries that display simultaneous over and under-reaction.

Given the spread of observations across all four quadrants, these findings firstly provide evidence that standard classifications about violations of FIRE in consensus forecasts are not ubiquitous across countries. The figure shows that there are fewer countries where the dominant bias is exclusively under or over-reaction, rather, across all quadrants, particularly in quadrant IV, countries seem to fall along the continuum of over and under-reaction, with some displaying stronger bias in either of the two directions.

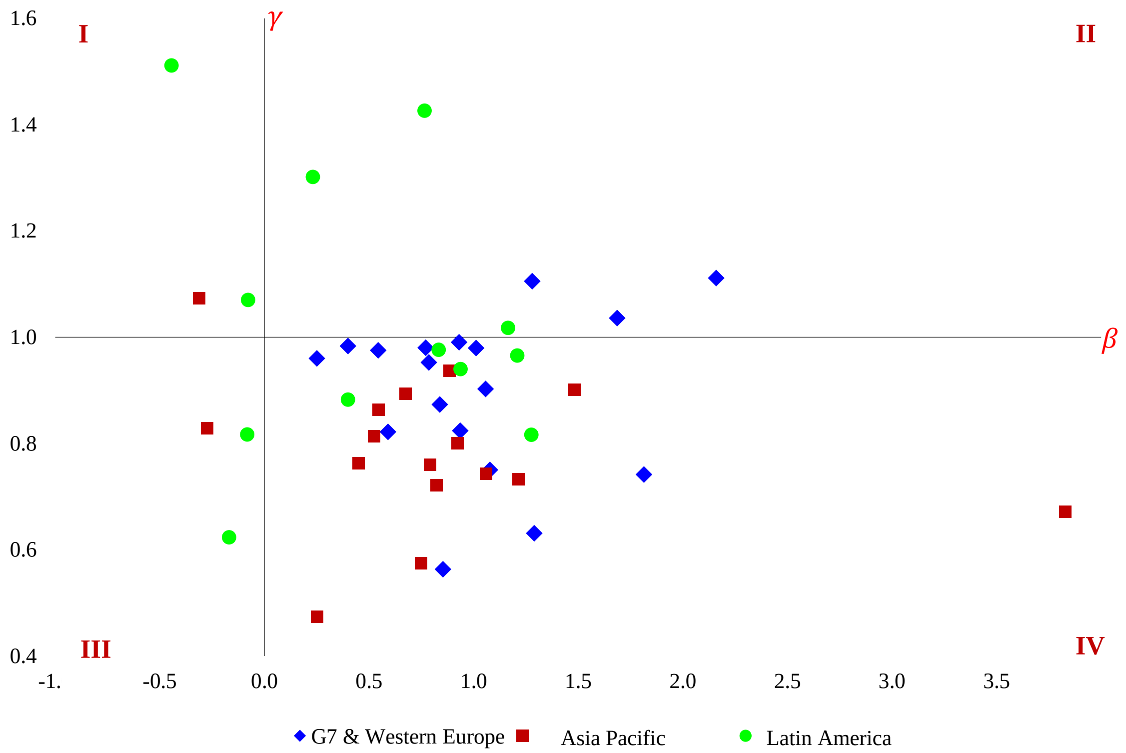
Figure (6) narrows these results by examining those countries where the results were simultaneously statistically significant for both the MZ and CG tests. While most countries appear to be clustered in quadrant IV, there is a wide variation in the results. Those with lower gamma coefficients, such as Norway and Belgium, display far stronger over-reaction than the other countries. On the other hand, Japan and Bolivia appear to be the most moderate with the lowest levels of over and under-reaction. Surprisingly, China, the outlier, exhibits strong bias in both directions, providing the most conflicting evidence of violations of FIRE.

Together, these findings point to the fact that estimates of over and under-reaction reflect

[19] This paper falls within the larger literature that explores how information inattention varies as inflation rates rise. See Branch and McGough (2018), Korenok et al. (2023) and Pfäuti (2024). It is also related to the literature that examines the role of experience effects in decision-making. See Malmendier and Nagel (2011).

the existence of a dominant bias that may be attenuated by the coexistence of bias in an opposing direction. The implication is that our observed estimates of under and over-reaction are also subject to measurement bias. In the context of this paper, I conjecture that the dominance of bias in a particular direction, to some extent, reflects factors other than those considered in the typical analysis of the data-generating process. That is, these results more likely reflect aspects of belief formation that are unique to a country's macroeconomic dynamics – a central argument of this paper. More specifically, they may reflect a country's unique historical experiences with inflation.

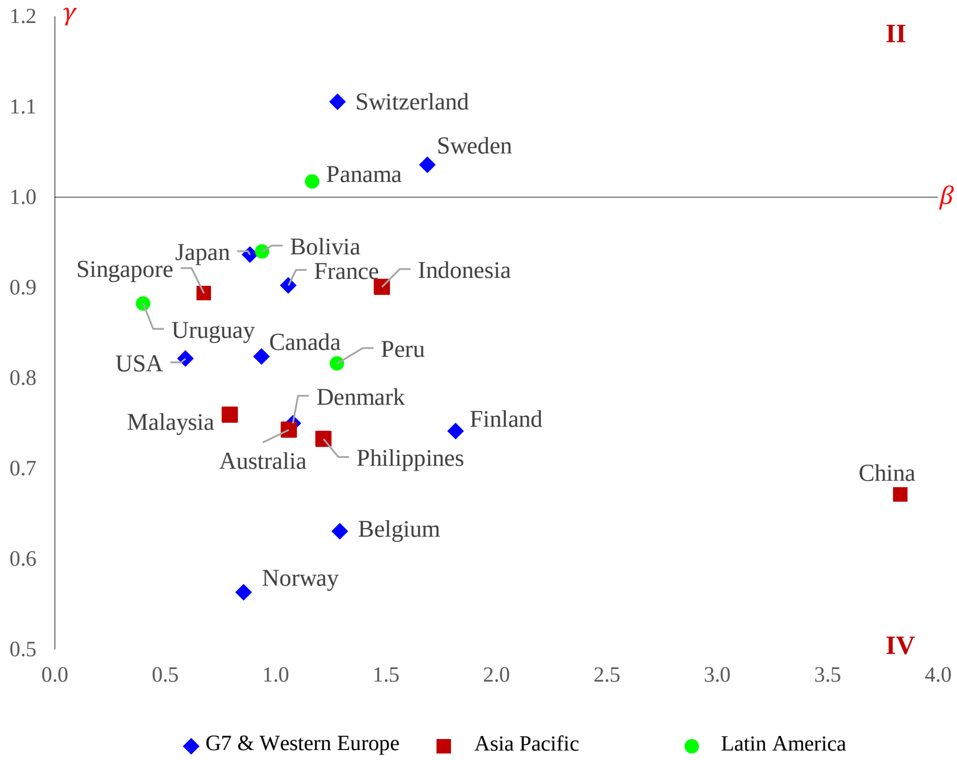
**Figure 5.** Estimated Cross-Country Coefficients




*Notes:* The figure shows the estimated $\gamma$ and $\beta$ coefficients from equations (1) and (2), respectively for each country color-coded by region. Negative values on the x-axis represent over-reaction and positive values represent under-reaction. On the y-axis, values greater than 1 represent forecasters' bias toward under-reaction and values less than 1, over-reaction

Though at odds with the literature, the coexistence of under and over-reaction in forecasts has been documented within the literature. Kohlhas and Walther (2021) using two different tests of FIRE, show that consensus forecasts of output from the Survey of Professional Forecasters (SPF) display simultaneous extrapolation bias (over-reaction) and under-reaction over the period 1970 - 2019. The authors present a model of asymmetric attention to account for these empirical findings. This paper differs from the approach of these authors as it presents the first comprehensive measurement, classification, and characterization of cross-country deviations from FIRE using a relatively standardized dataset of forecasts from professional forecasters. While evidence of the heterogeneity in the magnitude and direction of forecast bias across countries strengthens the case that a country-specific component drives violations of FIRE,

**Figure 6.** Estimated Cross-Country Coefficients - Statistically Significant Results




*Notes:* The figure shows the estimated $\gamma$ and $\beta$ coefficients from equations (1) and (2), respectively for each country color-coded by region. Negative values on the x-axis represent over-reaction and positive values represent under-reaction. On the y-axis, values greater than 1 represent forecasters' bias toward under-reaction and values less than 1, over-reaction

more broadly these results provide insight into the dynamics of the EFP. The findings speak loudly to the fact that there are aspects of belief formation yet to be explored, particularly in the cross-country context and calls for a closer examination of the parameters governing the data generating process, and the interaction of these parameters with country and region-specific macroeconomic dynamics.

