Supplemental Appendix for "Sovereign Debt, Default Risk, and the Liquidity of Government Bonds"

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Appendix B. Solution algorithm

It is well known that long-term debt models have convergence issues. To obtain convergence, I follow the literature and solve an approximate model that adds preference shocks to the default debt issue choices of the government as in Dvorkin et al. (2021) and Gordon (2019). Preference shocks are as small as needed to get convergence and do not significantly change government optimal choices. I describe the approximate model and the solution algorithm in appendix B.B1. Appendix B.B2 describes how simulations are computed. Replication files can be found in be found in Chaumont (2026).

B1. The approximate model

I first show the equations of the approximate model that I solve. As in Dvorkin et al. (2021) and Gordon (2019), the problem of the government consist on a discrete choice model with preference shocks for default and repayment, as well as preference shocks for each choice of debt for next period B'. Conditional on repayment, the sovereign chooses a level of debt for next period B' from a discrete set with N_b options $\mathcal{B} \equiv \{B_1, B_2, ..., B_{N_b}\}$. The problem of the government is given by

$$
\begin{aligned}
V(y, B, \varepsilon) &= \max_{\delta \in \{0,1\}} \left\{(1-\delta)V^R(y, B, \varepsilon) + \delta V^D(y, \varepsilon_D)\right\} \\
V^R(y, B, \varepsilon) &= \max_{B' \in \mathcal{B}} u(c) + \varepsilon_{n_{B'}} + \beta \mathbb{E}_{y'|y}\mathbb{E}_{\varepsilon'}\left[V(y', B', \varepsilon')\right] \\
s.t. : c &= y - [\lambda + (1-\lambda)z]\, B + q(y, B, B')\left[B' - (1-\lambda)B\right] \\
V^D(y, \varepsilon_D) &= u(h(y)) + \varepsilon_D + \beta \mathbb{E}_{y'|y}\mathbb{E}_{\varepsilon'}\left\{(1-\theta)\left[V^D(y', \varepsilon'_D)\right] + \theta\left[V(y', 0, \varepsilon')\right],\right\}
\end{aligned}
$$

where $\varepsilon \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_b}, \varepsilon_D\}$ is a vector with $N_b + 1$ specifying a utility value for defaulting, and for each choice of next period debt B'. It is assumed that ε is $i.i.d$ over time and has the following CDF

$$
F(\varepsilon) = \exp\left[-\left(\sum_{n_{B'}=1}^{N_b} \exp\left(-\frac{\varepsilon_{n_{B'}} - \mu}{\sigma}\right)\right) - \exp\left(-\frac{\varepsilon_D - \mu}{\sigma}\right)\right],
$$

where μ is the mean of the shocks and σ is the variance of the shocks. This is the Generalized Extreme Value distribution pioneered by McFadden (1978). What this means, is that if the government needs to choose a level of debt for the next period it would choose element $n^* \in 1, ..., N_b$ if the utility derived from B'_{n^*} is the largest among all choices.

For a given price schedule, I can determine the optimal bond supply from the government and default decisions. To compute the price of the bond we need to compute the net demand of bonds from investors that dealers channelize to the primary market. To compute investors' demand and supply of bonds we need to solve their respective problems. For each state $x = (y, B)$ at the beginning of the period and $s = (x, B')$, which is the information needed by an investor[1], the value of an investor without bonds is

$$
\begin{aligned}
I^0(s) \quad = \quad & \max_{f\in\bar{F}} \{\alpha\left(\theta(f)\right)\left[-q(s) - f + u_h + \frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')\right] \\
& + \frac{1-\alpha\left(\theta\left(f\right)\right)}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^0(s')\}.
\end{aligned}
$$

The value of holding a bond at the beginning of the period is,

$$
I^1(s) \quad = \quad \lambda + (1-\lambda)\left[z + \zeta J_\ell + (1-\zeta) J_h\right].
$$

where the values $J_i, i \in \{\ell, h\}$ are given by

$$
J_\ell = \max_{f\in\bar{F}} \left\{ \alpha\left(\theta(f)\right)\left[q(s) - f\right] + \left[1-\alpha\left(\theta(f)\right)\right]\left[u_\ell + \frac{\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')}{1+r}\right]\right\}
$$

$$
J_h = \max_{f\in\bar{F}} \left\{ \alpha\left(\theta(f)\right)\left[q(s) - f\right] + \left[1-\alpha\left(\theta(f)\right)\right]\left[u_h + \frac{\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')}{1+r}\right]\right\}
$$

where $s' = (y', B', B''(y', B', \varepsilon'))$.

