An Evolutionary Perspective on Updating Risk and Ambiguity Preferences

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August 2025

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High-Level Overview

- Evolutionary foundation for risk and ambiguity aversion.
- Use this new model to address several important questions:
  1. Updating non-expected-utility and ambiguity-averse preferences
  2. Model selection
  3. Link between risk and ambiguity preferences
  4. Self-randomization to hedge against ambiguity

Outline & Key Insights

1. Ambiguity and Risk closely associated with common and idiosyncratic uncertainty, respectively.
2. Evolutionary optimality generates preference for idiosyncratic over common uncertainty (Robson 1996).
3. **Adaptive Preferences**: Hidden actions (or phenotypic flexibility) lead to non-expected-utility preferences over both common and idiosyncratic uncertainty.
4. Preferences following information will be dynamically consistent but may violate consequentialism.

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Common Uncertainty and Ambiguity

- In many instances, the ambiguous factor is obviously common:
  - returns to financial assets
  - macroeconomic shocks
  - risk of natural disasters and climate change
- Other times, ambiguity takes the form of model uncertainty:
  - Model uncertainty: Risks faced by individuals are well-understood and idiosyncratic *conditional* on some common unknown and ambiguous model parameter (medical treatment efficacy, urn composition)
- Why might this connection exist?
  - One reason may be that idiosyncratic random variables can be studied using cross-sectional data, whereas aggregate variables by definition cannot.

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Preview: Aversion to Common Uncertainty

Choice between two actions for all individuals:

- Both have individual growth rate of either 2 or 4 (equal probability)
- Action A is idiosyncratic uncertainty
- Action B is common uncertainty (outcome perfectly correlated across individuals)

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Action A (idiosyncratic)

- Each individual's growth rate: either 2 or 4
- Population average: $\frac{1}{2}(2+4)=3$

Action B (common)

- Each individual's growth rate: either 2 or 4
- Population average: either 2 or 4

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- Each individual's growth rate: either 2 or 4
- Population average: $\frac{1}{2}(2+4) = 3$
- LR average population growth (over two periods): 9

Action B (common)

- Each individual's growth rate: either 2 or 4
- Population average: either 2 or 4
- LR average population growth (over two periods): $2 \times 4 = 8$

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Setting (without signals or random choice for now)

Two-dimensional state space:

- Ω — common (same ω for all individuals in the population)
- S — idiosyncratic (draws of s are independent, conditional on ω)

Uncertainty:

- $\mu \in \triangle(\Omega)$ — Common draw of ω for entire population with prob $\mu(\omega)$.
- $\nu_\omega \in \triangle(S)$ — Given ω, draws of s are independent with prob $\nu_\omega(s)$.

Acts: $f : \Omega \times S \to Z$

Evolutionary fitness function: $\psi : Z \to \mathbb{R}$ specifies the individual growth rate associated with each outcome.

Hidden Actions

state	act f	outcome	fitness function ψ	fitness
(ω, s)	$\longrightarrow$	$z = f(\omega, s)$	$\longrightarrow$	$\psi(z) = \psi(f(\omega, s))$

- Two dimensions of choice:
  1. Observable choice of act f (e.g., occupation, investments, vaccination)
  2. Unobservable hidden action (e.g., housing, other investments, social distance)
- We represent hidden actions in reduced form as the selection of a fitness function ψ from a feasible set Ψ.[1]

▸ Illustration of Multiple Fitness Functions

▸ Phenotypic Flexibility in Biology

[1] Derivation from more explicit model: If individuals take a hidden action $y \in Y$ and have a single fixed fitness function $\hat{\psi}(z, y)$ for outcome/action pairs, the resulting set of fitness functions in our model is $\Psi = \{\hat{\psi}(\cdot, y) : y \in Y\}$.

Evolution and the Adaptive Model

Theorem (Adaptive Model, without Signals)

If $\psi \in \Psi$ is chosen optimally, then the long-run growth rate (in logs) of the genotype from choosing act f is

$$V(f) = \sup_{\psi \in \Psi} \int_{\Omega} \ln\left(\int_S \psi(f(\omega, s))\, d\nu_\omega(s) \right) d\mu(\omega).$$

- Robson (1996) model is special case of $\Psi = \{\psi\}$ (discussed next).
- Our model adds (and demonstrates interactions between):
    1. Hidden actions
    2. Signals (discussed next)
    3. Random choice (end of talk)

Formulas with Random Choice

Intuition for Long-Run Growth Formula

- Conditional on ω, the average growth rate of a large subpopulation of individuals choosing (f, ψ) is approximately $\int_S \psi(f(\omega, s))\, d\nu_\omega(s)$.