This is not to say that we discard existing stylistic facts or past empirical findings. Indeed, the cluster of countries within quadrant IV of Figure (6) also suggests the existence of commonalities across countries in belief formation. That is, belief distortions driving departures from FIRE (*mistakes*) may simultaneously reflect the unique coexistence of heterogeneity in macroeconomic settings, with synchronization across countries and regions, reflecting the impact of the global transmission of macroeconomic shocks.

Ciccarelli and Mojon (2010), in their seminal paper on global inflation, come to a similar conclusion. The authors, in their analysis of inflation in 22 OECD countries from 1960 to 2008, found that approximately 70% of the variation in a country's domestic inflation was driven by the existence of a common global factor, with the implication that as little as 30% of the variation was being driven by country-specific factors.

This paper analyzes the extent to which departures from FIRE, driven by underlying belief distortions in the inflation expectations of professional forecasters, display co-movement. It attempts to deliver a measurement of both the cross-country and country-specific biases in inflation expectations. In the sections that follow, I use a well-known econometric approach to identify common trends in data — a Bayesian Dynamic Factor Model. Then in Section (4) I discuss the findings of this model and show that the strength of this factor varies over time and across regions.

# 3 A Bayesian Dynamic Factor Model

Given the evidence of heterogeneity in departures from FIRE across countries, I now examine the extent to which forecaster 'mistakes' may co-move across countries and the implications of this comovement for modeling the formation expectations and the formulation of effective monetary policy. To conduct this analysis I turn to a widely used class of models that decomposes a macroeconomic time series variable into components that are not directly observable by the econometrician - that is, *unobserved components models*. Included within this class of models are *state space models* where the observed time series depends linearly on a possibly unobserved state driven by a stochastic process. *Factor models* which distill information in large data-sets to a lower-dimensional set of (unobserved) factors, also fall within the class of state space models and can be modeled in a linear state space representation. These models assume that a set of $n$ observed variables depends linearly on $m$ unobserved common factors and on an individual or idiosyncratic component.[20]

This paper presents an empirical application of a *dynamic* factor model which is distinguished from a *static* factor model since the dynamic unobserved factors account for all intertemporal cross-correlations among variables.

## 3.1 Econometric Framework

Let $e_t = [e_{1t}, e_{2t}, ..., e_{nt}]', t = 1, ..., T, i = 1, ..., n$ denote a stationary $n \times 1$ vector of observable cross-country inflation forecast errors standardized to mean zero and unit variance at time $t$. Here, $e_t$ is the difference between realized inflation at time $t$ and predicted inflation at time *t-1* or $\pi_t - \pi^e_{t-1}$. A general representation of the factor model is given by:

$$e_t = \lambda f_t + u_t, \tag{3}$$

where $f_t$ is an $m \times 1$ vector of unobserved common factors which are believed to capture the co-movement of the cross-country panel of forecast errors and $\lambda$ is an $n \times m$ matrix of unknown factor loadings which determine the exact linear combination of the $m$ latent factors. Here, $\chi_t = \lambda f_t$ is referred to as the common component and $u_t = [u_{1t}, u_{2t}, ..., u_{n,t}]'$ as the idiosyncratic component or country-specific noise of the model. It is assumed that $u_t \sim iid\ N(0, \Sigma)$ where $\Sigma$

[20]Where $m < n$.

is set to be a diagonal matrix. This implies that $Eu_{it}u_{jt-s} = 0$ for $i \neq j$ or that the idiosyncratic shocks are uncorrelated. Therefore, $u_{jt}$ is a shock idiosyncratic to equation $i$ and $e_{it}$ is said to follow an *exact* factor model since all co-movements in the data arise from the latent factors plus an idiosyncratic shock.

Note that the above model is static since it does not allow for autocorrelation in the factors. For time series applications, however, the factors may be serially correlated and follow an autoregressive process of order $q$,

$$f_t = \Phi_1^f f_{t-1} + ... + \Phi_q^f f_{t-q} + \eta_t^f, \tag{4}$$

where, $\Phi_i^f$ is the $m \times m$ coefficient matrix corresponding to the $i$th lag of factor $f_t$ and $\eta_t^f = [\eta_{1t}^f, \eta_{2t}^f, ..., \eta_{n,t}^f]'$ is the error term. Note here that $\eta_t^f \sim iid\ N(0, \Sigma^f)$, where $\Sigma^f$ is assumed to be diagonal, that is $E\eta_{it}^f \eta_{it-s}^f = \sigma_f^2$ for $s = 0; 0$ otherwise.

Similarly, the evolution of the idiosyncratic errors may also follow an autoregressive process of order $p$,

$$u_t = \Phi_1 u_{t-1} + ... + \Phi_p u_{t-p} + \eta_t, \tag{5}$$

where, $\Phi_i$ is a matrix of autoregressive coefficients corresponding to the $i$th lag of $u_t$ and the innovations, similar to the factor innovations, are assumed to be zero mean, contemporaneously uncorrelated normal random variables, $\eta_t \sim iid\ N(0, \Sigma)$, where $\Sigma$ is set to be a diagonal matrix. Finally, note here that $E\eta_{it}^f \eta_{it-s} = 0\ \forall\ i, s$. That is, the idiosyncratic terms of the latent factor and the observables are contemporaneously uncorrelated normal random variables.

This allowance for serial correlation in the idiosyncratic errors and the latent factors is what makes the model dynamic. Together, equations (3) through (5) represent the dynamic factor model used to evaluate the inter-temporal co-movement of forecast errors across countries.[21] In the context of the linear Gaussian state space model, equation (3) represents the measurement equation and equations (4) and (5) the transition equations.

### 3.1.1 Identification Restrictions

It is well known that the dynamic factor model presented in equations (3) through (5) is not identifiable without further restrictions. It can be shown that pre-multiplying the dynamic factor by any arbitrary full-rank $m \times m$ matrix defines a new model which is observationally equivalent to the original model. For example, for any non-singular matrix $C$, $\lambda f_t = \lambda C C^{-1} f_t$, therefore, a model with factors $f_t$ and factor loadings $\lambda$ will give the same fit as one with factors $C^{-1} f_t$ and factor loadings $\lambda C$. As a result of this normalization issue, neither the signs nor the *scale* of the dynamic factors nor the associated factor loadings are separately identified.

Two additional restrictions are required for unique identification. Firstly, it is common to set the variance of $\eta_t^f$ to the identity matrix, as in Bai and Wang (2015). That is,

[21]This is the standard model put forward by Stock and Watson (1989) and applied in the context of international business cycles by Kose et al. (2003).

$$var \begin{bmatrix} \eta_{1t}^f \\ \eta_{2t}^f \\ \vdots \\ \eta_{nt}^f \end{bmatrix} = I_m.$$

This restriction addresses the *scale* issue. The second normalization restriction addresses the sign issue by setting $\lambda$ to be a lower triangular matrix with ones on the diagonal.[22] Appendix A.1 details the mathematical proof that these two restrictions are sufficient to uniquely identify the dynamic factor model presented in equations (3) through (5).

### 3.1.2 Selecting the Number of Dynamic Factors

Prior to the estimation of the dynamic factor model, an important consideration is the number of common latent factors to be included. This decision is largely approached via optimization of an *Information Criterion* to estimate the number of factors consistently. The objective is often to select the smallest number of latent trends without losing too much information.

