Monetary Policy, the Yield Curve, and the Repo Market

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ABSTRACT

This paper develops a preferred-habitat theory of the yield curve and the repo market that regards bonds as both investment assets and collateral. Habitat preferences for specific bonds introduce price differences between bonds with identical cash flows and generate special repo rates, namely collateralized borrowing rates significantly below the riskless rate. Special repo rates reduce arbitrageurs' short-selling activity, thus influencing their portfolio duration, the market price of interest rate risk, and the entire yield curve, over and above the valuation of specific bonds. This effect is consistent with the empirical evidence. Monetary policy recommendations are derived and illustrated by calibration.

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1 Introduction

Financial economists have long described the yields to maturity of riskless bonds as driven by the current interest rates in the money market and by a premium attached to the risk that such rates might change in the future. Traditionally, this characterization has emphasized the role of bonds as investment assets with varying maturities, priced in accordance with the exogenous dynamics of a unique money market interest rate. Following this approach, most theories of the yield curve have placed relatively less emphasis on the role of bonds as collateral in the market for repurchase agreements (repo), an important segment of the money market where interest rates reflect the quality of the collateral.[1]

The dual function of bonds—as investment assets as well as collateral—is key to interpreting several recent market episodes. One example is the European Central Bank (ECB)'s Asset Purchase Programme (APP), which exerted downward pressure on the yield curve. In this process, the ECB accumulated substantial holdings of specific Eurozone Treasury bonds, driving collateralized borrowing rates in the repo market to record lows. Thus, the APP had a considerable impact both on the yield curve and in the repo market.[2] The interconnection of these markets raises an important question: How does the yield curve interact with the repo market, where bonds serve as collateral for loans?

This paper presents a theory that integrates the yield curve and the repo market in which bonds serve both as investment assets and as collateral in repo agreements. Our theory operates under two main assumptions. First, demand forces generate price differences among bonds with identical cash flows by inducing a spread between the repo rates of generic and specific securities, known as repo specialness (Duffie, 1996). Second, through their influence on the market price of interest rate risk, demand forces affect the transmission of shocks across the yield curve (Vayanos and Vila, 2021). The resulting equilibrium provides a comprehensive characterization of bonds in terms of their yields and repo rates, both of which are determined endogenously by the workings of demand and supply forces.

The contribution of this paper is to show that repo specialness sheds light on the entire yield curve, above and beyond the valuation of specific bonds. We begin by reporting motivating evidence that aggregate repo specialness is positively associated with the yield spread between long- and short-term bonds. We then present a theory which accounts for the effects of repo specialness on the market price of interest rate risk. Our theory is consistent with the observation that bonds acquire special collateral value in the repo market when they are subject to short-selling pressure by arbitrageurs. Short-selling exposes arbitrageurs to interest rate risk, and their required compensation for risk shapes the yield curve. Thus, a broader message of the paper is that yield curve movements cannot always be understood solely in terms of the stance of monetary policy and the exogenous dynamics of a unique money market rate, but that the collateral specialness of bonds in the repo market is also a contributing element.

[1] The repo contract is a secured loan that consists of the spot sale of a bond combined with a forward agreement to repurchase the bond on a future day. The average daily volume of repo transactions outstanding globally, according to the Bank for International Settlements, is about $12 trillion, which hovers around 10% of the world's gross domestic product.

[2] See Arrata et al. (2020) and Corradin and Maddaloni (2020).

Our theory considers the valuation of riskless zero coupon bonds in capital and money markets. Interest rate risk is driven by the exogenous dynamics of the secured overnight financing rate. There are two groups of agents, preferred-habitat investors and risk-averse arbitrageurs. Preferred-habitat investors exhibit a downward-sloping demand for bonds with particular tenor and specific characteristics. An example of these habitat preferences is given by asset purchase programs, which are directed toward eligible bonds of various tenors, while excluding ineligible bonds of the same tenor. Another example of these habitat preferences is given by fixed-income funds targeting bonds with particular tenor and specific characteristics, such as index inclusion, green labels, or compliance with Islamic finance principles. The habitat preferences for specific characteristics introduce the possibility of differences in the yields to maturity and collateral values among bonds with identical cash flows.[3]

Arbitrageurs transmit the demand of preferred-habitat investors for specific bonds to the repo market and across the entire yield curve, by engaging in two investment strategies. First, they invest in convergence trade strategies that profit from the price difference between bonds with identical cash flows. Arbitrageurs' convergence trades ensure consistency between the price of a bond and its collateral value in the repo market. Second, arbitrageurs engage in risky carry trade strategies that profit from the misalignment between bonds' yields to maturity and the expected series of short-term interest rates in the repo market. Arbitrageurs' carry trades discipline the equilibrium relationship between the price of bonds of different tenors by determining the market price of interest rate risk. In implementing these investment strategies, arbitrageurs pay attention to repo rates. This is because arbitrageurs borrow bonds in the repo market and sell them outright to establish short positions, and vice versa to establish long positions. Notably, in clearing the bond market of the demand from preferred-habitat investors, arbitrageurs consistently assume short positions; consequently, arbitrageurs become exposed to risk by short-selling bonds. As arbitrageurs acquire greater exposure to risk, bonds face intensified short-selling pressure and a correspondingly stronger demand as collateral in the repo market. Simultaneously, the market price of interest rate risk in the bond market adjusts to compensate arbitrageurs for their greater risk exposure. Arbitrageurs thus forge a strong connection between the demand for collateral in the repo market and the pricing of risk across the entire yield curve.

In the capital market, arbitrageurs are crucial for the valuation of bonds. In the money market, arbitrageurs present a price-inelastic demand for collateral, as they require it to cover short positions. The degree to which preferred-habitat investors supply bonds as collateral in the repo market is thus crucial for equilibrium determination in both the bond and the repo markets. In the limiting case of preferred-habitat investors making their portfolio holdings entirely available in the repo market, the differences in the price and collateral repo rate between bonds with equivalent cash flows vanish and our theory converges to the framework of Vayanos and Vila (2021). In its general form, our theory contributes to endogenizing the money market and to address important market features, such as the presence of bonds with special collateral value (Duffie, 1996) and of price differences between bonds

[3]Cornell and Shapiro (1989) were among the first to document price differentials between bonds with identical cash flows.

with identical cash flows (Krishnamurthy, 2002). Adopting a preferred-habitat perspective uniquely positions our paper to evaluate the impact of repo specialness on the portfolio holdings of arbitrageurs and thus on the valuation of bonds across the yield curve. The main theoretical prediction of our paper is that the aggregate specialness of bonds in the repo market is positively associated with the term spread (the yield spread between long- and short-term bonds) across the entire yield curve, consistent with the evidence we document.

A unified theory of the yield curve and the repo market offers a powerful lens for conducting counterfactual analyses of quantitative monetary policy interventions. Existing theories of the transmission of monetary policy to the yield curve generally overlook its effect in the repo market; this omission is particularly consequential for central banks' asset purchases, which induce repo specialness in the targeted bonds (Corradin and Maddaloni, 2020). Our theory shows that, by inducing repo specialness, asset purchases generate a stronger impact on targeted securities and a more limited transmission to the broader yield curve. This prediction is consistent with the evidence in Lucca and Wright (2024), who document that in Australia, narrow asset purchases known as Yield Curve Control (YCC) affected only the class of bonds targeted by the central bank, generating local supply effects (i.e., price responses that are concentrated in the specific securities subject to demand shocks) without a global supply effect (i.e., changes in the pricing of a broad range of maturities along the yield curve).

Policymakers may draw on our model to control the strength of the local and global supply effects of their asset purchases, which are shaped by the degree of repo specialness. In equilibrium, repo specialness arises when bond purchases are withheld from the repo market. Policymakers aiming to generate global supply effects on the entire yield curve should supply their bonds on the repo market, to enable arbitrageurs to transmit the effect of asset purchases across the yield curve. Policymakers may also want asset purchases to selectively influence the price of specific bonds; for example, green bonds. Asset purchase of green bonds affect the price of these bonds relative to comparable ones only if their effects are strongly localized. To achieve this effect, policymakers can withhold the purchased bonds from the repo market, thereby enhancing their scarcity and allowing them to acquire special collateral value and trade at a premium relative to other bonds. This paper thus recommends that monetary policy be implemented in a coordinated manner across the bond and repo markets, as these are markets for the same securities.

The findings of this paper tie in well with the literature on the effects of demand forces on the yield curve (D'Amico and King, 2013; Greenwood and Vayanos, 2014). The canonical framework for this literature is found in Vayanos and Vila (2021), and provides the analytical structure to harmonize observed empirical findings with the received preferred-habitat theory (Culbertson, 1957; Modigliani and Sutch, 1966). Recent literature has integrated the preferred-habitat theory with macroeconomics (Ray et al., 2024; Jansen et al., 2024), the foreign exchange market (Greenwood et al., 2023; Gourinchas et al., 2025), the credit market (Costain et al., 2025), the interest rate swaps market (Hanson et al., 2024), and the mortgage-backed securities market (Malkhozov et al., 2016). In this paper, we do not extend the preferred-habitat theory to a new asset class, but rather to a different function of bonds: their role as collateral in the repo market. In a related inquiry, He et al. (2022) examine market dislocations in US Treasury bonds driven by flight-to-safety and flight-to-liquidity during crises such as the COVID-19 pandemic. In their model, intermediation frictions such as dealers' regulatory balance sheet constraints induce a spread between the overnight-index swap (OIS) rate and the General Collateral (GC) repo rate. Our paper differs in many respects, as it abstracts from intermediaries' balance sheet constraints and centers around the Special Collateral (SC) segment of the repo market, which is entirely absent from their paper. The SC segment of the repo market is an extremely important source of collateral for short sellers and warrants separate attention, as it exhibits a multiplicity of special repo rates that vary at the tenor and instrument level, generate price differentials among bonds with identical cash flows (Krishnamurthy, 2002), and respond endogenously to demand forces (D'Amico et al., 2018). SC repo transactions represent 87% of the repo market daily volume in the Euro Area (Arrata et al., 2020), and comprise the bulk of the bilateral US repo market, which accounts for 60% of the total daily repo volume in the US (Copeland et al., 2014). A key contribution of our paper is to uncover a novel asset pricing mechanism: through their impact on the arbitrageurs' portfolio, SC repo rates are positively associated with term spreads at all maturities and affect the transmission of monetary policy.

Methodologically, our point of departure from the preferred-habitat literature is to concentrate on habitat preferences for special bonds within maturity buckets, rather than for bonds with specific maturities.[4] Habitat preferences for special bonds can be motivated by investment mandates and liquidity considerations (Pasquariello and Vega, 2009). These habitat preferences induce differentials between otherwise equivalent securities and connect our analysis to Vayanos and Weill (2008), who consider a steady-state economy with search frictions and two assets paying identical cash flows. We abstract from search frictions and complement their stationary equilibrium from a dynamic perspective, which has the advantage of allowing for yield curve considerations.

In a foundational paper, Duffie (1996) shows that bond prices and the interest rate on the loans they collateralize are connected by an arbitrage restriction and demonstrates that the collateral value of bonds increases as arbitrageurs intensify their demand for collateral to sell a bond short. Other contributions in this area include Jordan and Jordan (1997), Buraschi and Menini (2002), Fisher (2002), Copeland et al. (2014), Martin et al. (2014), Mancini et al. (2016), and Roh (2022). D'Amico and Pancost (2022) estimate an affine econometric model of the yield curve that jointly accounts for the prices of Treasury securities and the corresponding repo rates. Their evidence from US markets highlights repo specialness as a risk factor that can explain asset pricing inconsistencies between bonds having similar cash flows. Our paper differs from theirs in two key aspects. First, we show that, in European markets, repo specialness not only explains price inconsistencies between bonds

[4]Naturally, bonds differ on many dimensions other than maturity. For example, Chen et al. (2022) study clientele effects on bond prices and repo rates in the context of Islamic bonds, and D'Amico et al. (2022) focus on Green premia, the yield differences between maturity-matched conventional and Green bonds.

with similar cash flows but also the relative pricing of bonds having *different* cash flows and maturity profiles, emphasizing its role as an explanatory variable for term spreads along the entire yield curve. Second, we rationalize this evidence through a structural model that derives the pricing of risk from first principles, rather than specifying the stochastic discount factor exogenously, allowing for a comprehensive examination of the equilibrium effects of special repo rates. Our theory, which we view as our main contribution, shows that special repo rates reduce arbitrageurs' short-selling activity, thus influencing their portfolio duration, the market price of interest rate risk, and the entire yield curve, over and above the valuation of specific bonds. To our knowledge, our paper is the first to present a dynamic equilibrium asset pricing theory formalizing the broader effect of special repo rates on the pricing of risk in the bond market and considering its implications for the transmission of monetary policy.

The remainder of the paper is organized as follows. Section 2 presents the motivating evidence. Section 3 develops a theory of the yield curve that integrates the bond and repo markets. Section 4 discusses its implications for monetary policy. Section 5 provides a calibration of the model to market data. Section 6 offers concluding remarks. All proofs are available in the Appendix.

2 Motivating Evidence

How does the yield curve interact with the repo market, where bonds serve as collateral for loans? In this section, we explore this question from an empirical perspective. For consistency with our theoretical framework, the empirical examination requires a large preferred-habitat investor operating in a market where repo specialness is both quantitatively significant and persistent. Accordingly, we analyze data from the German Treasury bond market during the ECB's APP period. The recognized status of German bonds (Bunds) as safe assets renders them some of the most valuable collateral in global markets (see, e.g., Mancini et al., 2016). Moreover, in Europe, bonds are issued "on tap," and repo specialness often persists throughout a bond's lifetime. Against this institutional background, the ECB's APP period provides a natural empirical counterpart to our preferred-habitat model of the yield curve and the repo market.

Our sample comprises the universe of Bunds between October 2014, the commencement of the APP, and July 2023, when the ECB ceased reinvesting redemptions from its APP portfolio.[5] Bond-level variables are obtained at the daily frequency from LSEG. From the universe of Bunds, we exclude inflation-linked securities and those for which fewer than 100 observations are available. We further restrict the sample to bonds with a duration between 0.1 and 30 years to ensure data quality. Bond yields to maturity data incorporate accrued interest and exhibit no discontinuities when coupons are paid. For each bond, we also obtain the daily volume-weighted average overnight repo rate by using tick-by-tick data recorded by BrokerTec, the largest electronic repo platform for German

[5] Source: https://www.ecb.europa.eu/mopo/implement/app/html/index.en.html.

Treasury bonds. Bond data are then matched with repo rate data at the ISIN level.

In our granular dataset, the main variables of interest are bond yields to maturity and repo rates, both of which are winsorized at the 1^{st} and 99^{th} percentiles to reduce the influence of outliers and quarter-end spikes. From an institutional perspective, repo contracts can be classified into two categories. For GC repo contracts, the primary purpose is to secure funding, and a range of bonds can be used interchangeably as collateral. For SC repo contracts, the lender seeks a specific bond, often to cover a short position. Competition for special bonds drives down the interest rate cash borrowers must pay, allowing owners of these bonds to borrow at more favorable terms, and enjoy a "repo specialness" remuneration that varies with the relative demand and supply for that particular bond. Empirically, we measure the repo specialness of the j^{th} bond on day t, Repo Specialness$_{jt}$, as the difference between the German GC repo rate index, GC Interest Rate$_t$, and the bond-specific volume-weighted repo rate, Interest Rate$_{jt}$.

$$\text{Repo Specialness}_{jt} = \text{GC Interest Rate}_t - \text{Interest Rate}_{jt}. \tag{1}$$

For each trading day, we compute a measure of aggregate specialness as the average across bonds of the product of bond duration and repo specialness,

$$\text{Aggregate Specialness}_t = \frac{1}{N_t}\sum_j D_{jt}\text{Repo Specialness}_{jt}, \tag{2}$$

where D_{jt} denotes the modified duration of bond j at time t, and N_t the number of valid securities at time t. As we will discuss in Section 3.10, the construction of this measure is guided by our theory.

For illustration, consider the Bund with ISIN DE0001135440. On October 1, 2014, the first day of our sample, this Bund has a modified duration of 6.15 years. As of this date, its total amount issued is €6 billion, its yield to maturity is 0.34%, and its annualized overnight repo rate is -0.20%. On the same day, the GC repo rate for German Bunds is -0.06%, implying a repo specialness of 0.14% for this particular Bund. For that day, the contribution of this bond to the aggregate specialness measure is $6.15 \cdot 0.14\% = 0.86\%$, while the value of the aggregate specialness measure across all bonds is 0.23%.

Following standard practices, we construct aggregate measures of yields and repo rates across duration buckets to capture the systematic relationship between bond maturity and market rates. The first bucket comprises bonds with durations between 0.75 and 1.25 years and serves as the benchmark for the 1-year segment of the curve. We construct six additional duration buckets, covering maturities from 2 to 30 years. For each bucket, we compute the yield spread relative to the 1-year segment of the curve, Term Spread$_t^\tau$, defined as the difference between the average yield in the τ-year bucket and the average yield in the 1-year bucket. We conduct this procedure using both daily and weekly data. The final dataset comprises 180 unique bond ISINs and 94,142 observations spanning 1,956 unique trading days over 452 unique weeks.

Summary statistics are reported in Table I. The panel is unbalanced, as some bonds trade throughout the full sample period, while others do not. The average bond duration is 7.2 years, with a standard deviation of 5.8 years, reflecting substantial cross-sectional dispersion across bonds that remains relatively stable over time.[6] Repo specialness is quantitatively significant, with a mean of 10 basis points and a standard deviation of 12 basis points. The aggregate specialness measure, which weights bond-level specialness by duration, has an average value of 61 basis points and a standard deviation of 49 basis points, reflecting a stronger contribution of long-duration bonds' repo specialness.

We examine this panel dataset to analyze the relationship between term spreads and aggregate repo specialness among bonds within different duration buckets. Figure 1 displays the time series of these variables alongside the GC interest rate. The figure suggests a strong positive correlation between the aggregate repo specialness and the term spread, statistically and economically significant across all buckets, peaking at 0.40 for the $[7{,}10)$ duration bucket even in unsmoothed data. As we discuss in Section 3.11, this feature of the data align with our theory. To formalize the statistical analysis, we follow the approach of Greenwood and Vayanos (2014) and estimate the following time series regression model, separately by duration bucket:

$$\text{Term Spread}_t^{\tau} = \beta_0 + \beta_1 \text{ Aggregate Specialness}_t + \varepsilon_t^{\tau}. \tag{3}$$

Table II presents the results from estimating this specification using weekly data. The data reveal a strong pattern of positive correlation between aggregate repo specialness and the term spread across all duration buckets, with the effect being generally more pronounced for higher duration buckets. Quantitatively, a one-standard deviation increase in aggregate specialness is associated with a $0.363 \cdot 0.49\% = 0.18\%$ increase of the term spread for the $[7{,}10)$ duration bucket, which is about one third of its unconditional average of 0.59%. This striking pattern cannot be attributed to variation in the average duration of outstanding bonds, which remains nearly constant in the time series of our sample. This finding is not easy to reconcile with existing equilibrium financial or macroeconomic theories of the yield curve that abstract from the repo market.[7]

To ensure robustness, we repeat the estimation in Panels B and C of Table II, separately by subsamples of bonds with high and low specialness. Specifically, bonds are classified as general if their repo specialness is below the cross-sectional median for that week, and as special if above.[8] The regression estimates remain strongly robust across these subsamples. Table III repeats the estimation at the daily frequency, with nearly identical estimates further confirming the robustness of the results. Unreported results confirm that the findings are robust to the duration buckets grouping and the specialness classification, and are not driven by any particular time period. Furthermore, this relationship is not

[6]The cross-sectional average duration of bonds remains stable over time, with a time-series standard deviation of only 0.34.

[7]Standard yield curve theories emphasizes the importance of macroeconomic factors, interest rate expectations, and market segmentation (Gürkaynak and Wright, 2012), and, more recently, demand and supply in the bond market (Greenwood and Vayanos, 2014); to our knowledge, none highlights the role of collateral specialness in the repo market.

[8]A statistical classification is necessary, as a bond may trade as both GC and SC within the same day in tick-by-tick data.

