Anatomy of the Phillips Curve: Micro Evidence and Macro Implications

Supplemental Appendix

L. Gagliardone M. Gertler S. Lenzu J. Tielens

OA.1. Theory derivations

In this section, we derive the key equations presented in Section I of the paper. We first derive the markup functions under two prominent frameworks featuring imperfect competition: Dynamic oligopoly with nested CES preferences and Kimball preferences. We then present the aggregation steps followed to derive the cost-based NKPC.

A. Derivation of the markup function

Dynamic oligopoly with nested CES preferences—Assume that there is a continuum of industries (indexed by i) and a finite number of firms N within each industry. Each firm is indexed by f (or j). Within each industry, firms compete à la Bertrand. In this environment, the price index for each industry P_{it} and the aggregate price index P_t are defined, respectively, as:

$$P_{it} := \left(\frac{1}{N}\sum_{f=1}^{N}(\varphi_{fit}P_{fit})^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \;;\; P_t := \left(\int_{i\in I}(\varphi_{it}P_{it})^{1-\sigma}di\right)^{\frac{1}{1-\sigma}},$$

where φ_{fit} is a firm-specific relative demand shifter (firm appeal), and φ_{it} is an industry-specific demand shifter (relative across industries). In what follows, the subscript i is dropped when redundant, and we normalize the steady-state price level to simplify the notation. The demand function for firm $f \in \mathcal{F}_i$ takes a nested CES form, with the elasticity of substitution across industries $\sigma > 1$ and the elasticity of substitution within industries $\gamma > \sigma$:

$$\mathcal{D}_{ft+\tau} = \left(\frac{\varphi_{ft+\tau}P^o_{ft}}{\varphi_{it+\tau}P_{it+\tau}}\right)^{-\gamma}\left(\frac{\varphi_{it+\tau}P_{it+\tau}}{P_{t+\tau}}\right)^{-\sigma}Y_{t+\tau}. \tag{OA.1}$$

Firms internalize the dynamic effect of their choices on the industry price index and on industry demand. Therefore, the residual elasticity of demand faced by firm f takes the following form:

$$\epsilon_{ft+\tau} := -\frac{\partial \ln \mathcal{D}_{ft+\tau}}{\partial \ln P^o_{ft}} = \gamma - (\gamma-\sigma)\frac{\partial p_{it+\tau}}{\partial p^o_{ft}}. \tag{OA.2}$$

We can further characterize the determinants of the residual demand elasticity. First, the price index of competitors of firm f is defined as:

$$P_{it}^{-f} := \left(\frac{1}{N-1}\sum_{j\neq f}^{N-1}(\varphi_{jit}P_{jit})^{1-\gamma}\right)^{\frac{1}{1-\gamma}}.$$

It follows that $P_{it}^{1-\gamma} = \frac{N-1}{N}\left(P_{it}^{-f}\right)^{1-\gamma} + \frac{1}{N}\left(\varphi_{ft}P_{ft}^{o}\right)^{1-\gamma}$. Next, we can express the derivative of the price index in period $t+\tau$ with respect to the firms' reset price in period t as follows:

$$\frac{\partial P_{it+\tau}}{\partial P_{ft}^{o}} = P_{it+\tau}^{\gamma}\left[\left(\frac{N-1}{N}\right)(P_{it+\tau}^{-f})^{-\gamma}\frac{\partial P_{it+\tau}^{-f}}{\partial P_{ft}^{o}} + \left(\frac{1}{N}\right)(\varphi_{ft})^{1-\gamma}(P_{ft}^{o})^{-\gamma}\right].$$

Multiplying both sides by $\frac{P_{ft}^{o}}{P_{it+\tau}}$, and defining the competitors' reaction function $\zeta_{ft+\tau} := \frac{\partial p_{it+\tau}^{-f}}{\partial p_{ft}^{o}}$, we obtain:

$$\begin{aligned}\frac{\partial p_{it+\tau}}{\partial p_{ft}^{o}} &= \zeta_{ft+\tau}\left(\frac{N-1}{N}\right)\left(\frac{P_{it+\tau}^{-f}}{P_{it+\tau}}\right)^{1-\gamma} + \frac{1}{N}\left(\frac{\varphi_{ft+\tau}P_{ft}^{o}}{P_{it+\tau}}\right)^{1-\gamma} \\ &= \zeta_{ft+\tau}(1-s_{ft+\tau}) + s_{ft+\tau},\end{aligned}$$

where $s_{ft+\tau} := \frac{1}{N}\frac{P_{ft}^{o}\mathcal{D}_{ft+\tau}}{P_{it}Y_{it+\tau}} = \frac{1}{N}\left(\frac{\varphi_{ft+\tau}P_{ft}^{o}}{P_{it+\tau}}\right)^{1-\gamma}$ denotes the within-industry revenue share of firm f, and $Y_{it+\tau} := \varphi_{it+\tau}^{\gamma-\sigma}\left(\frac{P_{it+\tau}}{P_{t+\tau}}\right)^{-\sigma}Y_{t+\tau}$ is the industry demand. Replacing the expression for $\frac{\partial p_{it+\tau}}{\partial p_{ft}^{o}}$ into Equation (OA.2), we have that the within-industry elasticity of demand faced by firm f is given by:

$$\epsilon_{ft+\tau} = \gamma - (\gamma-\sigma)\left[\zeta_{ft+\tau}(1-s_{ft+\tau}) + s_{ft+\tau}\right]. \tag{OA.3}$$

The intuition behind this expression is straightforward. The stronger the reaction of competitors to a firm's price change—captured by $\zeta_{ft+\tau}$—the lower the residual elasticity of demand. A low residual elasticity of demand, in turn, implies that the firm can sustain a higher markup in equilibrium. This result mirrors the one in the dynamic oligopoly environment in Wang and Werning (2022) and it nests a number of static environments featuring imperfectly competitive firms. In a static oligopoly, $\epsilon_{ft+\tau} = 0$ for $\tau > 0$. In Atkeson and Burstein (2008) static Nash oligopoly, $\epsilon_{ft+\tau} = 0$ for $\tau > 0$ and $\zeta_{ft+\tau} = 0$ for all τs. Under monopolistic competition, $N \to \infty$, which implies $\zeta_{ft+\tau} \to 0$ and $s_{ft+\tau} \to 0$.

We now use this result to derive the expression for the log-linearized desired markup in Equation (7) in the paper. As is standard, we log-linearize around a symmetric Nash steady state (Atkeson and Burstein, 2008).[1] Log-linearizing the elasticity in (OA.3) around the steady state, we obtain the steady state residual demand elasticity:

$$\epsilon = \gamma - (\gamma - \sigma)\frac{1}{N},$$

which corresponds to the expression in Atkeson and Burstein (2008). In this model, the desired markup is given by the Lerner index:

$$\mu_{ft+\tau} := \ln(\epsilon_{ft+\tau}/(\epsilon_{ft+\tau} - 1)).$$

Log-linearizing this expression and substituting the expression for steady-state residual demand elasticity, we obtain the expression for the log-linearized desired markup (in deviation from steady state) in Equation (7):

$$\mu_{ft+\tau} - \mu_f = -\Gamma\left(p^o_{ft} - p^{-f}_{it+\tau}\right) + u^\mu_{ft+\tau},$$

where $\mu_f = \mu$ for all fs in the symmetric steady state, $\Gamma := \frac{(\gamma-\sigma)(\gamma-1)}{\epsilon(\epsilon-1)}\frac{N-1}{N} > 0$ denotes the markup elasticity with respect to prices, and the firm-specific markup shock:

$$u^\mu_{ft} := -\frac{(\gamma-\sigma)(\gamma-1)}{\epsilon(\epsilon-1)}\ln\varphi_{ft} + \frac{\gamma-\sigma}{\epsilon(\epsilon-1)}\frac{N-1}{N}\zeta_{ft}, \tag{OA.4}$$

captures residual variation in the markup that depends on the demand shifters and changes in the slope of competitors' reaction function.

