Supplemental Appendix

The Economics of Discriminatory Job Reservations

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1 An example for which assumptions 1 through 4 hold

In this section I present an example of a production function G and a set of feasible partitions $\mathcal{F}$ that satisfy assumptions 1 through 4.

As in section 2.1, there are two groups of workers, dominant and oppressed. Let α_d and α_o be the measures of dominant and oppressed group workers.

There are N levels of tasks. Each worker inelastically supplies one unit of labor to one out of the set of level-0 tasks. Higher levels of tasks then consist of aggregates of lower level tasks. More specifically, each level-k task consists of an aggregate of level-(k-1) tasks.

In order to label tasks, I locate each level-k task within an $(N-k)$-dimensional unit cube. Let $(i_k, ..., i_{N-1})$ be the coordinates of a particular level-k task within the level-k unit cube $[0,1]^{N-k}$. Let $\ell_k(i_k, ..., i_{N-k})$ be the quantity produced of the level-k task with coordinate $(i_k, ..., i_{N-1})$. Since level-0 tasks are produced directly from raw labor supply, $\ell_k(i_0, ..., i_{N-1})$ is just the quantity of labor supplied to the level-0 task with coordinate $(i_0, ..., i_{N-1})$.

For all $k > 0$, the production function for each level-k task has the following CES form:

$$\ell_k(i_k, ..., i_{N-1}) = \left[\int_0^1 \ell_{k-1}(i_{k-1}, ..., i_{N-1})^{(\tau_{k-1}-1)/\tau_{k-1}} di_{k-1}\right]^{\tau_{k-1}/(\tau_{k-1}-1)} \tag{1}$$

That is, the quantity of each level k task is a CES aggregate of the level $k-1$ tasks that share the same final $N-k$ coordinates. Here τ_{k-1} is the elasticity of substitution between level $k-1$ tasks.

Aggregate labor L is just the level N aggregate task:

$$L = \ell_N = \left[\int_0^1 \ell_{N-1}(i_{N-1})^{(\tau_{N-1}-1)/\tau_{N-1}} di_{N-1}\right]^{\tau_{N-1}/(\tau_{N-1}-1)} \tag{2}$$

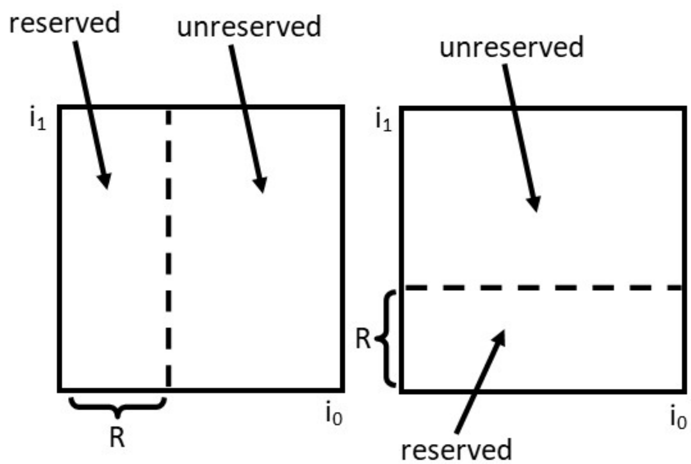
Final output is produced using aggregate labor L and the non-labor factor of production Z. The final production function is

$$Y = G(L, Z) \tag{3}$$

A partition $\mathcal{P}$ divides the set of level-0 tasks into sets of reserved and unreserved tasks. The set of feasible partitions $\mathcal{F}$ is the following. Let $\mathcal{R}$ be the set of coordinates of tasks in the set of reserved tasks generated by a partition $\mathcal{P}$. The partition $\mathcal{P}$ is feasible if and only if there exists $k < N$ and $R \in [0,1]$ such that $\mathcal{R} = \{(i_0, ..., i_k, ..., i_{N-1}) | i_k \leq R\}$.