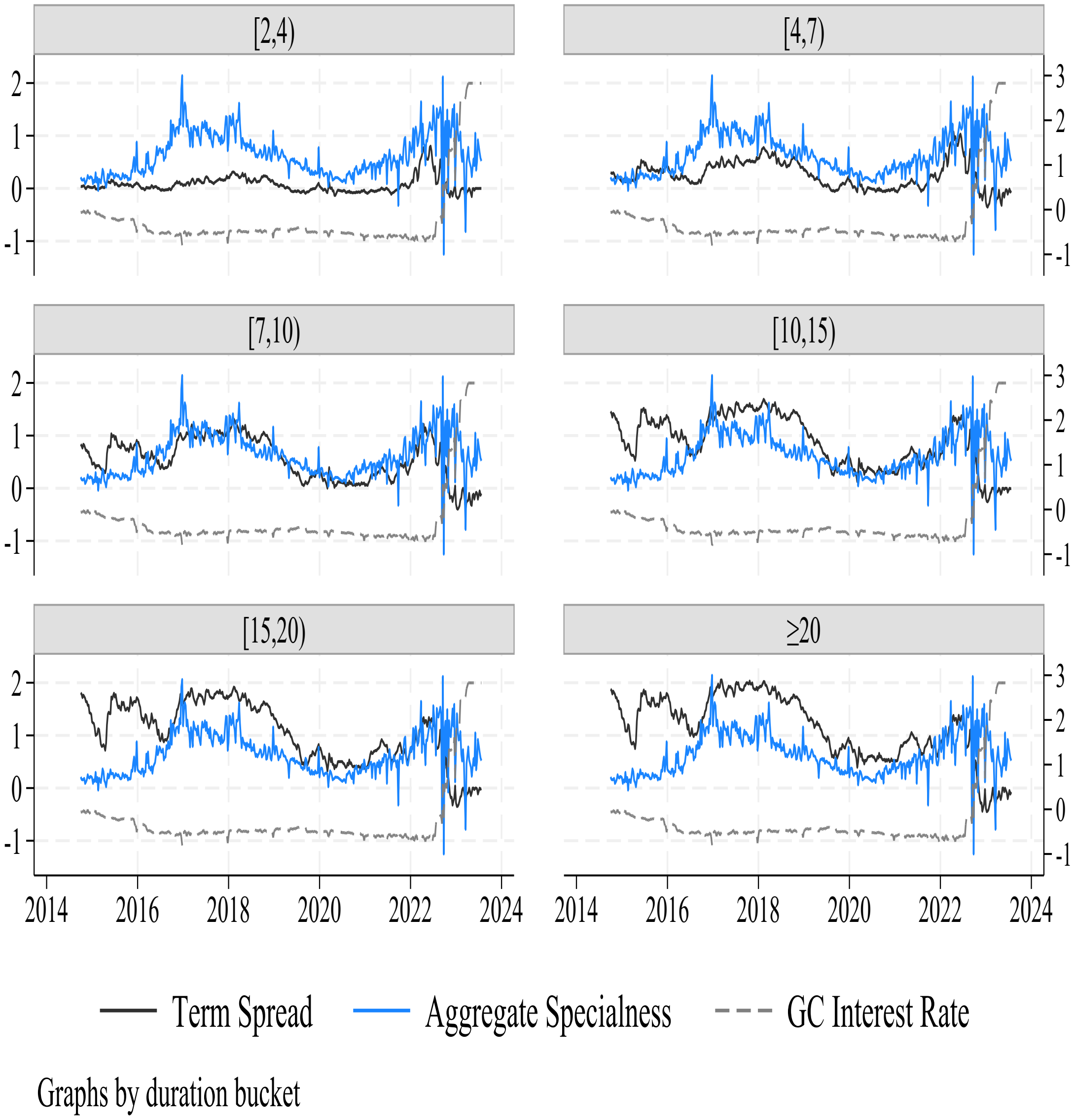

FIGURE 1: **Term spreads correlate with aggregate repo specialness.** This figure illustrates the term spreads of Bunds across several duration buckets (left y-axis), the aggregate repo specialness measure (left y-axis), and the German general collateral interest rate (right y-axis). Both y-axes are expressed in percentage points per annum. This figure is based on German Treasury weekly data from LSEG covering the period from October 2014 to July 2023.

a mechanical consequence of weighting bonds by their duration in constructing the aggregate specialness measure, since the measure explains term spreads across the entire maturity spectrum, not only at the long end of the curve. Overall, the data show a remarkably robust positive correlation between the aggregate repo specialness measure and term spreads across every segment of the yield curve.[9]

[9]This finding is consistent with Hu et al. (2013), who document empirically that price differences between bonds with equivalent cash flows help explaining term spreads. While our main contribution is theoretical, our empirical approach

We are not aware of any other paper that provides evidence that, above and beyond the effect of a bond's repo specialness on its own yield (see, e.g., Duffie, 1996), the *aggregate* repo specialness is systematically associated with term spreads across the entire yield curve. In terms of the underlying economic mechanism, term spreads reflect both expectations of future short rates and a risk premium. However, it remains implausible that the collateral value of bonds in repo agreements with overnight maturity contains information regarding future short rates. Moreover, as visible in Figure 1, variation in aggregate specialness does not predict changes in GC interest rates. Thereby, the novel empirical regularity we document strongly suggests that aggregate conditions in the repo market correlate with the risk premium of bonds in the capital market. Below, we develop a unified theory of the yield curve and the repo market to interpret this finding and to construct counterfactual scenarios for policy analysis.

3 The Model

The model is formulated in continuous time, denoted by t, and extends over an infinite horizon. It considers the valuation of riskless zero coupon bonds in a setting with two markets, the bond and the repo market, and two agents, preferred-habitat investors and arbitrageurs.

3.1 Bonds

Riskless zero-coupon Treasury bonds are indexed by their tenor, τ, and by their status, i. Bond status is either general, if $i=g$, or special, if $i=s$. General and special bonds of the same tenor have equivalent cash flows, but their prices can differ because of the demand effects of preferred-habitat investors detailed below. The bond price is denoted by P_{it}^{τ}. The continuously compounded yield to maturity is

$$y_{it}^{\tau} = -\frac{1}{\tau} \log P_{it}^{\tau}. \tag{4}$$

The short rate, r_t, is the limit of the yield to maturity when τ goes to zero. We take r_t as exogenous, and assume it follows a Vasicek process whose parameters have the usual interpretation.[10]

$$dr_t = \kappa_r (\overline{r} - r_t) dt + \sigma_r dv_t^r. \tag{5}$$

3.2 Repos

Repos are contracts where bonds are used as collateral to obtain overnight financing compensated at a repo rate.[11] The repo rate of a bond with status i and maturity τ is denoted by r_{it}^{τ}. The repo market

emphasizes that the repo market can be directly used to explain term spreads along the entire yield curve.

[10]The choice of a Gaussian innovations is standard. For a discussion of non-Gaussian models, see Berardi et al. (2021).

[11]We focus on overnight repo transactions, which attract by far the dominant proportion of volume, as the modeling of term repos would require the introduction of an additional index. As is standard in this literature, we abstract from collateral re-

is segmented and offers two distinct opportunities for investing cash, depending on whether the bond pledged by the counterparty as collateral is classified as general or special:

1. Receive a general bond ($i = g$) by entering an agreement that earns the GC repo rate, $r_{gt}^{\tau} = r_t$.

2. Receive a special bond ($i = s$), which is in elastic supply, and earn the SC repo rate, $r_{st}^{\tau} < r_t$.

Accordingly, the exogenous short-rate process in Equation (5) should be interpreted as describing the GC repo rate dynamics—for instance, the SOFR. By contrast, SC rates vary endogenously across tenors and over time, and are to be determined in equilibrium. While the GC secures a higher interest rate, GC contracts are not used for short-selling, as the cheapest-to-deliver option resides with the counterparty borrowing the cash. Thereby, arbitrageurs might want to borrow the SC bonds needed to meet any pending short-selling commitments, even at the cost of foregoing returns on their cash. As we will show in Section 3.9, the spread between GC and SC repo rates does not generate any arbitrage opportunities.

What makes special repo rates challenging to model by using exogenous short rate dynamics? The defining feature of special repo rates is their exposure to demand forces (Duffie, 1996). From a theoretical perspective on the yield curve, there is simply no room for demand pressure to directly impact the exogenously specified short rate process in Equation (5). Instead, in our proposed theory, the demand forces that affect bond prices contribute to the endogenous determination of special repo rates.

3.3 Preferred-Habitat Investors

Preferred-habitat investors such as central banks exhibit habitat preferences, which we allow to be a function of tenor, for bonds with specific characteristics. We define as *special* those bonds that are targeted by preferred-habitat investors, and index them through $i = s$; for instance, securities eligible for QE may be considered special. Conversely, we refer to bonds of all maturities for which the excess demand is permanently zero as general, and index them through their status $i = g$; one example is bonds that are not targeted by asset purchases. The demand of preferred-habitat investors is expressed net of the size of the issue supplied by the government, which is normalized to zero, without loss of generality. Building on Vayanos and Vila (2021), we define the excess demand Z_{it}^{τ} for bond i with tenor τ by

$$Z_{it}^{\tau} = \begin{cases} -\eta_{\tau} \log P_{it}^{\tau} - \theta_{\tau} & i = s, \\ 0 & i = g. \end{cases} \tag{6}$$

The demand slope, η_{τ}, and intercept, θ_{τ}, are constant over time but can depend on maturity, an assumption that could be relaxed without affecting the economic mechanisms at work. Under normal

hypothecation and repo haircuts. Collateral rehypothecation does not affect the mechanisms at work, so long as collateral reuse is partial, as is well understood empirically. An extension that incorporates repo haircuts is available upon request.

market conditions, Equation (6) describes the preferences of private investors for specific securities within maturity segments.[12] These habitat preferences are consistent with the empirical evidence that Treasury convenience premia exhibit discontinuities at specific annual maturities induced by clientele effects unrelated to fundamentals (Fleckenstein and Longstaff, 2024). In the context of QE, this formulation captures the purchases by central banks of targeted bonds relative to non-targeted bonds.

3.4 Arbitrageurs

Arbitrageurs such as hedge funds rely on repo financing to engage in trades that smooth out price differences which would otherwise arise in a segmented market equilibrium. Arbitrageurs establish a long (short) position by buying (selling) the bond outright in the spot market and finance (cover) that purchase (sale) by using the bond as collateral to enter an overnight (reverse) repo agreement.[13] For example, arbitrageurs would short-sell a bond that is overpriced as a result of the demand of preferred-habitat investors. To this end, arbitrageurs would lend their cash in a repo contract collateralized by that bond, and simultaneously sell the bond outright. At each point in time, arbitrageurs must either close the outright position or roll over the short-term repo contract. Arbitrageurs set their bond positions, denoted by X^{τ}_{it}, to maximize a mean-variance objective over instantaneous changes in their wealth, denoted by W_t.

$$\max_{\{X^{\tau}_{it}\}} \quad \frac{\mathbb{E}_t\left[dW_t\right]}{dt} - \frac{\gamma}{2}\frac{\mathbb{V}_t\left[dW_t\right]}{dt}. \tag{7}$$

Here, γ is a risk aversion parameter. Below, we characterize the arbitrageurs' budget constraint.

$$dW_t = r_t W_t dt + \underbrace{\int_0^{\infty} X^{\tau}_{gt}\left(\frac{dP^{\tau}_{gt}}{P^{\tau}_{gt}} - r_t\right)d\tau}_{\text{General bonds}} + \underbrace{\int_0^{\infty} X^{\tau}_{st}\left(\frac{dP^{\tau}_{st}}{P^{\tau}_{st}} - r^{\tau}_{st}\right)d\tau}_{\text{Special bonds}}. \tag{8}$$

As captured by the first term in dt, arbitrageurs' wealth earns at the instantaneous remuneration rate r_t offered by the GC rate, which is above the SC rates. The first integral on the right side of the equation is the marked-to-market value of the portfolio of general bonds net of their financing cost, the GC rate r_t. The second integral is the marked-to-market value of the portfolio of special bonds net of their financing costs, each represented by the respective SC repo rate r^{τ}_{st}.

Equation (8) is not the standard law of motion of wealth, which arises under the restriction of a unique short rate, $r^{\tau}_{it} = r_t \ \forall\ (i,\tau)$. Conceptually, this formulation differs from the textbook portfolio allocation problem where the short rate represents the opportunity cost of the risky assets. Here, the holdings of arbitrageurs are established by resorting to the repo market for collateralized lending. Unlike an opportunity cost interpretation, short-term interest rates thus represent the cost of the collat-

[12]As an example, private investors such as bond mutual funds often express demand for Green and Islamic bonds.
[13]For a detailed discussion of the repo market's role in facilitating arbitrage, see Fisher (2002).

eralized loan which repos the bond to finance the position, in the spirit of Tuckman and Vila (1992).

3.5 Bond Market Clearing

The bond market clearing condition is

$$Z_{it}^{\tau} + X_{it}^{\tau} = 0. \tag{9}$$

We observe that two dimensions of market clearing must hold at all times: bond demand and supply must coincide for each bond status, and for each bond tenor. On the one hand, because market clearing operates across bond status, the demand for general bonds ($i = g$) does not exceed their supply, arbitrageurs are active in equilibrium only in the market for special bonds ($i = s$). Of course, nothing prevents arbitrageurs from trading general bonds as well, so that in equilibrium these securities would be as profitable as special bonds from their perspective. On the other hand, because market clearing operates across bond tenor, arbitrageurs stand ready to meet the excess demand of preferred-habitat investors by short-selling τ-maturity special bonds and accepting the rollover risk associated with SC reverse repo agreements.[14] Thus, as we show in Section 3.8, higher activity from preferred-habitat investors increases repo specialness by reducing a bond's outstanding float and symmetrically raising arbitrageurs' demand for collateral in the repo market to short-sell the bond.[15]

3.6 General Bonds, Special Bonds

Two bond issues of the same tenor may differ in their exposure to demand pressure. In Treasury markets, bonds on special are regularly observed to be overpriced relative to bonds with identical cash flows. To highlight this distinction in our model, let us conjecture that the price process is exponentially affine in the short rate and in the demand of preferred-habitat investors.

$$-\log P_{it}^{\tau} = A_{\tau} r_t + B_{\tau} X_{it}^{\tau} + C_{\tau} \tag{10}$$

This conjecture, which will be formally verified in Section 3.11, is best interpreted in light of the bond market clearing condition, whereby $X_{st}^{\tau} = \eta_{\tau} \log P_{it}^{\tau} + \theta_{\tau}$ and $X_{gt}^{\tau} = 0$. Equation (10) allows bonds with identical cash flows to trade at different prices as a result of differing demand pressures, adding bond status as an incremental dimension to the models of the yield curve. This representation is our characterization of bond market segmentation. On the one hand, the exposure to the general interest rate, r_t, is common to maturity-matched general and special bonds. On the other hand, demand forces selectively exert a *local* price pressure on the bonds targeted by preferred-habitat investors. Importantly,

[14] As in Vayanos and Vila (2021), arbitrageurs may hedge duration risk by trading bonds of different maturities; in our setup, they could even engage in sophisticated convergence trades that go long a general bond and short a special bond of the same maturity. In both cases, market clearing prevents arbitrageurs from eliminating their aggregate risk exposure.

[15] Our approach is consistent with Banerjee and Graveline (2013), who decompose the premium of special Treasury bonds into higher prices encountered by long investors and increased borrowing costs for short sellers.

as we will show in Section 3.11, demand forces also exert a *global* influence on the equilibrium price of risk and play a central role in determining the pricing coefficients, A_τ, B_τ, and C_τ.

For convenience, we define $B_{i\tau} = B_\tau \mathbb{1}_{[i=s]}$, where $\mathbb{1}_{[i=s]} = 1$ if $i = s$ and 0 otherwise. We then write

$$-\log P_{it}^\tau = a_{i\tau} r_t + b_{i\tau} \theta_\tau + c_{i\tau}. \tag{11}$$

Here, we have used Equations (6) and (9), and defined $a_{i\tau} = \frac{A_\tau}{1+\eta_\tau B_{i\tau}}$, $b_{i\tau} = \frac{B_{i\tau}}{1+\eta_\tau B_{i\tau}}$, and $c_{i\tau} = \frac{C_\tau}{1+\eta_\tau B_{i\tau}}$. Equation (11) results from substituting preferred-habitat investors' demand function, which itself depends on bond price, into the conjectured price process. The coefficients $(a_{i\tau}, b_{i\tau}, c_{i\tau})$ are deterministic functions of bond status and maturity, and given i, they are in a one-to-one mapping with (A_τ, B_τ, C_τ).

3.7 The Optimization Problem of Arbitrageurs

To solve the arbitrageurs' problem, we begin by deriving the dynamics of bond prices through substitution of Equations (5) and (9) into Equation (11).[16]

$$\frac{dP_{it}^\tau}{P_{it}^\tau} = \mu_{it}^\tau dt - a_{i\tau} \sigma_r dv_t^r, \tag{12}$$

$$\mu_{it}^\tau \equiv \dot{a}_{i\tau} r_t + a_{i\tau} \kappa_r (r_t - \bar{r}) + \frac{1}{2} a_{i\tau}^2 \sigma_r^2 + \dot{b}_{i\tau} \theta_\tau + b_{i\tau} \dot{\theta}_\tau + \dot{c}_{i\tau}.$$

The notation $\dot{a}_{i\tau}$ represents the partial derivative of $a_{i\tau}$ with respect to time, t. In the expression above, μ_{it}^τ is the expected return from a bond. The volatility of bond returns is driven by innovations in the short rate, dv_t^r, with exposure that varies across maturities. We observe that Equation (12) describes the returns of both general and special bonds, as the coefficients $a_{i\tau}, b_{i\tau}$, and $c_{i\tau}$ depend on bond status. Substituting Equation (12) into the arbitrageurs' wealth dynamics in Equation (8), we obtain

$$dW_t = \left[W_t r_t + \int_0^\infty X_{gt}^\tau (\mu_{gt}^\tau - r_t) + X_{st}^\tau (\mu_{st}^\tau - r_{st}^\tau) d\tau \right] dt - \left[\int_0^\infty a_{g\tau} X_{gt}^\tau + a_{s\tau} X_{st}^\tau d\tau \right] \sigma_r dv_t^r.$$

Replacing the above expression into Equation (7),

$$\max_{\{X_{it}^\tau\}} W_t r_t + \int_0^\infty X_{gt}^\tau (\mu_{gt}^\tau - r_t) + X_{st}^\tau (\mu_{st}^\tau - r_{st}^\tau) d\tau - \frac{\gamma}{2} \left[\sigma_r^2 \int_0^\infty a_{g\tau} X_{gt}^\tau + a_{s\tau} X_{st}^\tau d\tau \right]^2.$$

The first-order condition (FOC) with respect to the position in the bond with tenor τ and status i is

$$\mu_{it}^\tau - r_{it}^\tau = -a_{i\tau} \lambda_t, \tag{13}$$

[16]To improve readability, only the equations to which there is a subsequent reference are numbered.

where

$$\lambda_t = -\gamma\sigma_r^2 \int_0^\infty a_{g\tau} X_{gt}^\tau + a_{s\tau} X_{st}^\tau d\tau. \tag{14}$$

Equation (13) expresses the arbitrageurs' optimal trade-off between risk and return for each bond. The left-hand side is the instantaneous expected return on a bond, μ_{it}^τ, net of the borrowing rate secured by that bond, r_{it}^τ. The right-hand side is the bond exposure to risk, measured by its sensitivity to the risk factor scaled by the market price of that risk. The risk factor is the instantaneous interest rate, r_t, and the sensitivity of a bond to the interest rate is the coefficient $a_{i\tau}$, which captures the bond duration. The market price of interest rate risk, λ_t, is expressed in Equation (14), and reflects the arbitrageurs' risk aversion multiplied by the exposure of their portfolio returns to interest rate risk.

The FOC closely resembles the familiar no-arbitrage condition underlying yield curve models, with an important distinction: the riskless rate, r_t, is replaced by the cross-section of general and special repo rates, r_{it}^τ. In standard models, the characterization of the yield curve by the absence of arbitrage is usually based on the restriction $r_{it}^\tau = r_t \ \forall \tau$, which cannot address any differences in the collateral quality of bonds. In contrast, our framework explicitly accounts for the function of bonds as collateral, and recognizes that arbitrageurs' optimality conditions apply to both general and special bonds. At all times, arbitrageurs require the FOC to hold uniformly across both maturities, τ, and collateral status, i.

Remark 1. *Arbitrageurs integrate the collateral specialness of bonds into their portfolio choice.*

In the repo market, interest rates vary at the bond level. This observation calls for a refinement of the classical principle whereby one should observe a constant (Sharpe) ratio between expected return of assets in excess of the risk-free rate and their standard deviation. Thus, we must adjust the Sharpe ratio for the bond-specific risk-free rate by replacing r_t with r_{it}^τ. This consideration is natural, once we recognize that special bonds generate an implicit stream of cash flows in the repo market.[17] This refinement follows directly from the arbitrageurs' FOC, from which we also deduce the following result, a useful ingredient for later analysis.

Lemma 1. *Bond yields satisfy the following decomposition.*

$$y_{it}^\tau = \underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^\tau r_{i,t+u}^{\tau-u} du\right]}_{\textit{Expected short rates}} - \underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^\tau a_{i,\tau-u}\lambda_{t+u} du\right]}_{\textit{Risk premium}} - \underbrace{\frac{1}{\tau}\left[\int_0^\tau a_{i,\tau-u}^2 \frac{\sigma_r^2}{2} du\right]}_{\textit{Convexity adjustment}}.$$

Proof. See Appendix A

Lemma 1 is a standard decomposition of bond yields into three terms (Greenwood et al., 2024), formulated in the context of our model. The first term represents expected short rates, which in our model vary with bond tenor and status; the second is a risk premium, capturing bond duration and

[17]This refinement could extend to equity assets, with securities lending rebate rates assuming the role of special repo rates.

expected price of the risk factor; and the third is a convexity adjustment, proportional to the squared bond duration and the volatility of the risk factor.