In each s, the optimal submarket choices for each type of investor are given by

$$
\begin{aligned}
\theta^0(s) \quad &= \quad \alpha'^{-1}\left(\frac{\gamma}{-q(s) + u_h + \frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')}\right) \times \mathbb{I}^0\{active\}, \\
\theta^1_\ell(s) \quad &= \quad \alpha'^{-1}\left(\frac{\gamma}{q(s) - u_\ell - \frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')}\right) \times \mathbb{I}^0_\ell\{active\} \\
\theta^1_h(s) \quad &= \quad \alpha'^{-1}\left(\frac{\gamma}{q(s) - u_h - \frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta\left(y', B', \varepsilon'\right)\right] I^1(s')}\right) \times \mathbb{I}^0_h\{active\},
\end{aligned}
$$

[1]Note that investors do not need to know the preference shock because they observe the choices of the government and the preference shocks for the government are *i.i.d.* over time.

where $\mathbb{I}_i^a\{active\}$ is an indicator function that takes value equal one when that type of investor is active in the secondary market. Investors are active in the secondary market whenever the gains from trade can at least compensate the entry cost for dealers, γ. Finally, knowing the optimal submarket choices of investors, we can determine the fraction of each type of investor that meets a dealer in the secondary market and compute the excess demand function in the primary market as

$$\begin{aligned} ED(s) \quad \equiv \quad & \alpha\left(\theta_h^0(s)\right)\left[\overline{I}-(1-\lambda)\,B\right]-\left[\max\left\{B',0\right\}-(1-\lambda)\,B\right] \\ & -\alpha\left(\theta_\ell^1(s)\right)(1-\lambda)\,\zeta B-\alpha\left(\theta_h^1(s)\right)(1-\lambda)\,(1-\zeta)\,B. \end{aligned}$$

I can pin down the unique price that is consistent with $q(s)ED(s)=0$. These steps provide me with an algorithm to solve the model. This algorithm begins with a guess of the price schedule, updates the policy functions of the government, solves for investors optimal submarket choice as a function of price $q\,(s)$ in each state $s\in\overline{S}$, and finally finds the price $q(s)$ consistent with zero excess demand in primary markets, for each $s\in\overline{S}$. Then, I used the resulting price schedule as a new guess and iterate until all equilibrium objects converge.

B2. Model simulation

I simulate the model over $T=500,000$ periods. Then, I burn the $T_1=1,000$ initial periods. Find all episodes of length $T=69$ periods where the 69th period is a default episode and none of the previous 99 periods are default periods. I discard the first $T_0=30$ periods and keep 69 periods before default. Length T is chosen to be 69 because I use data on Greek GDP from 1995Q1 and default happens 68 quarters later in 2012Q1. Since after re-gaining access to international financial markets the government re-enters with $B=0$, I choose to discard T_0 periods before the beginning of each replica of the economy so that I let the model reach the ergodic set. Then, I compute the summary statistics for each of them. Finally, I average over these episodes and report the averages of the moments.

Appendix C. Data description

All data citations, can be found in Chaumont (2026). I collect the following time series for Greek economy.

National accounts. — Quarterly time series from 1995Q1 - 2023Q4 for consumption, exports, imports, and GDP are obtained from Eurostats. I use seasonal adjusted and calendar adjusted chain-linked (2010) in million euros. (Eurostat data series: namq_10_gdp, Eurostat (2024a)).

Debt and investment position. — Data on net international investment position (as % of GDP) for the period 2003Q4 - 2012Q4 and Government consolidated gross debt (as % of GDP) for the period 2000Q1 - 2012Q4 are obtained from the Eurostats. (Eurostat data series: bop_iip6_q, Eurostat (2018*a*), and gov_10q_ggdebt, Eurostat (2018*b*)).