Figure 6 shows an example of two different feasible partitions with $N = 2$ levels.

Figure 6: Reserved and unreserved tasks



This figure shows two different feasible partitions with $N = 2$ levels. Level-0 tasks are indexed by (i_0, i_1). The figure on the left shows a partition such that all level-0 tasks with coordinate $i_0 \leq R$ are reserved, while the figure on the right shows a partition such that all level-0 tasks with coordinate $i_1 \leq R$ are reserved.

Suppose that a partition is chosen such that all level-0 tasks with k coordinate $i_k \leq R$ are reserved. Suppose also that $R > \alpha_d/(\alpha_d + \alpha_o)$, so that the constraint that oppressed group workers cannot choose reserved jobs binds. Then all dominant group workers choose reserved tasks and all oppressed group workers choose unreserved tasks. All tasks enter symmetrically into the production function. The law of one price for tasks thus implies that within the sets of reserved and unreserved tasks, the amount of labor allocated to each task is the same. Thus, $\ell_0(i_0, ..., i_k, ..., i_{N-1}) = \alpha_d/R$ if $i_k \leq R$, and $\ell_0(i_0, ..., i_k, ..., i_{N-1}) = \alpha_o/(1 - R)$ if $i_k \geq R$. Plugging these quantities into (1) and solving iteratively yields that aggregate labor is given by:

$$L = \left[R \left(\frac{\alpha_d}{R} \right)^{(\tau_k - 1)/\tau_k} + (1 - R) \left(\frac{\alpha_o}{1 - R} \right)^{(\tau_k - 1)/\tau_k} \right]^{\tau_k/(\tau_k - 1)} \tag{4}$$

Thus the reduced form production function can be written as $Y = F(L, Z)$ where L is given by (4), for any pair (R, σ) such that $R \in [\alpha_d/(\alpha_d + \alpha_o), 1]$ and such that $\sigma \in \{\tau_0, ..., \tau_{N-1}\}$. The set of feasible values of σ is thus $\mathcal{S} = \{\tau_0, ..., \tau_{N-1}\}$. As mentioned in section 2.1, the set $\mathcal{S}$ is discrete.

I do not know if it is possible to construct an example that satisfies assumptions 1 through 4 and such that $\mathcal{S}$ is continuous.

2 Conditions under which assumption 5 holds

I show if F is the CES production function, then part 3 of assumption 5 holds if and only if the elasticity of substitution between labor and the non-labor factor θ satisfies $\theta > 1 - s_L$, where s_L is the labor share of income.

Proposition 8. *Let*

$$F(L, Z) = \left(\beta L^{(\theta-1)/\theta} + (1-\beta) Z^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)} \tag{5}$$

Then $(\partial^2 F/\partial L^2)L + \partial F/\partial L > 0$ *if and only if* $\theta > 1 - s_L$.

Proof. Write the labor share as:

$$s_L \;=\; \frac{L\,F_L}{F}, \tag{6}$$

Some algebra shows that:

$$\frac{F_{LL}}{F_L} = \frac{s_L - 1}{\theta\, L} \qquad \Longrightarrow \qquad F_{LL}L + F_L = F_L\Big(\frac{s_L - 1}{\theta} + 1\Big) = F_L\,\frac{s_L + (\theta - 1)}{\theta}\,. \tag{7}$$

Since $F_L > 0$,

$$F_{LL}L + F_L > 0 \quad \Longleftrightarrow \quad s_L + (\theta - 1) > 0 \quad \Longleftrightarrow \quad \theta > 1 - s_L. \tag{8}$$

□

3 Conditions under which assumption 6 holds

I show that assumption 6 holds if F is the CES production function.