The equilibrium in the bond market is a set of bond prices such that the market clears and the arbitrageurs maximize their objective function given the demand of the preferred-habitat investors. It can be obtained by replacing the bond market clearing condition (Equation (9)) into the market price of risk (Equation (14)), to then derive the equilibrium bond pricing coefficients and verify the conjecture in Equation (10). We defer a detailed discussion of the market price of risk to Section 3.10, and present the analytical expression for bond prices in Section 3.11. For clarity of exposition, we first characterize the equilibrium in the repo market and its key implications for bond valuation.

3.8 Equilibrium in the Repo Market

3.8.1 Collateral Demand

The demand for bonds in the repo market depends on whether the collateral is classified as GC or SC. GC bonds are not in high demand, as investors have no need to borrow them to cover short-selling positions. By contrast, arbitrageurs present a demand for SC bonds in the repo market; indeed, arbitrageurs short-sell the specific bonds that are targeted by the demand of preferred-habitat investors in the bond market by borrowing them in the repo market. Consequently, arbitrageurs have an outstanding obligation to deliver SC bonds in the repo market, and they present a demand for SC bonds that is inelastic to their repo specialness. These considerations motivate the following specification for the demand for bonds as collateral in the repo market, denoted by D_{it}^{τ}.

$$D_{it}^{\tau} = -X_{it}^{\tau}. \tag{15}$$

In the above equation, the demand for collateral in the repo market depends on whether the bond is sought by arbitrageurs to meet short-selling commitments, $-X_{it}^{\tau}$; by bond market clearing, the portfolio of the arbitrageurs is the opposite of the excess demand of preferred-habitat investors in the capital market, $-X_{it}^{\tau} = Z_{it}^{\tau}$. In the capital market, preferred-habitat investors exhibit zero demand for general bonds ($Z_{gt}^{\tau} = 0$) and a positive excess demand for special bonds ($Z_{gt}^{\tau} = -\eta_{\tau} \log P_{it}^{\tau} - \theta_{\tau}$). Accordingly, in the repo market arbitrageurs exhibit no demand for GC bonds and a positive demand for SC bonds.

3.8.2 Collateral Supply and Securities Lending

The supply of bonds in the repo market depends on whether the collateral is classified as GC or SC, as well as on the bond's repo specialness, l_{it}^{τ}, which is defined as the difference between GC and SC rates of bonds with a given tenor, and endogenously determined by market forces. Namely,

$$l_{it}^{\tau} = r_t - r_{it}^{\tau}. \tag{16}$$

The supply of GC bonds in the repo market is perfectly elastic with respect to repo specialness, as these bonds are substitutable; any positive repo specialness would elicit an unbounded supply response. The supply of SC bonds in the repo market is imperfectly elastic to repo specialness, as these bonds represent a scarce resource; accordingly, an increase in repo specialness induces only a finite supply response. Hence, the holders of SC bonds are entitled to receive a greater compensation to supply additional units of the special security (Duffie, 1996).[18]

The affine term structure specification that we adopt in this paper implies an affine relationship between the supply of bonds in the repo market and their repo specialness; we denote by $\mathcal{E}_i$ the inverse of the elasticity of supply to repo specialness.[19] GC bonds have perfectly elastic supply ($\mathcal{E}_g = 0$), whereas SC bonds exhibit imperfectly elastic supply ($\mathcal{E}_s > 0$). Crucially, preferred-habitat investors may affect the supply of collateral by lending their asset purchases in the repo market; we denote the quantity of their securities lending by $Q_{it}^{\tau} \leq Z_{it}^{\tau}$. The supply of bonds as collateral in the repo market, denoted by S_{it}^{τ}, is given by

$$S_{it}^{\tau} = Q_{it}^{\tau} + \mathcal{E}_i^{-1} l_{it}^{\tau}. \tag{17}$$

Thus, the supply of collateral in the repo market, S_{it}^{τ}, depends on repo specialness, l_{it}^{τ}, and on the inverse of the supply elasticity, $\mathcal{E}_i^{-1}$; moreover, it depends on the securities lending of preferred-habitat investors, Q_{it}^{τ}. Attention thus turns to the role of preferred-habitat investors, who *may or may not* supply their bonds in the repo market. For instance, central banks may or may not make their QE portfolio of bonds available for borrowing against cash through a Securities Lending Facility (SLF). We denote by ϕ_{it}^{τ} the securities lending activity of preferred-habitat investors, defined as the proportion of bonds lent relative to their holdings. Specifically, $\phi_{st}^{\tau} = Q_{st}^{\tau}/Z_{st}^{\tau}$. To avoid division by zero, we set $\phi_{gt}^{\tau} = 0$. As we will demonstrate in Section 3.11, the securities lending activity of preferred-habitat investors ϕ_{it}^{τ} is the single parameter in our model that causes it to differ from a traditional preferred-habitat model.

3.8.3 Equilibrium and Repo Specialness

The equilibrium in the repo market is characterized by a set of repo rates such that the market clears given the demand of collateral by arbitrageurs (Equation (15)) and the supply of collateral by preferred-habitat investors (Equation (17)). The intersection of collateral demand and supply clears the market and characterizes the equilibrium repo specialness, which is given by

$$l_{it}^{\tau} = Z_{it}^{\tau}(1 - \phi_{it}^{\tau})\mathcal{E}_i. \tag{18}$$

[18] The amount of bonds outstanding is fixed by the size of the issue, and buy-and-hold investors such as pension funds and insurance companies (Maddaloni and Roh, 2021; Ballensiefen et al., 2023; Coen et al., 2024), as well as central banks during quantitative easing operations (Arrata et al., 2020), typically exhibit limited participation in the repo market.

[19] This framework could be generalized; for instance, a quadratic term structure model would give rise to a quadratic collateral supply curve.

Accordingly, the repo specialness of GC bonds is always equal to zero; namely, $l_{gt}^{\tau}=0$. By contrast, the repo specialness of SC bonds is weakly positive; namely, $l_{st}^{\tau}\geq 0$.[20]

In the market for SC bonds, the level of repo specialness that results from the asset purchases of preferred-habitat investors, Z_{st}^{τ}, is determined by their securities lending in the repo market, Q_{st}^{τ}. Appendix Figure IA.I illustrates the equilibrium in the SC segment of the repo market as a function of preferred-habitat investors' participation in securities lending. In the *absence* of securities lending by preferred-habitat investors ($\phi_{st}^{\tau}=0$), their asset purchases induce a scarcity of bonds in the repo market and gives rise to a proportional repo specialness ($l_{st}^{\tau}=\mathcal{E}_s Z_{st}^{\tau}$), consistent with the empirical findings of Arrata et al. (2020) and other studies documenting that QE generates repo specialness. By contrast, in the *presence* of securities lending by preferred-habitat investors, their securities portfolio becomes available in the repo market, inducing a rightward shift of the collateral supply curve. In the limiting case, as preferred-habitat investors lend their entire securities portfolio ($\phi_{st}^{\tau}=1$), repo specialness is fully eliminated ($l_{st}^{\tau}=0$). Our equilibrium characterization of the repo market thus provides a tractable framework to analyze the effects of new monetary policy tools that operate through the repo market, such as the SLF.

3.9 Relative Bond Prices and Repo Rates

Below, we discuss the equilibrium relationship between bond prices and repo rates, which is given by

$$P_{it}^{\tau}=\exp\left(-A_{\tau}r_t-B_{i\tau}X_{it}^{\tau}-C_{\tau}\right)=\mathbb{E}_t^{*}\left[\exp\left(-\int_0^{\tau} r_{i,t+u}^{\tau-u}du\right)\right]. \tag{19}$$

In the first equality of Equation (19), we use the affine representation of the bond price. In the second equality, we denote by $\mathbb{E}_t^{*}$ the expectations taken under the risk-neutral measure, and equate the bond price to the risk-adjusted expected present discounted value of its appropriate interest rates (Duffie and Kan, 1996).[21] Accordingly, the notional principal at maturity is priced using the appropriate series of short rates: GC rates for general bonds, and SC rates for special bonds.

Next, consider the relative price and repo rate of general and special bonds with identical cash flows. As arbitrageurs can establish convergence trades, any discrepancies between the price of two bonds with identical cash flows that are not justified by repo financing costs would give rise to arbitrage opportunities (Duffie, 1996). In the absence of such opportunities, any price differences between bonds with identical cash flows induced by the demand of preferred-habitat investors must be mirrored in repo specialness, which captures arbitrageurs' collateral demand in the money market.[22]

[20] The first statement follows from $Z_{gt}^{\tau}=Q_{gt}^{\tau}=\mathcal{E}_i=0$; the second from $Z_{gt}^{\tau}\geq Q_{gt}^{\tau}$ and $\mathcal{E}_s>0$.

[21] The coefficients A_{τ}, $B_{i\tau}$, and C_{τ} project the current value of the risk factors on the risk-adjusted rational expectations forecast of their future conditional realizations, incorporating their level and persistence into market quotes.

[22] In line with this reasoning, D'Amico et al. (2018) employ data on the volume of reverse repo contracts as an empirical measure of excess demand in the bond market.

Lemma 2. *In equilibrium, the price ratio of special to general bonds with equivalent cash flows is*

$$\frac{P^{\tau}_{st}}{P^{\tau}_{gt}} = \underset{\substack{\uparrow \\ \textit{Collateral Value}}}{\exp\left(B_{s\tau} Z^{\tau}_{st}\right)} = \underbrace{\mathbb{E}^{*}_{t}\left[\exp\left(-\int_{0}^{\tau} r^{\tau-u}_{s,t+u} du\right)\right]}_{\textit{SC Repo Rates}} \underbrace{\mathbb{E}^{*}_{t}\left[\exp\left(-\int_{0}^{\tau} r_{t+u} du\right)\right]^{-1}}_{\textit{GC Repo Rates}}.$$

Proof. See Appendix B.

In equilibrium, the relative price of special and general bonds with equivalent cash flows (on the left-hand side of the first equality) is equal to the ratio of the cost of replicating the two positions through a series of overnight repo contracts, in expected risk-adjusted terms (on the right-hand side of the second equality). Given that $r^{\tau}_{t} = r^{\tau}_{it} + l^{\tau}_{it}$, an increase in repo specialness, l^{τ}_{t}, raises the price difference between general and special bonds with equivalent cash flows. Due to the affine structure, $B_{s\tau}$ represents the linear projection of current demand pressure, Z^{τ}_{it}, onto the entire stream of repo specialness attached to the specific bond. Lemma 2 thus establishes that the targeted demand pressure of habitat investors induces a price premium between bonds with equivalent cash flows. The log-price premium of special bonds, $B_{s\tau} Z^{\tau}_{st}$, which we refer to as the *collateral value* of special bonds in the capital market, is the risk-adjusted present value of the entire stream of repo specialness attached to the specific bond conditional on the current level of demand for that bond.

Lemma 2 is consistent with Buraschi and Menini (2002), Cherian et al. (2004), and D'Amico and Pancost (2022), who also suggest that repo specialness must be included in the pricing of special bonds across every maturity. However, in this paper, repo specialness is endogenously determined by the interaction of arbitrageurs and preferred-habitat investors such as central banks; moreover, as we show below, it affects the market price of interest rate risk.

3.10 *The Market Price of Risk*

In this section, we explicitly solve for the market price of risk in our model. Imposing the market clearing condition (Equation (9)) into the expression for the market price of risk (Equation (14)), we obtain

$$\lambda_t = \gamma \sigma^2_r \int_{0}^{\infty} a_{s\tau} [\eta_\tau (A_\tau r_t + B_{s\tau} X^{\tau}_{st} + C_\tau) - \theta_\tau] d\tau. \tag{20}$$

The market price of risk reflects arbitrageurs' risk aversion and the interest rate risk of their portfolio. By market clearing, the arbitrageurs' portfolio depends on the demand of preferred-habitat investors. Suppose that the demand from preferred-habitat investors rises. To accommodate their demand, arbitrageurs short-sell more bonds and carry their trades by investing the proceeds at the series of short rates. Greater carry trade activity heightens the arbitrageurs' exposure to interest rate risk, inducing them to require a higher compensation for risk. The required compensation is an increase in the market price of risk, which raises the price of long-term bonds and makes short-selling at these higher prices more attractive to arbitrageurs. This adjustment leads to a decline in the yield to maturity on long-term bonds, resulting into a *flattening* of the yield curve. Thus, as is standard in the preferred-habitat literature, the market price of risk and the yield on long-term bonds are negatively related.

A distinct feature of our theory is that, by market clearing, arbitrageurs solely hold special bonds. As a consequence, the market price of risk reflects the interest rate risk of the returns of the arbitrageurs' portfolio of special bonds. Importantly, however, the market price of interest rate risk affects the pricing of both general and special bonds, governing the *transmission* of the demand of preferred-habitat investors from special bonds to the yield curve of general bonds not directly targeted by their demand.

Remark 2. *Both general and special bonds are exposed to the same market price of interest rate risk.*

In preparation for our next key result, it is useful to recall that in our theory bonds serve two roles. First, as investment opportunities that generate cash flows in the bond market. Second, as means of collateral that generate cash flows in the repo market. (The collateral value of special bonds, $B_{s\tau}Z_{st}^{\tau}$, was derived in Lemma 2). To appreciate the relative contribution of these two components to the market price of risk, replace the bond market clearing condition ($X_{it}^{\tau}+Z_{it}^{\tau}=0$) into Equation (20), and rearrange terms as follows.

$$\lambda_t^{\tau}=\underbrace{\gamma\sigma_r^2\int_0^{\infty}a_{s\tau}\left[\eta_{\tau}\left(A_{\tau}r_t+C_{\tau}\right)-\theta_{\tau}\right]d\tau}_{\text{Investment-value duration}}-\underbrace{\gamma\sigma_r^2\int_0^{\infty}a_{s\tau}\eta_{\tau}B_{s\tau}Z_{st}^{\tau}d\tau}_{\text{Collateral-value duration}}. \tag{21}$$

Equation (21) decomposes the market price of risk into two terms. The first term is the interest rate sensitivity of the present value of the cash flows generated by the arbitrageurs' portfolio in the *bond market*, which we label "investment-value duration." The second term is the interest rate sensitivity of the present value of the cash flows generated by the arbitrageurs' portfolio in the *repo market*, which we label "collateral-value duration." It can be observed that the collateral-value duration of the arbitrageurs' portfolio returns reflects an aggregate measure of the collateral specialness of bonds across tenors weighted by bond duration.

In the preferred-habitat theory, the market price of risk is endogenously related to the duration of the arbitrageurs' portfolio returns (Vayanos and Vila, 2021). The literature has thus far emphasized this insight in the context of bonds as investment assets that carry an investment-value duration. The distinct function of bonds as collateral assets highlights an important channel in the arbitrageurs' exposure to risk that has not been documented before: the collateral-value duration of their portfolio. Intuitively, the collateral-value duration of the arbitrageurs' portfolio captures the interest rate sensitivity of the arbitrageurs' repo financing costs incurred over the lifetime of their carry trades. This new channel forges a strong connection between the collateral specialness of bonds and the pricing of risk across the yield curve.[23]

[23] Empirical support for this prediction comes from Fontaine and Garcia (2012), who employ price difference between

By formalizing this insight, Equation (21) provides the theoretical foundation for Equation (2), which constructs the empirical measure Aggregate Specialness$_t$ by weighting each bond's specialness by its duration. Duration weights, denoted by $a_{s\tau}$ in the model, capture the sensitivity of each arbitrageurs' bond collateral-value to interest rate risk, and thus its effect on the market price of risk.

Remark 3. *The market price of interest rate risk reflects both bonds' investment-value duration and their collateral-value duration.*

Assessing the impact of shocks on the pricing of risk thus requires joint consideration of the bond and repo markets. For example, an increase in the demand of preferred-habitat investors simultaneously generates two effects: (i) In the bond market, arbitrageurs take on greater carry trade positions. With greater carry trade activity, the market price of risk rises, and the yield curve flattens. As shown in Equation (21) this effect is induced by the investment-value duration of the arbitrageurs' portfolio. (ii) In the repo market, arbitrageurs' short-selling pressure endogenously increases the collateral specialness of bonds, raising short-selling costs. With greater short-selling costs, the arbitrageurs' carry trade activity diminishes, the market price of risk falls, and the yield curve steepens. As shown in Equation (21), this effect is induced by the collateral-value duration of the arbitrageurs' portfolio. Quantitatively, for realistic parameter values, the investment-value duration effect in (i) dominates, implying that an increase in preferred-habitat demand raises the market price of risk and flattens the yield curve overall. However, the effect in (ii) dampens the transmission of preferred-habitat demand to the market price of risk and thus to the yields of long-term bonds.

In a nutshell, endogenous short-selling costs transmit part of the demand from preferred-habitat investors for bonds to higher special collateral values in the repo market, in turn influencing the arbitrageurs' portfolio choice and the global pricing of risk in the yield curve. This leads to important implications for the conduct of monetary policy, which we discuss in Section 4.1.

3.11 Combined Equilibrium in the Bond and in the Repo Market

In this section, we derive analytically the equilibrium bond prices and repo rates. We first examine a benchmark equilibrium in the absence of repo specialness, which obtains when preferred-habitat investors lend their entire securities portfolio in the repo market at the GC rate ($\phi^{\tau}_{st}=1$). Equation (18) implies that no repo specialness emerges under this equilibrium.

Proposition 1. Equilibrium with Securities Lending ($\phi^{\tau}_{st}=1$).

$$a_{i\tau}=\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}, \qquad b_{i\tau}=0, \qquad c_{i\tau}=\kappa_r^*\overline{r}^*\int_0^\tau a_{i\tau}d\tau-\frac{\sigma_r^2}{2}\int_0^\tau a_{i\tau}^2 d\tau.$$

pair of bonds with equivalent cash flows but different age characteristics to construct a funding liquidity factor for the cross section of bond returns.

The coefficients $a_{i\tau}$,$b_{i\tau}$, and $c_{i\tau}$, together with bond status i, uniquely determine A_τ,B_τ, and C_τ, verifying the conjecture for the bond price process in Equation (10). The scalars $(\kappa_r^*,\bar{r}^*)$, which are the counterparts of $(\kappa_r,\bar{r})$ under the risk-neutral measure, are defined by

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2 \int_0^\infty \eta_\tau a_{s\tau}^2 d\tau, \qquad \kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\theta_\tau - \eta_\tau c_{s\tau}\right] d\tau.$$

The short rate, r_t, is unique and there is no repo specialness, $l_{it}^\tau = 0$. The market price of risk, $\lambda_t = \gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\eta_\tau\left(A_\tau r_t + C_\tau\right) - \theta_\tau\right] d\tau$, solely reflects the investment-value duration of arbitrageurs' portfolio.

Proof. See Appendix C.

This equilibrium reduces to Vayanos and Vila (2021), where the money market is exogenous and the short-term interest rate is unique. However, despite its foundational nature, this equilibrium cannot account for the pervasive specialness observed in repo markets or the price dispersion among bonds with identical cash flows. Moreover, this equilibrium is silent on the sensitivity of special repo rates and bond specialness to the demand of preferred-habitat investors, a relationship that is strongly supported by empirical evidence (D'Amico et al., 2018; Arrata et al., 2020; Corradin and Maddaloni, 2020). Finally, this equilibrium cannot explain the relationship of aggregate repo specialness and the pricing of different maturity segments along the yield curve, which we have documented in Section 2.

To address these features of the data, we consider a more comprehensive equilibrium, which obtains when preferred-habitat investors do *not* lend their securities portfolio in the repo market ($\phi_{st}^\tau = 0$). Equation (18) implies the endogenous emergence of repo specialness in this equilibrium.

Proposition 2. Equilibrium absent Securities Lending ($\phi_{st}^\tau = 0$).

$$a_{i\tau} = \frac{1 - e^{-\kappa_r^*\tau}}{\kappa_r^*}, \qquad b_{i\tau} = \frac{\mathcal{E}_i(1-g_\tau)(1-e^{-\int\bar{\theta}_\tau d\tau})}{\bar{\theta}_\tau}, \qquad c_{i\tau} = \kappa_r^*\bar{r}^* \int_0^\tau a_{iu} du - \frac{\sigma_r^2}{2}\int_0^\tau a_{iu}^2 du$$

The coefficients $a_{i\tau}$,$b_{i\tau}$, and $c_{i\tau}$, together with bond status i, uniquely determine A_τ,B_τ, and C_τ, verifying the conjecture for the bond price process in Equation (10). The deterministic functions of bond tenor $\bar{\theta}_\tau$ and g_τ are defined in the Appendix. The scalars $(\kappa_r^*,\bar{r}^*)$ are defined by

$$\kappa_r^* = \kappa_r + \eta_\tau\mathcal{E}_i + \gamma\sigma_r^2 \int_0^\infty \eta_\tau a_{s\tau}^2 d\tau, \qquad \kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right] d\tau.$$

There exists a cross-section of repo market rates, $r_{it}^\tau = r_t - l_{it}^\tau$, whose repo specialness is given by

$$l_{it}^\tau = \mathcal{E}_i Z_{it}^\tau.$$

The market price of risk, $\lambda_t = \gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\eta_\tau\left(A_\tau r_t + B_{s\tau}X_{st}^\tau + C_\tau\right) - \theta_\tau\right] d\tau$, incorporates both the investment-value and collateral-value duration of the portfolio of arbitrageurs.