Finally, log-linearizing the industry price index and ignoring constants, we obtain:

$$p_{it} = \frac{N-1}{N}p^{-f}_{it} + \frac{1}{N}(\ln\varphi_{ft} + p^o_{ft}).$$

Substituting this expression in Equation (6) for the markup and rearranging we obtain the dynamic pricing equation in Equation (8):
(OA.5)

$$p^o_{ft} = (1-\beta\theta)\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}(\beta\theta)^\tau\left((1-\Omega)(mc^n_{ft+\tau} + \mu_f) + \Omega p^{-f}_{it+\tau} + (1-\Omega)u^\mu_{ft+\tau}\right)\right\},$$

[1] The symmetry assumption is standard in the literature (e.g., Midrigan (2011) and Alvarez and Lippi (2014)), which eases the notation but is largely immaterial for our estimation purposes. Relaxing this assumption would imply firm-specific steady-state demand elasticities, ϵ_f. In this case, the estimates of the parameters of our pricing equations should be interpreted as average across firms. The assumption of Nash steady state, also standard in the literature, implies that $\zeta_{j,\tau} = 0$ in the steady state for all js and τs. This comes with some loss of generality, but two points can be made. First, as shown by Wang and Werning (2022), one can write a "behavioral" model with the weaker assumption that $\mathbb{E}\{\zeta_{j,\tau}\} = 0$ for all js and τs, which delivers, under specific values for the elasticities σ and γ, a pass-through of shocks to marginal cost into prices qualitatively similar to that produced by the Nash model. Second, these considerations also apply to our empirical analysis, as we directly estimate the parameters (Γ, in particular) rather than the underlying elasticities.

where, as in the paper, $\Omega := \frac{\Gamma}{1+\Gamma}$. This parameter denotes the relative weight on the competitors' price index (p_{it}^{-f}) and captures the importance of strategic complementarities. When Ω is close to one, firms are not strategic and only look at their marginal cost when resetting prices. In particular, $\Omega \to 0$ as $N \to \infty$, which is the monopolistic competition case. The error term in Equation (8) is:

$$u_{ft} := (1-\beta\theta)(1-\Omega)\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} u^{\mu}_{ft+\tau} \right\}, \tag{OA.6}$$

which is therefore a firm-specific demand shock that depends on the expected discounted future demand shocks.

Monopolistic competition with Kimball preferences—Assume that the industry output Y_{it} is produced by a unitary measure of perfectly competitive firms using a bundle of differentiated intermediate inputs Y_{ft}, $f \in i$. The bundle of inputs is assembled into final goods using the Kimball aggregator:

$$\int_0^1 \Upsilon\left(\frac{Y_{ft}}{Y_{it}}\right) df = 1,$$

where $\Upsilon(\cdot)$ is strictly increasing, strictly concave, and satisfies $\Upsilon(1) = 1$. Taking as given the industry demand Y_{it}, each firm minimizes costs subject to the aggregate constraint:

$$\min_{Y_{ft}} \int_0^1 \varphi_{ft} P_{ft} Y_{ft} df \qquad \text{s.t.} \int_0^1 \Upsilon\left(\frac{Y_{ft}}{Y_{it}}\right) df = 1,$$

where φ_{fit} is a firm-specific relative demand shifter (firm appeal). Denoting by ψ the Lagrange multiplier of the constraint, the associated first-order condition is:

$$\varphi_{ft} P_{ft} = \psi \Upsilon'\left(\frac{Y_{ft}}{Y_{it}}\right) \frac{1}{Y_{it}}. \tag{OA.7}$$

Define implicitly the industry price index P_{it} as:

$$\int_0^1 \phi\left(\Upsilon'(1) \frac{\varphi_{ft} P_{ft}}{P_{it}}\right) df = 1,$$

where $\phi := \Upsilon \circ (\Upsilon')^{-1}$. Evaluating the first-order condition (OA.7) at symmetric prices, $\varphi_{ft} P_{ft} = P_{it}$, we get $\psi = \frac{P_{it} Y_{it}}{\Upsilon'(1)}$. Replacing for ψ, we obtain:

$$\frac{\varphi_{ft} P_{ft}}{P_{it}} = \frac{1}{\Upsilon'(1)} \Upsilon'\left(\frac{Y_{ft}}{Y_{it}}\right). \tag{OA.8}$$

Therefore, the demand function faced by firms when resetting prices is:

$$\mathcal{D}_{ft+\tau} = \left[(\Upsilon')^{-1}\left(\Upsilon'(1)\frac{\varphi_{ft}P^o_{ft}}{P_{it+\tau}}\right)\right]\left(\frac{P_{it+\tau}}{P_{t+\tau}}\right)^{-\sigma} Y_{t+\tau}.$$

Taking logs of Equation (OA.1.A) and differentiating, we obtain the following expression for the residual elasticity of demand:

$$\epsilon_{ft+\tau} := -\frac{\partial \ln \mathcal{D}_{ft+\tau}}{\partial \ln P^o_{ft}} = -\frac{\Upsilon'\left(\frac{Y_{ft+\tau}}{Y_{it+\tau}}\right)}{\Upsilon''\left(\frac{Y_{ft+\tau}}{Y_{it+\tau}}\right)\cdot\left(\frac{Y_{ft+\tau}}{Y_{it+\tau}}\right)}. \tag{OA.9}$$

We now use this result to derive the expression for the log-linearized desired markup in Equation (7) in the paper, under monopolistic competition with Kimball preferences. As above, for ease of exposition, we focus on the symmetric steady state. Denote the steady-state residual demand elasticity by $\epsilon = -\frac{\Upsilon'(1)}{\Upsilon''(1)}$. Then the derivative of the residual demand elasticity $\epsilon_{ft+\tau}$ in (OA.9) with respect to $\frac{Y_{ft+\tau}}{Y_{it+\tau}}$, evaluated at the steady state, is given by:

$$\epsilon' = \frac{\Upsilon'(1)\left(\Upsilon'''(1)+\Upsilon''(1)\right)-\left(\Upsilon''(1)\right)^2}{\left(\Upsilon''(1)\right)^2} \leq 0. \tag{OA.10}$$

The equation above holds with equality if the elasticity is constant (e.g., under CES preferences). Also in this model, the desired markup is given by the Lerner index. Log-linearizing the Lerner index around the steady state and using Equation (OA.10), we have that, up to a first-order approximation, the log-markup (in deviation from the steady state) is equal to:

$$\mu_{ft+\tau} - \mu_f = \frac{\epsilon'}{\epsilon(\epsilon-1)}\left(y_{ft+\tau} - y_{it+\tau}\right)$$

Finally, log-linearizing the demand function (OA.1.A) and using it to replace the log difference in output, we obtain:

$$\mu_{ft+\tau} - \mu_f = -\Gamma\left(p^o_{ft} - p_{it+\tau}\right) + u^\mu_{ft},$$

where, in the case of Kimball preferences, the sensitivity of the markup to the relative price is given by $\Gamma := \frac{\epsilon'}{\epsilon(\epsilon-1)}\frac{1}{\Upsilon''(1)}$ and $u^\mu_{ft} := -\Gamma\varphi_{ft}$ captures residual variation in the markup that depends on the demand shifters. Note that, because there is a continuum of firms within an industry, we have that $p_{it+\tau} = p^{-f}_{it+\tau}$ without loss of generality. Substituting into the pricing equation in (6) and rearranging leads to the expression in Equation (7) in the paper. Finally, following the same steps as the previous section, we obtain $\Omega := \frac{\Gamma}{1+\Gamma}$ and the corresponding mapping to the dynamic pricing equation in Equation (8).