Within the empirical literature, among the most widely used approaches to factor selection, are the Bai and Ng (2002) criterion for static factor models where the minimization of the information criteria is based on a trade-off between the quality of the adjustment of the model to the data and the risk of over-adjustment.

In the context of dynamic factor models, the Bai and Ng (2007) builds on the Bai and Ng (2002) criteria by first taking the optimal number of factors selected as given. The authors then estimate a VAR*(p)* on these factors using the Bayes Information Criterion (BIC) and then as a final step, use the Bai and Ng (2007) criteria to obtain the optimal number of dynamic factors.

Finally, Stock and Watson (2005) and Amengual and Watson (2007) show that Bai and Ng (2002) criteria can be used to estimate the number of dynamic factors.

### 3.1.3 Estimation

Following Stock and Watson (1993, 1989) the classical approach to this system of equations has been to apply statistical techniques which make use of the Kalman filter to estimate model parameters and the Kalman smoother to extract an estimate of the latent factor, however, there are a few drawbacks to this method. Firstly, when $n$ is large, this method can be computationally demanding, and secondly, since the factors are random variables when $n$ is finite, estimators will not be consistent.

[22] There are several variations to address the normalization issue in this model. For example, to address the *scale* issue, Sargent and Sims (1977) and Stock and Watson (1993, 1989) assume that $\sigma_f^2$ is equal to a constant, which is quite similar to the approach taken in this paper. Further, it is common to identify the signs by requiring one of the factor loadings to be positive as in Otrok and Whiteman (1998) or to require $\lambda$ to be a lower triangular matrix with strictly positive diagonal elements as specified in Bai and Wang (2015).

In the more recent literature (see Otrok and Whiteman (1998)), these equations have also been estimated using Bayesian inference which makes use of *posterior simulation*. Here, a Monte Carlo Markov chain (MCMC) procedure can be employed to generate samples from a given target distribution. In the empirical application of this paper, the distribution of interest is the *joint posterior distribution* of the model parameters and latent factors given the observed data, that is, $p(\Theta, f|\text{data})$. Here $\Theta$ represents the parameters, $\lambda, \Sigma, \Sigma^f, \phi_i^f, i = 1, ..., q$, and $\phi_i, i = 1, ..., p$.

Given the large cross-sectional data-set used in this paper, however, sampling from the joint posterior distribution directly is complex and even prohibitive. The Gibbs sampler, an MCMC sampling algorithm, addresses this problem by first expressing this distribution in terms of its complete set of conditional distributions and then iteratively sampling from the individual *posterior conditional distributions*.[23] Casella and George (1992) show that under weak conditions, this iterative sampling converges to drawing from the joint posterior distribution directly as the number of sampling draws, $j \to \infty$. With this approach, the complexity of sampling directly from the joint posterior distribution is simplified via a series of smaller, more manageable problems.[24]

As an example of how the Gibbs sampling methodology cycles through conditional distributions, starting with an initial (specified) value, $f^0$, a sample is drawn from the posterior distribution of the parameters conditional on the factor, $p(\Theta|f^0, data)$, which produces a drawing, $\Theta^1$. Next, a sample is drawn from the distribution of the latent factor, conditional on the parameters, $p(f|\Theta^1, data)$ which produces a drawing, $f^1$. This completes one Gibbs sample or the first step of the *Markov Chain*.[25]

Continuing this process generates a *Gibbs sequence* of random variables, $f^0, \Theta^1, f^1, ...\Theta^j, f^j$. That is, once the initial value $f^0$ is specified, the remainder of the sequence is obtained iteratively by alternately generating, at each step, drawings from,

$$\begin{aligned} \Theta^j &\sim p(\Theta|f^{j-1}, data) \\ f^j &\sim p(f|\Theta^j, data), \end{aligned} \tag{6}$$

for $j = 1, ..., J$. Once the samples are obtained, the sample mean and median of the posterior distributions represent the traditional point estimates, while the percentiles can be used as confidence intervals.

[23]Based on the assumption that the joint distribution can be characterized by its complete set of conditional distributions. Usually applied to standard continuous distributions such as Normal, $t$, Beta or Gamma or discrete distributions such as Binomial, Multinomial, or Dirichlet.

[24]Chan et al. (2019) present a comprehensive introductory example of the Gibbs sampling routine.

[25]Note that the order of sampling, that is, which conditional is sampled first or second does not matter, what matters is that a sample is drawn from each of the unknown blocks. In the application presented in this paper, there are two blocks (parameters and the cross-country factor). Kose et al. (2003) present an application of the Gibbs sampler with four blocks in their application of dynamic factor analysis to international business cycles with a multi-level factor structure.

# 4 The Cross-Country Dynamic Factor: A Model of Synchronized 'Mistakes'

Figure (7) presents evidence of the existence of a cross-country common component in forecast errors among professional forecasters from 2001 - 2020. It is represented by the median posterior distribution of the first latent factor estimated from equations (3) through (5) that is,

$$e_t = \lambda f_t + u_t, \tag{7}$$

where $f_t$ and $u_t$ follow autoregressive processes of orders $q = 1$ and $p = 1$, respectively.[26] The fluctuations in the factor indeed reflect the major economic events throughout this study, such as the 2007 to 2008 Global Financial Crisis (GFC), and the recent COVID-19 pandemic with the factor appearing to increase in strength and statistical significance during macroeconomic shocks. This assertion is consistent with the findings of Borio and Filardo (2007). The authors show that the relevance of global inflation is time-varying and becomes more relevant during economic shocks. An interesting extension of this paper is to assess the increased relevance of this factor coming out of the pandemic, as more data becomes available.

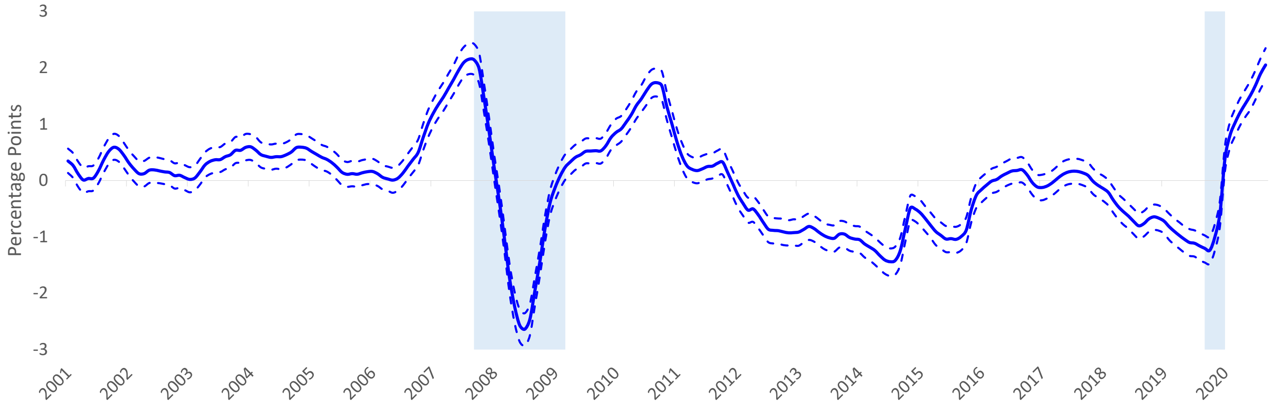
One can also observe that forecasters' predictions oscillate between under-prediction of inflation when the factor is positive, and over-prediction when the factor is negative. Interestingly, the factor displays differences in the direction of forecasters' bias leading up to and during the crises presented. For example, the period leading up to the GFC reflects a bias toward under-prediction whereas one can observe a bias toward over-prediction leading up to the COVID-19 pandemic. Additionally, there appears to be a strong '*reversal*' in the bias after the respective crises, with forecaster predictions emerging from over to under-prediction. Notably, the speed of the '*reversal*' was faster emerging out of the COVID-19 pandemic, with forecasters' predictions biased toward under-prediction at a level that had not been seen since its peak in 2010. This may reflect their views that inflationary pressures would ease, even in the short-term, or be an adjustment to their overly pessimistic views during the pandemic. It may also suggest the existence of well-anchored long-term inflation expectations compared to other periods of economic disruptions.