- Annualized growth rate over a sequence of realized $\omega_1, \ldots \omega_n$:

$$\prod_{t=1}^{n} \left(\int_S \psi(f(\omega, s))\, d\nu_{\omega_t}(s) \right)^{1/n}.$$

- Taking the logarithm of this expression gives:

$$\frac{1}{n} \sum_{t=1}^{n} \ln \left(\int_S \psi(f(\omega, s))\, d\nu_{\omega_t}(s) \right) \xrightarrow{a.s.} \int_\Omega \ln \left[\int_S \psi(f(\omega, s))\, d\nu_\omega(s) \right] d\mu(\omega).$$

- When ψ chosen optimally, long-run growth rate (in logs) from choosing f is:

$$V(f) = \sup_{\psi \in \Psi} \int_\Omega \ln \left[\int_S \psi(f(\omega, s))\, d\nu_\omega(s) \right] d\mu(\omega).$$

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Smooth Model (Static)

$$V(f) = \sup_{\psi \in \Psi} \int_{\Omega} \ln \left(\int_S \psi(f(\omega, s))\, d\nu_{\omega}(s) \right) d\mu(\omega).$$

Robson (1996) benchmark — $\Psi = \{\psi\}$ (no hidden actions):

$$V(f) = \int_{\Omega} \ln \left(\int_S \psi(f(\omega, s))\, d\nu_{\omega}(s) \right) d\mu(\omega).$$

- Issue-preference model (Nau (2006), Ergin and Gul (2009))
- For acts $f(s)$ that only depend on s, smooth model (Klibanoff, Marinacci, and Mukerji (2005))
- Even if individuals form subjective priors over common factors, they are treated differently than idiosyncratic risk.

Illustration

These ideas apply to all our examples: macroeconomic variables, natural disasters, medical treatments, or composition of the urn.

Example (Ellsberg experiment)

Ellsberg urn: 1 **black** ball, 2 balls that are either **red** or **yellow**.

Bets: B, R, BY, RY

Payoffs

	b	r	y
B	1	0	0
R	0	1	0
BY	1	0	1
RY	0	1	1

Typical preferences

$$B \succ R$$
$$BY \prec RY$$

Violation of Savage's Sure-Thing Principle!

Illustration

Example (Ellsberg experiment, continued)

- Each individual has independent draw: $S = \{b, r, y\}$
- Suppose urn composition is common for all individuals:
  - $\omega_1 = (b, r, r)$, $\omega_2 = (b, r, y)$, $\omega_3 = (b, y, y)$
  - $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and $\mu(\omega_i) = \frac{1}{3}$

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$$V(B) = \ln\left[\tfrac{1}{3}\right] > \tfrac{1}{3}\ln\left[\tfrac{2}{3}\right] + \tfrac{1}{3}\ln\left[\tfrac{1}{3}\right] + \tfrac{1}{3}\ln[0] = V(R)$$

$$V(BY) = \tfrac{1}{3}\ln\left[\tfrac{1}{3}\right] + \tfrac{1}{3}\ln\left[\tfrac{2}{3}\right] + \tfrac{1}{3}\ln[1] < \ln\left[\tfrac{2}{3}\right] = V(RY)$$

- Note: Self-randomization is not optimal in this example for these choice sets $\{B, R\}$ and $\{BY, RY\}$ (but would be optimal for others, e.g., $\{R, Y\}$).

What do Hidden Actions add?

- The previous example of the smooth involved the special case of no hidden actions (a single fitness function): $\Psi = \{\psi\}$.

- So what is the role of Hidden Actions (or Phenotypic Flexibility) in our model?
  1. Adding Realism: Individuals often have actions that are unobservable to the modeler (or unobserved adaptations in the context of evolutionary biology).

  2. Expanding our Scope: By incorporating hidden actions (non-singleton Ψ), *observed* preferences over acts f could take many different forms:
     - Rank-dependent utility.
     - Versions of variational preferences.
     - Models combine ambiguity aversion and non-expected-utility for risk.