Proof. See Appendix D.

Proposition 2 highlights two levels at which the collateral function of bonds affects the equilibrium: (i) Locally, the collateral value of bonds induces price differences among bonds with equivalent cash flows, and influences their repo specialness, l_{it}^{τ}; (ii) Globally, the collateral-value duration of the arbitrageurs' portfolio returns influences the market price of risk and thereby the risk-neutral parameters governing the interest rate process, $(\kappa_r^*, \bar{r}^*)$, and affects the entire yield curve, over and above the valuation of specific bonds.

This characterization is our more general result, and presents several important elements of novelty. The proposition jointly describes *two yield curves*: one for general bonds ($i = g$), and one for special bonds ($i = s$), capturing price differences between bonds with identical cash flows through the $b_{i\tau}$ coefficients. These coefficients are parametrized by the elasticity of collateral supply,[24] and equal to zero for general bonds, which are inelastically supplied in the repo market, and positive for special bonds, which are elastically supplied in the repo market.[25] Consistent with the absence of arbitrage opportunities, special bonds command a price premium over general bonds that reflects their current and expected special collateral value in the repo market. It merits attention that special repo rates are explicitly linked to the demand of preferred-habitat investors. Indeed, rather than restricting the repo rates of all securities to a common exogenous short rate, this equilibrium generates *endogenous repo specialness*, given by l_{it}^{τ}.

Even more interestingly, within this equilibrium, the collateral-value duration of the arbitrageurs' portfolio returns influences the *market price of risk*, λ_t, along the lines discussed in Section 3.10. The market price of risk, in turn, fully determines the difference between the parameter governing interest rate dynamics under the physical probability measure, $(\kappa_r, \bar{r})$, and under the risk-neutral probability measure, $(\kappa_r^*, \bar{r}^*)$. Hence, through its effect on the market price of risk, the collateral-value duration of the arbitrageurs' portfolio returns exerts global effects on the pricing coefficients of *both special and general bonds* and, thus, their entire yield curves. This result stands in contrast to earlier contributions, in which repo specialness is reflected solely in the yield to maturity of each individual security.

Proposition 2 nests the more traditional Proposition 1 as a special case which obtains by setting $\phi_{st}^{\tau} = 1$, a restriction corresponding to the standard preferred-habitat model where bonds serve only as investment opportunities, the secured financing rate is exogenous, and the collateral is general.[26]

Corollary 1. *In the equilibrium of Proposition 2, an increase in aggregate specialness, defined as the duration-weighted average of repo specialness across maturities, raises term spreads across the entire yield curve. Let Term Spread$_{it}^{\tau} \equiv y_{it}^{\tau} - r_{it}^{\tau}$ and Aggregate Specialness$_t \equiv \int_0^{\infty} a_{i\tau} l_{it}^{\tau} d\tau$. Then, for*

[24]To build intuition, consider that in discrete time the recursion for the a_{it} coefficients would begin from an initial value of one, as one-period general bonds held to maturity yield r_t. Analogously, in discrete time the recursion for the b_{it} coefficients would begin from an initial value of $\mathcal{E}_i$, as one-period special bonds deliver an incremental cash flow of $\mathcal{E}_i Z_{it}^{\tau}$.

[25]Formally, $\mathcal{E}_g = 0 \Rightarrow b_{g\tau} = 0 \; \forall \tau$, whereas $\mathcal{E}_s > 0 \Rightarrow b_{s\tau} > 0 \; \forall \tau$.

[26]Beyond the cases $\phi_{st}^{\tau} = 1$ (Proposition 1) and $\phi_{st}^{\tau} = 0$ (Proposition 2), the equilibrium could be derived for any $\phi_{st}^{\tau} \in [0,1]$.

some increasing function f,

$$\textit{Term Spread}_{it}^{\tau} = f(\textit{Aggregate Specialness}_t). \tag{22}$$

Proof. See Appendix E.

Corollary 1 provides theoretical guidance for interpreting the empirical findings in Section 2, where we document empirically that term spreads present a robust positive association with the duration-weighted aggregate repo specialness. The intuition for this result is that the aggregate specialness of bonds has a positive relationship with the collateral-value duration of the arbitrageurs' portfolio returns and thus a negative relationship with the market price of risk (Equation 21). A decline in the market price of risk, holding expected short rates constant, raises the yield on bonds with long durations relative to bonds with short duration, thereby increasing term spreads (as illustrated by the risk premium component of bond yields in Lemma (1)). Essentially, repo specialness discourages the arbitrageurs' carry trade activity, reducing their required compensation for risk and steepening the yield curve. Theory and empirical evidence thus concur that aggregate repo specialness conveys information about the market price of interest rate risk and term spreads across the entire yield curve.

In our theory, repo specialness is completely silent about the arbitrageurs' portfolio investment-value duration, but conveys direct information about its collateral-value duration. Comparing Propositions 1 and 2 reveals that the absence of repo specialness may result either from the absence of preferred-habitat investors or from their decision to lend their bond inventories through the repo market. However, the *presence* of repo specialness in market data unambiguously reflects heightened costs of short-selling, which reduce the carry trade activity of arbitrageurs, lessens the magnitude of their exposure to interest rate risk, reduces the market price of risk, steepens the yield curve, and raises term spreads. This insight is substantiated by the empirical evidence we uncover in European markets, which emphasizes the importance of repo specialness in explaining term spreads across all segments of the yield curve.

4 Monetary Policy, Bond Yields, and Repo Rates

The theory we advance offers detailed insights regarding the transmission of quantitative monetary policy interventions, both to the yield curve and in the repo market, as we discuss below.

4.1 Quantitative Easing and Securities Lending

The purchase of bonds by the central bank through QE raises the demand for long-term bonds. In our model, we formalize this by defining a function of maturities, $QE_{\{\tau\}} = \{-\Delta\theta_\tau\}_0^\infty$, that collects the increase in the demand intercepts of preferred-habitat investors for special bonds at each maturity. (Recall that θ_τ is the negative of the intercept of the preferred-habitat demand). A shift in the demand intercept at a specific maturity is denoted by QE_τ. A QE intervention corresponds to a positive shift in the function $QE_{\{\tau\}}$ corresponding to asset purchases for at least one maturity. This quantitative monetary policy intervention, while leaving the short rate unchanged, affects both the yield curve and the repo market. First, consider the impact of QE on the yield curve. Within the preferred-habitat framework, QE operates by lowering the risk-adjusted long-run mean of the interest rate, $\overline{r}^*$, relative to its counterpart under the physical measure, $\overline{r}$. This mechanism is consistent with the interpretation of policymakers. As pointed out by Philip Lane, Member of the ECB's Executive Board,

> "In purchasing long-dated assets, a central bank takes duration risk off private hands, which translates into lower term premia and long-term interest rates."
>
> Philip Lane, New York, October 11, 2022.

In our theory, QE entails a threefold effect on bond yields. First, it absorbs duration risk from the market, by rendering the risk exposure of arbitrageurs more negative, which leads to a global supply effect that compresses long-term interest rates (of both special and general bonds and across the entire yield curve). Second, it also leads to a scarcity of bonds in the repo market, which raises repo specialness, as is now well understood empirically, and brings about a local supply effect by impacting the yields of special bonds directly targeted by QE, which decline by more than the yields of general bonds. There is, however, a third equilibrium effect: the higher price of special bonds leads preferred-habitat investors to endogenously reduce their bond holdings, given that their demand is downward sloping in bond prices. This reduction is met in equilibrium by a portfolio rebalancing from the arbitrageurs, who scale back their carry trade positions, thus dampening the initial reduction in long-term interest rates. The implication is that, *ceteris paribus*, the stronger is the localized effect on the specialness of targeted bonds, the weaker is the global supply effect of asset purchases on risk premia and long-term interest rates.

Importantly, the effects of QE on bond yields can be modulated by central bank policy in the repo market. When central banks establish the SLF to lend on the repo market the bonds purchased via QE, they prevent the increase in repo specialness induced by the scarcity of bonds, facilitating the arbitrageurs' global transmission of asset purchases to long-term interest rates. Our theory thus offers a new and actionable perspective, whereby the global effects of QE on long-term general bond yields depend not only on the scale of asset purchases, but also on how the central bank manages the acquired portfolio in the repo market. Specifically, whether the central bank makes the purchased securities available for lending influences the transmission of QE to the yields of long-term bonds, as shown analytically in the next result.[27]

Lemma 3. ***The SLF and the transmission of QE.*** *The effect of QE on the yield of long-term general bonds is stronger if asset purchases are paired with the SLF. Formally, let* $\Delta y^\tau_{gt}(QE_{\{\tau\}},\phi^\tau_{st})$ *denote*

[27]We consider the absolute value of the yield variation induced by QE, because a QE intervention induces a decline in the yields of long-term bonds and we are interested in the magnitude of this effect.

the variation in the yield of long-term general bonds induced by $QE_{\{\tau\}}$. Then,

$$\underbrace{\left|\Delta y_{gt}^{\tau}(QE_{\{\tau\}},\phi_{st}^{\tau}=1)\right|}_{\text{Effect of QE with SLF } (\phi_{st}^{\tau}=1)}=\frac{\gamma\sigma_r^2}{\tau}\int_0^{\infty}a_{s\tau}QE_{\tau}d\tau\geq\frac{\gamma\sigma_r^2}{\tau}\int_0^{\infty}a_{s\tau}(1-\eta_{\tau}b_{s\tau})QE_{\tau}d\tau=\underbrace{\left|\Delta y_{gt}^{\tau}(QE_{\{\tau\}},\phi_{st}^{\tau}=0)\right|}_{\text{Effect of QE without SLF } (\phi_{st}^{\tau}=0)}.$$

Proof. See Appendix F.

Lemma 3 compares the impact of a given quantity of asset purchases on bond yields under two alternative scenarios: when the central bank lends out its securities portfolio and when it does not. When the central bank lends out its securities portfolio, the equilibrium corresponds to that without repo specialness in Proposition 1. In this equilibrium, QE is entirely absorbed as an increase in arbitrageurs' risk exposure, with each targeted bond contributing a decline in long-term interest rates of $\frac{1}{\tau}\gamma\sigma_r^2 a_{s\tau}$. In contrast, when the central bank does not lend its securities portfolio in the repo market, the equilibrium corresponds to that in Proposition 2 where QE endogenously generates repo specialness. In this equilibrium, the effect of QE is only partly reflected in the arbitrageurs' risk exposure, as repo specialness induces a collateral premium and reduces the downward-sloping demand of preferred-habitat investors by a factor of $\eta_{\tau}b_{s\tau}$, thereby dampening the transmission of QE to long-term interest rates of *general* bonds. In Section 5.4.3, we will illustrate this result by calibration.

Differently from other theories of the yield curve, our model also accounts for the impact of QE on the repo market. In the absence of securities lending, a shift in the demand of preferred-habitat investors such as QE increases the repo specialness of the targeted bonds (see Equation (18)). This effect is consistent with the evidence reported by Arrata et al. (2020) and Corradin and Maddaloni (2020), who demonstrate that ECB asset purchases induce scarcity in targeted bonds, leading to a significant increase in their repo specialness. To illustrate this effect, Online Appendix Figure IA.II documents that the volume-weighted average repo specialness rises with the proportion of bonds held by the ECB under its QE program (top panel). The proportion of bonds held by the ECB is also associated with a rise in the noise measure of Hu et al. (2013) capturing yield curve fitting errors (bottom panel), exposing price differences among bonds with identical cash flows.

According to our theory, the central bank can mitigate the repo specialness arising from QE by introducing the SLF, thereby shifting the equilibrium from that described in Proposition 2 to the one in Proposition 1. More precisely, our theory predicts that the extension of a SLF under which bonds are lent at the GC rate reduces repo specialness, thereby narrowing local price differentials between bonds with equivalent cash flows and generating a global flattening of the yield curve. To integrate theory and data, it is useful to consider that on December 15, 2016, the ECB introduced a SLF that enabled investors to borrow its securities portfolio against cash. This event is marked by the dashed vertical line in the Internet Appendix Figure IA.II. Consistent with the model's implications, both repo specialness and yield curve fitting errors stabilized following the SLF's introduction, though they remained positive. This is partly explained by the ECB's choice to lend its securities at a rate 30 basis points below the GC rate, rather than at the GC rate itself.

Internet Appendix Figure IA.III further corroborates the model's implications by plotting the bond-by-bond yield of each German Bund in our sample against its maturity around December 15, 2016. Each panel in the figure corresponds to a trading day around the SLF's implementation, allowing for a visual inspection of changes in the yield curve. While the short end of the yield curve remains anchored at -0.9%, the yield on long-term bonds gradually declines from approximately 1.2% four days before the event to around 1% four days after. Correspondingly, the term spread between the two ends of the curve declines from approximately 2.1% to 1.9%, a reduction of about 10%. This movement reflects a flattening of the yield curve around the SLF's implementation, consistent with our model.[28]

4.2 *Local Supply Effects*

A merit of our theory is its ability to generate pronounced yield curve local supply effects, consisting of relative price anomalies between bonds with similar cash flows (D'Amico and King, 2013). Our theory points out that such relative price anomalies are not inconsistent with the absence of arbitrage, provided they reflect a price premium attached to bonds with special collateral value in the repo market. This insight is consequential for asset purchases, which generate specialness in the targeted bonds, endogenously lowering their repo rate and yield to maturity. If one were to estimate the yield curve without distinguishing between general and special bonds, this collateral specialness would manifest as an anomalously low yield for that maturity segment, relative to neighboring maturity segments not subject to specialness. Asset purchases in our model with endogenous repo specialness can thus generate far more pronounced local supply effects than those implied by standard preferred-habitat models of the yield curve. This approach makes the model particularly well suited to analyze YCC, where the central bank targets specific segments of the bond market and local supply effects are especially important (see Lucca and Wright, 2024, Figure 5). Section 5.4.1 illustrates by calibration local supply effects under YCC.

4.3 *The Trade-off between Local and Global Supply Effects*

Another merit of our theory is to uncover a novel trade-off between local supply effects of quantitative policy interventions and global supply effects on risk premia that impact the entire yield curve. Given a fixed amount of asset purchases, two scenarios (or a combination thereof) may unfold:

1. The asset purchase portfolio is withheld by the central bank. In this case, asset purchases induce a specialness of targeted bonds in the repo market, lowering their specific yields and generating local supply effects. As shown by Lemma 3, global supply effects become less pronounced.

[28]Fleming et al. (2010), D'Amico and Pancost (2022), and Pelizzon et al. (2025) present empirical evidence on the impact of the SLF on bonds' collateral value. Unlike our paper, none of these studies considers the effect of SLF on term spreads.

2. The asset purchase portfolio is lent by the central bank in the repo market. In this case, asset purchases do not induce specialness of the targeted bonds in the repo market, weakening local supply effects. As shown by Lemma 3, global supply effects become more pronounced.

An extreme case of scenario 1) is the YCC, which induces local supply effects without affecting the slope of the yield curve. In the extreme theoretical case where the central bank targets a single maturity, the intervention induces a Dirac mass at that maturity without affecting the market price of risk, which reflects the integral of asset purchases across maturities. Scenario 2) corresponds to broad-based QE in combination with the SLF, which does not induce repo specialness. In the extreme case where the central bank lends its entire securities portfolio in the repo market without requiring any specialness, the equilibrium converges to Proposition 1, and the global effects of asset purchases through the market price of risk are maximized.

These considerations highlight a simple trade-off. *Ceteris paribus*, an increase in the demand for a bond is either reflected in a lower yield of the specific bond or distributed across the entire yield curve through a change in the market price of risk that is common across assets, resulting into a flatter yield curve. As discussed in the introduction, policymakers may draw on this trade-off to control the strength of the local and global supply effects of their quantitative policy interventions. For example, policymakers can withhold their asset purchases of green bonds from the repo market to allow bonds targeted by Green QE to trade at a premium relative to other bonds.

The same trade-off is at work with quantitative tightening (QT), which, in our theory, corresponds to a downward shift in the intercept of the demand curve of preferred-habitat investors.[29] In some cases, policymakers utilize QT to reduce the size of the portfolio of the central bank, without aiming to influence the slope of the yield curve. In this case, our findings suggests that the central bank should not supply its securities portfolio as collateral on the repo market. Alternatively, policymakers may want to utilize QT to influence the slope of the yield curve. Central banks with this objective should supply their remaining portfolio of asset purchases as collateral on the repo market to facilitate the activity of arbitrageurs. Generally speaking, our theory recommends that monetary policy be implemented in a coordinated manner across the bond and repo markets.

5 Calibration

The aim of our calibration exercise is to evaluate the model's capacity to accurately represent market data and to assess the quantitative impact of counterfactual scenarios induced by quantitative monetary policy interventions affecting bond and repo markets.

[29] In the preferred-habitat framework, QT is typically interpreted as a symmetric reversal of QE. Kaminska et al. (2025) depart from this assumption and consider the state-dependent transmission of balance sheet monetary policies.

5.1 *Measurement*

The structural estimation of our model involves the combined fitting of the general and special yield curves in the bond market. These curves are not directly available, as data providers typically offer interpolated yield curves without distinguishing between bonds based on their specialness. Thus, we must analyze the market at the disaggregated level of individual bond yields.

To interpolate the yield curve, we employ a spline method that represents yields as a linear combination of piecewise polynomial functions, producing a smooth yield curve. To prevent overfitting, the optimal degree of smoothness is determined by maximum likelihood estimation. Illustrative examples of the interpolation procedure are presented in Appendix Figure IA.IV. We apply this method to obtain general and special yield curves from the granular bond-level dataset described in Section 2. Bonds with repo specialness above the weekly median are classified as special, and the remainder as general. The yields of each group are interpolated separately over annual maturities from 1 to 30 years, repeating the procedure for each day-group with more than 40 observations.

We then average the resulting daily fitted yield curves over the full sample period. Figure 2 presents the average yield curves for general bonds ($i = g$) and special bonds ($i = s$). We observe that the average yields of the interpolated curves are consistent, maturity by maturity, with the summary statistics of the bucketed data in Panel B of Table I. However, relative to the bucketed data, the bond-level interpolated data in Figure 2 enable a clear distinction between general and special bond yields after controlling for maturity. The yield curve interpolated from general bonds (black solid line) lies entirely above the curve interpolated from special bonds (blue solid line), clearly indicating the role of specialness.