Interest rates and spreads. — Interest rates data is collected from Eurostats and Bloomberg. Interest rate spreads is calculated as the difference in the annual interest rates between Greek and German long term government yields in Eurostats. Long term debt yields are composed from central government bonds with residual maturity of around 10 years. Computing interest rate spreads using generic central government bonds from Greece and Germany collected from bloomberg results in almost identical time series. Daily bid and ask prices are collected from Bloomberg using generic central government bonds. I compute quarterly time series using average of active days in each quarter. I use time series for bid and ask prices for 10 year bonds. Daily bid and ask prices for 5 year bonds have some missing values but quarterly time series results in almost identical bid-ask spreads as for 10-year bonds. (Eurostat series: irt_lt_mcby_q, Eurostat (2024*b*). Bloomberg data series: GTGRD10Y Govt and GTDEM10Y Govt, Bloomberg Finance L.P. (2020)).

Secondary market volumes and turnover rates. — Information on secondary market trade volumes is obtained from the electronic secondary securities market (HDAT) available at the Bank of Greece website Bank of Greece (2024).[2] Monthly traded volumes are available from January 2001. Quarterly time series are calculated as the sum of monthly traded volumes.

Appendix D. Details on the calibration

D1. Endowment process

The endowment process is estimated running the following AR(1) process on GDP cycle:

$$y_t - \bar{y} \quad = \quad \rho_y(y_{t-1} - \bar{y}) + \varepsilon_t, \text{ with } \varepsilon_t \sim N(0, \eta_y).$$

The GDP cycle is obtained after de-trending the quarterly real seasonal and calendar adjusted GDP using an HP-filter parameter $\lambda = 50,000$. The large HP-filter parameter aims to capture the long cycle observed in Greek GDP data. Results are robust to alternative de-trending but a recalibration of the model is needed.

[2]https://www.bankofgreece.gr/Pages/en/Markets/HDAT/statistics.aspx

D2. *Calculating utility from holding bonds*

I calibrate preferences bonds u_h and u_ℓ such that they satisfy to conditions: (i) that ex-ante expected utility from holding the asset is zero for a given ζ, and (ii) such that $u_h - u_\ell$ targets the average bid-ask spread in the data. Condition (i) imposes the following restriction

$$\begin{aligned} 0 &= u_h + \sum_{j=1}^{\infty} \left\{ \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)\zeta\left(1-\alpha\left(\theta_{\ell,j}\right)\right) \right]^j u_\ell \right\} \\ &= \sum_{j=0}^{\infty} \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \sum_{j=1}^{\infty} \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)\zeta\left(1-\alpha\left(\theta_{\ell,j}\right)\right) \right]^j u_\ell . \end{aligned}$$

Although I use the constraint above to calibrate the model, we can develop further intuition assuming that the probability of default is more or less constant around the targeted average default probability $\bar{\delta}$. In such case, the previous expression is approximately equal to

$$\begin{aligned} 0 &\approx \sum_{j=0}^{\infty} \left[\frac{1-\bar{\delta}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \sum_{j=1}^{\infty} \left[\frac{1-\bar{\delta}}{1+r} (1-\lambda)\zeta(1-\bar{\alpha}) \right]^j u_\ell \\ &= \frac{(1+r)\,u_h}{1+r-(1-\bar{\delta})(1-\lambda)(1-\zeta)} + \frac{(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})u_\ell}{1+r-(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})} \end{aligned}$$

$$\Longrightarrow \frac{(1+r)}{1+r-(1-\bar{\delta})(1-\lambda)(1-\zeta)} u_h \approx -\frac{(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})}{1+r-(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})} u_\ell ,$$

where $\bar{\alpha} \equiv \alpha\left(\theta_{\ell,j} \middle| \Pr[\delta = 1] = \bar{\delta}\right)$ is the probability of matching for a low type investor conditional on a default probability equal to $\bar{\delta}$, which would also be a constant if default risk is constant.