Proposition 9. *Let*

$$F(L, Z) = \left(\beta L^{(\theta-1)/\theta} + (1-\beta) Z^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)} \tag{9}$$

Then

$$-\frac{\partial^3 F/\partial L^3}{\partial^2 F/\partial L^2} L \le 1 - \frac{\partial^2 F/\partial L^2}{\partial F/\partial L} L \tag{10}$$

Proof. Define

$$A \;=\; \beta\, L^{\frac{\theta-1}{\theta}}, \quad B \;=\; (1-\beta)\, Z^{\frac{\theta-1}{\theta}}, \quad U \;=\; A+B,$$

so

$$F(L,Z) \;=\; U^{\frac{\theta}{\theta-1}}.$$

Some algebra yields:

$$\begin{aligned}
F_L &= U^{\frac{1}{\theta-1}}\;\beta\, L^{-1/\theta},\\
F_{LL} &= U^{\frac{2-\theta}{\theta-1}}\;\frac{\beta}{\theta}\, L^{-1/\theta-1}\,\big((1-\theta)\,A \;+\; B\big),\\
F_{LLL} &= U^{\frac{3-2\theta}{\theta-1}}\;\frac{\beta}{\theta^2}\, L^{-1/\theta-2}\,\Big[(1-\theta)\big((2-\theta)\,A+B\big) \;+\; (2-\theta)\,B\Big].
\end{aligned}$$

The labor share s_L is:

$$s_L \;=\; \frac{F_L\, L}{F} \;=\; \frac{\beta\, L^{(\theta-1)/\theta}}{A+B} \;=\; \frac{A}{U}, \quad 1-s_L \;=\; \frac{B}{U}.$$

A little algebra shows

$$\begin{aligned}
\frac{L\,F_{LL}}{F_L} &\;=\; -\,\frac{\theta-1}{\theta}\,(1-s_L),\\
\frac{L\,F_{LLL}}{F_{LL}} &\;=\; -\,\frac{\theta-1}{\theta}\,(1-s_L) \;-\; 1 \;+\; \frac{(\theta-1)^2}{\theta^2}\,\frac{A\,B}{U^2}.
\end{aligned}$$

Plug these into

$$1 \;-\; \frac{L\,F_{LL}}{F_L} \;+\; \frac{L\,F_{LLL}}{F_{LL}} \;=\; 1 \;-\; \Big[-\tfrac{\theta-1}{\theta}(1-s_L)\Big] \;+\; \Big[-\tfrac{\theta-1}{\theta}(1-s_L)-1+\tfrac{(\theta-1)^2}{\theta^2}\,\tfrac{AB}{U^2}\Big].$$

Everything but the last term collapses, leaving

$$1 \;-\; \frac{L\,F_{LL}}{F_L} \;+\; \frac{L\,F_{LLL}}{F_{LL}} \;=\; \frac{(\theta-1)^2}{\theta^2}\,\frac{A\,B}{\big(A+B\big)^2} \;=\; \frac{(\theta-1)^2}{\theta^2}\,\frac{\beta\,(1-\beta)\,L^{\frac{\theta-1}{\theta}}\,Z^{\frac{\theta-1}{\theta}}}{\Big(\beta\,L^{\frac{\theta-1}{\theta}}+(1-\beta)\,Z^{\frac{\theta-1}{\theta}}\Big)^2}.$$

Hence

$$1 \;-\; \frac{L\,F_{LL}}{F_L} \;+\; \frac{L\,F_{LLL}}{F_{LL}} > 0,$$

which rearranges to

$$-\frac{L\,F_{LLL}}{F_{LL}} \;<\; 1 \;-\; \frac{L\,F_{LL}}{F_L}.$$

□

4 Conditions under which assumption 7 holds

I show that if F is the CES function, then part 2 of assumption 7 holds if and only if the elasticity of substitution θ between labor and the non-labor factor is such that $\theta < 1$.

Proposition 10. *Let*

$$F(L,Z) = \left(\beta L^{(\theta-1)/\theta} + (1-\beta)Z^{(\theta-1)/\theta}\right)^{\theta/(\theta-1)} \tag{11}$$

Then

$$-\frac{\partial^3 F/\partial Z\partial L^2}{\partial^2 F/\partial Z\partial L} < -\frac{\partial^2 F/\partial L^2}{\partial F/\partial L} \tag{12}$$

if and only if $\theta < 1$.