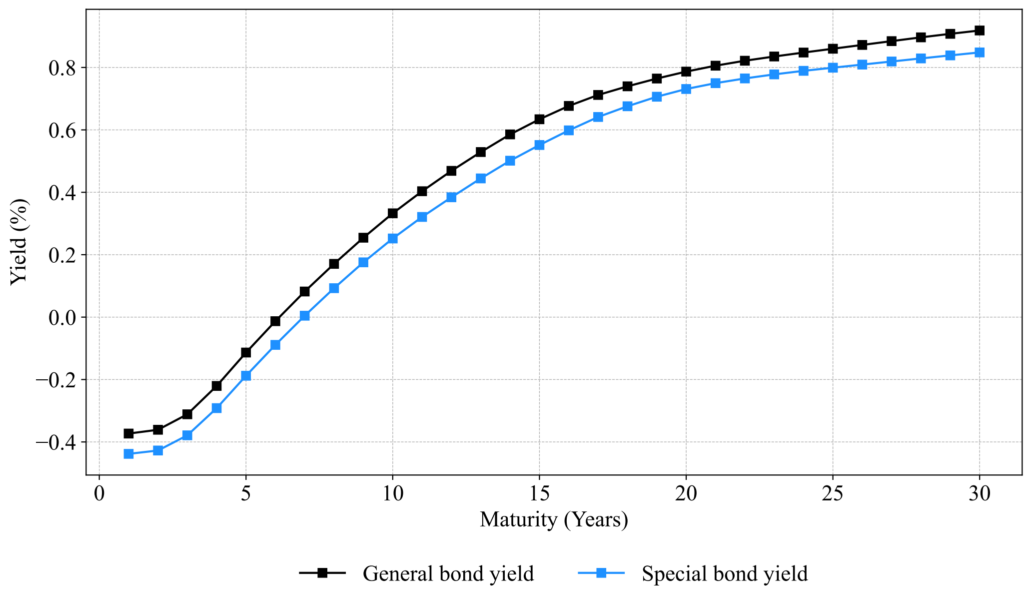

FIGURE 2: **Yield Curve Measurement.** This figure presents the yield curves of general and special bonds. The curves are constructed as the average of daily yield curves interpolated using German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

5.2 *Quantitative Model*

Figure 2 reveals two salient features of the data. First, we observe that the average spread over time between the general and the special yield curves is approximately constant across maturities. A possible explanation for this empirical regularity is that German Treasury bonds are regularly retapped, and their specialness often persists for the entire life of the bond. Consequently, on average, a 10-year bond yield exhibits a similar degree of specialness as a 9-year bond yield, after adjusting for maturity. This observation suggests that in our sample the demand of preferred-habitat investors for special bonds is constant across maturities. Hence, we assume that the intercept and slope of their demand function are constant across maturities and given by the scalars θ and η. This specification provides a reasonable benchmark, given that in our model habitat preferences are directed toward specific bonds among those with equivalent cash flows.[30]

Second, we observe that the interpolated yield curves feature two inflection points, which cannot be adequately captured by a one-factor affine model. Therefore, following the approach of Costain et al. (2025), we enhance our preferred-habitat model for quantitative realism. Specifically, we now allow for a multivariate representation, modeling the short rate as a linear combination of independent factors:

$$r_t = \delta_0 + \delta_1^\top R_t. \tag{23}$$

Here, δ_0 is a scalar, δ_1 a vector of weights, and R_t a vector of latent factors with dynamics given by

$$dR_t = K_r\left(\overline{R} - R_t\right)dt + \Sigma dV_t^r. \tag{24}$$

We consider two factors, defining the 2×1 vectors R_t, $\overline{R}$, and V_t^r and the 2×2 matrices K_r and Σ as

$$R_t = \begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix}, \quad \overline{R} = \begin{bmatrix} \overline{r}_1 \\ \overline{r}_2 \end{bmatrix}, \quad V_t^r = \begin{bmatrix} v_{1t}^r \\ v_{2t}^r \end{bmatrix}, \quad K_r = \begin{bmatrix} \kappa_{1r} & 0 \\ 0 & \kappa_{2r} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{1r} & 0 \\ 0 & \sigma_{2r} \end{bmatrix}.$$

We conjecture that there exist coefficients $\mathscr{A}_\tau = [\mathscr{A}_{1\tau}, \mathscr{A}_{2\tau}]^\top$, $\mathscr{B}_\tau$, and $\mathscr{C}_\tau$ such that bond prices satisfy

$$-\log P_{it}^\tau = \mathscr{A}_\tau^\top R_t + \mathscr{B}_\tau X_{it}^\tau + \mathscr{C}_\tau. \tag{25}$$

The solution of our earlier one-factor model generalizes to this multifactor model, as detailed in Internet Appendix B. We emphasize that all our results extend to this generalization, which we use in our calibration henceforth.

[30] Stochastic demand factors could be included; however, Figure 2 suggests their relevance in our sample would be limited. Additionally, the demand slope could be specified as a piecewise-constant function on a finite partition of tenors; namely, $\eta_\tau = \eta_h$, for $\tau \in [\tau_h, \tau_{h+1})$. We found limited evidence in favor of such refinement in our sample.

5.3 Parameter Estimation

The baseline calibration corresponds to the equilibrium where preferred-habitat investors do not lend their bond holdings in the repo market ($\phi_{it}^{\tau}=0$). The quantitative version of the model is characterized by two risk factors and twelve key parameters: the interest rate parameters, $\delta_0,\delta_1,\kappa_{1r},\kappa_{2r},\overline{r}_1,\overline{r}_2,\sigma_{1r},\sigma_{2r}$; the risk-aversion coefficient of arbitrageurs, γ; the slope and intercept parameters of the demand of preferred-habitat investors, η and θ; and the elasticity of the supply of bonds as collateral in the repo market, $\mathcal{E}_s$. The latent factors determining the short rate, R_t, are unobservable; without loss of generality, we set δ_0 as the average GC rate in our sample, equal to -0.36% per annum, and assume δ_1 equal to a 2×1 vector of ones, which implies $r_t=\delta_0+r_{1t}+r_{2t}$. We collect the remaining parameters into a 1×10 vector, Θ, which we estimate via a two-step Generalized Method of Moments (GMM) procedure with optimal weighting matrix. Specifically, we minimize the statistic

$$J(\Theta)=m(\Theta)^{\top}\hat{\Omega}^{-1}m(\Theta), \tag{26}$$

where $m(\Theta)$ is the $1\times n$ vector of moment conditions residuals and $\hat{\Omega}$ is the $n\times n$ sample covariance matrix of the moment conditions. The moment conditions are specified as the differences between model-implied and observed average yields at annual maturities from 1 to 30 years, separately for both general and special bonds; thus, $n=60$. To ensure robustness, we initialize the optimization from multiple starting values and employ various optimization methods.

At the estimated parameters $\hat{\Theta}$, the Root Mean Squared Error equals $\sqrt{\frac{1}{n}m(\hat{\Theta})^{\top}m(\hat{\Theta})}=0.00026$. This value is below the median Bund bid-ask spread of 0.0003 reported by De Roure et al. (2025), indicating that our model does a remarkably good job in simultaneously fitting both the general yield curve and the special yield curve. The model fit of the data is illustrated in Figure 3.

The parameter estimates are presented in Table IV. The estimated speeds of mean reversion reflect a high persistence of the short-rate processes; namely, $\kappa_{1r}=0.0510$ (half-life of shocks 13.59 years) and $\kappa_{2r}=0.0487$ (half-life of shocks 14.23 years). Consistently, the daily autocorrelation coefficient of one-year yields is 0.9998, confirming that short rates exhibit near unit-root behavior in the data. As both factors are initialized at zero, the initial value of the short rate is $\delta_0=-0.0036$. The long-run means of the factors, estimated at $\overline{r}_1=0.0688$ and $\overline{r}_2=0.0694$, generate a gradual increase in the conditional expectation of the short rate, reproducing the upward slope of the yield curve. The diffusion parameters are estimated at $\sigma_{1r}=0.0127$ and $\sigma_{2r}=0.0151$. Thus, r_1 acts as a relatively stable process, whereas r_2 is somewhat more volatile; their interaction helps to generate two inflection points in the yield curve, consistent with Figure 2. These estimates are generally comparable with those reported in Costain et al. (2025), who also calibrate a preferred-habitat model to German Treasury yields.

Turning to the demand of preferred-habitat investors, we estimate the intercept parameter at $\theta=-0.0152$, corresponding to a positive excess demand for special bonds, and the slope at $\eta=0.0213$. These parameters quantify the strength and sensitivity of preferred-habitat demand, implying that the

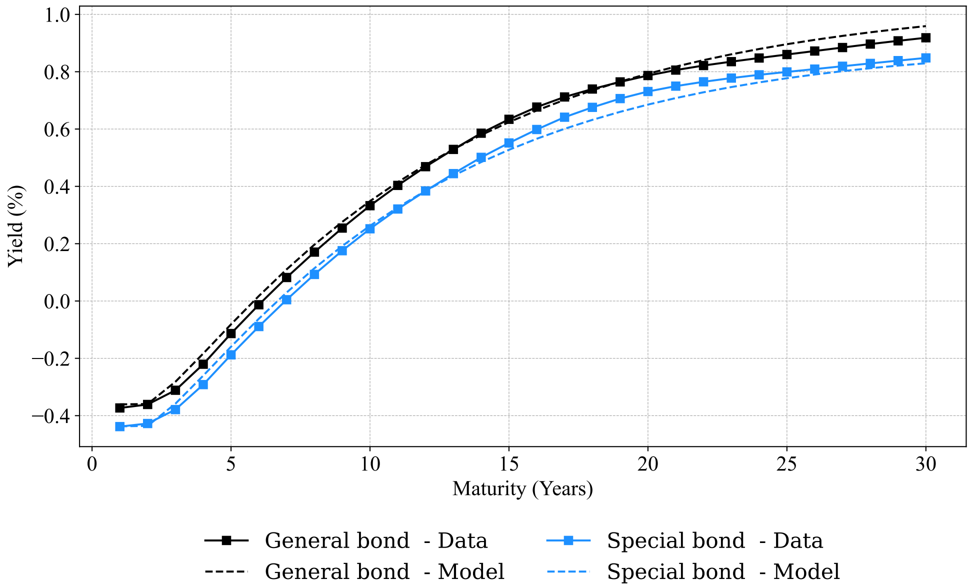

FIGURE 3: **Model Fit.** This figure compares the yields implied by the quantitative model against the yield curves of general and special bonds constructed using German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

demand for special bonds exceeds their supply by 1.5% of the issue and is relatively price inelastic. These values match the observed spread between general and special bond yields after controlling for maturity.[31] The arbitrageur risk-aversion coefficient is estimated at $\gamma = 1.7825$, between the value of 0.0108 reported by Costain et al. (2025) and the range from 6.78 to 33.9 reported by Vayanos and Vila (2021), and well above the risk-neutral benchmark $\gamma = 0$. The elasticity of supply of special collateral in the repo market is estimated at $\mathcal{E}_s = 0.0511$; namely, a 1% increase in the demand of preferred-habitat investors for a specific security reduces its repo rate by 5.11 basis points. Consistent with this estimate, Corradin and Maddaloni (2020) document that a 1% shock to the demand for a bond in the repo market is associated with a decline of its repo rate of 5 basis points. This value is well above the benchmark case $\mathcal{E}_s = 0$, which would describe a perfectly elastic supply of special bonds in the repo market, returning the model to the standard environment of Proposition 1 in which specialness premia vanish and bonds with identical cash flows are priced identically.

Figure 4 presents our baseline model calibration. For convenience, the top panel of the figure reproduces the model-implied yield curves of general bonds (black solid line) and special bonds (blue solid line) from Figure 3. The yields of general bonds can be replicated through rolling over GC repo contracts, and the yields of special bonds through rolling over SC repo contracts (Lemma 2). Thus, the yield curves of general and special bonds must be considered in combination with the instantaneous in-

[31] The demand of preferred-habitat investors is estimated in reduced form from yield data, with parameter values summarizing the empirical relation between bond specialness and excess demand, without describing the proportion of bonds held by the central bank. For perspective, under the ECB's asset purchase programmes the Bundesbank held around 24% of the outstanding volume of Bunds in 2018 and 36% in 2022. Source: Bundesbank July 2018 and May 2024 Monthly Reports.

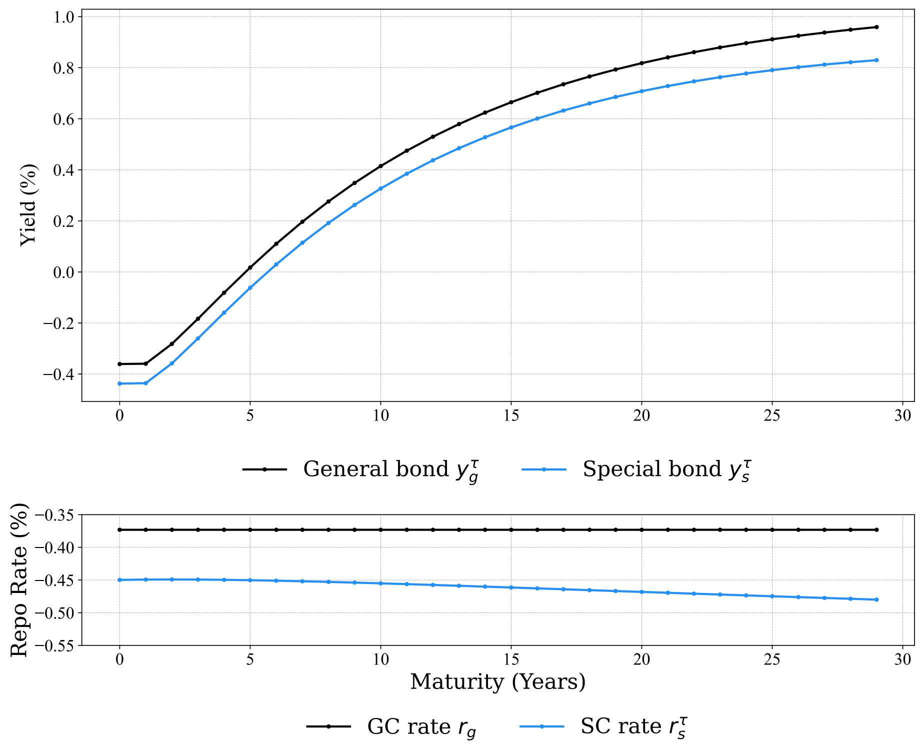

FIGURE 4: **Baseline Calibration.** The top panel of the figure shows the calibrated yield curves of general and special bonds, and the bottom panel shows the calibrated repo market interest rates of general and special bonds of different maturities. Black solid lines represent general bonds and blue solid lines represent special bonds. This figure is based on a calibration of the model to German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

terest rates associated with these bonds. Accordingly, the bottom panel of Figure 4 presents the GC rate (black solid line), which is constant across maturity, and the SC rates (blue solid line), which decline with maturity. The SC rates are implied by replacing the parameter estimates into the equilibrium expression for repo specialness, l_{st}^τ, in Proposition 2. The median repo specialness implied by the model, equal to 0.09% per annum, is comparable to its sample average of 0.10%; moreover, long-term bonds feature moderately lower repo rates than short-term bonds, as in Table I. It is reassuring that the model reproduces these features of the data, which were not explicitly targeted in the estimation. Overall, the quantitative model calibration convincingly replicates the market observed bond yields and repo rates.

5.4 Monetary Policy Interventions

Given the capacity of the calibrated model to accurately represent the data, we can employ it to perform counterfactual analyses. Below, we examine the transmission of two unconventional monetary policy tools: (i) asset purchases targeting the yields of specific bonds at specific maturities (YCC) and (ii) broad-based asset purchases across the entire yield curve (QE). To highlight the role of the repo market, we also consider the effect of combining the two aforementioned bond market monetary policy tools with a third policy tool: (iii) securities lending facility in the repo market (SLF).

5.4.1 *Yield Curve Control*

The YCC is a monetary policy tool consisting of bond market asset purchases targeting the yields of specific bonds at specific maturities. It has been used by central banks around the world, including the United States in 1942, Japan in 2016, and Australia in 2020. To illustrate this policy tool in the context of our model, Figure 5 presents a counterfactual scenario where the demand of preferred-habitat investors for the 10-year special bond has an intercept, $\theta_{10} = 2 \cdot \theta$, double in magnitude than that for other maturities, whose intercepts remain at the baseline value. This targeted demand pressure captures the structural intervention of central banks to control yields at specific maturities. As can be seen from the top panel of Figure 5, this intervention induces a decline in the yield of the targeted security. Relative to the baseline calibration where $y_s^{10} = 0.33\%$, this scenario results into $y_s^{10} = 0.25\%$, i.e., about a 24% decline in the 10-year yield of the targeted security. In other words, targeted demand induces a localized kink in the yield curve of special bonds, without any other meaningful effects on the yield curve. This pattern closely mirrors the experience with YCC in Australia, where the central bank bought approximately 61% of the outstanding volume of a specific target bond, dislocating its yield from other financial market instruments while leaving the rest of the curve largely unchanged (Lucca and Wright (2024), Figure 5).

The bottom panel of Figure 5 illustrates the impact of YCC in the repo market. Relative to the baseline calibration, where special repo rates are constant in the cross-section, the 10-year tenor SC that is more aggressively targeted by the intervention becomes highly special; namely, $r_s^{10} = -0.53\%$, relative to a baseline value of 0.46%. Thus, the model is capable of generating spikes in the repo specialness of bonds at specific maturities and capturing a positively skewed distribution of repo specialness whose mean lies above the median. Importantly, special repo rates respond endogenously to demand forces. In the model, YCC generates strong demand pressure that raises a bond's price and lowers its yield—thus, as arbitrageurs increase their short-selling activity to meet the central bank's willingness to target a specific bond, that security goes on special in the repo market. As a result, YCC gives rise to strong *local supply* effects on the yield curve of special bonds and the corresponding special repo rates.

Beyond YCC interventions, kinks in the yield curve similar to the one illustrated in this calibration are a pervasive feature of market data.[32] The ability of our framework to generate this non-monotonic pattern highlights an important advantage vis-à-vis equilibrium models of the yield curve and even econometric interpolation techniques in the spirit of Nelson and Siegel (1987).

[32] Demand targeted to specific bonds may also reflect portfolio constraints on investors, institutional features of derivatives markets, Treasury auction reopenings, or short squeezes (Nyborg and Strebulaev, 2003).

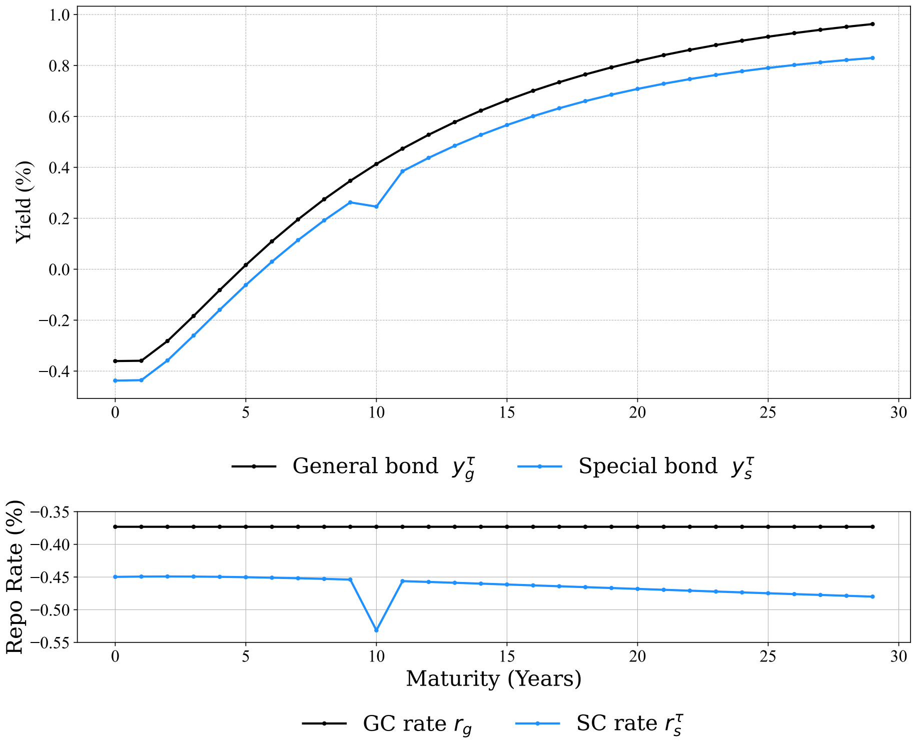

FIGURE 5: **Yield Curve Control.** This figure presents a counterfactual analysis of asset purchases targeted to specific bonds, conducted relative to the baseline calibration illustrated in Figure 4. The top panel of the figure shows the calibrated yield curves of general and special bonds, and the bottom panel shows the calibrated repo market interest rates of general and special bonds of different maturities. Black solid lines represent general bonds and blue solid lines represent special bonds. This figure is based on a calibration of the model to German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

5.4.2 Quantitative Easing

Broad-based QE is a monetary policy tool consisting of bond market asset purchases distributed across maturities along the entire yield curve. To illustrate this policy tool in the context of our model, Figure 6 presents a scenario where the demand of preferred-habitat investors' intercept is homogeneously shocked across the entire yield curve at the new value $\tilde{\theta} = 1.15 \cdot \theta$. This broad demand pressure captures a structural intervention of central banks corresponding to a 15% increment in the excess demand for bonds, equally pronounced across every maturity. For clarity of exposition, in this scenario we have muted differences in excess demand across maturities and the resulting local supply effects.

The top panel of Figure 6 shows the yield curve of general bonds in the baseline scenario (black solid line) and in the QE scenario (black dotted line), alongside the yield curve of special bonds in the baseline scenario (blue solid line) and in the QE scenario (blue dotted line). As can be observed, broad-based QE induces a global flattening of the yield curve for both general and special bonds, reducing 10-year yields by about 0.10% in annualized terms. Special bond yields are somewhat more responsive to QE than general bond yields, but this differential reaction is far less pronounced than in the YCC scenario. Indeed, unlike YCC, QE exerts a broad impact on the entire yield curve of both general and special bonds. That is, the effects of purchases of special bonds $(i=s)$ extend to the yield curve of general bonds that are not directly targeted $(i=g)$, as QE influences the market price of interest rate risk, common to both general and special bonds. Along the yield curves, QE exerts its strongest impact on long-term bond yields, which are more sensitive to changes in the market price of interest rate risk, inducing a reduction in yields peaking at about -0.20% per annum for 30-year bonds.

The bottom panel of of Figure 6 shows the level of GC repo rates (black solid line) alongside the level of SC repo rates in the baseline scenario (blue solid line) and in the QE scenario (blue dotted line). Naturally, QE does not exert any effect on the GC rate, which follows the exogenous process specified in Equation (5). However, QE does induce a decline in SC repo rates of about 1 basis point, comparable to the magnitude reported by D'Amico et al. (2018); moreover, consistent with their empirical evidence, repo rates of short-term securities are relatively more responsive to QE. This counterfactual highlights that, in our model, QE impacts both the bond market and the repo market, as documented by a growing body of empirical evidence.