B. *Aggregation and the cost-based New Keynesian Phillips curve*

Suppose $N < \infty$ and order firms in each industry from 1 to N.[2] The aggregate price index (in log-linear terms) is:

$$p_t = \int_{i \in I} \left(\frac{1}{N} \sum_{f=1}^{N} p_{fit} \right) di,$$

(In the paper, we dropped the industry subscript for ease of notation.) Denote by $A^{\star}_{ft}$ for $f \in \{1, \ldots, N\}$ the set of industries in which the f-th firm can adjust. The price index can then be rewritten as:

$$p_t = \frac{1}{N} \sum_{f=1}^{N} \left(\int_{i \in I / A^{\star}_{ft}} p_{fit-1} di + \int_{i \in A^{\star}_{ft}} p^{o}_{fit} di \right),$$

where we are using the fact that firms that cannot adjust set their price to their $t-1$ level, whereas firms that can adjust set it to the optimal reset price.

Since $A^{\star}_{ft}$ has measure $1-\theta$, and the identity of firms that adjust is an i.i.d. draw from the total population of firms, using the law of large numbers for each $f = \{1, \ldots, N\}$ across industries we have that:[3]

$$\frac{1}{N} \sum_{f=1}^{N} \int_{i \in I / A^{\star}_{ft}} p_{fit-1} di = \theta \int_{i \in I} \left(\frac{1}{N} \sum_{f=1}^{N} p_{fit-1} \right) di = \theta p_{t-1}$$

and

$$\frac{1}{N} \sum_{f=1}^{N} \int_{i \in A^{\star}_{ft}} p^{o}_{fit} di = (1-\theta) \int_{i \in I} \left(\frac{1}{N} \sum_{f=1}^{N} p^{o}_{fit} \right) di.$$

Defining the average reset price in the economy:

$$p^{o}_t := \int_{i \in I} \left(\frac{1}{N} \sum_{f=1}^{N} p^{o}_{fit} \right) di,$$

[2]Letting $N \to \infty$, all results hold under Kimball preferences. Note also that the same argument goes through with minor modifications, but heavier notation, for $N_i \neq N$ for a non-zero measure of industries. In general, heterogeneity of the parameters can be accommodated by repeating the same argument for each group of homogeneous industries with non-zero measure and then taking weighted averages of different industries. See, e.g., Appendix C2 in Wang and Werning (2022).

[3]The i.i.d. assumption implies that: $\int_{i \in B \subseteq [0,1]} p_{fit} di = Pr(B) \int_{i \in I} p_{fit} di$. Notice also that $\int_{i \in [0,1]} \left(\frac{1}{N} \sum_{f=1}^{N} p^{-f}_{it} \right) di = \int_{i \in [0,1]} \left(\frac{1}{N} \sum_{f=1}^{N} \left[\frac{N}{N-1} p_{it} - \frac{1}{N-1} p_{fit} \right] \right) di = p_t$.

we obtain equation Equation (9) in the paper:[4]

$$p_t = \theta p_{t-1} + (1-\theta)p_t^o.$$

Next, we replace the aggregate reset price, p_t^o, with an expression that depends on aggregate marginal costs and prices. Using the definition of firm-level marginal cost in Equation (5), and allowing for arbitrary returns to scale ($\nu_{ft} \lesseqgtr 0$), we obtain the following expression for the logarithm of firm-level nominal marginal cost:

$$mc_{fit}^n = c_{it} + a_{fit} + \nu_{ft} y_{fit}.$$

The average marginal cost in the industry is $mc_{it}^n := \frac{1}{N}\sum_{f=1}^N mc_{fit}^n$, implying:

$$mc_{it}^n = c_{it} + a_{it} + \nu y_{it}.$$

where we defined aggregate returns to scale $\nu := \frac{1}{N}\sum_{f=1}^N \nu_{ft}$ (same for all t and constant across industries, for simplicity). Combining the two equations above and subtracting the (log) industry price index from both sides, we obtain an expression that relates real marginal costs to cost shifters and output:

$$mc_{fit} = mc_{it} + (a_{fit} - a_{it}) + \nu(y_{fit} - y_{it}).$$

We use the demand function to express the log output deviation, $y_{fit} - y_{it}$, in terms of log prices. In the case of CES preferences (see Equation (OA.1)), we obtain:

$$mc_{fit} = mc_{it} + (a_{fit} - a_{it}) - \gamma\nu(p_{fit}^o - p_{it}) - \gamma\nu \ln \varphi_{fit},$$

where γ denotes the within-industry elasticity of substitution.[5]

We then proceed with the following steps: we first manipulate Equation (OA.5) to express the reset price in recursive form, then decompose firm-level nominal marginal cost into firm-level real marginal cost and the industry price index prices, and finally use Equation (OA.1.B) to replace for firm-level real marginal cost:

$$\begin{aligned} p_{fit}^o &= (1-\beta\theta)\left((1-\Omega)(mc_{fit}^n + \mu_f) + \Omega p_{it}^{-f} + (1-\Omega)u_{fit}^{\mu}\right) + \beta\theta\mathbb{E}_t p_{fit+1}^o \\ &= (1-\beta\theta)\Theta\left((1-\Omega)\widehat{mc}_{it} + \Omega p_{it}^{-f} + (1-\Omega)(1+\gamma\nu)p_{it} + (1-\Omega)u_{fit}^{\mu}\right) \\ &\quad + \beta\theta\mathbb{E}_t p_{fit+1}^o + (1-\beta\theta)\Theta(1-\Omega)\Big(a_{fit} - a_{it} - \gamma\nu\ln\varphi_{fit}\Big)\ , \end{aligned}$$

where $\Theta := \frac{1}{1+\gamma\nu(1-\Omega)}$ captures macroeconomic complementarities due to aggregate returns to scale in production. By averaging across firms and industries, we

[4]Notice that, up to a first-order approximation around the symmetric steady state, $p_t = \theta p_{t-1} + (1-\theta)p_t^o$ also holds with Kimball preferences.