Proof. Let $s_L = F_L L/F$ be the labor share of income. Some algebra shows that:

$$\frac{F_{LLZ}}{F_{LZ}} - \frac{F_{LL}}{F_L} = \frac{1-\theta}{\theta}\frac{s_L}{L} \tag{13}$$

Thus

$$-\frac{F_{LLZ}}{F_{LZ}} < -\frac{F_{LL}}{F_L} \iff \theta < 1 \tag{14}$$

□

5 Proofs not included in the main text

For the following proofs it is helpful to reproduce equations (3), (9), and (12) from the main text:

$$w_d(\alpha_d, \alpha_o, R, \sigma) = \frac{\partial F}{\partial L}\left(L\frac{R}{\alpha_d}\right)^{1/\sigma} \tag{15}$$

$$MRTS = \frac{\partial L/\partial \alpha_d}{\partial L/\partial \alpha_o} = \left[\frac{R}{(1-R)}\frac{\alpha_o}{\alpha_d}\right]^{1/\sigma} \tag{16}$$

$$\frac{\partial w_d}{\partial R} = \underbrace{\frac{\partial^2 F}{\partial L^2}\frac{\partial L}{\partial R}\left(L\frac{R}{\alpha_d}\right)^{1/\sigma}}_{competition\ effect} + \underbrace{\frac{1}{\sigma}\frac{\partial F}{\partial L}\left(L\frac{R}{\alpha_d}\right)^{(1-\sigma)/\sigma}\frac{L}{\alpha_d}}_{wage\ premium\ effect} + \underbrace{\frac{1}{\sigma}\frac{\partial F}{\partial L}\left(L\frac{R}{\alpha_d}\right)^{(1-\sigma)/\sigma}\left(\frac{\partial L}{\partial R}\frac{R}{\alpha_d}\right)}_{complementarity\ effect} \tag{17}$$

5.1 Proof of proposition 2

I begin by proving a lemma:

Lemma 2. *Assumption 6 implies the following statement. Suppose that $\partial F/\partial L > 0$ and $\partial^2 F/\partial L^2 < 0$. Given a constant k, if*

$$\frac{\partial^2 F}{\partial L^2}L + k\frac{\partial F}{\partial L} \geq 0, \tag{18}$$

then

$$\frac{\partial^3 F}{\partial L^3}L + (1+k)\frac{\partial^2 F}{\partial L^2} \geq 0 \tag{19}$$

Proof. Suppose that

$$F_{LL}L + kF_L \geq 0, \tag{20}$$

Then

$$k \geq -\frac{LF_{LL}}{F_L} \tag{21}$$

By assumption 6,

$$-\frac{LF_{LLL}}{F_{LL}} \leq 1 - \frac{LF_{LL}}{F_L} \tag{22}$$

Putting these results together yields

$$-\frac{LF_{LLL}}{F_{LL}} \leq 1 + k \tag{23}$$

which rearranges to

$$LF_{LLL} + (1+k)F_{LL} \geq 0 \tag{24}$$

as was to be shown. $\square$

Now, by proposition 1, $\sigma = \underline{\sigma}$ at the optimum. Set $\sigma = \underline{\sigma}$ and rearrange (17) to get:

$$\frac{\partial w_d}{\partial R} = \frac{\partial L}{\partial R}\frac{R}{\alpha_d}\left(L\frac{R}{\alpha_d}\right)^{(1-\underline{\sigma})/\underline{\sigma}}\left\{\frac{\partial^2 F}{\partial L^2}L + \frac{1}{\underline{\sigma}}\frac{\partial F}{\partial L}\left[1 + \frac{1}{\partial L/\partial R}\frac{L}{R}\right]\right\} \tag{25}$$