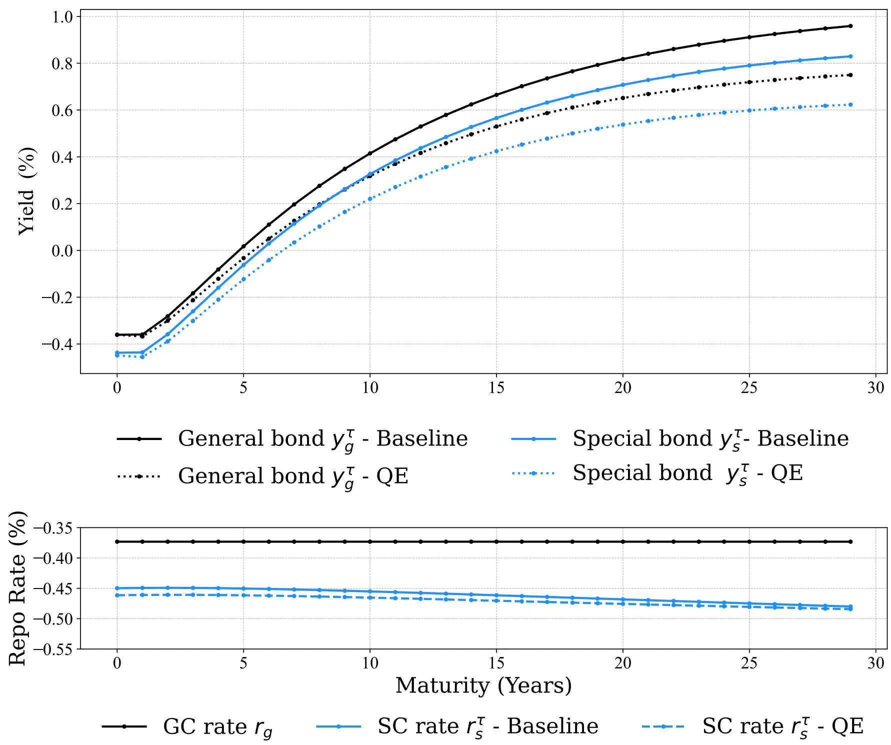

FIGURE 6: **Quantitative Easing.** This figure presents a counterfactual analysis of quantitative easing implemented uniformly across maturities, conducted relative to the baseline calibration illustrated in Figure 4. The top panel of the figure shows the calibrated yield curves of general and special bonds, and the bottom panel shows the calibrated repo market interest rates of general and special bonds of different maturities. Black solid lines represent general bonds and blue solid lines represent special bonds. This figure is based on a calibration of the model to German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

Furthermore, a qualitative implication of our model is that policymakers can fine-tune the time profile of their asset purchases to be impactful for the yield of long-term bonds while minimizing distortions on the repo market. Specifically, policymakers can conduct a given quantity of asset purchases and minimize specialness in the repo market by implementing it over time in a series of interventions. On the one hand, forward-looking bond prices respond to the expected path of asset purchases; on the other hand, special repo rates reflect the demand and supply of collateral in the repo market at each point in time. This suggests that, compared with a one-time intervention, policymakers can implement QE in a manner that smooths distortions in the repo market over time. This is generally consonant with the practice of the major central banks, including the ECB, the Bank of Japan, and the Fed, over the past decade.[33] We are not aware of other models of the yield curve that are able to rationalize this aspect of the implementation of QE.

5.4.3 Securities Lending Facility

The SLF is a monetary policy tool through which the central bank temporarily lends securities from its portfolio in the repo market to reduce repo specialness. To illustrate this policy tool within our model, we examine counterfactual scenarios in which bonds purchased through either YCC or QE interventions are subsequently lent on the repo market via the SLF. For simplicity, we model the SLF as a shift from the equilibrium without securities lending ($\phi^{\tau}_{st} = 0$) to that with full securities lending ($\phi^{\tau}_{st} = 1$). As shown analytically, the SLF returns the equilibrium of Proposition 2 to that of Proposition 1, eliminating the presence of specialness in the repo market.

Fist, we consider the effects of YCC in combination with the SLF. By eliminating repo specialness, the SLF also removes price differences between bonds with identical cash flows, which are absent in Proposition 1. In effect, the SLF offsets the local supply effects induced by YCC. Since the YCC scenario does not generate any global supply effects, a counterfactual analysis of the SLF relative to the YCC intervention in Figure 5 simply restores the model to the baseline calibration shown in Figure 4. Second, we consider the effects of QE in combination with the SLF. As we next show, combining QE with the SLF amplifies the effect on long-term yields relative to QE alone (Lemma 3). For clarity of exposition, we illustrate this effect with reference to the yield curve of general bonds—namely, the risk-free rates of general interest.

Figure 7 plots the calibrated yield curve of general bonds under three alternative scenarios. First, the "Baseline" scenario reproduces the yields on general bonds in the baseline calibration in Figure 4, where $\theta = -0.0152$. Second, the "QE without SLF" scenario reproduces the yields of general bonds in the counterfactual scenario in Figure 6, where the central bank expands its securities portfolio to $\tilde{\theta} = 1.15 \cdot \theta$, while withholding its securities portfolio from the repo market ($\phi^{\tau}_{st} = 0$). Third, the "QE with SLF" scenario plots the yields of general bonds when the central bank conducts the same

[33] From the FAQ on the Public Sector Purchase Program available on the ECB website: "The need to preserve smooth market functioning calls for the necessary amount of purchases at yields below the Deposit Facility Rate [special bonds] to be distributed over time, rather than abruptly changing the sectors of the yield curve where asset purchases take place."

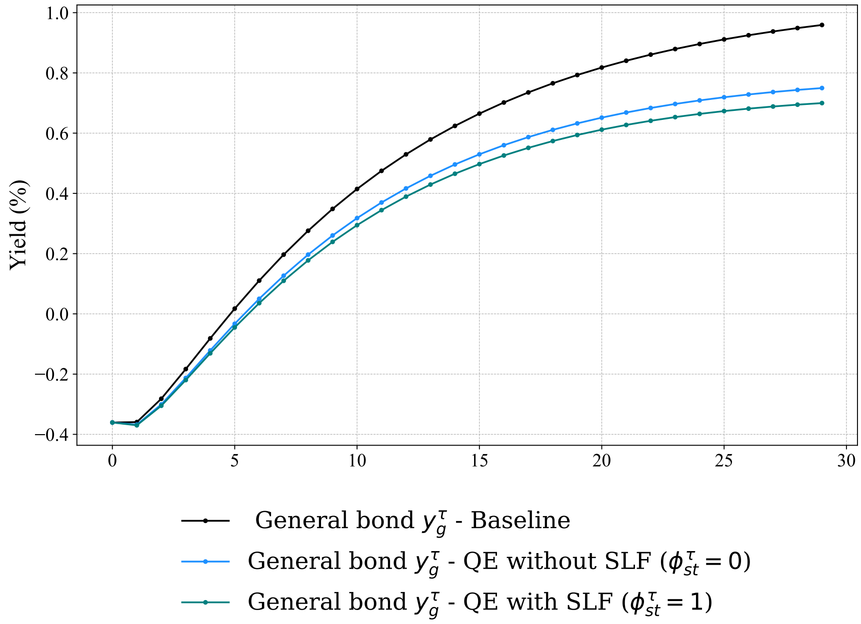

FIGURE 7: **Quantitative Easing and the Securities Lending Facility.** This figure presents the calibrated yield curve of general bonds under three alternative scenarios. The first scenario (black solid line) is the baseline calibration, illustrated in Figure 4. The second scenario (blue solid line) is the QE intervention conducted in the absence of securities lending, illustrated in Figure 6. The third scenario (green solid line) is the QE intervention conducted in the absence of securities lending. Note: as this figure reports yield curves for general bonds, the corresponding repo rates coincide with the GC rate across all maturities and scenarios. This figure is based on a calibration of the model to German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

QE intervention, but in combination with the SLF. In this third scenario, the central bank again sets $\tilde{\theta} = 1.15 \cdot \theta$. Moreover, the central bank sets $\phi_{st}^\tau = 1$, posting its entire securities portfolio as collateral on the repo market without requiring any special repo rate for lending its securities. This third scenario corresponds to the equilibrium in Proposition 1, as the SLF eliminates repo specialness and the resulting price differences between bonds with equivalent cash flows.

Since Figure 7 reports yield curves for general bonds under alternative scenarios, the corresponding repo rates coincide with the GC rate across all maturities in all three scenarios, making it unnecessary to plot them separately, unlike the previous figures that included yield curves for special bonds. For reference, the special repo rates in the "Baseline" scenario are shown in the bottom panel of Figure 4; in the "QE without SLF" scenario, they are reported in the bottom panel of Figure 6; and in the "QE with SLF" scenario, special repo rates coincide with the GC rate.

A comparison between the three scenarios in Figure 7 highlights that, given the quantity of asset purchases, QE combined with the SLF exerts a stronger effect on the yield curve of general bonds than QE alone. Specifically, introducing the SLF in combination with QE induces a further 2.5 basis points decline in the yield of 10-year bonds relative to the "QE without SLF" scenario. Thus, as was derived analytically in Lemma 3, the SLF maximizes the impact of QE on long-term yields.

The flattening of the Bund yield curve around the introduction of the SLF, documented in Internet Appendix Figure IA.III, provides direct empirical support for this result.

As an important insight, this counterfactual analysis highlights that dysfunctional repo markets impair the transmission of QE to the yield of long-term bonds. The key mechanism at work is that repo specialness acts as an important friction, curbing the scale of carry trades by arbitrageurs. Substantial levels of repo specialness induce arbitrageurs to hold more conservative portfolios, reducing their exposure to interest rates and their required compensation for bearing interest rate risk in the form of a flatter yield curve. Thus, our analysis suggests that the SLF reinforces the transmission of QE to long-term rates. This statement may seem counterintuitive: after all, QE involves an exchange of cash for bonds with the private sector, while the SLF involves an exchange of bonds for cash with the private sector—therefore, one might think that the SLF acts in the opposite direction of QE. However, the two policy tools play a distinct role: QE shifts the *duration risk* of the purchased bonds between the private sector and the central bank, while the SLF maintains *collateral availability* in repo markets, facilitating the activity of arbitrage. As our analysis underscores, the yields of long-term bonds are shaped by both the exposure of investors to bond duration risk and the availability of collateral in the repo market. Thus, QE in the bond market is more effective when combined with the SLF in the repo market.

6 Conclusion

Standard models used in macroeconomics and finance generally explain the yield curve by considering agents' decisions to invest in bonds with different maturities. This approach regards bonds as investment opportunities but abstracts from their important role as collateral.

This paper develops a dynamic equilibrium model in which bonds serve both as investment opportunities and as collateral. The model considers two markets, the bond and the repo markets, and two types of agents, investors with preferences for specific bonds and risk-averse arbitrageurs. This heterogeneity gives rise to price differences between bonds with identical cash flows that affect the collateral specialness of bonds in the repo market. This effect arises when preferred-habitat investors purchase bonds, but refrain from offering them on the repo market. Repo specialness raises the cost of carry trades for arbitrageurs and influences their portfolio duration and, therefore, the market price of interest rate risk and the transmission of shocks across the yield curve. The equilibrium predicts and accounts for the positive association between aggregate repo specialness and term spreads in the data.

The paper offers important and actionable recommendations for the conduct of monetary policy. To maximize the transmission of monetary policy to the yield curve, policymakers should modulate the quantity of bonds available as collateral in the repo market. Policymakers aiming to generate global supply effects on the entire yield curve should supply their bonds on the repo market, and enable arbitrageurs to transmit the effect of asset purchases across the yield curve. Policymakers willing to influence the price of specific bonds, such as green bonds, can withhold the purchased bonds from the repo market, thereby allowing them to acquire special collateral value and trade at a premium relative to other bonds. This paper thus recommends that monetary policy be implemented in a coordinated manner across the bond and repo markets. Future research could incorporate our model into a macroeconomic setup in which interest rates interact with real economic activity.

A Proof of Lemma 1

From Arbitrageurs' FOC in Equation (13),

$$\mu_{it}^{\tau}-r_{it}^{\tau}=\tau y_{it}^{\tau}-(\tau-dt)\mathbb{E}_t[y_{it+dt}^{\tau-dt}]+\frac{1}{2}\mathbb{V}_t[y_{it+dt}^{\tau-dt}]-r_{it}^{\tau}=-a_{i\tau}\lambda_t. \tag{A.1}$$

Iterating forward this condition yields

$$y_{it}^{\tau}=\underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^{\tau} r_{i,t+u}^{\tau-u}du\right]}_{\text{Expected short rates}}+\underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^{\tau} \mu_{i,t+u}^{\tau-u}-r_{i,t+u}^{\tau-u}du\right]}_{\text{Risk premium}}-\underbrace{\frac{1}{\tau}\left[\int_0^{\tau} a_{i,\tau-u}^2\frac{\sigma_r^2}{2}du\right]}_{\text{Convexity adjustment}}.$$

Substituting again $\mu_{it}^{\tau}-r_{it}^{\tau}=-a_{i\tau}\lambda_t$ completes the proof. *Q.E.D.*

B Proof of Lemma 2

By substituting Equation (10) into the affine representation in Equation (19), we obtain the price of general and special bonds, since $X_{gt}^{\tau}=0$ for general bonds whose status is $i=g$:

$$P_{gt}^{\tau}=\mathbb{E}_t^*\left[e^{-\int_0^{\tau} r_{t+u}du}\right]=e^{-A_\tau r_t-C_\tau},$$
$$P_{st}^{\tau}=\mathbb{E}_t^*\left[e^{-\int_0^{\tau} r_{s,t+u}^{\tau-u}du}\right]=e^{-A_\tau r_t-B_\tau X_{st}^{\tau}-C_\tau}.$$

The result follows by taking the price ratio of the general bond P_{gt}^{τ} to the special bond P_{st}^{τ}, and by observing that the bond market clearing condition requires $Z_{it}^{\tau}+X_{it}^{\tau}=0$. *Q.E.D.*

C Proof of Proposition 1

Arbitrageurs' optimality condition in the bond market is

$$\mu_{it}^{\tau}-r_{st}^{\tau}=a_{i\tau}\gamma\sigma_r^2\int_0^{\infty} a_{g\tau}X_{gt}^{\tau}+a_{s\tau}X_{st}^{\tau}d\tau. \tag{C.2}$$

Substituting the values of μ_{it}^{τ} and r_{st}^{τ}, we derive the following relationship.

$$\dot{a}_{i\tau}r_t + a_{i\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a_{i\tau}^2\sigma_r^2 + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - r_{st}^{\tau} = a_{i\tau}\gamma\sigma_r^2\int_0^\infty a_{g\tau}X_{gt}^{\tau} + a_{s\tau}X_{st}^{\tau}d\tau.$$

From the repo market equilibrium with securities lending, we have $l_{it}^{\tau} = 0$, so that $r_{st}^{\tau} = r_t$. From the bond market equilibrium, we have $X_{gt}^{\tau} = 0$ and $X_{st}^{\tau} = -Z_{st}^{\tau} = \theta_\tau - \eta_\tau\left(a_\tau r_t + b_\tau\theta_\tau + c_\tau\right)$. Therefore,

$$\dot{a}_{i\tau}r_t + a_{i\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a_{i\tau}^2\sigma_r^2 + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - r_t = a_{i\tau}\gamma\sigma_r^2\int_0^\infty a_\tau\left[\theta_\tau - \eta_\tau\left(a_{s\tau}r_t + b_{s\tau}\theta_\tau + c_{s\tau}\right)\right]d\tau.$$

The above equation must hold for all values of r_t. By matching the coefficients in r_t, we derive the following first-order linear ordinary differential equation (ODE).

$$\dot{a}_{i\tau} + a_{i\tau}\kappa_r - 1 = -a_{i\tau}\gamma\sigma_r^2\int_0^\infty \eta_\tau a_{s\tau}^2 d\tau. \tag{C.3}$$

Moreover, the terms that are independent of the risk factors must add up to zero, implying

$$\dot{c}_{i\tau} = a_{i\tau}\kappa_r\overline{r} - \frac{1}{2}a_{i\tau}^2\sigma_r^2 - \dot{b}_{i\tau}\theta_\tau - b_{i\tau}\dot{\theta}_\tau + a_{i\tau}\gamma\sigma_r^2\int_0^\infty a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right]d\tau.$$

Since at maturity a bond repays the principal notional, we have $A_0 = B_0 = C_0 = 0$, which implies that the initial conditions for the ODEs are $a_{i0} = b_{i0} = c_{i0} = 0$. We follow Vayanos and Vila (2021) and proceed in two steps. First, we take the integrals as given and solve the above equations as linear ODEs. Second, we require that the solution is consistent with the value of the integrals. This procedure results into

$$a_{i\tau} = \frac{1 - e^{-\kappa_r^*\tau}}{\kappa_r^*}, \qquad b_{i\tau} = 0, \qquad c_{i\tau} = \kappa_r^*\overline{r}^*\int_0^\tau a_{i,u}du - \frac{\sigma_r^2}{2}\int_0^\tau a_{i,u}^2 du,$$

where the scalars $(\kappa_r^*, \overline{r}^*)$ are defined by

$$\kappa_r^* = \kappa_r + \gamma\sigma_r^2\int_0^\infty \eta_\tau a_{s\tau}^2 d\tau, \tag{C.4}$$

$$\kappa_r^*\overline{r}^* = \kappa_r\overline{r} + \gamma\sigma_r^2\int_0^\infty a_{s\tau}\left[\theta_\tau - \eta_\tau c_{s\tau}\right]d\tau. \tag{C.5}$$

Finally, it can be observed that the coefficients $(a_{i\tau}, b_{i\tau}, c_{i\tau})$, together with bond status i, uniquely determine the coefficients (A_τ, B_τ, C_τ), verifying the conjecture that the price is an affine function of r_t and X_{it}^{τ}. *Q.E.D.*

D Proof of Proposition 2

Arbitrageurs' optimality condition in the bond market is

$$\mu_{it}^{\tau} - r_{st}^{\tau} = a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g\tau}X_{gt}^{\tau} + a_{s\tau}X_{st}^{\tau}d\tau. \tag{D.6}$$

Substituting the values of μ_{it}^{τ} and r_{st}^{τ}, we derive the following relationship.

$$\dot{a}_{i\tau}r_t + a_{i\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a_{i\tau}^2\sigma_r^2 + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - r_t + l_{it}^{\tau} = a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g\tau}X_{gt}^{\tau} + a_{s\tau}X_{st}^{\tau}d\tau.$$

From the repo market equilibrium without the securities lending, we have $l_{it}^{\tau} = \mathcal{E}_s Z_{st}^{\tau}$. From the bond market equilibrium, we have $X_{gt}^{\tau} = 0$ and $X_{st}^{\tau} = -Z_{st}^{\tau} = \theta_\tau - \eta_\tau\left(a_\tau r_t + b_\tau\theta_\tau + c_\tau\right)$. Therefore,

$$\begin{aligned}
&\dot{a}_{i\tau}r_t + a_{i\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a_{i\tau}^2\sigma_r^2 + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - r_t - \mathcal{E}_i\left[\theta_\tau - \eta_\tau\left(a_{i\tau}r_t + b_{i\tau}\theta_\tau + c_{i\tau}\right)\right] \\
&= a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} a_\tau\left[\theta_\tau - \eta_\tau\left(a_{s\tau}r_t + b_{s\tau}\theta_\tau + c_{s\tau}\right)\right]d\tau.
\end{aligned} \tag{D.7}$$

Equation (D.7) must hold for all values of r_t. By matching the coefficients in the risk factors, we derive the following ODE.

$$\dot{a}_{i\tau} + a_{i\tau}\left(\kappa_r + \eta_\tau\mathcal{E}_i\right) - 1 = -a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} \eta_\tau a_{s\tau}^2 d\tau. \tag{D.8}$$

Moreover, the terms that are independent of the risk factors must add up to zero, implying

$$\begin{aligned}
\dot{c}_{i\tau} &= a_{i\tau}\kappa_r\overline{r} - \frac{1}{2}a_{i\tau}^2\sigma_r^2 - \dot{b}_{i\tau}\theta_\tau - b_{i\tau}\dot{\theta}_\tau + \mathcal{E}_i(\theta_\tau - \eta_\tau(b_{i\tau} + c_{i\tau})) \\
&\quad + a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right]d\tau.
\end{aligned} \tag{D.9}$$