[5]A similar expression holds under monopolistic competition with Kimball preferences. In this case, γ is replaced with the corresponding elasticity of relative output to relative prices, $1/\Upsilon''(1)$.

have that the aggregate reset price is given by:

$$p_t^o = (1-\beta\theta)\left((1-\Omega)\Theta\widehat{mc}_t + p_t\right) + \beta\theta\mathbb{E}_t p_{t+1}^o + \frac{\theta}{1-\theta}u_t,$$

where $u_t := \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\Omega)\Theta\int_{i\in I}\left(\frac{1}{N}\sum_{f=1}^{N} u_{fit}^{\mu}\right) di$ is an aggregate cost-push shock and $\left(a_{fit}-a_{it}+\gamma\nu\ln\varphi_{fit}\right)$ is such that $\int_{i\in I}\left(\frac{1}{N}\sum_{f=1}^{N}(a_{fit}-a_{it}+\gamma\nu\ln\varphi_{fit})\right) di = 0$. This follows from the i.i.d. assumption on price adjustments, which implies that the average productivity of resetting firms coincides with the unconditional average. Subtracting p_t from both sides and using the log-linearized price index:

$$\begin{aligned} p_t^o - p_t &= (1-\beta\theta)(1-\Omega)\Theta\widehat{mc}_t + \beta\theta(\mathbb{E}_t p_{t+1}^o - p_t) + \frac{\theta}{1-\theta}u_t \\ \Rightarrow \frac{\theta}{1-\theta}\pi_t &= (1-\beta\theta)(1-\Omega)\Theta\widehat{mc}_t + \beta\theta\mathbb{E}_t\left(\frac{\theta}{1-\theta}\pi_{t+1} + \pi_{t+1}\right) + \frac{\theta}{1-\theta}u_t \end{aligned}$$

Rearranging one obtains the marginal cost-based Phillips curve:

$$\pi_t = \lambda\Theta\ \widehat{mc}_t + \beta\mathbb{E}_t\pi_{t+1} + u_t$$

where $\lambda := \frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\Omega)$ is the slope. The equation above highlights that macroeconomic complementarities also mediate the pass-through of marginal cost to prices via Θ. Under the assumption of constant aggregate returns to scale, we have that $\Theta = 1$, and the Phillips curve simplifies to Equation (11). This condition is exactly met when $\nu = 0$, but also when $\Omega = 1$.

OA.2. Data and measurement

A. Data sources and data cleaning

We use information from PRODCOM to compute the quarterly change in product- and firm-level prices and to define the boundaries of markets (industries) in which firms compete. PRODCOM is a large-scale survey commissioned by Eurostat and administered in Belgium by the national statistical office. The survey is designed to cover at least 90% of domestic production value within each manufacturing industry (4-digit NACE codes) by surveying all firms operating in the country with (a) a minimum of 20 employees or (b) total revenue above 4.5 million Euros (European Commission 2014). Firms are required to disclose, on a monthly basis, product-specific physical quantities (e.g., volume, kg., m^2, etc.) of production sold and the value of production sold (in Euros) for all their manufacturing products. Products are defined in PRODCOM by an 8-digit PC code (e.g., 10.83.11.30 is "Decaffeinated coffee, not roasted", 10.83.11.50 is "Roasted coffee, not decaffeinated", and 10.83.11.70 is "Roasted decaffeinated coffee"). Industries are defined by the first four digits of the product codes (e.g., 10.83 is "Processing of tea and coffee"). Sectors are defined by the first two digits of the product codes (e.g., AC is "Manufacture of food products, beverages, and tobacco products"). The sector definitions follow the NACE Rev.2 classification. For the most part, the industry definition also follows Rev.2. The only exceptions are industries under NACE Rev.1 (i.e., before 2008) that do not directly map into a NACE Rev.2 industry; these are assigned a fictitious, unique 4-digit industry code.

In the raw data, there are approximately $4,000$ product headings distributed across 13 manufacturing sectors. The PC product codes have been revised several times between 1999 and 2019, with a substantial overhaul in 2008. We use the conversion tables provided by Eurostat (n.d.) and firm-specific information on firms' product baskets to harmonize the 8-digit product codes across consecutive quarters and harmonize 4-digit industry codes over time.[6] In most cases, the conversion tables provide a unique mapping of the 8-digit product codes across consecutive years. In a limited number of cases, the mapping is many-to-one, one-to-many, or many-to-many. The many-to-one mapping is straightforward, while the one-to-many and many-to-many mappings could be problematic. We are able to handle most of these cases using information on the basket of products produced by each firm.[7] In a limited number of cases (less than 0.1% of the sample), we do not have sufficient information to resolve the uncertainty regarding the mapping. We drop these observations from the sample. Table OA.1 reports the list of manufacturing sectors and their 2-digit PC codes.

We aggregate monthly information at the quarterly level and construct product-level prices (unit values) by dividing product-level sales by product-level quantities sold. As explained in the paper, we are interested in domestic prices, i.e., prices charged by producers in Belgium. PRODCOM does not require firms to separately report production and sales to domestic and international customers. Therefore, we recover domestic values and quantities sold by combining information from PRODCOM with data on firms' product-level exports (quantities and sales) available through Belgian Customs (for extra-EU trade) and the Intrastat Inquiry (for intra-EU trade).[8] We use the official conversion tables provided by Eurostat to map the CN product code classification used in the international trade data to the PRODCOM product code classification.[9] In most cases, the CN-to-PC conversion involves either a one-to-one or many-to-one mapping, which poses no issues. We drop observations that involve one-to-many and many-to-many mappings. These account for less than 5% of the observations and production

[6] The harmonization of the industry code essentially consists of harmonizing the NACE Rev.1 industry, used before 2008, to the NACE Rev.2 industry codes, used from 2008.

[7] For example, consider a case where the official mapping indicates that product 11.11.11.11 in year t could map to either 22.22.22.21 or 22.22.22.22 in year $t+1$. Suppose two firms, f_1 and f_2, report in period t sales of product 11.11.11.11 in year t. If f_1 reports *only* sales of 22.22.22.21 and f_2 *only* reports sales of 22.22.22.22 in year $t+1$ we infer that we should map 11.11.11.11 to 22.22.22.21 for the former and 11.11.11.11 to 22.22.22.22 for the latter.

[8] In constructing our measure of domestic sales, we address issues related to carry-along trade, which might overstate the amount of production by firms that import products destined for immediate sales.

[9] The first six digits of the CN product classification codes correspond to the World HS classification system.

value.

We apply the following filters and data manipulations to the PRODCOM data set. First, we retain firms' observations in a given quarter only if there was positive production reported for at least one product in that quarter. This avoids large jumps in quarterly values due to non-reporting for some months by certain firms. In the rare cases when a firm reports positive values but quantities are missing, we impute the quantity sold from the average value-to-quantity ratio in the months where both values and quantities are reported. Second, we require firms to file VAT declarations and Social Security declarations (as explained below); these two data sources are needed to measure firms' marginal costs.