Define

$$\xi(R,\sigma) = 1 + \frac{1}{\partial L/\partial R}\frac{L}{R} \tag{26}$$

Define

$$k(R,\sigma) = (1/\sigma)\xi(R,\sigma) \tag{27}$$

Define

$$\zeta(R,\sigma) = \frac{\partial^2 F}{\partial L^2}L + \frac{1}{\sigma}\frac{\partial F}{\partial L}\xi(R) \tag{28}$$

Suppose first that $\zeta(R,\underline{\sigma}) < 0$ for all $R < 1$. Then $\partial w_d/\partial R > 0$ for all $R < 1$, since $\partial L/\partial R < 0$. So the unique optimal value of R is $R^* = 1$.

Now suppose that $\zeta(R,\underline{\sigma}) \geq 0$ for some R. Differentiating $\zeta(R,\sigma)$ with respect to R yields:

$$\frac{\partial \zeta}{\partial R} = \frac{\partial L}{\partial R}\left[\frac{\partial^3 F}{\partial L^3}L + (1 + k(R,\sigma))\frac{\partial^2 F}{\partial L^2}\right] + \frac{\partial k}{\partial R}\frac{\partial F}{\partial L} \tag{29}$$

Differentiating L with respect to R shows that $\partial L/\partial R < 0$, which implies that $\xi(R,\sigma) < 1$. Since $\zeta(R,\underline{\sigma}) \geq 0$, $(\partial^2 F/\partial L^2)L + k(R^*,\underline{\sigma})(\partial F/\partial L) \geq 0$. Thus by lemma 2, the first term of (29) is non-negative. Differentiating k with respect to R shows that the second term of (29) is positive. Thus $\partial\zeta/\partial R > 0$. Since $\zeta(R,\underline{\sigma})$ approaches $-\infty$ as R approaches $\alpha_d/\alpha_d + \alpha_o$, and since $\zeta(R,\underline{\sigma})$ is continuous, there exists exactly one value of R such that $\zeta(R,\underline{\sigma}) = 0$. Let R^* be the value of R such that $\zeta(R,\underline{\sigma}) = 0$. Since ζ is strictly increasing in R, w_d is strictly increasing in R for all $R < R^*$ and strictly decreasing in R for all $R > R^*$, which implies that R^* is the unique optimal value of R.

5.2 Proof of proposition 3

I begin with a lemma:

Lemma 3. *Assumption 7 implies the following statement. Given a constant k, if*

$$\frac{\partial^2 F}{\partial L^2} L + k \frac{\partial F}{\partial L} \geq 0, \tag{30}$$

then

$$\frac{\partial^3 F}{\partial Z \partial L^2} + k \frac{\partial^2}{\partial L \partial Z} > 0. \tag{31}$$

Proof. Suppose that

$$F_{LL} L + k F_L \geq 0 \tag{32}$$

Then

$$k \geq -\frac{L F_{LL}}{F_L} \tag{33}$$

By assumption 7,

$$-\frac{F_{LLZ}}{F_{LZ}} < -\frac{F_{LL}}{F_L} \tag{34}$$

Putting these results together yields

$$-\frac{F_{LLZ}}{F_{LZ}} < k \tag{35}$$

Since $F_{LZ} > 0$, this expression rearranges to

$$F_{LLZ} + k F_{LZ} > k \tag{36}$$

This completes the proof. $\square$

Suppose first that $\zeta(R, \underline{\sigma}) < 0$ for all $R < 1$. Then $\partial w_d / \partial R > 0$ for all $R < 1$, so the unique optimal value of R is $R^* = 1$. In this case w_d does not depend on σ so $dR^*(\underline{\sigma})/d\underline{\sigma} = 0$.