While the factor in Figure (7) reflects the comovement of forecast errors across all forty-six countries in this study, given the heterogeneity in results observed in Section (2) of this paper, I turn to examining the results by region. Figure (8) shows that all three regional factors are statistically significant throughout this analysis. A parallel can be seen between the factor for the G7 and Western European Region and the factor representing all countries in Figure (7). This suggests that this region makes a substantial contribution to the cross-country factor or indicates that the factor is heavily influenced by the macroeconomic trends of these economies.

The results also reflect wide deviations leading up to, during, and after periods of economic

[26]Note that this analysis begins from 2001, the latest date in the data set when data for all countries are available as estimation requires a balanced panel.

**Figure 7.** Cross-Country Dynamic Factor in Forecast Errors



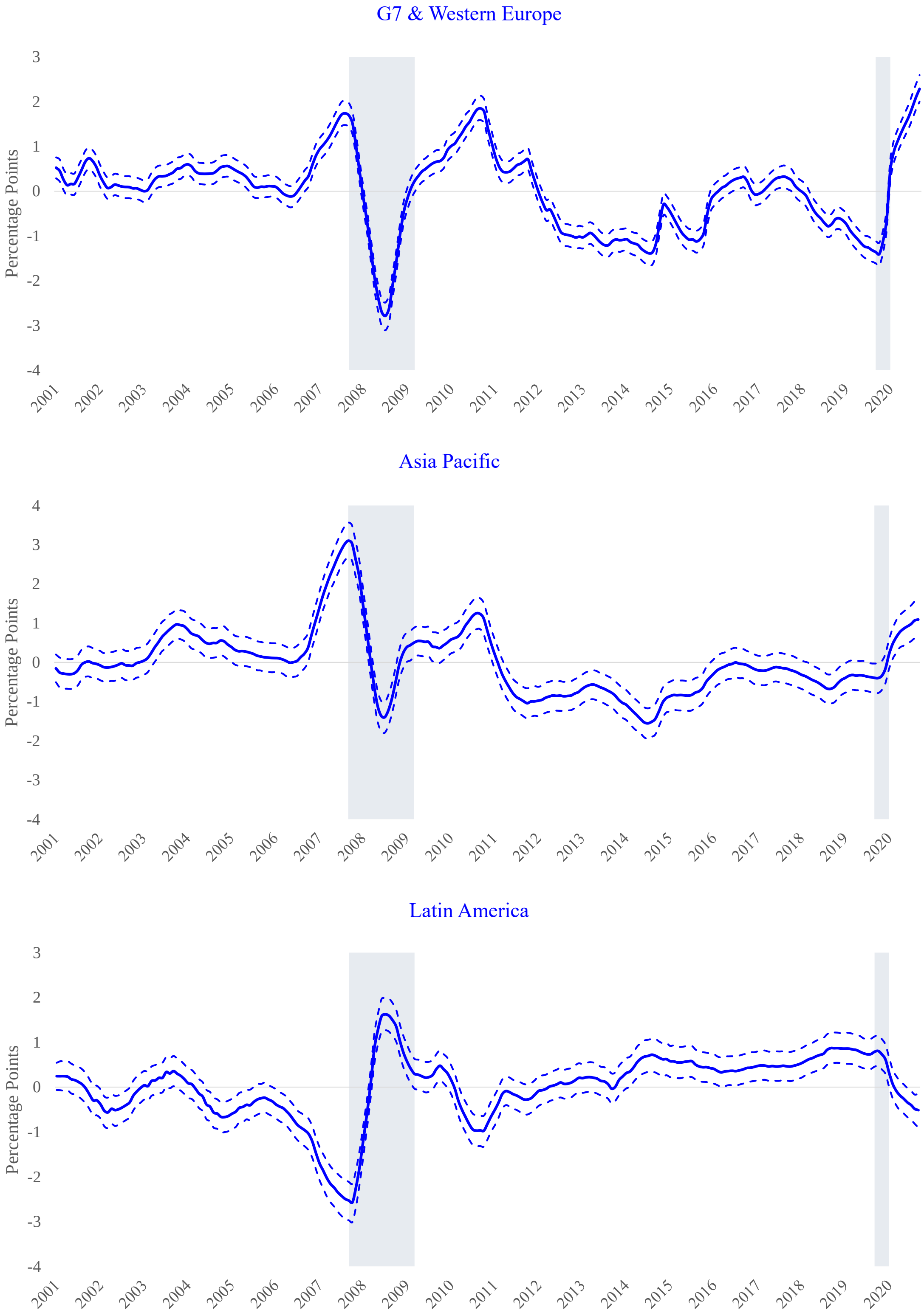

*Notes:* The figure shows the estimated dynamic latent factor in monthly forecast errors across 46 countries from Jan. 2001 - Dec. 2020. Light blue broken lines represent the 95% confidence interval. Forecast errors have been standardized to mean zero and unit variance. Shaded areas denote NBER recession dates. For all countries n = 247.

distress with significant asymmetry in the relevance of the factor around the time of the GFC. For example, within the G7 & Western Europe Region, the influence of the cross-country factor on domestic forecast errors was significantly stronger *during* the crisis period than the periods leading up to and after the crises. This was less so for the Asia Pacific and Latin American regions. For countries within the Asia Pacific region, however, leading up to the GFC, the factor played a significant role in domestic cross-country forecast errors compared to during the crisis – more so than across the other regions.

The results are similar for Latin America, but appear to go in the opposite direction. That is, the factor appears to be stronger and biased in the direction of over-prediction leading up to the crisis and weaker but biased in the direction of under-prediction during the crisis. This is consistent with Section (2) of this paper, where we see countries within the Latin American Region displaying departures from FIRE in magnitude and directions that varied significantly from countries within other regions.

While the data for the Covid-19 pandemic is limited given the period of the study, it is clear that the significance of the factor was strongest for the G7 & Western European countries. An interesting observation is that for both the G7 & Western European and Asia Pacific regions, the period leading up to the crisis was characterized by over-prediction of inflation while throughout the crisis, under-prediction of inflation was the dominant tendency. This is consistent with our experience. That is, forecasters foresaw neither the depth nor breadth of the impact of inflation resulting from the COVID-19 crisis. Many, including policymakers in the US and around the world, believed that inflation was transitory, leading to a delay in monetary policy tightening and possibly a deeper, more prolonged effect of inflation – the effects of which would only be revealed as research is conducted into the causes and consequences of inflation during the Covid-19 pandemic. Early research, including a 2022 Congressional Research Services Report analyzing inflation in the US (see Labonte and Weinstock (2022)) as well as the 2024 NBER and Brookings research paper by Bernanke and Blanchard's paper analyzing

**Figure 8.** Cross-Country Dynamic Factor in Forecast Errors by Region




*Notes:* The figure shows the estimated dynamic latent factor in monthly forecast errors across 46 countries from Jan. 2001 - Dec. 2020 by region. Light blue broken lines represent the 95% confidence interval. Forecast errors have been standardized to mean zero and unit variance. Shaded areas denote NBER recession dates. For all countries n = 247.

pandemic era inflation in 11 economies (see Blanchard and Bernanke (2024)), confirm that policymakers' initial view of rising inflation was that it would quickly subside to pre-pandemic levels. Professional forecasters appeared to be no different, but more strikingly, they share similar beliefs across countries and regions.

Before concluding this section, it must be noted that the results for countries within the Latin American region again appear to go in an opposing direction compared to the countries within the first two regions. Interestingly, leading up to the crisis forecasters were predisposed to under-predicting inflation, while throughout the crisis they over-predicted inflation, this turn was less statistically significant given the wider confidence bands around the factor. This movement from under to over prediction may reflect the region's historical experiences with high inflation. That is, with rising inflation due to the pandemic, forecasters were more quick to recognize the potential for inflation to surge compared to forecasters in the other two regions.

Together, these findings strengthen the case for greater consideration of the dynamics of variations in macroeconomic settings across countries and regions, on the formation of expectations. In the next section, I continue this analysis by measuring the relative contribution of these factors to country and region-specific departures from FIRE and show numerically that many of the observations about the factor across countries and regions are confirmed.