We use the customs microdata to obtain information on international competitors selling manufacturing products in Belgium. For each domestic firm, the merged Customs-Intrastat data reports the quantity purchased (in kg) and sales (converted to Euros) of different manufacturing products (about 10,000 distinct CN product headings) purchased by Belgian firms from each foreign country. As is standard when dealing with customs data, we define a foreign competitor as a foreign country-domestic buyer pair. For each foreign competitor, we aggregate the product-level sales and quantity sold at the quarterly level (the reporting is monthly in the raw data) and compute quarterly prices (unit values) by taking the ratio of the two.[10]

We leverage data from two administrative sources to measure firms' total production (turnover) and variable production costs on a quarterly basis. Belgian firms file VAT declarations to the tax authority that contain information on the total sales of the enterprise as well as information on purchases of raw materials and other goods and services that entail VAT-liable transactions, including domestic and international transactions. The coverage of the VAT declarations is almost universal, with a limited number of exceptions that affect the reporting of sole proprietorships and self-employed individuals, and therefore mostly do not apply to the firms surveyed by PRODCOM.[11] We obtain information on employment and labor costs (wage bill) from the Social Security declarations filed quarterly by each Belgian firm with the Department of Social Security of Belgium.

We sum firms' quarterly expenditures on intermediates and labor to obtain a measure of total variable costs, which we use to construct firms' marginal costs.

[10]Some CN codes change over time (although to a lesser extent than PC codes). We use the official conversion tables, available on the Eurostat website, to map CN product codes across consecutive years. We make adjustments only if the code change is one-to-one between two years. We do not account for changes in PC codes that involve splitting into multiple codes or multiple PC codes combining into one code. Effectively, these changes in the PC codes are treated as if new products are generated.

[11]Enterprises file their VAT declarations online, either monthly or quarterly, depending on certain size-based thresholds. Small enterprises (turnover $\leq$ 2.5 million Euros excl. VAT) can choose to file either monthly or quarterly. Large enterprises file monthly. In the case of multiple plants or establishments under one VAT identifier, the declaration is filed as a single document for that VAT identifier. We aggregate all monthly declarations to the quarterly level. At this reporting frequency, VAT declarations tend to reflect the sales of output produced in the previous quarter. For this reason, we use one-quarter leads in VAT declarations to construct the measure of firm-level value added used in the regressions discussed in Section VI.

Table OA.1—List of manufacturing sectors

Sector	Sector definition	NACE Rev.2 2-digits codes
CA	Food products, beverages and tobacco products	10–12
CB	Textiles, apparel, leather and related products	13–15
CC	Wood and paper products, and printing	16–18
CE	Chemicals and chemical products	20
CG	Rubber and plastics products, and other non-metallic mineral products	22–23
CH	Basic metals and fabricated metal products, except machinery and equipment	24–25
CJ	Electrical equipment	27
CK	Machinery and equipment n.e.c.	28
CL	Transport equipment	29–30

Note: This table reports the list of manufacturing sectors in our sample and the corresponding 2-digit NACE Rev.2 codes.

We multiply these costs by the ratio of total manufacturing sales (from PRODCOM) to total sales (from the VAT declarations) to adjust for the fact that some firms also have production outside manufacturing.[12]

Sample selection—We apply the following data-cleaning steps to address missing values and outliers. (i) We focus on manufacturing industries defined by the NACE Rev.2 industry codes from 10 to 30, excluding from our sample all PRODCOM product headings that correspond to mining, quarrying, and industrial services related to other manufacturing activities. (ii) We drop observations referring to firms whose sales from manufacturing products (as measured in PRODCOM) are lower than seventy percent of total firm-level sales (as reported in the VAT declarations). This ensures that our sample includes firms whose real activity is primarily, if not entirely, in manufacturing. (iii) As is standard, we exclude firms that operate in the "Coke and refined petroleum products" sector and the "Pharmaceuticals, medicinal chemical, and botanical products" sector, whose output prices are frequently privately bargained or determined in international markets. We drop firms operating in the "Computer, electronic and optical products" and "Electrical equipment" sectors due to the small number of domestic producers operating in these sectors. Finally, we exclude firms operating in the "Other manufacturing and repair and installation of machinery and equipment" sector, a residual grouping consisting of firms producing diverse and varied products for which it is difficult to define an appropriate set of competitors. (iv) We keep only observations for which we are able to compute product-level price index, the

[12]As mentioned below, we conservatively drop observations referring to firms whose manufacturing sales are less than seventy percent of total sales. In the remaining sample, the ratio has a mean of 0.94 and a median of 0.97, confirming PRODCOM's extensive coverage.

corresponding quantity index, competitors' price index, and marginal costs. (v) We drop observations for which the quarter-to-quarter change of either the firm-level price index or marginal costs is greater than 100% in absolute value. (vi) For each firm-industry pair that enters our dataset intermittently, we only retain the longest continuous time period. This approach ensures that each time series used in the estimation is gapless, avoiding the need to interpolate data and make assumptions about prices and marginal costs when information is missing. (vii) Finally, we exclude firm-industry pairs with a continuous time series shorter than 8 quarters from our dataset. This minimum duration requirement helps mitigate potential Nickell bias in our model estimation.

B. Construction of price indices

We construct a set of indices that capture price changes in manufacturing goods at various levels of aggregation (firm-industry, firm, industry, individual manufacturing sector, and whole manufacturing sector).

Firm-industry price index—The main variable of interest is the price of domestically sold manufacturing products at the firm-industry level, P_{ft}, for both domestic and foreign producers. We construct this variable using information on price changes at the most disaggregated level allowed by the data.

Due to repeated revisions of PRODCOM product codes, a consistent 8-digit product code taxonomy does not exist across the entire sample period. Therefore, we compute the sequence of price changes across consecutive time periods (t and $t+1$) by mapping the product codes at $t+1$ to their corresponding codes at t, aggregating them at the firm-industry level, and recovering the time series of the firm-industry price index (in levels) by concatenating quarterly price changes.

Specifically, denote by $\mathcal{P}_{ft}$ the set of products manufactured by firm f and by P_{pt} the price (unit value) of a given product $p \in \mathcal{P}_{ft}$. We first compute the gross price change for each product, P_{pt}/P_{pt-1}. In doing so, we utilize the official PRODCOM harmonization tables to account for changes in product codes between consecutive quarters and eliminate product-level observations with abnormally large price fluctuations within a given quarter ($P_{pt}/P_{pt-1} > 3$ or $P_{pt}/P_{pt-1} < 1/3$) to mitigate the impact of outliers. We then construct a Törnqvist index measuring the quarterly firm-industry price changes:

$$P_{ft}/P_{ft-1} = \prod_{p \in \mathcal{P}_{ft}} (P_{pt}/P_{pt-1})^{\bar{s}_{pt}}, \tag{OA.1}$$

where $\bar{s}_{pt}$ is a Törnqvist weight computed as the average of the sale shares between t and $t-1$: $\bar{s}_{pt} := \frac{s_{pt}+s_{pt-1}}{2}$.[13] Finally, the time series of firm-industry price levels,

[13]This index accounts for the presence of multi-product firms by averaging across products produced by the same firm in a given industry. The Törnqvist weights give larger weights to those products that account for a larger share of the firm's turnover.