We further require that the optimality holds for both general and special bonds. It can be observed that Lemma 2 implies $b_{g\tau} = 0\ \forall\tau$. Thereby, we separate Equation (D.9) into two ODEs,

$$\dot{b}_{s\tau} + b_{s\tau}\overline{\theta}_\tau = \mathcal{E}_s\left(1 - \eta_\tau c_\tau\theta_\tau^{-1}\right) \tag{D.10}$$

$$\dot{c}_{i\tau} = a_{i\tau}\kappa_r\overline{r} - \frac{1}{2}a_{i\tau}^2\sigma_r^2 + a_{i\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right]d\tau, \tag{D.11}$$

where we have defined $\overline{\theta}_\tau = \dot{\theta}_\tau(1 + \eta_\tau\mathcal{E}_s)/\theta_\tau$. We thus have a system of three linear first-order ODEs. Since at maturity a bond repays the principal notional, we have $A_0 = B_0 = C_0 = 0$, which implies that the initial conditions for the ODEs are $a_{i0} = b_{i0} = c_{i0} = 0$. We again proceed in two steps. First, we take the integrals as given and solve the above equations as linear ODEs. Second, we require that the solution is consistent with the value of the integrals. This procedure results into

$$a_{i\tau} = \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}, \qquad b_{i\tau} = \frac{\mathcal{E}_i(1-g_\tau)(1-e^{-\int\overline{\theta}_\tau d\tau})}{\overline{\theta}_\tau}, \qquad c_{i\tau} = \kappa_r^*\overline{r}^*\int_0^\tau a_{i,u}du - \frac{\sigma_r^2}{2}\int_0^\tau a_{i,u}^2 du,$$

where the scalars $(\kappa_r^*, \overline{r}^*)$ are defined by

$$\kappa_r^* = \kappa_r + \eta_\tau \mathcal{E}_i + \gamma\sigma_r^2 \int_0^\infty \eta_\tau a_{s\tau}^2 d\tau, \tag{D.12}$$

$$\kappa_r^*\overline{r}^* = \kappa_r\overline{r} + \gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right]d\tau. \tag{D.13}$$

Here, the deterministic function g_τ is defined by

$$g_\tau = e^{-\int\overline{\theta}_\tau d\tau}\int \frac{\eta_\tau c_{i\tau}}{\theta_\tau} e^{\int\overline{\theta}_\tau d\tau} d\tau.$$

Finally, it can be observed that the coefficients $(a_{i\tau}, b_{i\tau}, c_{i\tau})$, together with bond status i, uniquely determine the coefficients (A_τ, B_τ, C_τ), verifying the conjecture that the price is an affine function of r_t and X_{it}^τ. *Q.E.D.*

E Proof of Corollary 1

In the equilibrium absent securities lending (Proposition 2), repo specialness equals $l_{it}^\tau = Z_{it}^\tau \mathcal{E}_i$. By definition, general bonds carry no repo specialness. As a consequence, we rewrite the duration-weighted average specialness in the repo market as Aggregate Specialness$_t = \int_0^\infty a_{s\tau}\mathcal{E}_s Z_{st}^\tau d\tau$. Thus, aggregate repo specialness is positively related to the duration-weighted collateral value premium of special bonds, $\int_0^\infty a_{s\tau} B_{s\tau} Z_{st}^\tau d\tau$, given that $B_{s\tau}$ is an increasing function of $\mathcal{E}_s$. Moreover, it can be observed that the aggregate duration-weighted collateral value premium of special bonds is positively related to the collateral-value duration of the arbitrageurs' portfolio and inversely related to the market price of risk, according to Equation (21) reproduced for convenience below.

$$\lambda_t^\tau = \underbrace{\gamma\sigma_r^2 \int_0^\infty a_{s\tau}\left[\eta_\tau\left(A_\tau r_t + C_\tau\right) - \theta_\tau\right]d\tau}_{\text{Investment-value duration}} - \underbrace{\gamma\sigma_r^2 \int_0^\infty a_{s\tau}\eta_\tau B_{s\tau} Z_{st}^\tau d\tau}_{\text{Collateral-value duration}}. \tag{E.14}$$

Thereby, the market price of risk is a decreasing function of aggregate repo specialness, namely, $\lambda_t = h(\text{Aggregate Specialness}_t)$ for some decreasing function h. We next consider the relationship between the market price of risk and bond yields. Given Lemma 1,

$$\text{Term Spread}_{it}^{\tau} = \underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^{\tau} r_{i,t+s}^{\tau-s} ds\right]}_{\text{Expected short rates}} - \underbrace{\frac{1}{\tau}\mathbb{E}_t\left[\int_0^{\tau} a_{i,\tau-s}\lambda_{t+s} ds\right]}_{\text{Risk premium}} - \underbrace{\frac{1}{\tau}\left[\int_0^{\tau} a_{i,\tau-s}^2 \frac{\sigma_r^2}{2} ds\right]}_{\text{Convexity adjustment}} - r_{it}^{\tau}.$$

a lower market price of risk, λ_t, raises the term spread across the entire yield curve in proportion to the duration of the bonds considered, $a_{i\tau}$, which is increasing with maturity. Thus, the market price of risk is negatively related to term spreads. Then, term spreads are positively related to aggregate specialness, so that for some increasing function f, $\text{Term Spread}_{it}^{\tau} = f(\text{Aggregate Specialness}_t)$. *Q.E.D.*

F Proof of Lemma 3

The analytical expression for the yield of general bonds is

$$\begin{aligned} y_{g,t}^{\tau} &= \frac{1}{\tau}\left(a_{g,\tau} r_t + c_{g,\tau}\right) \\ &= \frac{1}{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} r_t + \kappa_r^* \bar{r}^* \int_0^{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right) d\tau - \frac{\sigma_r^2}{2}\int_0^{\tau}\left(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*}\right)^2 d\tau\right). \end{aligned} \tag{F.15}$$

Equation (F.15) shows the relationship between the yield of general bonds and κ_r^* and $\bar{r}^*$, the counterparts of κ_r and $\bar{r}$ under the risk-neutral measure. From Proposition 2, we observe that the risk-adjusted speed of mean reversion, κ_r^*, is influenced by the preferred-habitat demand elasticity, α_τ, the elasticity of collateral supply $\mathcal{E}_i$, the arbitrageurs' risk aversion, γ, and the volatility of the interest rate, σ_r, but not by the demand intercept, θ_τ. Given that κ_r^* does not depend on θ_τ, neither does the coefficient $a_{i\tau}$. QE is an exogenous shock to demand, $QE_{\{\tau\}}$, that does not affect the short rate, r_t. Thus, QE solely affects the yields of general bonds via its effect on $\bar{r}^*$, the risk-adjusted long-run mean of the interest rate, which impacts the coefficient $c_{i\tau} = \kappa_r^*\bar{r}^*\int_0^{\tau} a_{i,u} du - \frac{\sigma_r^2}{2}\int_0^{\tau} a_{i,u}^2 du$ only through the product $\kappa_r^*\bar{r}^*$. Next, examine $\kappa_r^*\bar{r}^*$. In Proposition 1,

$$\kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2\int_0^{\infty} a_{s\tau}\left[\theta_\tau - \eta_\tau c_{s\tau}\right] d\tau.$$

In Proposition 2,

$$\kappa_r^*\bar{r}^* = \kappa_r\bar{r} + \gamma\sigma_r^2\int_0^{\infty} a_{s\tau}\left[\theta_\tau - \eta_\tau(b_{s\tau}\theta_\tau + c_{s\tau})\right] d\tau.$$

Given these considerations, the statement follows by comparing the effects of QE on general bond yields across the equilibria characterized in Propositions 1 and 2. Analytically, the purchase of a bond at each maturity induces an effect $\frac{\partial y_{gt}^{\tau}}{\partial\theta_{\tau}}=\frac{1}{\tau}\frac{\partial\kappa_{\tau}^{*}\bar{r}^{*}}{\partial\theta_{\tau}}$. This effect differs across the two equilibria; specifically,

$$\left|\frac{\partial y_{gt}^{\tau}(\phi_{st}^{\tau}=1)}{\partial\theta_{\tau}}\right|=\frac{1}{\tau}\gamma\sigma_{r}^{2}a_{s\tau}\geq\frac{1}{\tau}\gamma\sigma_{r}^{2}a_{s\tau}(1-\eta_{\tau}b_{s\tau})=\left|\frac{\partial y_{gt}^{\tau}(\phi_{st}^{\tau}=0)}{\partial\theta_{\tau}}\right|. \tag{F.16}$$

The former obtains under Proposition 1 and the latter under Proposition 2. The variation induced by QE_{τ} in the yield of general bonds, Δy_{gt}^{τ}, equals the integral across maturities of the pointwise variation in the yield induced by asset purchases at each maturity.

$$\Delta y_{gt}^{\tau}(QE_{\{\tau\}},\phi_{st}^{\tau})=\int_{0}^{\infty}-\frac{\partial y_{gt}^{\tau}(\phi_{st}^{\tau})}{\partial\theta_{\tau}}QE_{\tau}d\tau.$$

Integrating Equation (F.16) across maturities,

$$\underbrace{\left|\Delta y_{gt}^{\tau}(QE_{\{\tau\}},\phi_{st}^{\tau}=1)\right|}_{\text{Effect of QE with SLF }(\phi_{st}^{\tau}=1)}=\frac{\gamma\sigma_{r}^{2}}{\tau}\int_{0}^{\infty}a_{s\tau}QE_{\tau}d\tau\geq\frac{\gamma\sigma_{r}^{2}}{\tau}\int_{0}^{\infty}a_{s\tau}(1-\eta_{\tau}b_{s\tau})QE_{\tau}d\tau=\underbrace{\left|\Delta y_{gt}^{\tau}(QE_{\{\tau\}},\phi_{st}^{\tau}=0)\right|}_{\text{Effect of QE without SLF }(\phi_{st}^{\tau}=0)}.$$

The inequality follows because, in the absence of the SLF ($\phi_{st}^{\tau}=0$), $b_{s\tau}$ is positive, reflecting the price premium associated with special bonds in the equilibrium with endogenous specialness.

Q.E.D.

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TABLE I: **Summary Statistics**

The table reports summary statistics for the sample, which consists of the universe of German bonds from October 2014 to July 2023, as recorded by LSEG. The sample is composed of 1,956 unique days and 452 unique weeks. Repo rates are from Brokertec. The yield to maturity and the interest rate variables are expressed in percentage points per annum.

PANEL A: DAILY DATA

	Observations	Mean	Median	Std Dev	Skewness	Min	Max
Duration (years)	94,142	7.245	5.595	5.836	1.424	0.750	30.000
Amount issued (€ billions)	94,142	4.910	5.000	1.060	-0.534	0.511	8.000
Yield to maturity	94,142	-0.028	-0.239	0.742	1.460	-0.882	2.499
GC repo rate	94,142	-0.360	-0.494	0.518	4.599	-0.850	2.830
Bond-level repo rate	94,142	-0.464	-0.571	0.534	4.099	-1.250	2.717
Repo specialness	94,142	0.105	0.073	0.124	1.749	-0.210	0.647
Aggregate specialness	94,142	0.610	0.521	0.494	1.092	-1.372	3.747

PANEL B: DAILY DATA BY DURATION BUCKET

		Yield to maturity			Repo rate		
Duration Bucket	Observations	Mean	Median	Std Dev	Mean	Median	Std Dev
[0.75, 1.25]	1,956	-0.452	-0.653	0.616	-0.433	-0.554	0.510
[2, 4)	1,956	-0.387	-0.569	0.620	-0.455	-0.567	0.511
[4, 7)	1,956	-0.185	-0.289	0.608	-0.469	-0.580	0.517
[7, 10)	1,956	0.135	0.159	0.613	-0.507	-0.604	0.530
[10, 15)	1,956	0.480	0.542	0.643	-0.450	-0.583	0.521
[15, 20)	1,956	0.673	0.782	0.648	-0.482	-0.595	0.540
$\geq$20	1,956	0.782	0.899	0.621	-0.476	-0.590	0.532

PANEL C: WEEKLY DATA BY DURATION BUCKET

		Yield to maturity			Repo rate		
Duration Bucket	Observations	Mean	Median	Std Dev	Mean	Median	Std Dev
[0.75, 1.25]	452	-0.283	-0.645	0.880	-0.312	-0.554	0.766
[2, 4)	452	-0.210	-0.538	0.874	-0.341	-0.568	0.768
[4, 7)	452	-0.022	-0.268	0.828	-0.348	-0.580	0.774
[7, 10)	452	0.268	0.196	0.776	-0.382	-0.603	0.788
[10, 15)	452	0.593	0.588	0.767	-0.324	-0.581	0.777
[15, 20)	452	0.764	0.834	0.737	-0.351	-0.592	0.798
$\geq$20	452	0.860	0.939	0.694	-0.348	-0.586	0.788

TABLE II: BOND TERM SPREADS AND AGGREGATE REPO SPECIALNESS (WEEKLY DATA)

The table reports estimates from the time series regression in Equation (3), relating bond term spreads across several duration buckets to the general collateral interest rate and the aggregate repo specialness times bond duration. Regressions are estimated separately by subsample. For each week, bonds are classified as general if their repo specialness is below the cross-sectional median, and as special if above. The relevant subsamples are then aggregated into duration buckets. This table is based on German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023. Robust standard errors are reported in parentheses. One, two, and three asterisks denote statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

PANEL A: ALL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.165***	0.294***	0.396***	0.404***	0.365***	0.366***
(SE)	(0.021)	(0.033)	(0.047)	(0.057)	(0.069)	(0.076)
Constant	-0.032**	0.074***	0.300***	0.620***	0.816***	0.911***
(SE)	(0.012)	(0.021)	(0.031)	(0.039)	(0.046)	(0.049)
Observations	452	452	452	452	452	452
R^2	0.20	0.22	0.17	0.11	0.07	0.06

PANEL B: GENERAL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.101***	0.227***	0.363***	0.314***	0.265***	0.233**
(SE)	(0.019)	(0.031)	(0.046)	(0.052)	(0.062)	(0.078)
Constant	0.009	0.134***	0.387***	0.724***	0.909***	1.012***
(SE)	(0.012)	(0.019)	(0.029)	(0.033)	(0.039)	(0.044)
Observations	444	446	446	436	416	408
R^2	0.11	0.16	0.16	0.09	0.05	0.03

PANEL C: SPECIAL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.135***	0.232***	0.272***	0.289***	0.297***	0.195**
(SE)	(0.020)	(0.031)	(0.044)	(0.048)	(0.068)	(0.065)
Constant	-0.015	0.096***	0.334***	0.655***	0.929***	0.991***
(SE)	(0.012)	(0.021)	(0.031)	(0.035)	(0.049)	(0.046)
Observations	405	405	405	380	353	396
R^2	0.17	0.18	0.11	0.10	0.09	0.03

TABLE III: BOND TERM SPREADS AND AGGREGATE REPO SPECIALNESS (DAILY DATA)

The table reports estimates from the time series regression in Equation (3), relating bond term spreads across several duration buckets to the general collateral interest rate and the aggregate repo specialness times bond duration. Regressions are estimated separately by subsample. For each day, bonds are classified as general if their repo specialness is below the cross-sectional median, and as special if above. The relevant subsamples are then aggregated into duration buckets. This table is based on German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023. Robust standard errors are reported in parentheses. One, two, and three asterisks denote statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

PANEL A: ALL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.106***	0.211***	0.308***	0.323***	0.312***	0.326***
(SE)	(0.008)	(0.015)	(0.023)	(0.027)	(0.030)	(0.032)
Constant	0.000	0.138***	0.398***	0.734***	0.933***	1.034***
(SE)	(0.005)	(0.010)	(0.015)	(0.018)	(0.021)	(0.021)
Observations	1956	1956	1956	1956	1956	1956
R^2	0.15	0.18	0.16	0.11	0.08	0.08

PANEL B: GENERAL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.098***	0.205***	0.354***	0.288***	0.221***	0.166***
(SE)	(0.008)	(0.015)	(0.026)	(0.028)	(0.032)	(0.039)
Constant	-0.006	0.139***	0.407***	0.778***	1.029***	1.106***
(SE)	(0.005)	(0.010)	(0.017)	(0.019)	(0.022)	(0.024)
Observations	1915	1937	1922	1672	1587	1396
R^2	0.17	0.17	0.18	0.09	0.04	0.02

PANEL C: SPECIAL BONDS

	Term Spread$_t^\tau$					
	[2,4)	[4,7)	[7,10)	[10,15)	[15,20)	$\geq$20
Aggregate Specialness$_t$	0.092***	0.166***	0.187***	0.240***	0.292***	0.215***
(SE)	(0.010)	(0.016)	(0.021)	(0.030)	(0.035)	(0.035)
Constant	0.006	0.138***	0.394***	0.673***	0.935***	1.006***
(SE)	(0.005)	(0.009)	(0.013)	(0.018)	(0.023)	(0.021)
Observations	1431	1437	1437	1275	1288	1296
R^2	0.11	0.13	0.07	0.08	0.08	0.04

TABLE IV: **Calibration**

The table reports the model parameter annualized estimates from the two-step GMM procedure with an optimal weighting matrix for the quantitative model described in Section 5.2. The model is calibrated using German Treasury bond-level data from LSEG covering the period from October 2014 to July 2023.

Parameter	Interpretation	Value
δ_0	Constant component of r_t	-0.0037
κ_{1r}	Mean reversion of r_{1t}	0.0510
$\overline{r}_1$	Long-run mean of r_{1t}	0.0688
σ_{1r}	Diffusion of r_{1t}	0.0127
κ_{2r}	Mean reversion of r_{2t}	0.0487
$\overline{r}_2$	Long-run mean of r_{2t}	0.0694
σ_{2r}	Diffusion of r_{2t}	0.0151
$\mathcal{E}_s$	Special collateral supply slope	0.0511
η	Preferred-habitat demand slope	0.0213
θ	Preferred-habitat demand intercept	-0.0152
γ	Arbitrageur risk aversion	1.7825

Internet Appendix
“Monetary Policy, the Yield Curve, and the Repo Market”

Ruggero Jappelli, Loriana Pelizzon, and Marti G. Subrahmanyam.

Internet Appendix A: Additional Figures

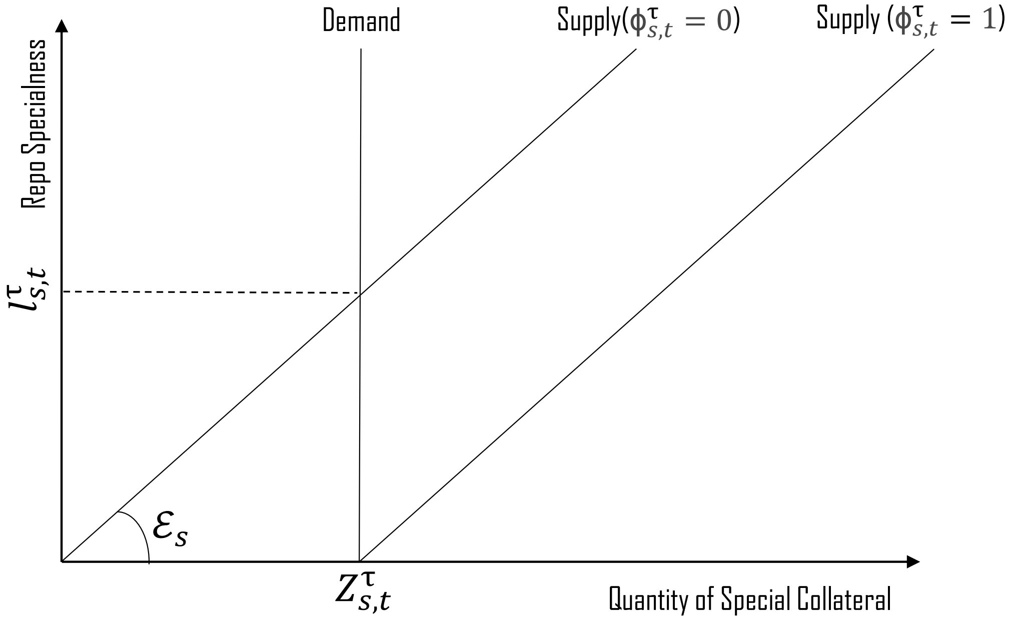

FIGURE IA.I: **Demand and supply of special collateral.** The figure illustrates the functioning of the market for repurchase agreements collateralized by special bonds. The horizontal axis shows the quantity of collateral, and the vertical axis shows repo specialness. The demand curve is price-inelastic because of the commitment of short sellers to deliver the specific issue. The supply, by contrast, is elastic, as holders of special collateral bonds require greater compensation to pledge additional units of the security on the repo market. The figure depicts how increasing securities lending by preferred-habitat investors, as ϕ^{τ}_{st} ranges from 0 (no lending) to 1 (full lending), induces a rightward shift of the supply of collateral.