D3. *Computing turnover rates*

The turnover rate is 78% in HDAT in Greece is per quarter. This includes transactions between dealers and investors as well as interdealer transactions. To calculate the turnover rate in the model we have to compute all the transactions that happen in secondary markets between dealers and investors and also interdealers transactions in primary markets. This is not exactly what happens in reality as some dealers hold inventories and do not need to trade with other dealers and some other trades occur through a long chain of dealer to dealer transactions. We will assume that the number of transactions in the model approximates the amount of transactions in the data. In the model, we can calculate the amounts of transactions in both the primary and secondary market as follows.

In the primary market, whenever the government issues debt it is purchased by a dealer. Then, the number of transactions in the primary market is given by

$$\max\left\{B',0\right\}-(1-\lambda)\,B.$$

Now, in the secondary markets, the following trades occur between a dealer and an investor:

$$\begin{aligned}
\ell-type\text{ investors to dealer} &: \alpha\left(\theta_\ell^1\right)\zeta\,(1-\lambda)\,B\\
\text{Potential } h-type\text{ investors to dealer} &: \alpha\left(\theta_h^1\right)(1-\lambda)\,(1-\zeta)\,B\\
\text{Dealer to old buyer} &: \alpha\left(\theta_h^0\right)(\bar{I}-B)\\
\text{Dealer to new buyer} &: \alpha\left(\theta_h^0\right)\lambda B.
\end{aligned}$$

Since we do not model the amounts of trades in the interdealers market this is the minimum amount of transactions in secondary markets. The maximum amount of transactions adds to these transactions the maximum amount of possible trades between dealers in the interdealers market, so we have to clean the turnover rate in the data to remove those trades. This is the sum of the following trades.

$$\begin{aligned}
\text{Primary buyers to selling dealers} &: \max\left\{B',0\right\}-(1-\lambda)B\\
\text{Secondary buyers to selling dealers} &: \alpha\left(\theta_\ell^1\right)\zeta\,(1-\lambda)\,B+\alpha(\theta_h^1)(1-\lambda)(1-\zeta)B.
\end{aligned}$$

Since primary market clears we can simplify this calculation using total purchases by investors,

$$\text{Total investors' purchases}:\alpha(\theta_h^0)(\bar{I}-B)+\alpha(\theta_h^0)\lambda B=\alpha(\theta_h^0)\left[\bar{I}-(1-\lambda)B\right].$$

Then, the maximum amount of transactions in secondary markets, given that interdealers market is competitive, is given by

$$Vol^{\max}=2\left[\alpha(\theta_h^0)(\bar{I}-B)+\alpha(\theta_h^0)\lambda B\right]+\alpha\left(\theta_\ell^1\right)\zeta(1-\lambda)B+\alpha\left(\theta_h^1\right)(1-\lambda)\,(1-\zeta)\,B.$$

In reality the minimum amount of trades in secondary markets is given by

$$Vol^{\min}=\alpha(\theta_h^0)(\bar{I}-B)+\alpha(\theta_h^0)\lambda B+\alpha(\theta_\ell^1)\zeta(1-\lambda)B+\alpha(\theta_h^1)(1-\lambda)(1-\zeta)B.$$

This happens when the same dealer is connecting both the investor selling and the investor buying and just acting as a bridge. As mentioned before, the number of transactions in reality could be lower or higher than Vol as long chains of dealers would be require to transfer one bond from an investor to another one. Li and Schurhoff (2018) find that for municipal bonds in United States, the average chain involves 1.5 dealers. So, we can compute an intermediate amount of trades in secondary markets as

$$Vol^{mean} = 1.5\left[\alpha(\theta_h^0)(\bar{I}-B)+\alpha(\theta_h^0)\lambda B\right]+\alpha\left(\theta_\ell^1\right)\zeta(1-\lambda)B+\alpha\left(\theta_h^1\right)(1-\lambda)(1-\zeta)B,$$

where the last term in the right hand side is usually zero because $h-type$ investors do not sell their bonds in the secondary market in equilibrium. I use $Vol = Vol^{mean}$ to compute the model's implied turnover rate as

$$\text{Turnover rate} = \frac{Vol}{B}.$$

D4. Alternative values for $\bar{I}$

This appendix shows that increasing the parameter $\bar{I}$ does not significantly change the results compared to the baseline calibration. This is because the amount of potential demand $\bar{I}$ is sufficiently large in the baseline calibration and it does not bind in any significant way the amount of debt issued by the government in equilibrium. If course, reducing the parameter $\bar{I}$ can affect the results as debt issuance might start being bound by the potencial world demand for sovereign bonds. Table D1 shows how the main moments change as we change $\bar{I}$. For larger values of $\bar{I}$ a small re-calibration of the model would restore the baseline moments.