Now suppose that $\zeta(R, \underline{\sigma}) = 0$ for some $R < 1$. Then R^* is the value of R such that $\zeta(R^*, \underline{\sigma}) = 0$. Differentiating $\zeta(R, \sigma)$ with respect to Z yields:

$$\frac{\partial \zeta}{\partial Z} = \frac{\partial^3 F}{\partial L^2 \partial Z} L + k(R, \sigma) \frac{\partial^2 F}{\partial L \partial Z} \tag{37}$$

Lemma 3 implies that $\partial \zeta / \partial Z > 0$, which implies that $\partial^2 w_d / \partial R \partial Z < 0$ at $R = R^*$. Therefore, R^* is strictly decreasing in Z.

The second part of proposition 3 follows from the observation that the wage ratio w_d/w_o is strictly increasing in R from (16).

5.3 Proof of proposition 4

I begin with a lemma:

Lemma 4. *Suppose that* $L_1 \leq L_2$. *Let*

$$g(Z) = \frac{\partial F(L_1, Z)/\partial L}{\partial F(L_2, Z)/\partial L} \tag{38}$$

Then $g'(Z) \leq 0$, *with strict inequality if* $L_1 < L_2$.

Proof. Applying the quotient rule and rearranging shows that

$$g'(Z) = \frac{F_L(L_1, Z) F_L(L_2, Z)}{[F_L(L_2, Z)]^2} \left[\frac{F_{LZ}(L_1, Z)}{F_L(L_1, Z)} - \frac{F_{LZ}(L_2, Z)}{F_L(L_2, Z)} \right] \tag{39}$$

If $L_1 = L_2$, then it follows that $g'(Z) = 0$.

Let

$$h(L, Z) = \frac{F_{LZ}(L, Z)}{F_L(L, Z)} \tag{40}$$

Then we have

$$\frac{\partial}{\partial L} h(L, Z) = \frac{F_L(L, Z) F_{ZLL}(L, Z) - F_{LZ}(L, Z)}{[F_L(L, Z)]^2} \tag{41}$$

So

$$\text{sign}(h_L) = \text{sign} \left(\frac{F_{ZLL}}{F_{LZ}} - \frac{F_{LL}}{F_L} \right) \tag{42}$$

By assumption 7, $F_{ZLL}/F_{LZ} > F_{LL}/F_L$, which implies that $h_L > 0$. If $L_1 < L_2$, then $g'(Z) < 0$, completing the proof. □

The dominant group prefers to impose discrimination if $(1 - c)w_d \geq w$, that is, if

$$\frac{(\partial F(L(R^*), Z)/\partial L) \left(L(R^*) \frac{R^*}{\alpha_d} \right)^{1/\sigma}}{\partial F(\alpha_d + \alpha_o, Z)/\partial L} \geq \frac{1}{1 - c} \tag{43}$$

Let

$$\Delta(R, Z) = \frac{(\partial F(L(R), Z)/\partial L) \left(L(R) \frac{R}{\alpha_d} \right)^{1/\sigma}}{\partial F(\alpha_d + \alpha_o, Z)/\partial L} \tag{44}$$

Choose $\underline{Z}$ and $\bar{Z}$ such that $\underline{Z} < \bar{Z}$. For any $R > \alpha_d + \alpha_o$, $L(R) < \alpha_d + \alpha_o$, so by lemma 4, $\Delta(R^*(\bar{Z}), \underline{Z}) > \Delta(R^*(\bar{Z}), \bar{Z})$. Since $R^*(\bar{Z})$ is not optimal for $Z = \underline{Z}$, $\Delta(R^*(\underline{Z}), \underline{Z}) \geq \Delta(R^*(\bar{Z}), \underline{Z})$. Putting these inequalities together yields

$$\Delta(R^*(\underline{Z}), \underline{Z}) > \Delta(R^*(\bar{Z}), \bar{Z}) \tag{45}$$

which implies that $d\Delta(R^*(Z), Z)/dZ < 0$.