We'll explore some of these special cases *later in the talk*... but first we illustrate the implications of our model for *information and updating*.

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Ellsberg Example with Signals

Adding information to the Ellsberg example:

- Ex-ante preferences: $B \succ R$ and $BY \prec RY$.
- Information: learn whether yellow (y) or not yellow ($\neg y$)
- Ex-post preferences?

Two normatively appealing properties:

- **Dynamic Consistency:** Ex ante plans for how actions will respond to information are actually followed ex post.
- **Consequentialism:** Ex post choices are not influenced by outcomes that would have obtained on some unrealized event.

An Unavoidable Tension

Issue: Except in special cases, consequentialism and dynamic consistency are incompatible:

- Ellsberg + dynamic consistency requires:

$$V(B) > V(R) \implies V(B|\neg y) > V(R|\neg y)$$
$$V(BY) < V(RY) \implies V(BY|\neg y) < V(RY|\neg y)$$

- Consequentialism requires:

$$V(B|\neg y) > V(R|\neg y) \iff V(BY|\neg y) > V(RY|\neg y)$$

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- Can have "Ellsberg+DC" or "Ellsberg+Consequentialism" but not all three.
  - General issue: spans all ambiguity preferences and all updating rules.
  - Shows limits of "rectangularity" condition of Epstein and Schneider (2003)

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Evolutionary Answer: Ex ante objective of the genotype is to maximize long-run fitness; same ex post objective $\implies$ **dynamic consistency**.

Setting with Signals and Plans (no random choice)

Information:

- Σ — space of private signals.
- $\mu \in \triangle(\Omega)$ — Common draw of ω with prob $\mu(\omega)$, same as before.
- $\nu_\omega \in \triangle(S \times \Sigma)$
  - Given ω, draws of s and σ are independent with prob $\nu_\omega(s, \sigma)$.
  - Signal σ is informative about (ω, s)

Plans specify path through the decision tree:

Signal-Contingent Action Plan: $(f_\sigma)_{\sigma \in \Sigma}$ or (f_σ) for short.

- Choose act f_σ after signal realization σ.

Signal-Contingent Adaptation Plan: $(\psi_\sigma)_{\sigma \in \Sigma}$ or (ψ_σ) for short.

- Assigns fitness function ψ_σ after signal realization σ.

Main Results: Ex Ante Plans

Theorem (Adaptive Model, with Signals)

If $(\psi_\sigma)_{\sigma\in\Sigma}$ is chosen optimally, then the long-run growth rate (in logs) of the genotype from choosing plan $(f_\sigma)_{\sigma\in\Sigma}$ is

$$V((f_\sigma)) = \sup_{(\psi_\sigma)\in\Psi^\Sigma} \int_\Omega \ln\left(\int_{S\times\Sigma} \psi_\sigma(f_\sigma(\omega,s))\, d\nu_\omega(s,\sigma)\right) d\mu(\omega).$$

▸ Formulas with Random Choice

Main Results: Dynamic Consistency

Theorem (Dynamic Consistency)

Evolutionarily optimal ex ante preferences are represented by $V((f_\sigma))$. Evolutionarily optimal ex post preferences given plan (f_σ) and signal realization $\bar{\sigma}$ are represented by:

$$V(g|\bar{\sigma},(f_\sigma)) = \sup_{(\psi_\sigma)\in\Psi^\Sigma} \int_\Omega \ln\left(\nu_\omega(\bar{\sigma}) \int_S \psi_{\bar{\sigma}}(g(\omega,s))d\nu_\omega(s|\bar{\sigma}) + \int_{S\times\Sigma\setminus\{\bar{\sigma}\}} \psi_\sigma(f_\sigma(\omega,s))d\nu_\omega(s,\sigma)\right) d\mu(\omega).$$

Thus, evolutionarily optimal choice is dynamically consistent.