P_{ft}, is constructed by concatenating the Törnqvist index:

$$P_{ft} = P_{f0} \prod_{\tau=t_f^0+1}^{t} \left(P_{f\tau}/P_{f\tau-1}\right). \tag{OA.2}$$

Here t_f^0 denotes the firm-industry's base year. That is the first quarter when f appears in our data. P_{f0} is the level of the price index in the base year. We construct P_{f0} as follows. First, using product-level prices, we compute the average price of the products produced by f in period t_f^0 relative to the average price of products in the industry in the same period, and multiply this relative price by the aggregate price index $P_{t_f^0}$:

$$P_{f0} = P_{t_f^0} \cdot \frac{\prod_{p \in \mathcal{P}_{ft_f^0}} \left(P_{pt_f^0}\right)^{s_{pt_f^0}}}{\prod_{p \in \mathcal{F}_{it_f^0}} \left(P_{pt_f^0}\right)^{s_{pt_f^0}}}. \tag{OA.3}$$

The aggregate price index is initialized to one in the first quarter of our sample (1999:Q1) and is constructed recursively by concatenating a Törnqvist index of industry-level prices. As discussed in the paper, the choice of the firm-industry price index is immaterial for our empirical analysis, as level effects are absorbed by the firm-industry fixed effects included in our empirical specifications.

Firm price index—As discussed in the paper, the vast majority of firms in our data operate in only one (4-digit) industry, implying that the firm-industry price index, P_{ft}, and the firm price index, $\bar{P}_{ft}$, coincide. However, in a limited number of cases, it becomes necessary to construct a firm's price index that aggregates across different firm-industry price indices. In doing this, we construct the firm-level price index $\bar{P}_{ft}$ following a method similar to the one described above. Specifically, we construct a Törnqvist index that aggregates across price changes of the individual (4-digit) industry bundles $i \in I_f$ produced by firm f in quarter t: $\bar{P}_{ft}/\bar{P}_{ft-1} = \prod_{i \in I_f} (P_{fit}/P_{fit-1})^{\bar{s}_{fit}}$, with Törnqvist weights defined as $\bar{s}_{fit} := (s_{fit} + s_{fit-1})/2$, where s_{fit} is the share of sales of industry i in the firms' total sales (across manufacturing industries). To obtain the price index in levels, $\bar{P}_{ft}$, we concatenate the quarterly price changes starting from a base year. The procedure to construct the base years is the same as the one discussed above (Equation OA.3). Note that for single-industry firms the price index $\bar{P}_{ft}$ coincides with the firm-industry price index P_{ft} in Equation (OA.2).

Competitors price index—Using a similar approach, we construct the competitors' price index for each domestic firm. We start by computing quarterly price changes: $P_{it}^{-f}/P_{it-1}^{-f} = \prod_{k \in \mathcal{F}_i/f} (P_{kt}/P_{kt-1})^{\bar{s}_{kt}^{-f}}$, with $\bar{s}_{kt}^{-f} := \frac{1}{2}\left(\frac{s_{kt}}{1-s_{ft}} + \frac{s_{kt-1}}{1-s_{ft-1}}\right)$. Here s_{kt-1} is a Törnqvist weight, averaging the residual revenue share of industry competitors (excluding firm f) at times t and $t-1$. We concatenate these changes starting from a base year, which is the geometric average of firm sales in the industry. Note that the set of domestic competitors for each Belgian producer, denoted in the paper by $\mathcal{F}_i$, includes not only other Belgian manufacturers operating in the same industry, but also foreign manufacturers that belong to the same industry and sell to Belgian customers.

Industry, sector, and aggregate price index—We construct the industry-level, sector-level, and aggregate (manufacturing) price indices by aggregating quarterly firm-level price changes. The formula to construct the percentage change in these price indices is analogous to the one in Equation (OA.1), where now the Törnqvist weights assigned to each firm-industry price change, P_{ft}/P_{ft-1}, capture the (weighted) average market shares of the firm in its own industry, sector, or manufacturing, respectively. Once again, the levels of the indices are constructed by concatenating quarterly changes in their respective Törnqvist indices.

C. *Estimation of firm-level technical efficiency (TFPQ)*

We construct our productivity index, log-technical productivity (TFPQ), as a residual from a gross-output production function. In logarithms:

$$y_{ft} = TFPQ_{ft} + f\big(l_{ft}, k_{ft}, m_{ft}; \boldsymbol{\vartheta}\big),$$

where y_{ft} denotes firm-level physical output produced and $f(\cdot)$ the log-gross output production function aggregating labor (l_{ft}), capital (k_{ft}), and intermediate inputs (m_{ft}). $\boldsymbol{\vartheta}$ is a vector collecting the parameters that pin down the elasticities of output with respect to different inputs.

We use the estimates $\boldsymbol{\vartheta}$ obtained from Lenzu et al. (2025*a*), which recover this parameter vector via a production function estimation.[14] Below we outline the assumptions and steps behind the estimation procedure and refer to the original paper for additional details.

Setup and estimation routine—We assume that a firm's technical efficiency is the sum of two components $TFPQ_{ft} := \omega_{ft} + z_{ft}$. The first, ω_{ft}, represents the persistent component of productivity. It is observable by the firm when it makes production decisions at the beginning of each period t and it evolves as a first-order Markov process. The second component, z_{ft}, is a non-persistent shock, realized after period-t input and pricing decisions have been made.

Denote by I_{ft} the firm's information at the beginning of period t. Exploiting the Markovian nature of the persistent component of TFPQ, we have that $\omega_{jt} = h(\omega_{jt-1}) + \xi_{jt}$, where $h(\omega_{jt-1}) = \mathbb{E}\left[\omega_{jt} \mid \omega_{jt-1}\right]$ and ξ_{jt} is an unanticipated productivity "innovation" such that $\mathbb{E}\left[\xi_{jt} \mid \mathcal{I}_{jt-1}\right] = 0$.

[14] The production function estimates are provided by Lenzu et al. (2025*b*). The approach builds on the two-stage estimation routine in Gandhi, Navarro and Rivers (2020), augmented to control for differences in output quality and market power in the product market.

The estimation routine for TFPQ consists of two steps.

The first step of the estimation strategy is based on a transformation of the firm's first-order condition for intermediate inputs, which relates observed input shares for intermediate inputs (s^m_{ft}) to the elasticity of output for intermediate inputs and firm-level markup chosen by the firm:

$$s^m_{ft} = \left(\frac{\partial}{\partial m_{ft}} f(k_{ft}, l_{ft}, m_{ft}; \cdot)\right) - \mu_{ft} - z_{ft}, \tag{OA.4}$$

where, as in our theoretical framework, the desired log-markup μ_{ft} is given by a Lerner index that depends on the elasticity of demand at the optimum, $\epsilon_{ft} = \left(\frac{\partial Y_{ft}}{\partial P_{ft}} \frac{P_{ft}}{Y_{ft}}\right)$.

As in our theoretical model, we assume an invertible demand system that generates the following residual demand function for goods produced by firm f:

$$\mathcal{D}_{ft} := d(P_{ft}, P_{it}, \varphi_{ft}) Y_{it},$$

which implies that quantity demanded depends on the firm's price P_{ft}, a firm-specific demand shock φ_{ft} (assumed to be multiplicative), and a vector of industry-time specific shifters α_{it} capturing industry expenditures. Using this result, we can approximate ϵ_{ft} as a function of the firm's price and industry and time shifters (p_{ft}, α_{it}) and re-write Equation (OA.4) as:

$$s^m_{ft} = \ln\left(\frac{\partial}{\partial m_{ft}} f(k_{ft}, l_{ft}, m_{ft}; \cdot)\right) - \mu(p_{ft}, \alpha_{it}) - z_{ft} \tag{OA.5}$$

$$= n\left(k_{ft}, l_{ft}, m_{ft}, p_{ft}, \alpha_{it}\right) - z_{ft}, \tag{OA.6}$$

Thus, following Blum et al. (2024), we recover the ex-post productivity shock, $\widehat{z}_{ft}$, and a term combining the output elasticity of intermediate inputs and the desired markup by estimating a regression of intermediate input shares on a polynomial in $(k_{ft}, l_{ft}, m_{ft}, p_{ft})$ and fixed effects. Note that, as in Ackerberg, Caves and Frazer (2015), the output elasticity of intermediate inputs is recovered in the second stage, along with the rest of the production function.