Table D1—The role of secondary market's trade frictions and flows

Moments	Baseline ($\bar{I}=5$)	$\bar{I}=7.5$	$\bar{I}=10$	$\bar{I}=15$
Mean bond spread (%)	3.42	3.38	3.37	3.37
Std. dev. bond spread (%)	2.18	2.17	2.17	2.18
Debt to output (%)	125	126	126	126
Mean bid–ask spread (bps.)	76	76	76	75
Bonds turnover rate (%)	78	78	78	78

Note: The table reports the values of targeted moments as the parameter governing potential demand for sovereign bonds, $\bar{I}$, increases.

Appendix E. Interpreting the primary and secondary markets

In reality, the market for sovereign bonds has a primary market, where only authorized primary dealers are allowed to participate. For Greece, there is a list of around 20-24 primary dealers that renews every year but is very stable over time. Primary dealers are the largest international banks worldwide and some of the largest domestic banks (often owned by foreign banks). These primary dealers are not only the banks that are allowed to purchase bonds in the primary auctions, but they are also required to behave as market makers and intermediate transactions in the secondary market. The role of market makers is to provide bid and ask quotes for sovereign bonds and be ready to trade at those prices. Secondary market participants are smaller banks, pension funds, hedge funds, and private investors in general who do not have access to the primary market. For these and more details about the roles of primary dealers in primary and secondary markets see the following links: [2015, see pages 99-117], 2025).

In the model, the dealers aim to resemble the market access of the primary dealers, who can both trade bonds in the primary market and provide intermediation services in the secondary markets. That is, a good way to think about these intermediaries is to think about large banks that participate in primary auctions, such as JP Morgan, Golman Sachs, BNP Paribas or Deutsche Bank. As in reality, primary dealers not only have access to primary and secondary markets, but also serve as market makers in the secondary market.

The centralized market in the model aims to resemble a combination of the primary market and the interdealer market. These markets are highly competitive and are modeled as perfectly competitive markets. Only primary dealers (and the government) can access the centralized market. The decentralized market aims to resemble the over-the-counter secondary market, where transactions are bilateral (mostly between a dealer and a smaller investor without access to the primary market), time-consuming, and costly (as can be seen in the spreads between the bid and ask prices, which represent the fees charged by intermediaries). Smaller investors, who do not qualify to be primary dealers, can only trade bonds in the secondary market by meeting a counterpart who wants to trade in the opposite direction. This counterpart in the model are dealers. Although it is true that small investors could trade with each other without an intermediary, I rule out this possibility in the model because in reality almost all of these transactions are intermediated by market makers.

I model trading frictions in secondary markets using directed search. This assumption can be interpreted as follows. In reality, dealers (market makers) post bid and ask quotes in the secondary market. Those dealers that offer the lowest ask prices are the first to sell bonds to other investors, while those dealers that offer the highest bid prices are the first ones to buy bonds from other investors. Directed search captures these features. Dealers competitively choose to quote bid or ask prices for bonds, after paying an entry cost. This the marginal cost of producing transactions in the secondary market. It captures wages, market research efforts, the opportunity cost of time, and/or inventory management costs for the primary dealers operating as market makers in the bond market. In addition, in reality, primary dealers are required by regulation to operate as market makers and those requirements impose additional costs on dealers. All these costs are not formally modeled and γ simply captures them in reduced form. In the model, the absolute value between the quoted price and the interdealer price represents the fee charge by a dealer (and the name of the submarket). As in reality, those dealers who charge lower fees (those who quote lower ask prices or higher bid prices) trade with higher probability (or faster). Consistent with directed search, Li and Schurhoff (2019) document that, for the case of US municipal bonds, there is a systematic price dispersion in fees charged by different dealers and that those dealers charging higher fees provide more liquidity immediacy to investors.

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