

Internet Appendix

A Debt Sustainability Accounting

Following [Hall and Sargent \(2011\)](#), we can impute variation in the debt-to-output ratio of a country to its history of output growth, inflation, and interest rates. Let G_t denote nominal government spending before interest expenses on the debt, T_t denote nominal government tax revenue. We start from the flow government budget constraint in a deterministic environment:

$$G_t - T_t + D_{t-1}R_t^D = D_t, \quad (\text{A1})$$

where R_t^D denotes the gross return on the entire portfolio of marketable debt D_{t-1} . By iterating backwards, we obtain the following expression for the debt-to-output ratio. The debt-to-output ratio can be stated today as a function of cumulative past returns $R_{t-j,t}^D$, cumulative past nominal growth $X_{t-j,t}$, as well as past primary deficit/output ratios $\frac{G_{t-j}-T_{t-j}}{Y_{t-j}}$:

$$\frac{D_t}{Y_t} = \sum_{j=0}^t \left(\frac{G_{t-j} - T_{t-j}}{Y_{t-j}} \right) \frac{R_{t-j,t}^D}{X_{t-j,t}} + \frac{R_{0,t}}{X_{0,t}} \frac{D_{-1}}{Y_{-1}},$$

where $R_{t-j,t}^D = \prod_{k=1}^j R_{t-j+k}^D$ and $X_{t-j,t} = \prod_{k=1}^j X_{t-j+k}$.

When r^D , the real rate of return on debt, is lower than x , the real growth rate of the economy, the government can sustain steady-state deficits ($G > T$) while maintaining a constant debt-to-GDP ratio. This can be seen clearly in the steady state version of government budget constraint: $\frac{x-r^D}{1+x} \frac{D}{Y} = \frac{G-T}{Y}$. In our sample period, the average rate of real GDP growth is -0.03%, while the average real return on the Japanese government bond portfolio is 1.30%. The Japanese central government is not in the $r^D < x$ region where it can just roll over steady-state deficits at an interest rate that is below the growth rate of the economy ([Blanchard, 2019](#); [Mehrotra and Sergeyev, 2021](#)).

However, if we start from the consolidated budget constraint that takes into account the returns on financial assets,

$$G_t - T_t + D_{t-1}R_t^D - A_{t-1}R_t^A = D_t - A_t \equiv ND_t$$

where R_t^A denotes the gross return on the portfolio of assets, the net debt-to-output ratio ND_t/Y_t can be stated as a function of cumulative past debt returns, cumulative past nominal growth X_{t-j} , primary deficit/output ratios $\frac{G_{t-j}-T_{t-j}}{Y_{t-j}}$, past asset position/output ratios $\frac{A_{t-j}}{Y_{t-j}}$,

as well as past cumulative excess returns between R_t^A and R_t^D :

$$\frac{ND_t}{Y_t} = \sum_{j=0}^t \left(\frac{G_{t-j} - T_{t-j}}{Y_{t-j}} \right) \frac{R_{t-j,t}^D}{X_{t-j,t}} - \sum_{j=0}^t \frac{A_{t-j}}{Y_{t-j}} \frac{R_{t-j,t}^A - R_{t-j,t}^D}{X_{t-j,t}} + \frac{R_{0,t}^D}{X_{0,t}} \frac{ND_{-1}}{Y_{-1}},$$

where $R_{t-j,t}^A = \prod_{k=1}^j R_{t-j+k}^A$.

The steady state version of budget constraint is modified as:

$$\frac{r^D - x}{1+x} \frac{D}{Y} = \frac{T - G}{Y} + \frac{r^A - r^D}{1+x} \frac{A}{Y},$$

where r^A denotes the real rate of return on its assets. Even if the real growth rate is lower than the real rate of return on debt, $r^D > x$, the consolidated government can run steady-state deficits ($G > T$) with a constant debt-to-GDP ratio, provided that both the real returns on its asset position, r^A exceed the government's funding costs, and the asset position itself, A , are sufficiently large.

B No-Carry-Trade Counterfactual

We conduct a counterfactual exercise to evaluate the impact of the carry trade strategy on Japan's balance sheet. Specifically, we assume that instead of investing in equities and foreign assets, the Japanese government allocates these holdings exclusively to 6-month Japanese Treasury bills. We proceed in two steps. First, using balance sheet data and the estimated return on each balance sheet item, we back out the net flow of funds into or out of asset type i using the following formula:

$$F_{t,t+1}^i = A_{t+1}^i - R_{t,t+1}^i A_t^i, \quad (\text{A2})$$

where A^i and F^i denote the stock and flow of asset i , respectively.

Second, we assume that equities, foreign equities, and foreign bonds earn only the return on Japanese 6-month Treasury bills. The hypothetical asset position is then calculated as:

$$A_{t+1}^i = F_{t,t+1}^i + R_{t,t+1}^{TB} A_t^i, \quad (\text{A3})$$

where R^{TB} denotes the return on 6-month Japanese Treasury bills. The calculation begins with the initial data point at the end of 1997, and from there uses the estimated fund flows and T-bill returns to compute the hypothetical asset position for each asset. This step is iterated forward until the hypothetical asset positions are obtained for all periods. Since the return on the liability side remains unchanged, the liability position is unaffected.

The Figure [A1](#) presents the net liability position under both the actual data and the

counterfactual scenario. It is evident that the gap widens substantially after 2012, reflecting the large carry trade position implemented by the Japanese public sector.

Figure A1: No-Carry-Trade Net Liability Counterfactual