## 4.1 The Contribution of the Cross-Country Factor

While the presence of a global factor reflects commonalities among forecasters, additional insight can be gained by assessing the contribution of this cross-country component to country-specific forecast errors. To examine this, I begin with a measure of the relative contribution of the cross-country factor to variations in the forecast errors in each country. Here, I decompose the variance of each observable into the fraction that is due to the cross-country factor (or common unobserved component) and the fraction driven by the idiosyncratic or country-specific component. In an approach similar to Kose et al. (2003), the variance of the observable forecast errors due to the cross-country factor $i$ can be written as:

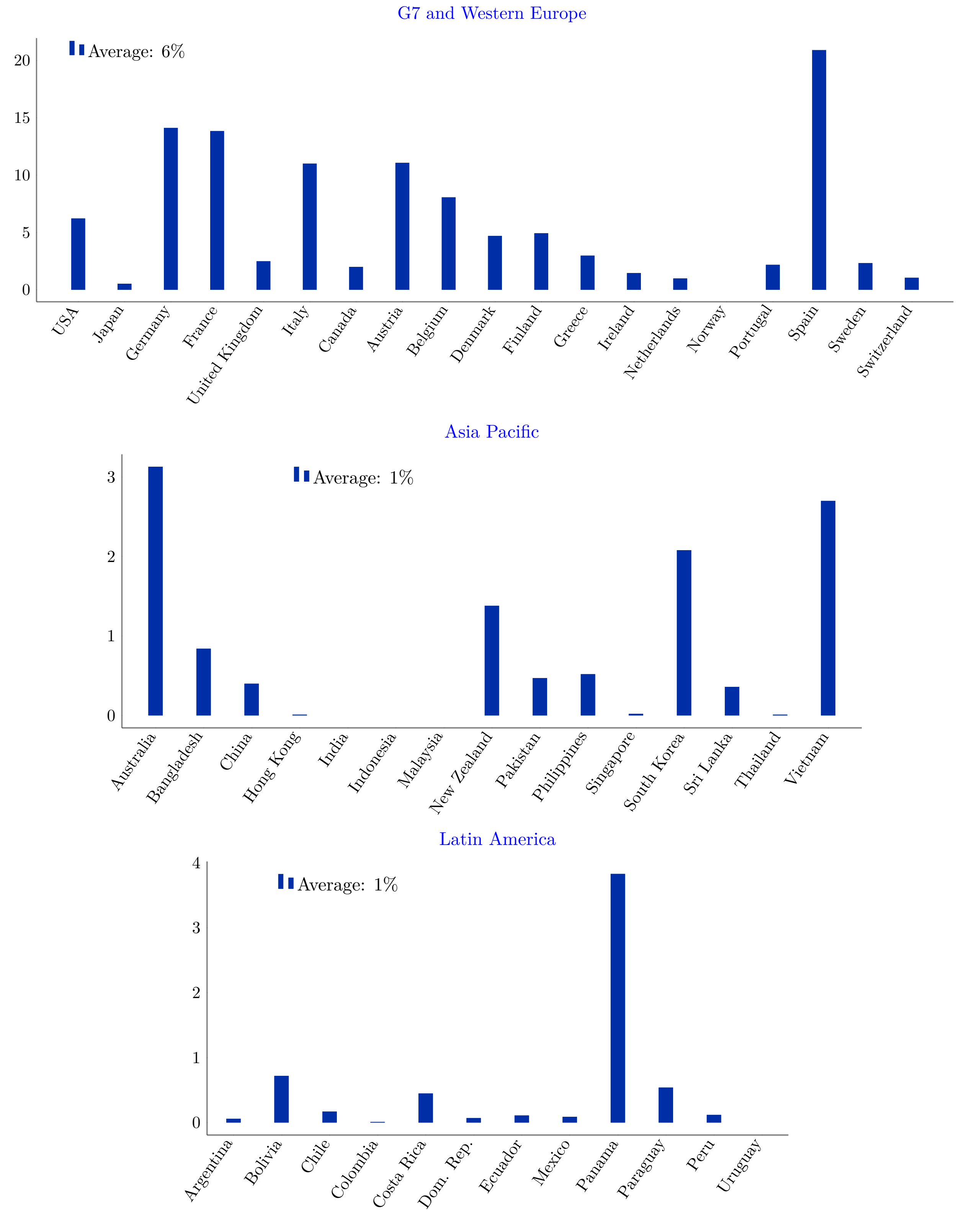
$$var(e_{it}^h) = (\lambda_i^h)^2 var(f_t^h) + var(u_{it}^h), \tag{8}$$

where the fraction of volatility attributed to the cross-country factor for each country is measured as,

$$var(f_{it}^h) = \frac{(\lambda_i^h)^2 var(f_t^h)}{var(e_{it}^h)} \ .$$

Figure (9) presents the variance shares attributable to the cross-country latent factors for each country. Note here that the fraction of the variance of the observable (forecast errors) due to the idiosyncratic or country-specific component is simply $1 - var(e_{it}^h)$.

**Figure 9.** Variance Decomposition by Region




*Notes:* The figure shows the contribution of the cross-country dynamic latent factor to each country's forecast errors over the period 2001 to 2020 in percentage points.

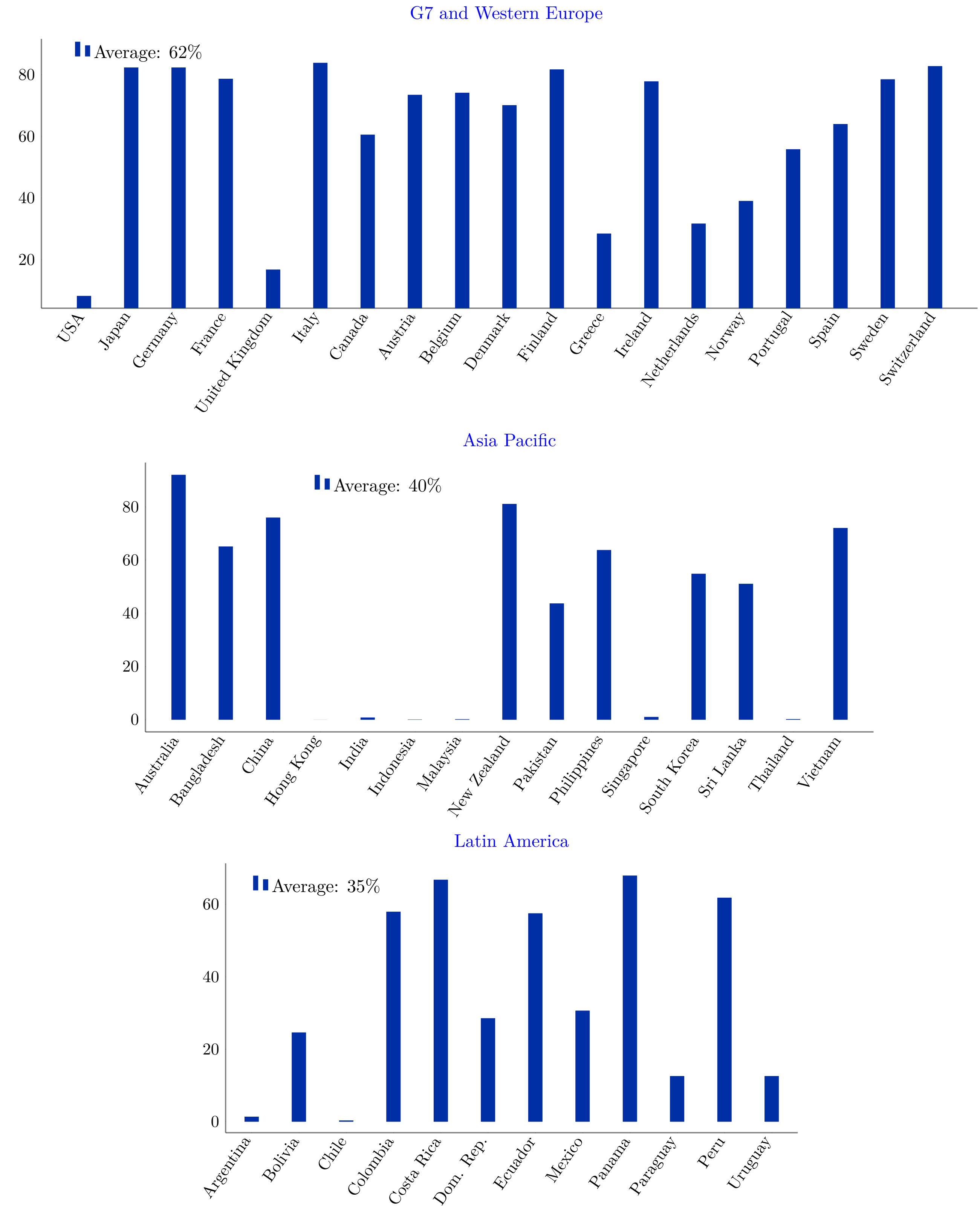
Across all countries, on average the factor explains 3% of the total variation in forecast errors. It is most relevant across countries within the G7 and Western Europe region where the factor contributes as much as 6% to the variability in forecast errors. Notably, for countries such as Germany, France, Italy, Spain, and Austria, the factor explains more than 10% of the variability of each country's forecast errors. The implication is that while the role of domestic macroeconomic conditions appears to dominate expectations formation across most countries, there is a compelling argument to be made for the role of region-specific factors, particularly when countries share a common currency union such as the Euro area. The results also indicate that the factor has less impact in the Asia Pacific and Latin American regions, suggesting that country-specific dynamics play a larger role in forecast errors or deviations from FIRE.