The second step of the estimation procedure recovers the production function and productivity. Define firm output net of the ex-post productivity shock as $\mathcal{Y}_{ft} \equiv y_{ft} - z_{ft} = f(k_{ft}, l_{ft}, m_{ft}; \cdot) + \omega_{ft}$. We recover the empirical counterpart of this as a difference between the firm's output and the $\hat{z}_{ft}$ recovered from the step of the estimation procedure in (OA.6).

Exploiting the Markovian property of ω_{ft}, we can re-write $\mathcal{Y}_{ft}$ as:

$$\mathcal{Y}_{ft} = \underbrace{h\left(\mathcal{Y}_{ft-1} - f(k_{ft-1}, l_{ft-1}, m_{ft-1}; \cdot)\right)}_{h(\omega_{ft-1})} + f(k_{ft}, l_{ft}, m_{ft}; \cdot) + \xi_{ft}.$$

where $h(\omega_{ft-1})$ is a control function that controls for the persistency of productivity. As is standard (Ackerberg, Caves and Frazer (2015); Gandhi, Navarro and Rivers (2020)), we model the control function as polynomial in inputs, $h(\omega_{ft-1}) = \sum_{0<a\leq 3}\psi_a \omega_{ft-1}^a$, and construct the following recursive equation:

$$\mathcal{Y}_{ft} = f(k_{ft}, l_{ft}, m_{ft}; \cdot) + \sum_{0<a\leq 3} \psi_a \left(\mathcal{Y}_{ft-1}(\boldsymbol{\psi}, \boldsymbol{\vartheta}) - f(k_{ft-1}, l_{ft-1}, m_{ft-1}; \cdot)\right)^a .$$

We need to address possible bias arising from the fact that firm-level output is measured in quantity but, as we explain below, firm-level inputs are measured as expenditures deflated by their respective industry price indices.[15] To do so, we follow the approach in De Loecker et al. (2016) and augment the regression equation with a second control function of (output) prices and market shares to correct for the bias to obtain the following estimating equation:

(OA.7)

$$\begin{aligned}\mathcal{Y}_{ft} =& f(k_{ft}, l_{ft}, m_{ft}; \cdot) - cf(p_{ft}, s_{it}; \boldsymbol{\phi}) + \\ & \sum_{0<a\leq 3} \psi_a \left(\mathcal{Y}_{ft-1}(\boldsymbol{\psi}, \boldsymbol{\vartheta}) - f(k_{ft-1}, l_{ft-1}, m_{ft-1}; \cdot) - cf(p_{ft-1}, s_{it-1}; \boldsymbol{\phi})\right)^a ,\end{aligned}$$

which depends on the production parameter vector $\boldsymbol{\vartheta}$–the object of interest—and the ancillary parameter vectors $\boldsymbol{\psi}$ and $\boldsymbol{\phi}$.

We estimate Equation (OA.7) via GMM. As is standard, we assume that capital is pre-determined at the beginning of each period t, while labor and intermediate inputs are flexibly chosen period-by-period. Under these assumptions $(k_{ft}, l_{jt}, \mathcal{Y}_{ft-1})$ are orthogonal to ξ_{ft} and they can be used as instruments for themselves. However, since l_{ft}, m_{ft}, p_{ft} and s_{ft} are chosen in period t, they are correlated with ξ_{ft}. We therefore use their one-year lag l_{ft-1}, m_{ft-1}, p_{ft-1} and s_{ft-1} as instruments.

The following set of moment conditions identifies parameters $(\boldsymbol{\vartheta}, \boldsymbol{\psi}, \boldsymbol{\phi})$:

$$\mathbb{E}[\xi_{ft} \cdot k_{ft}^{\tau_k} l_{ft-1}^{\tau_l} m_{ft-1}^{\tau_m}] = 0 \qquad \mathbb{E}[\xi_{ft} \cdot \mathcal{Y}_{ft-1}^a] = 0 \qquad \mathbb{E}[\xi_{ft} \cdot p_{ft}^{\tau_p} s_{ft}^{\tau_s}] = 0$$

where $0 \leq \tau_k + \tau_l + \tau_m \leq 2$, $0 \leq \tau_p + \tau_s \leq 2$, and $\tau_k, \tau_l, \tau_m, \tau_p, \tau_s \geq 0$.

Measurement and implementation—We perform the production function estimation using yearly data. To construct our baseline TFPQ instrument, we assume firms' technologies can be approximated by a Cobb-Douglas production function, allowing output elasticities to vary between sectors (2-digit NACE):

(OA.8) $$TFPQ_{ft} = y_{ft} - \widehat{\vartheta}_l \cdot l_{ft} - \widehat{\vartheta}_k \cdot k_{ft} - \widehat{\vartheta}_m \cdot m_{ft}.$$

[15]See De Loecker et al. (2016) for a discussion of the sources and implications of this bias.

TABLE OA.2—ESTIMATES OF OUTPUT ELASTICITIES AND RETURNS TO SCALE

Sector	Output elasticities			Returns to scale	
	Labor (ϑ_l)	Intermediates (ϑ_m)	Capital (ϑ_k)	Short-run ($\vartheta_l + \vartheta_m$)	Long-run ($\vartheta_l + \vartheta_m + \vartheta_k$)
CA	0.214	0.026	0.752	0.966	0.992
CB	0.200	0.028	0.764	0.964	0.992
CC	0.230	0.033	0.731	0.960	0.994
CE	0.217	0.039	0.720	0.937	0.976
CG	0.219	0.008	0.768	0.987	0.996
CH	0.188	0.028	0.776	0.963	0.991
CJ	0.177	0.036	0.783	0.960	0.996
CK	0.247	0.054	0.684	0.932	0.986
CL	0.243	0.036	0.725	0.968	1.004
Aggregate	0.212	0.026	0.753	0.965	0.992

Note: This table reports the production function estimates for different sectors. The first three columns report the estimated output elasticities of labor, intermediate inputs, and capital. The last two columns report the short-run and long-run returns to scale. Each row refers to a different manufacturing sector. The last row is the average of the sectoral estimates.

where $Y_{ft} = \frac{(PY)_{ft}}{\bar{P}_{ft}}$ is the firm-level quantity index constructed by deflating firm-level sales by the firm-level price index. We measure labor services using the total quarterly wage bill from the Social Security dataset and intermediate costs using the quarterly expenses in materials and services reported in firms' VAT declarations. We construct a quarterly measure of capital services following the perpetual inventory method. To do so, we use data on the book value of fixed assets from the firms' Annual Accounts to initialize the capital series and data on quarterly investments in fixed tangible assets from firms' VAT declarations to recursively update the series.[16] We deflate labor, capital, and intermediate inputs using the corresponding industry-level deflators from National Bank of Belgium (2022*d*).