Let $c(Z)$ be the value of c such that $\Delta(R^*(Z), Z) = 1/(1 - c(Z))$. Since $d\Delta(R^*(Z), Z)/dZ < 0$, $\partial c/\partial Z < 0$, completing the proof.

5.4 Proof of proposition 5

From (16), the wage ratio w_d/w_o is increasing in R. Differentiating L with respect to R shows that L is decreasing in R. Therefore, if assumption 8 holds, then increasing R both increases the wage ratio w_d/w_o and increases the total payment to labor, so increasing R must increase the wage w_d. So it is optimal to set R as large as possible, that is, $R = 1$. If $R = 1$ then the wage w_d is the same for all finite values of σ, so any finite value of σ is optimal.

5.5 Proof of proposition 6

By assumption 1, $dw/d\alpha_o < 0$.

Differentiate (15) with respect to α_o and apply the envelope theorem to get:

$$\frac{dw_d^*}{d\alpha_o} = \frac{\partial w_d}{\partial \alpha_o} = \frac{\partial L}{\partial \alpha_o} \frac{R}{\alpha_d} \left(L \frac{R}{\alpha_d} \right)^{(1-\sigma)/\sigma} \left[\frac{\partial^2 F}{\partial L} L + \frac{1}{\sigma} \frac{\partial F}{\partial L} \right] \tag{46}$$

Recall again the expression for $\xi(R)$ derived in (26). Since $\partial L/\partial R < 0$ for all $\sigma < \infty$, $\xi(R) < 1$. Using this fact, comparing (46) with (25) shows that whenever (25) is equal to zero, (46) is strictly greater than 0. If $R^* < 1$, then (25) is equal to zero at $R = R^*$. Therefore, if $R^* < 1$, $dw_d/d\alpha_o > 0$.

If $R^* = 1$ then it is straightforward to verify that $dw_d/d\alpha_o = 0$.

5.6 Proof of proposition 7

The sign of (46) is the same as the sign of the expression within the square brackets in (46). By assumption 5, for σ sufficiently large, the expression in the square brackets is negative. Therefore

$\partial w_d/\partial\alpha_o < 0$ for sufficiently large σ.

6 Simulation robustness checks

Figure 7: Simulation robustness, $\theta = 0.5$

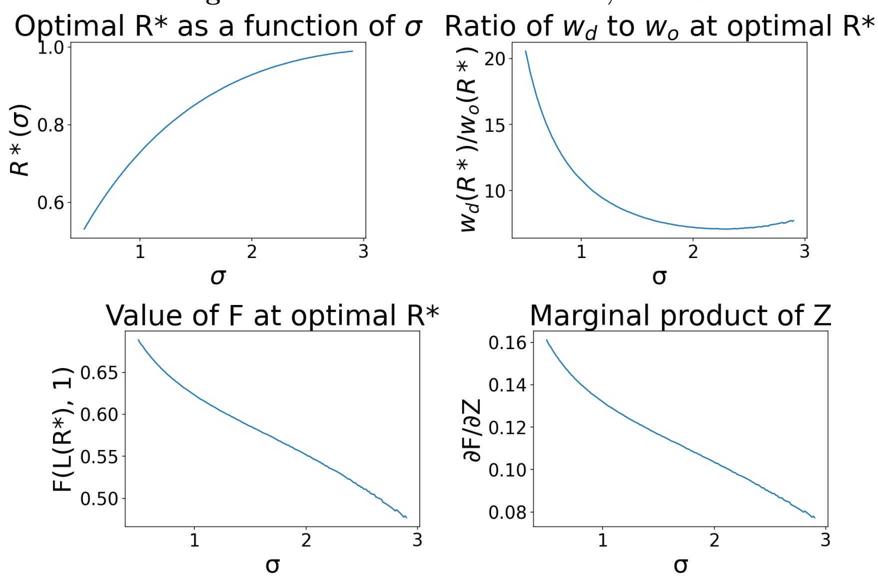


Figure 8: Simulation robustness, $\theta = 0.9$