It is important to note that these results represent a summary measure of the contribution of the factor to domestic forecast errors over the entire period of the study and, therefore, may vary when examined over different periods. To analyze variations in the relative contribution of the factor, I conduct a similar analysis over selected time intervals, including the period of the Great Recession, which I examine from 2007 to 2009. Figure (10) reveals that during this period, the factor's average contribution to country-specific forecast errors was 62% across the G7 & Western European region, 40% in the Asia Pacific region, and 35% within the Latin American region. Figure (11) presents the results of the analysis for the entire period of the study analyzed over three-year intervals from 2001 to 2018 and a two-year interval for the final period 2019 to 2020.[27] Firstly, it reveals that the relative importance of the factor is significantly more pronounced over shorter horizons. As expected, the contribution of the factor to domestic forecast errors rose throughout the Great Recession (here 2007 to 2009), but surprisingly, over the time intervals leading up to the COVID-19 pandemic, from 2012 to 2018, across all three regions, the importance of the factor was significantly higher than during the financial crisis. Secondly, while the relative contribution of the factor before the crisis was larger for the G7 & Western Europe region, from 2012 to 2018, the cross-country factor was more relevant to Latin America.

These findings add a new dimension to the role of the *global* or in the context of this paper the *cross-country* factor to country-specific dynamics. Historically, within the literature, we have examined contribution of the factor to variability in domestic conditions over the entire period of studies making summary statements about findings such as in Ciccarelli and Mojon (2010) where the authors note that as much as 70% of the variability of country-specific inflation for the US is accounted for by the global factor over the period of their study (1960 to 2008). Here, I show that the results are highly sensitive to time periods over which they are examined. While the limitation of this approach is that the shorter horizons may contain more noise than the longer horizons, there is still a noteworthy implication for monetary policy which often targets inflation and inflation expectations over similarly short horizons. That is, across countries, forecast errors and by extension, departures from FIRE are driven by an aggregate

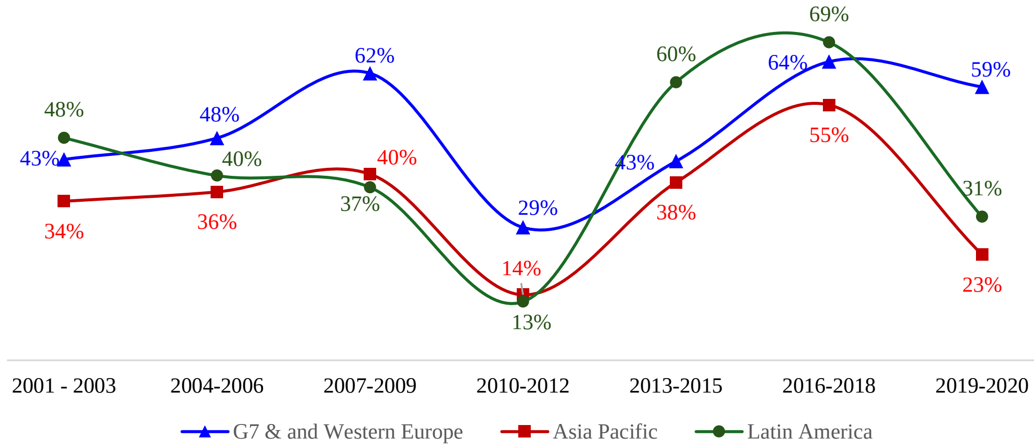
[27] Each three-year interval contains 36 observations per country, whilst the two-year interval contains 24 observations per country.

**Figure 10.** Variance Decomposition: Global Financial Crisis




*Notes:* The figure shows the contribution of the cross-country dynamic latent factor to each country's forecast errors over the period 2007 to 2009 in percentage points.

**Figure 11.** Variance Decomposition by Time Interval




*Notes:* The figure shows contribution of the cross-country dynamic factor to each country's forecast errors in three year intervals organized by region.

component at the international level that is significant and which remains largely unaccounted for within the literature. As policymakers' reaction functions incorporate inflation expectations, an implicit factor is the role of the cross-country factor in these expectations, even in the short to medium term.

In concluding this section, I note that the existence of the latent factor in the forecast errors across countries brings to the fore the role of forecaster biases in the formation of expectations and how these biases, which comove internationally, bear upon forecasters' predictions in a country-specific context. The challenge for the central banker is to assess the transmission of biases through the expectations channel and its relevance for country-specific outcomes in the EFP. It suggests that monetary policy formulation as it relates to the role of inflation expectations hinges on the ability of policymakers to understand more deeply the role of macroeconomic dynamics across these economies in understanding and interpreting how professional forecasters and other agents form expectations.

# 5 Conclusion

The findings outlined in Sections (2) through (4) present a consistent yet heterogeneous perspective on cross-country inflation expectations. The overwhelming message from the tests of rational expectations is that while forecasters across countries make predictions that violate the assumptions of full-information rational expectations they do so in a distinctively heterogeneous manner, to the extent that traditional models which account for these departures are unable to sufficiently address the findings. In other words, there is no one-size-fits-all model when examining the expectation formation process across countries. The results of the dynamic factor analysis admit a similar story. We observe compelling evidence of the existence of a cross-country latent factor in forecast errors , however, upon close examination, there is significant heterogeneity driving these results.

Undoubtedly, over the years, there has been significant progress in our understanding of the expectation formation process and in particular how we account for departures from full-information rational expectations. As discussed throughout this paper, models of sticky information and inattentiveness Mankiw and Reis (2002) Mankiw and Reis (2010), rational inattention Sims (2003), adaptive learning, Evans and Honkapohja (2001), memory Malmendier and Nagel (2016) and diagnostic expectations Bordalo et al. (2020) to name a few, have emerged as some of the leading approaches to accounting for forecaster 'mistakes'.

It is difficult, however, to overlook the fact that researchers have yet to settle on a unified approach to explaining key findings. This paper sits at the core of this issue. While we observe commonalities, the breadth and depth of the heterogeneity in findings often stand in the way of the broad adoption of our models. It is not surprising, therefore, that foundational macroeconomics classes still espouse the underlying tenets of full-information rational expectations as the bedrock upon which macroeconomic modeling is built.

Progress toward a more unified approach, perhaps in the form of a more parsimonious model to account for departures from full-information rational expectations, begins with the recognition of the need for flexibility in analyzing expectations. It also calls for an even more comprehensive analysis of the expectations across agents and countries. This is the key insight of this paper and the premise upon which much of the analysis rests.