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Revisiting the Smooth Model Ellsberg Example:

	$\nu_\omega(\neg y)\nu_\omega(r\|\neg y)$	$\nu_\omega(y)$	$\nu_\omega(\neg y)\nu_\omega(b\|\neg y)$
$\omega_1 = (b, r, r)$	$1 \cdot \frac{2}{3} = \frac{2}{3}$	0	$1 \cdot \frac{1}{3} = \frac{1}{3}$
$\omega_2 = (b, r, y)$	$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$
$\omega_3 = (b, y, y)$	$\frac{1}{3} \cdot 0 = 0$	$\frac{2}{3}$	$\frac{1}{3} \cdot 1 = \frac{1}{3}$

Maximizing fitness of the genotype (when ex ante plan is RY):

$$V(R|\neg y, RY) = \sum_\omega \mu(\omega) \ln\Big(\underbrace{\nu_\omega(\neg y)}_{\substack{\text{fraction}\\ \text{getting}\\ \text{signal } \neg y}} \; \underbrace{\nu_\omega(r|\neg y)}_{\substack{\text{average fitness}\\ \text{from } R \text{ after}\\ \text{signal } \neg y}} + \underbrace{\nu_\omega(y)}_{\substack{\text{fraction}\\ \text{getting}\\ \text{signal } y}} \; \underbrace{1}_{\substack{\text{fitness}\\ \text{from } Y\\ \text{after } y}}\Big)$$

$$> \sum_\omega \mu(\omega) \ln\Big(\nu_\omega(\neg y)\; \nu_\omega(b|\neg y) + \nu_\omega(y)\; 1\Big) = V(B|\neg y, RY)$$

Understanding Violations of Consequentialism

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Maximizing fitness of the genotype (when ex ante plan is B):

$$V(R|\neg y, B) = \sum_\omega \mu(\omega) \ln\Big(\nu_\omega(\neg y)\,\nu_\omega(r|\neg y)\Big)$$

$$< \sum_\omega \mu(\omega) \ln\Big(\nu_\omega(\neg y)\,\nu_\omega(b|\neg y)\Big) = V(B|\neg y, B)$$

Evolutionary Insight: Individuals must consider complementarity with others of same genotype getting signal y when own signal is $\neg y$.

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    - *Example:* Rank-Dependent Utility (no common uncertainty)
    - Combining Ambiguity Aversion and Non-EU Risk Preferences
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Special Cases (of Static Model)

Return to the Static Model (for expositional simplicity):

$$V(f) = \sup_{\psi \in \Psi} \int_\Omega \ln\left(\int_S \psi(f(\omega, s))\, d\nu_\omega(s) \right) d\mu(\omega).$$

- No hidden actions — $\Psi = \{\psi\}$ (Robson (1996) benchmark):

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  - Nests many other non-EU functional forms as special cases

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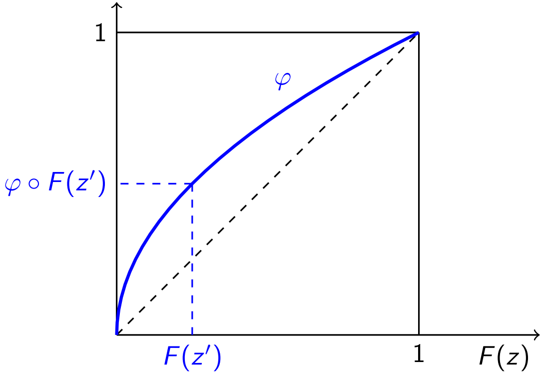
Refresher: Rank-Dependent Utility

- Rank-dependent utility is given by a utility function $u : Z \to \mathbb{R}$ and a probability distortion function $\varphi : [0, 1] \to [0, 1]$.
- Utility of a cumulative distribution function (cdf) F on $Z \subset \mathbb{R}$ is

$$\int_Z u(z)\, d(\varphi \circ F)(z).$$



Rank-Dependent Utility (Static)

- To show RDU is nested within our Adaptive Model, we now allow hidden actions but shut down correlation: $\Omega = \{\omega\}$.
- So drop ω from the value function:

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Theorem (Sadowski and Sarver (2024))

Given $u : Z \to \mathbb{R}$ *and* $\varphi : [0,1] \to [0,1]$ *non-decreasing and concave, there exists a set* Ψ *such that*

$$V(f) = \ln \int_Z u(z)\, d(\varphi \circ F_{f,\nu})(z)$$

where $F_{f,\nu}$ *is the cumulative distribution of act* f *given measure* ν

$$F_{f,\nu}(z) = \int_S \mathbf{1}[f(s) \leq z]\, d\nu(s).$$

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  - Nests many other non-EU functional forms (Sadowski and Sarver (2024)).
  - Can we embed these non-EU functional forms in the general model with common uncertainty?