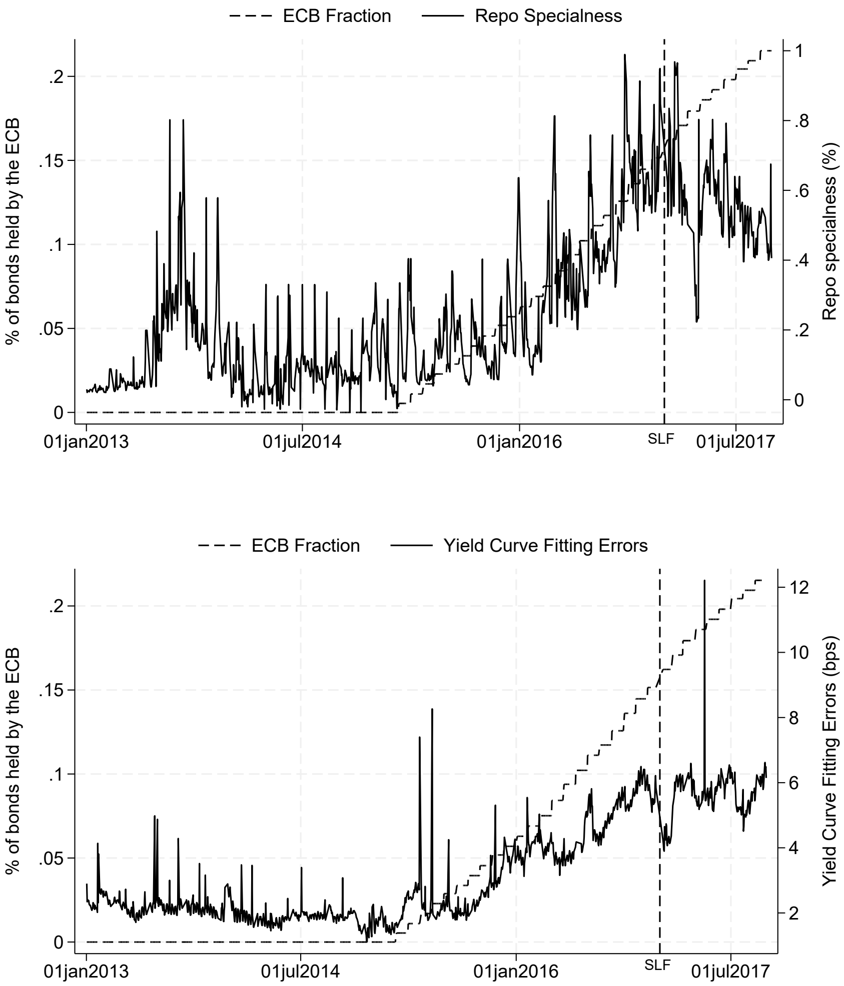

FIGURE IA.II: **Asset Purchases, Repo Specialness, and Yield Curve Fitting Errors**. The top panel shows the proportion of bonds held by the ECB (left y-axis) and the volume-weighted average repo specialness of German treasury bonds (right y-axis). The bottom panel shows the proportion of bonds held by the ECB (left y-axis) and the Hu et al. (2013) German treasury yield curve fitting errors (right y-axis). The dashed vertical line corresponds to the implementation of the cash-collateralized SLF. The data are from Pelizzon et al. (2025).

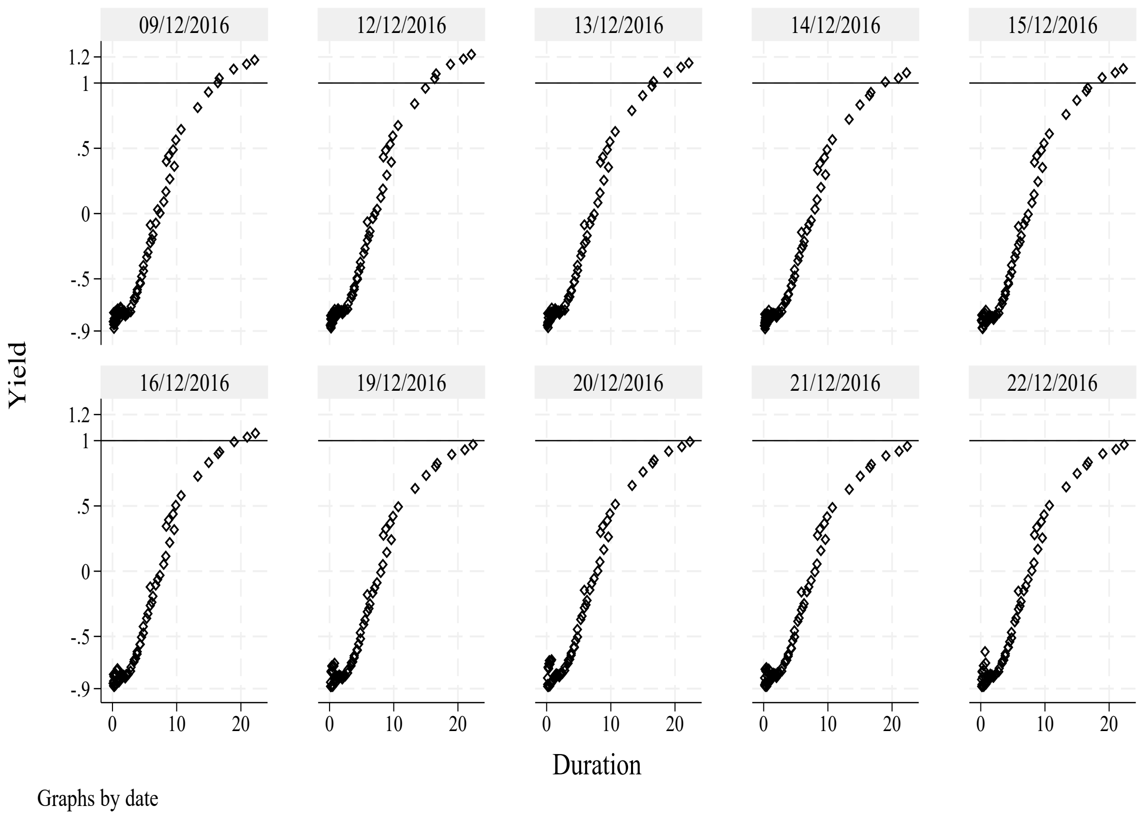

FIGURE IA.III: **Securities Lending Facility and the Yield Curve**. The figure plots the yields of the entire universe of German Treasury bonds covered in our sample against their durations. The plots are shown separately for each trading day surrounding the introduction of the Securities Lending Facility (SLF) on December 15, 2016. The solid horizontal line indicates a yield level of 1 percent per annum. Bond-level data are sourced from LSEG.

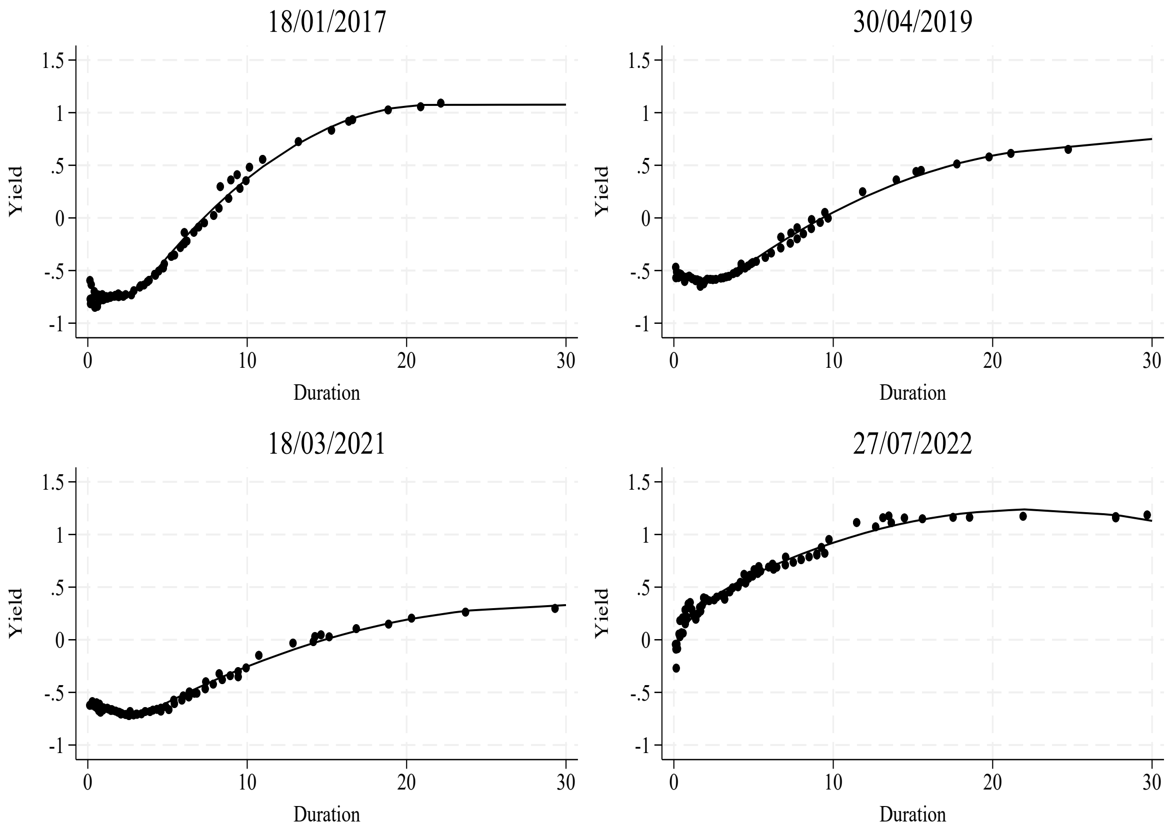

FIGURE IA.IV: **Examples of Interpolated Yield Curves and Market Observed Bond Yields**. The figure plots the yields of German Treasury bonds against their durations for four dates chosen to illustrate different market conditions. In each panel, the black dots represent the observed yields for each bond, while the solid line is the yield curve estimated using the spline methodology described in Section 5. The horizontal axis represents the bond's duration and the vertical axis represents its yield. Bond-level data are sourced from LSEG.

Internet Appendix B: Quantitative Model

Here, we characterize the equilibrium when the short rate is a linear combination of independent factors,

$$r_t = \delta_0 + \delta_1^\top R_t. \tag{IA.1}$$

We want to show that there exist deterministic functions of bond tenor $\mathscr{A}_\tau = [\mathscr{A}_{1\tau}, \mathscr{A}_{2\tau}]^\top$, $\mathscr{B}_\tau$, and $\mathscr{C}_\tau$ such that bond prices satisfy

$$-\log P_{it}^\tau = \mathscr{A}_\tau^\top R_t + \mathscr{B}_\tau X_{it}^\tau + \mathscr{C}_\tau. \tag{IA.2}$$

As in Equation (11), we define $a_{i\tau} = [a_{1i\tau}, a_{2i\tau}]^\top = \left[\frac{\mathscr{A}_{1\tau}}{1+\eta_\tau \mathscr{B}_{i\tau}}, \frac{\mathscr{A}_{2\tau}}{1+\eta_\tau \mathscr{B}_{i\tau}}\right]^\top$, $b_{i\tau} = \frac{\mathscr{B}_{i\tau}}{1+\eta_\tau \mathscr{B}_{i\tau}}$, and $c_{i\tau} = \frac{\mathscr{C}_\tau}{1+\eta_\tau \mathscr{B}_{i\tau}}$. By replacing Equation (9) into Equation (25), we then rearrange the guess in Equation (IA.2) as follows.

$$-\log P_{it}^\tau = a_{i\tau}^\top R_t + b_{i\tau}\theta_\tau + c_{i\tau}. \tag{IA.3}$$

With this notation in place, we replace Equations (23) and (24) into Equation (25) and apply Ito's lemma, obtaining the dynamics of bond prices.

$$\frac{dP_{it}^\tau}{P_{it}^\tau} = \mu_{it}^\tau dt - a_{i\tau}^\top \Sigma dV_t^r, \tag{IA.4}$$

$$\mu_{it}^\tau \equiv \dot{a}_{i\tau}^\top R_t + a_{i\tau}^\top K_r(R_t - \overline{R}) + \frac{1}{2} a_{i\tau}^\top \Sigma\Sigma a_{i\tau} + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau}.$$

By symmetry of the covariance matrix, $\Sigma^\top = \Sigma$. It can be observed that the model retains the same structural form as in the one-factor case. Replacing the bond price dynamics into the problem of the arbitrageurs yields

$$\max_{\{X_{it}^\tau\}} W_t r_t + \int_0^\infty X_{gt}^\tau(\mu_{gt}^\tau - r_t) + X_{st}^\tau(\mu_{st}^\tau - r_{st}^\tau) d\tau - \frac{\gamma}{2}\left[\int_0^\infty X_{gt}^\tau a_{g\tau} + X_{st}^\tau a_{s\tau} d\tau\right]^\top \Sigma\Sigma \left[\int_0^\infty X_{gt}^\tau a_{g\tau} + X_{st}^\tau a_{s\tau} d\tau\right].$$

Arbitrageurs' FOC in the bond market is given by the following expression.[34]

$$\mu_{it}^\tau - r_{it}^\tau = \gamma a_{i\tau}^\top \Sigma\Sigma \left[\int_0^\infty X_{gt}^\tau a_{g\tau} + X_{st}^\tau a_{s\tau} d\tau\right]. \tag{IA.5}$$

Substituting the values of μ_{it}^τ and $r_{it}^\tau = \delta_1^\top R_t - l_{it}^\tau$, and imposing the bond market clearing condition, we derive the following relationship describing the equilibrium in the bond market.

$$\begin{aligned}
&\dot{a}_{i\tau}^\top R_t + a_{i\tau}^\top K_r(R_t - \overline{R}) + \frac{1}{2} a_{i\tau}^\top \Sigma\Sigma a_{i\tau} + \dot{b}_{i\tau}\theta_\tau + b_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - \delta_1^\top R_t + l_{it}^\tau \\
&= \gamma a_{i\tau}^\top \Sigma\Sigma \int_0^\infty X_{gt}^\tau a_{g\tau} + X_{st}^\tau a_{s\tau} d\tau \\
&= \gamma a_{i\tau}^\top \Sigma\Sigma \int_0^\infty \left[\theta_\tau - \eta_\tau\left(a_{s\tau}^\top R_t + b_{s\tau}\theta_\tau + c_{s\tau}\right)\right] a_{s\tau} d\tau.
\end{aligned} \tag{IA.6}$$

Consider the equilibrium in the repo market. When preferred-habitat investors lend their entire bond portfolio in the repo market, our model reverts to the benchmark preferred-habitat theory. Thereby, we consider the equilibrium in the absence of securities lending. From Equation (18), repo specialness is given by

$$l_{it}^\tau = \mathcal{E}_i Z_{it}^\tau = \mathcal{E}_i\left[\eta_\tau\left(a_{i\tau}^\top R_t + b_{i\tau}\theta_\tau + c_{i\tau}\right) - \theta_\tau\right]. \tag{IA.7}$$

[34] Equation (IA.5) is the counterpart in our model of Lemma 2 in Vayanos and Vila (2021).

We next replace Equation (IA.7) into Equation (IA.6) to obtain the combined bond and repo market equilibrium:

$$\dot{a}_{i\tau}^\top R_t + a_{i\tau}^\top K_r(R_t - \overline{R}) + \frac{1}{2}a_{i\tau}^\top \Sigma\Sigma a_{i\tau} + \dot{\mathscr{b}}_{i\tau}\theta_\tau + \mathscr{b}_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} - \delta_1^\top R_t + \mathcal{E}_i\left[\eta_\tau\left(a_{i\tau}^\top R_t + \mathscr{b}_{i\tau}\theta_\tau + c_{i\tau}\right) - \theta_\tau\right]$$
$$= \gamma a_{i\tau}^\top \Sigma\Sigma \int_0^\infty \left[\theta_\tau - \eta_\tau\left(a_{s\tau}^\top R_t + \mathscr{b}_{s\tau}\theta_\tau + c_{s\tau}\right)\right] a_{s\tau} d\tau. \tag{IA.8}$$

In the quantitative model, the intercept of preferred-habitat investors' demand is constant over maturities; thereby, $\eta_\tau = \eta$. We require Equation (IA.8) to hold for all values of the risk factors, R_t. By matching the coefficients in the risk factors, we derive the following nonhomogeneous linear matrix ODE.

$$\dot{a}_{i\tau} + M a_{i\tau} - \delta_1 = 0. \tag{IA.9}$$

Here, M is the 2×2 matrix given by

$$M \equiv K_r^\top + I\mathcal{E}_i\eta + \gamma\int_0^\infty \eta a_{s\tau} a_{s\tau}^\top d\tau \Sigma\Sigma.$$

I is the 2×2 identity matrix. Equation (IA.9) is the counterpart of Equation (36) in Vayanos and Vila (2021). Next, by matching the coefficients that are independent of the risk factors in Equation (IA.8), we obtain the following fist-order linear ODE.

$$-K_r\overline{R} + \frac{1}{2}a_{i\tau}^\top \Sigma\Sigma a_{i\tau} + \dot{\mathscr{b}}_{i\tau}\theta_\tau + \mathscr{b}_{i\tau}\dot{\theta}_\tau + \dot{c}_{i\tau} + \mathcal{E}_i\left[\eta\left(\mathscr{b}_{i\tau}\theta_\tau + c_{i\tau}\right) - \theta_\tau\right]$$
$$= \gamma a_{i\tau}^\top \Sigma\Sigma \int_0^\infty \left[\theta_\tau - \eta\left(\mathscr{b}_{s\tau}\theta_\tau + c_{s\tau}\right)\right] a_{s\tau} d\tau. \tag{IA.10}$$

Finally, we require that the optimality holds for both general and special bonds. Hence, we separate Equation (IA.10) into two ODEs,

$$\dot{\mathscr{b}}_{s\tau} + \mathscr{b}_{s\tau}\overline{\theta}_\tau = \mathcal{E}_s\left(1 - \eta c_\tau \theta_\tau^{-1}\right) \tag{IA.11}$$
$$\dot{c}_{i\tau} = a_{i\tau}^\top K_r\overline{R} - \frac{1}{2}a_{i\tau}^\top \Sigma\Sigma a_{i\tau} + \gamma a_{i\tau}^\top \Sigma\Sigma \int_0^\infty \left[\theta_\tau - \eta\left(\mathscr{b}_{s\tau}\theta_\tau + c_{s\tau}\right)\right] a_{s\tau} d\tau. \tag{IA.12}$$

Equation IA.12 is the counterpart of Equation (D.10), and Equation IA.12 is the counterpart of Equation (38) in Vayanos and Vila (2021). Following their approach, we first regard the integrals as constants, and solve Equations (IA.9), (IA.11), and (IA.12) as linear ODEs with constant coefficients. We then require the solution to be consistent with the value of the integrals. Since at maturity a bond repays the principal notional, $\mathscr{A}_0 = [0,0]$ and $\mathscr{B}_0 = \mathscr{C}_0 = 0$, which implies that the initial conditions for the ODEs are $a_{i0} = [0,0]$ and $\mathscr{b}_{i0} = c_{i0} = 0$.

Next, we solve the initial value problems outlined. Suppose that the matrix M has two distinct eigenvalues, ν_1 and ν_2, which represent the bivariate extension of the scalar κ_r^* in the one-factor model. Let D be a diagonal matrix with such eigenvalues on the main diagonal, and P be a matrix of eigenvector, so that $M = P^{-1}DP$. Multiplication of Equation (IA.9) from the left by P and integration with initial condition $a_{i0} = [0,0]$ yields

$$a_{i\tau} = P^{-1}\begin{bmatrix} \frac{1-e^{-\nu_1\tau}}{\nu_1} & 0 \\ 0 & \frac{1-e^{-\nu_2\tau}}{\nu_2} \end{bmatrix} P\delta_1^\top. \tag{IA.13}$$

Since K_r, I, and Σ are diagonal matrices, the matrix M is diagonal as well. Thereby, $P = I$, which implies

$$a_{i\tau} = \left[\frac{1-e^{-\nu_1\tau}}{\nu_1}, \frac{1-e^{-\nu_2\tau}}{\nu_2}\right]^\top. \tag{IA.14}$$

In the quantitative model, the intercept of preferred-habitat investors' demand is constant over maturities, which implies $\dot{\theta}_\tau = \overline{\theta}_\tau = 0$. Hence, the solution to Equation (D.10) with initial condition $\mathscr{b}_{i\tau} = 0$ is given by

$$\mathscr{b}_{i\tau} = \mathcal{E}_i \int_0^\tau (1 - \eta c_{iu} \theta^{-1}) du. \tag{IA.15}$$

The solution to Equation (IA.12) with initial condition $c_{i\tau} = 0$ is given by

$$c_{i\tau} = \left[\int_0^\tau a_{iu}^\top du \right] \chi - \frac{1}{2} \int_0^\tau a_{iu}^\top \Sigma \Sigma a_{iu} du, \tag{IA.16}$$

where χ is the 2×1 vector given by

$$\chi \equiv K_r \overline{R} + \gamma \Sigma \Sigma \int_0^\infty \left[\theta - \eta \left(\mathscr{b}_{s\tau} \theta + c_{s\tau} \right) \right] a_{s\tau} d\tau. \tag{IA.17}$$

Equations (IA.16) and (IA.17) are the counterparts of Equations (41) and (42) in Vayanos and Vila (2021). Moreover, χ represents the bivariate extension of the product $\kappa_r^* \overline{r}^*$ in the one-factor model. Finally, it can be observed that the coefficients $(a_{i\tau}, \mathscr{b}_{i\tau}, c_{i\tau})$, together with bond status i, uniquely determine the coefficients $(\mathscr{A}_\tau, \mathscr{B}_\tau, \mathscr{C}_\tau)$, verifying the conjecture that the price is an affine function of R_t and X_{it}^τ. *Q.E.D.*