# Appendix A

## A.1 Proof of Identification of the Dynamic Factor Model

The objective of this mathematical proof is to show that the two identification restrictions outlined in section 3.1.1 uniquely identify the factor loadings and dynamic factors outlined in the dynamic factor model specified in equations (3) through (5).

In a similar approach as Bai and Wang (2015) Bai and Wang (2015), let $C$ be a full-rank $m \times m$ rotation matrix. Left-multiply the dynamic factor from equation (3) by $C$ and right-multiply the matrix of unknown factor loadings, $\lambda$, by $C^{-1}$. This rotation defines the new factor $\tilde{f}_t = Cf_t$ which allows us to restate equation (4) as,

$$\tilde{f}_t = \Phi_1^f \tilde{f}_t + ... + \Phi_q^f \tilde{f}_{t-q} + C\eta_t^f. \tag{9}$$

After the rotation, the measurement equation (3) becomes,

$$e_t = \lambda C^{-1} \tilde{f}_t + u_t. \tag{10}$$

To establish identification, it must be that the only $C$ that should be acceptable is a diagonal matrix with either 1 or -1 on the diagonal, in which case, both the factors and factor loadings are identified up to a sign change.

From equation (9) the first normalization requirement implies that $var\ (C\eta_t^f) = I_m$. This implies that,

$$CC' = I_m, \tag{11}$$

or that $C$ is an orthogonal matrix.[28]

To address the second normalization requirement that $\lambda C^{-1}$ be a lower triangular matrix with ones on the diagonal, let

$$\lambda = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \lambda_{m1} & \dots & \dots & 1 \\ \vdots & \dots & \dots & \vdots \\ \lambda_{n1} & \dots & \dots & \lambda_{nm} \end{bmatrix} \text{ and } C^{-1} = \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{m1} & \dots & c_{mm} \end{bmatrix}.$$

[28]The implication here is also that $CC^{-1} = I_m$ and therefore, $C' = C^{-1}$

Carrying out the operation $\lambda C^{-1}$ gives us,

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \lambda_{m1} & \dots & \dots & 1 \\ \vdots & \dots & \dots & \vdots \\ \lambda_{n1} & \dots & \dots & \lambda_{nm} \end{bmatrix} \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{m1} & \dots & c_{mm} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda^*_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \lambda^*_{m1} & \dots & \dots & 1 \\ \vdots & \dots & \dots & \vdots \\ \lambda^*_{n1} & \dots & \dots & \lambda^*_{nm} \end{bmatrix}. \tag{12}$$

Equation (12) shows that the $n \times m$ matrix, $\lambda C^{-1}$, is a lower triangular matrix where $\lambda^*_{ij} = 0$ $\forall\ j > i$ and assuming that $\lambda^*_{ii} \neq 0$, $i = 1, ..., m$. A further implication of equation (11) is that $C^{-1}(C^{-1})' = I_m$. Hence, multiplying both sides of equation (12) by $(C^{-1})'$ we have,

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \lambda_{m1} & \dots & \dots & 1 \\ \vdots & \dots & \dots & \vdots \\ \lambda_{n1} & \dots & \dots & \lambda_{nm} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \lambda^*_{21} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ \lambda^*_{m1} & \dots & \dots & 1 \\ \vdots & \dots & \dots & \vdots \\ \lambda^*_{N1} & \dots & \dots & \lambda^*_{nm} \end{bmatrix} \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{m1} & \dots & c_{mm} \end{bmatrix}, \tag{13}$$

from which we can deduce that $c_{ij} = 0\ \forall i, j$ when $i > j$. This proves therefore that $C^{-1}$ is a diagonal $m \times m$ matrix. Further, since $C^{-1}(C^{-1})' = I_m$, it must be that $c_{ii} = 1$ for all $i = 1, ..., m$, so that the rotation matrix C is also a diagonal matrix with either 1 or -1 on the diagonal. This proves, therefore, that the dynamic factors and the associated factor loadings are identified up to a sign change.

Finally, note that the normalization assumption that $\lambda^*_{ii} = 1$, $i = 1, ..., m$, imposes a positive sign, therefore, both the dynamic factors and the associated factor loadings are fully identified. **Q.E.D**.

# References


**Afrouzi, Hassan, Spencer Yongwook Kwon, Augustin Landier, Yueran Ma, and David Thesmar**, “Overreaction and Working Memory,” Technical Report w27947, National Bureau of Economic Research October 2020.

**Amengual, Dante and Mark W Watson**, “Consistent Estimation of the Number of Dynamic Factors in a Large N and T Panel,” *Journal of Business & Economic Statistics*, January 2007, *25* (1), 91–96.

**Angeletos, George-Marios, Zhen Huo, and Karthik A. Sastry**, “Imperfect Macroeconomic Expectations: Evidence and Theory,” *NBER Macroeconomics Annual*, 2021, *35* (1), 1–86. Publisher: University of Chicago Press.

**Bai, Jushan and Peng Wang**, “Identification and Bayesian Estimation of Dynamic Factor Models,” *Journal of Business & Economic Statistics*, 2015, *33* (2), 221–240. Publisher: [American Statistical Association, Taylor & Francis, Ltd.].

**_ and Serena Ng**, “Determining the Number of Factors in Approximate Factor Models,” *Econometrica*, 2002, *70* (1), 191–221.

**_ and _** , “Determining the Number of Primitive Shocks in Factor Models,” *Journal of Business & Economic Statistics*, 2007, *25* (1), 52–60.

**Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer**, “Extrapolation and bubbles,” *Journal of Financial Economics*, August 2018, *129* (2), 203–227.

**Bernanke, Ben S.**, “Federal Reserve Policy in an International Context,” *IMF Economic Review*, 2017, *65* (1), 5–36. Publisher: [Palgrave Macmillan Journals, International Monetary Fund].

**Bianchi, Francesco and Leonardo Melosi**, “Modeling the Evolution of Expectations and Uncertainty in General Equilibrium,” *International Economic Review*, 2016, *57* (2), 717–756.

**_ , Sydney C. Ludvigson, and Sai Ma**, “Belief Distortions and Macroeconomic Fluctuations,” *American Economic Review*, July 2022, *112* (7), 2269–2315.

**Blanchard, Olivier J. and Ben S. Bernanke**, “An Analysis of Pandemic-Era Inflation in 11 Economies,” May 2024.

**Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer**, “Diagnostic Expectations and Credit Cycles,” *The Journal of Finance*, 2018, *73* (1), 199–227. Publisher: [American Finance Association, Wiley].

**_ , _ , and _** , “Overreaction and Diagnostic Expectations in Macroeconomics,” *Journal of Economic Perspectives*, August 2022, *36* (3), 223–244.

— , — , **Yueran Ma, and Andrei Shleifer**, "Overreaction in Macroeconomic Expectations," *American Economic Review*, September 2020, *110* (9), 2748–2782.

**Borio, Claudio E. V. and Andrew J. Filardo**, "Globalisation and Inflation: New Cross-Country Evidence on the Global Determinants of Domestic Inflation," May 2007. Place: Rochester, NY Type: SSRN Scholarly Paper.

**Branch, William A. and Bruce McGough**, "Chapter 1 - Heterogeneous Expectations and Micro-Foundations in MacroeconomicsWe are grateful to the editors, Cars Hommes and Blake LeBaron, as well as Guillaume Rocheteau and three anonymous referees for many helpful comments.," in Cars Hommes and Blake LeBaron, eds., *Handbook of Computational Economics*, Vol. 4 of *Handbook of Computational Economics*, Elsevier, January 2018, pp. 3–62.

**Camara, Santiago, Lawrence Christiano, and Hüsnü Dalgic**, "The International Monetary Transmission Mechanism," in "NBER Macroeconomics Annual 2024, volume 39," University of Chicago Press, July 2024.

**Casella, George and Edward I. George**, "Explaining the Gibbs Sampler," *The American Statistician*, August 1992, *46* (3), 167.