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Dual Formulas (without Signals or Random Choice)

Theorem (Duality)

Suppose the set Ψ *is convex, pointwise bounded above, and closed in the topology of pointwise convergence, and fix* μ *and* ν*. Then,*

$$\begin{aligned} V(f) &= \sup_{\psi\in\Psi} \int_\Omega \ln\left(\int_S \psi(f(\omega,s))\,d\nu_\omega(s)\right) d\mu(\omega) \\ &= \inf_{q\in M(\mu_\Omega)} \left[\ln\left(\sup_{\psi\in\Psi} \int_\Omega \int_S \psi(f(\omega,s))\,d\nu_\omega(s)\,dq(\omega)\right) + R(\mu \,\|\, q)\right] \end{aligned}$$

- $R(p \,\|\, q)$ is relative entropy, $M(\mu)$ is measures with finite entropy.

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$$V(f) = \sup_{\psi \in \Psi} \int_\Omega \ln\left(\int_S \psi(f(\omega,s))\, d\nu_\omega(s)\right) d\mu(\omega)$$
$$= \inf_{q \in M(\mu_\Omega)} \left[\ln\left(\sup_{\psi\in\Psi} \int_\Omega \int_S \psi(f(\omega,s))\, d\nu_\omega(s)\, dq(\omega)\right) + R(\mu \,\|\, q)\right]$$

- $R(p \,\|\, q)$ is relative entropy, $M(\mu)$ is measures with finite entropy.
- Step 1—Entropy Duality:

$$V(f) = \sup_{\psi \in \Psi} \int_\Omega \ln\left(\int_S \psi(f(\omega,s))\, d\nu_\omega(s)\right) d\mu(\omega)$$
$$= \sup_{\psi\in\Psi} \inf_{q \in M(\mu)} \left[\ln\left(\int_\Omega \int_S \psi(f(\omega,s))\, d\nu_\omega(s)\, dq(\omega)\right) + R(\mu \,\|\, q)\right]$$

- Step 2—von Neumann–Sion–Tuy Minimax:

$$V(f) = \inf_{q \in M(\mu)} \left[\ln\left(\sup_{\psi\in\Psi} \int_\Omega \int_S \psi(f(\omega,s))\, d\nu_\omega(s)\, dq(\omega)\right) + R(\mu \,\|\, q)\right]$$

Dual Formulas: Implications

- Can apply additional duality to inner term giving, for example, RDU.

$$V(f) = \inf_{q \in M(\mu)} \left[\ln \left(\underbrace{\sup_{\psi \in \Psi} \int_\Omega \int_S \psi(f(\omega, s))\, d\nu_\omega(s)\, dq(\omega)}_{\text{RDU as special case}} \right) + R(\mu \,\|\, q) \right]$$

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- Random choice: If individuals self-randomize with distribution $\rho(f)$ over acts:

$$V(\rho) = \inf_{q \in M(\mu)} \left[\ln \left(\underbrace{\mathbb{E}_\rho \left[\underbrace{\sup_{\psi \in \Psi} \int_\Omega \int_S \psi(f(\omega, s))\, d\nu_\omega(s)\, dq(\omega)}_{\text{RDU as special case}} \right]}_{\text{non-EU functional inside expectation given } \rho(f)} \right) + R(\mu \,\|\, q) \right]$$

- Exogenous randomization (e.g., AA mixtures of acts): In contrast, if ρ is instead an exogenous randomization (with late resolution), then using RDU to illustrate:

$$V(\rho) = \inf_{q \in M(\mu)} \left[\ln \left(\text{RDU distortion applied to } \mu,\ q, \text{ and } \rho \right) + R(\mu \,\|\, q) \right]$$

Takeaway: For non-EU risk preferences, random choice hedges ambiguity better.

Dual Formulas: Incorporating Signals

- One of our main results shows that similar dual formulas apply in the case of signals. ▸ Details and Theorem
- This duality allows us to apply our insights on dynamically consistent updating to other:
  - Ambiguity preferences
  - Non-expected-utility risk preferences
  - Hybrid models that combine ambiguity aversion and non-EU for risk.
- It also facilitates comparison to other dynamically consistent models of ambiguity and updating (Hanany and Klibanoff (2007, 2009)).