**Chan, Joshua, Gary Koop, Dale J. Poirier, and Justin L. Tobias**, "Bayesian Econometric Methods," August 2019. Publication Title: Higher Education from Cambridge University Press.

**Ciccarelli, Matteo and Benoît Mojon**, "GLOBAL INFLATION," *The Review of Economics and Statistics*, 2010, *92* (3), 524–535.

**Coibion, Olivier and Yuriy Gorodnichenko**, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, August 2015, *105* (8), 2644–2678.

**Croushore, Dean and Tom Stark**, "A real-time data set for macroeconomists," *Journal of Econometrics*, 2001, *105* (1), 111–130. Publisher: Elsevier.

— **and** — , "A Real-Time Data Set for Macroeconomists: Does the Data Vintage Matter?," *The Review of Economics and Statistics*, 2003, *85* (3), 605–617.

**Eusepi, Stefano and Bruce Preston**, "Expectations, Learning, and Business Cycle Fluctuations," *American Economic Review*, October 2011, *101* (6), 2844–2872.

**Evans, George W. and Seppo Honkapohja**, *Learning and Expectations in Macroeconomics*, Princeton University Press, 2001.

**Farmer, Leland E., Emi Nakamura, and Jón Steinsson**, "Learning about the Long Run," *Journal of Political Economy*, October 2024, *132* (10), 3334–3377. Publisher: The University of Chicago Press.

**Fernandez, Adriana, Evan F. Koenig, and Alex Nikolsko-Rzhevskyy**, "Real-Time Historical Dataset Enhances Accuracy of Economic Analyses, Volume 7, Number 6," July 2012.

**Fuster, Andreas, David Laibson, and Brock Mendel**, "Natural Expectations and Macroeconomic Fluctuations," *Journal of Economic Perspectives*, December 2010, *24* (4), 67–84.

**Gabaix, Xavier**, "A Sparsity-Based Model of Bounded Rationality *," *The Quarterly Journal of Economics*, November 2014, *129* (4), 1661–1710.

**Galí, Jordi and Mark Gertler**, *International Dimensions of Monetary Policy*, University of Chicago Press, 2010.

**Giovanni, Julian Di, Şebnem Kalemli-Özcan, Alvaro Silva, and Muhammed A. Yıldırım**, "Pandemic-Era Inflation Drivers and Global Spillovers," Staff Reports (Federal Reserve Bank of New York), Federal Reserve Bank of New York November 2023. Series: Staff Reports (Federal Reserve Bank of New York).

**Greenwood, Robin and Andrei Shleifer**, "Expectations of Returns and Expected Returns," *The Review of Financial Studies*, 2014, *27* (3), 714–746. Publisher: [Oxford University Press, The Society for Financial Studies].

**Grossman, Gene M. and Elhanan Helpman**, "Globalization and Growth," *American Economic Review*, May 2015, *105* (5), 100–104.

**Ha, Jongrim, M. Ayhan Kose, and Franziska Ohnsorge**, "One-stop Source: A Global Database of Inflation," July 2021.

**Kohlhas, Alexandre N. and Ansgar Walther**, "Asymmetric Attention," *American Economic Review*, September 2021, *111* (9), 2879–2925.

**Korenok, Oleg, David Munro, and Jiayi Chen**, "Inflation and Attention Thresholds," *The Review of Economics and Statistics*, November 2023, pp. 1–28.

**Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman**, "International Business Cycles: World, Region, and Country-Specific Factors," *American Economic Review*, September 2003, *93* (4), 1216–1239.

**Labonte, Marc and Lida R. Weinstock**, *Inflation in the U.S. economy: causes and policy options* number R47273. In 'CRS report.', Washington, District of Columbia: Congressional Research Service, 2022.

**Li, Thomas H., Juan M. Londono, and Sai Ma**, "The Global Transmission of Inflation Uncertainty," January 2025.

**Malmendier, Ulrike and Stefan Nagel**, "Depression Babies: Do Macroeconomic Experiences Affect Risk Taking?*," *The Quarterly Journal of Economics*, February 2011, *126* (1), 373–416. Publisher: Oxford Academic.

_ **and** _ , "Learning from Inflation Experiences *," *The Quarterly Journal of Economics*, February 2016, *131* (1), 53–87.

**Mankiw, N. Gregory and Ricardo Reis**, "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *The Quarterly Journal of Economics*, November 2002, *117* (4), 1295–1328.

_ **and** _ , "Chapter 5 - Imperfect Information and Aggregate SupplyWe are grateful to students at Columbia University and Faculdade de Economia do Porto for sitting through classes that served as the genesis for this survey, and to Stacy Carlson, Benjamin Friedman, John Leahy, and Neil Mehrotra for useful comments.," in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3, Elsevier, January 2010, pp. 183–229.

_ **,** _ **, and Justin Wolfers**, "Disagreement about Inflation Expectations," *NBER Macroeconomics Annual*, January 2003, *18*, 209–248. Publisher: The University of Chicago Press.

**Maćkowiak, Bartosz and Mirko Wiederholt**, "Optimal Sticky Prices under Rational Inattention," *The American Economic Review*, 2009, *99* (3), 769–803.

**Mincer, Jacob A. and Victor Zarnowitz**, "The Evaluation of Economic Forecasts," in "Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance," NBER, 1969, pp. 3–46.

**Miranda-Agrippino, Silvia and Hélène Rey**, "U.S. Monetary Policy and the Global Financial Cycle," *Review of Economic Studies*, 2020, *87* (6), 2754–2776.

**Otrok, Christopher and Charles H. Whiteman**, "Bayesian Leading Indicators: Measuring and Predicting Economic Conditions in Iowa," *International Economic Review*, 1998, *39* (4), 997–1014. Publisher: [Economics Department of the University of Pennsylvania, Wiley, Institute of Social and Economic Research, Osaka University].

**Pfäuti, Oliver**, "Inflation—Who Cares? Monetary Policy in Times of Low Attention," *Journal of Money, Credit and Banking*, 2024.

**Rogoff, Kenneth**, "Globalization and Global Disinflation," 2003. Paper prepared for the Federal Reserve Bank of Kansas City conference on "Monetary Policy and Uncertainty: Adapting to a Changing Economy" Jackson Hole, WY, August 29, 2003.

**Sargent, Thomas and C.A. Sims**, "Business Cycle Modeling Without Pretending to Have Too Much a Priori Economic Theory," in "New Methods in Business Cycle Research," Federal Reserve Bank of Minneapolis, October 1977, pp. 45–109.

**Sheffrin, Steven M.**, *Rational Expectations*, Cambridge University Press, June 1996.

**Sims, Christopher**, "Implications of Rational Inattention," *Journal of Monetary Economics*, April 2003, *50*, 665–690.

**Souleles, Nicholas S.**, "Expectations, Heterogeneous Forecast Errors, and Consumption: Micro Evidence from the Michigan Consumer Sentiment Surveys," *Journal of Money, Credit and Banking*, 2004, *36* (1), 39–72. Publisher: Blackwell Publishing.

**Stock, James and Mark Watson**, "New Indexes of Coincident and Leading Economic Indicators," NBER Chapters, National Bureau of Economic Research, Inc 1989.

_ **and** _ , *Business Cycles, Indicators and Forecasting*, University of Chicago Press, 1993. Pages: 95-156.

**Stock, James H. and Mark W. Watson**, "Implications of Dynamic Factor Models for VAR Analysis," July 2005.

**Weber, Michael, Bernardo Candia, Hassan Afrouzi, Tiziano Ropele, Rodrigo Lluberas, Serafin Frache, Brent Meyer, Saten Kumar, Yuriy Gorodnichenko, Dimitris Georgarakos, Olivier Coibion, Geoff Kenny, and Jorge Ponce**, "Tell Me Something I Don't Already Know: Learning in Low- and High-Inflation Settings," *Econometrica*, 2025, *93* (1), 229–264.

**Woodford, Michael**, "Imperfect common knowledge and the effects of monetary policy," *Knowledge, information, and expectations in modern macroeconomics : in honor of Edmund S. Phelps*, 2003.