Summary and Conclusions

Model Ingredients

- Evolution of preferences approach adapted from evolutionary biology
- Hidden actions (or phenotypic flexibility)

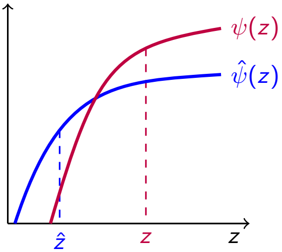
Implications

- Non-expected-utility and ambiguity-averse preferences
- Dynamic choice:
  - Dynamic consistency: ALWAYS
  - Consequentialism: SOMETIMES
- Combining ambiguity aversion, non-EU risk preferences, signals, and random choice: EASY (after our duality results)
- Random choice as a hedging mechanism:
  - Self-randomization hedges: ALWAYS
  - Exogenous randomization (mixtures) hedges: SOMETIMES

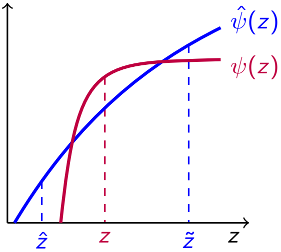
Additional Material

Hidden Actions: Illustration

1. Observable choice of act f (e.g., occupation, investments, vaccination)
2. Unobservable hidden action modeled as choice of $\psi \in \Psi$ (e.g., housing, other investments, social distance)



Example: Hidden preparations for specific range of outcomes



Example: Hidden insurance or liquidity decisions

◂ Back to Hidden Actions

Phenotypic Flexibility in Evolutionary Biology

Phenotypic Plasticity: Genotype constrains but does not fully determine the phenotype/traits.

Genotype $\longrightarrow$ Phenotype/traits

Phenotypic plasticity includes:

- **Developmental Plasticity:** Irreversible variation in the traits of individuals that results from processes during development as a consequence of variation in the environment.
- **Phenotypic Flexibility:** When environmental conditions change rapidly [...], individuals that can show [...] reversible transformations in behaviour, physiology and morphology might incur a selective advantage. There are now several studies documenting substantial but reversible phenotypic changes within adult organisms.

(Piersma and Drent (2003))

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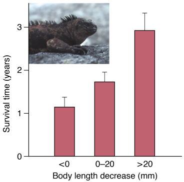
Examples of Phenotypic Flexibility

- Many species of amphibious fish adjust to life on land with rapid changes to muscle tissue, breathing organs, skin properties. (Wright and Turko (2016))

- Individual marine iguanas can decrease body length by up to 20% (68mm). Shrinkage appears to be a strategic response to food scarcity. (Wikelski and Thom (2000))



Source: Piersma and Drent (2003)

- In humans makeup of muscle tissue responds to functional demands (more or less active lifestyle). (Flück (2006))

Phenotype

A phenotype consists of a fitness function ψ together with a decision rule for choosing between acts (i.e., preferences).

Back to Hidden Actions

Long-Run Growth with Random Choice (no Signals)

- Random choice of act: $\rho(f)$ where $\rho \in \triangle_s(\mathcal{F})$
- Random choice of ψ given f: $\tau(\psi|f)$ where $\tau \in (\triangle_s(\Psi))^{\mathcal{F}}$

Theorem (Adaptive Model with Self-Randomization)

If $\tau \in (\triangle_s(\Psi))^{\mathcal{F}}$ is chosen optimally, then the long-run growth rate of the genotype from random choice rule ρ is

$$V(\rho) = \sup_{\tau \in (\triangle_s(\Psi))^{\mathcal{F}}} \int_{\Omega} \ln\left(\int_S \mathbb{E}_{\tau \otimes \rho}\left[\psi(f(\omega, s))\right] d\nu_\omega(s) \right) d\mu(\omega)$$

where

$$\mathbb{E}_{\tau \otimes \rho}\left[\psi(f(\omega, s))\right] = \int_{\mathcal{F}} \int_{\Psi} \psi(f(\omega, s))\, d\tau(\psi|f)\, d\rho(f).$$

◂ Back to (Deterministic) Theorem

Long-Run Growth with Random Choice and Signals

- Random choice of act given σ: $\rho_\sigma(f)$ where $\rho \in (\triangle_s(\mathcal{F}))^\Sigma$
- Random choice of ψ given σ and f: $\tau_\sigma(\psi|f)$ where $\tau \in (\triangle_s(\Psi))^{\Sigma\times\mathcal{F}}$

Theorem (Adaptive Model with Self-Randomization)

If $\tau \in (\triangle_s(\Psi))^{\Sigma\times\mathcal{F}}$ is chosen optimally, then the long-run growth rate of the genotype from random choice rule ρ is

$$V(\rho) = \sup_{\tau\in(\triangle_s(\Psi))^{\Sigma\times\mathcal{F}}} \int_\Omega \ln\left(\int_S \mathbb{E}_{\tau_\sigma\otimes\rho_\sigma}\left[\psi(f(\omega,s))\right] d\nu_\omega(s,\sigma)\right) d\mu(\omega)$$

where

$$\mathbb{E}_{\tau_\sigma\otimes\rho_\sigma}\left[\psi(f(\omega,s))\right] = \int_{\mathcal{F}}\int_\Psi \psi(f(\omega,s))\, d\tau_\sigma(\psi|f)\, d\rho_\sigma(f).$$

Back to (Deterministic) Theorem

Dual Formulas with Random Choice and Signals

- Random choice of act given σ: $\rho_\sigma(f)$ where $\rho \in (\triangle_s(\mathcal{F}))^\Sigma$
- For any $q \in \triangle(\Omega)$, define measure $\nu \otimes q$ on $\Omega \times S \times \Sigma$ to have marginal q on Ω and conditional distribution ν_ω on $S \times \Sigma$:

$$(\nu \otimes q)(E) = \int_\Omega \int_{S\times\Sigma} \mathbf{1}[(\omega, s, \sigma) \in E]\, d\nu_\omega(s,\sigma)\, dq(\omega).$$

Theorem (Duality)

Suppose the set Ψ is convex, pointwise bounded above, and closed in the topology of pointwise convergence, and fix μ and ν. Then,

$$V(\rho) = \inf_{q \in M(\mu)} \left[\ln\left(\int_\Sigma U(\rho_\sigma, q, \sigma)\, d(\nu \otimes q)(\sigma) \right) + R(\mu \,\|\, q) \right]$$

where

$$U(\rho_\sigma, q, \sigma) = \mathbb{E}_{\rho_\sigma}\left[\sup_{\psi \in \Psi} \int_{\Omega \times S} \psi(f(\omega, s))\, d(\nu \otimes q)(\omega, s|\sigma) \right].$$

Back to Discussion

References

Epstein, L. G. and M. Schneider (2003). **"Recursive Multiple-Priors".** In: *Journal of Economic Theory* 113, pp. 1–31.

Ergin, H. and F. Gul (2009). **"A Theory of Subjective Compound Lotteries".** In: *Journal of Economic Theory* 144, pp. 899–929.

Flück, M. (2006). **"Functional, Structural and Molecular Plasticity of Mammalian Skeletal Muscle in Response to Exercise Stimuli".** In: *Journal of Experimental Biology* 209, pp. 2239–2248.

Hanany, E. and P. Klibanoff (2007). **"Updating Preferences with Multiple Priors".** In: *Theoretical Economics* 2, pp. 261–298.

Hanany, E. and P. Klibanoff (2009). **"Updating Ambiguity Averse Preferences".** In: *The B.E. Journal of Theoretical Economics* 9.

Klibanoff, P., M. Marinacci, and S. Mukerji (2005). **"A Smooth Model of Decision Making Under Ambiguity".** In: *Econometrica* 73, pp. 1849–1892.

Nau, R. F. (2006). **"Uncertainty Aversion with Second-Order Utilities and Probabilities".** In: *Management Science* 52, pp. 136–145.

Piersma, T. and J. Drent (2003). **"Phenotypic Flexibility and the Evolution of Organismal Design".** In: *Trends in Ecology & Evolution* 18, pp. 228–233.

Robson, A. J. (1996). **"A Biological Basis for Expected and Non-Expected Utility".** In: *Journal of Economic Theory* 68, pp. 397–424.

Sadowski, P. and T. Sarver (2024). **"Adaptive Preferences: An Evolutionary Model of Non-Expected Utility and Ambiguity Aversion".** In: *Journal of Economic Theory* 218, p. 105840.

Wikelski, M. and C. Thom (2000). **"Marine Iguanas Shrink to Survive El Niño".** In: *Nature* 403, p. 37.

Wright, P. A. and A. J. Turko (2016). **"Amphibious Fishes: Evolution and Phenotypic Plasticity".** In: *Journal of Experimental Biology* 219, pp. 2245–2259.