Table OA.2 presents the estimates of the output elasticities and returns to scale for individual manufacturing sectors and for the aggregate economy. The latter is the average of the sectoral estimates. SR-RTS are the sum of the elasticities of variable inputs (labor and intermediates). Consistent with the findings in previous studies, our estimates indicate returns to scale in the ballpark of unity for most sectors and, therefore, in the aggregate.[17]

Alternative production functions and variable SR-RTS—We construct an alternative productivity index by modeling firms' technologies with a Translog produc-

[16]The Annual Accounts data are sources from National Bank of Belgium (2022*a*). Depreciation rates are sourced from Lenzu and Manaresi (2018).

[17]See Gandhi, Navarro and Rivers (2020) and Lenzu et al. (2025*a*) and references therein.

tion function:

$$TFPQ_{ft}^{Translog} = y_{ft} - \sum_{x=l,k,m} \widehat{a}_x \cdot x_{ft} - \sum_{x=l,k,m} \sum_{x'=l,k,m} \widehat{a}_{xx'}/2 \cdot x_{ft} x'_{ft}.$$

As in our baseline estimation, we estimate the production function parameters $\widehat{a}$s separately for each sector and recover firm-time specific input elasticities, $\widehat{\vartheta}_{x,ft} = \widehat{a}_x + \widehat{a}_{xx'}/2 \cdot x'_{ft} + \widehat{a}_{xx'} \cdot x_{ft}$ of capital, labor, and intermediate inputs.

Using these, we obtain an estimate of firm-time specific short-run returns to scale, $\widehat{\vartheta}_{l,ft}+\widehat{\vartheta}_{m,ft}$, and construct a measure of marginal cost that directly accounts for this variation, $MC_{ft}^n = AVC_{ft}^{(1+\widehat{\nu}_{ft})}$, where $\widehat{\nu}_{ft} = (\widehat{\vartheta}_{l,ft} + \widehat{\vartheta}_{m,ft})^{-1} - 1$. These variables are used to perform the robustness exercises reported in Table 3.

OA.3. Adjustment of TFPQ for variable capacity utilization

As a robustness exercise for our marginal cost instrument, we construct two "purified" TFPQ indices that account for variability in capacity utilization. Toward this purpose, we gather supplementary data on capacity utilization from the Business Survey administered by the National Bank of Belgium (National Bank of Belgium (2022*b*)). The survey covers a subset of our data, asking firms to report the percentage of their production capacity utilized in any given quarter. After matching the Business Survey to our final regression sample, we are able to gather information on capacity utilization for 485 firms and 18,422 firm-industry-quarter observations.

Construction of TFPQ adjusted for capacity utilization—Because information on capacity utilization is available for only a small subset of our dataset, we rely on predictive regressions to estimate firm-level capacity when it is not directly observable. First, for firms where capacity utilization data is available for some periods, missing values are imputed using the capital-to-labor ratio, intermediates-to-labor ratio, intermediates-to-sales ratio, current and past investments in physical assets, percentage change in the firm's sales, percentage change in the firm's wage bill, firm fixed effects, and industry-by-time fixed effects:

$$\begin{aligned}\widehat{CU}_{ft} = {} & \underbrace{\beta_1}_{-0.042} (K_{ft}/WL_{ft}) + \underbrace{\beta_2}_{0.091} (PM_{ft}/WL_{ft}) + \underbrace{\beta_3}_{-0.045} (PM_{ft}/PY_{ft}) + \\ & \underbrace{\beta_4}_{0.004} (\ln Invest_{ft}) + \underbrace{\beta_5}_{0.006} (\ln Invest_{ft-1}) + \underbrace{\beta_6}_{0.006} (\Delta \ln PY_{ft}) + \\ & \underbrace{\beta_7}_{0.059} (\Delta \ln WL_{it}) + \iota_f + \iota_{it}.\end{aligned}$$

For firms that never appear in the Business Survey sample, we impute capacity utilization using information on the firm's capital-to-labor ratio, intermediates-to-labor ratio, intermediates-to-sales ratio, current and past investments in physical assets, percentage change in industry sales, percentage change in the firm's sales, percentage change in the firm's wage bill, a second-order polynomial in the firm's age, and industry-by-calendar quarter fixed effects:[18]

$$\begin{aligned}\widehat{CU}_{ft} = {} & \underbrace{\beta_1}_{-0.018}(K_{ft}/WL_{ft}) + \underbrace{\beta_2}_{0.050}(PM_{ft}/WL_{ft}) + \underbrace{\beta_3}_{-0.043}(PM_{ft}/PY_{ft}) + \\ & \underbrace{\beta_4}_{0.007}(\ln Invest_{ft}) + \underbrace{\beta_5}_{0.006}(\ln Invest_{ft-1}) + \underbrace{\beta_6}_{0.010}(\Delta \ln PY_{it}) + \\ & \underbrace{\beta_7}_{0.018}(\Delta \ln PQ_{ft}) + \underbrace{\beta_8}_{0.065}(\Delta \ln WL_{ft}) + \underbrace{\beta_9}_{0.001}(age_{ft}) + \underbrace{\beta_{10}}_{-0.001}\left(age^2_{ft}\right) + \iota_{qi}.\end{aligned}$$

We then construct two "purified" TFPQ instruments by adjusting either capital or both capital and labor for capacity utilization. The capital- and capital-labor adjusted TFPQ measures are constructed as in Equation (OA.8) but scaling the measure of capital and labor in the production function as follows: $k^{CU}_{ft} = \ln(\widehat{CU}_{ft} \cdot K_{ft})$ and $l^{CU}_{ft} = \ln(\widehat{CU}_{ft} \cdot L_{ft})$.

Falsification tests using money and oil shocks—We perform the following falsification test to assess the sensitivity of our purified technical productivity measure (adjusting both capital and labor for capacity utilization).

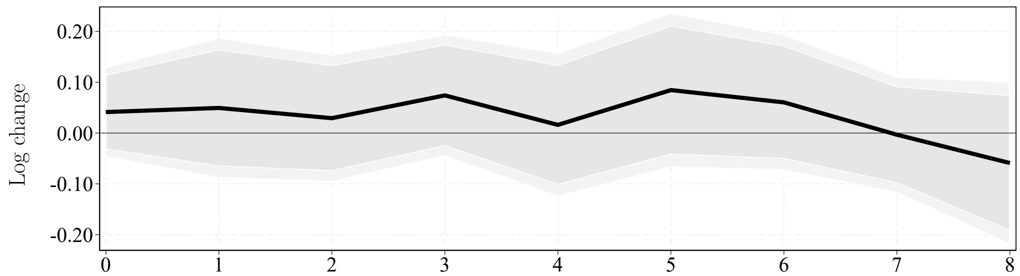
We aggregate our firm-level measure and check whether this variable responds to high-frequency monetary shocks and oil shocks. The idea is that these high-frequency demand and supply shocks may trigger adjustments in capacity utilization, but—to the extent that our firm-level productivity index has been adjusted for this variation—the aggregate "purified" TFPQ index should reflect technical productivity, which is a slow-moving variable. To test this, we estimate the following local linear projection models:

$$\begin{aligned}TFPQ_{t+h} - TFPQ_{t-1} &= b^{OS}_0 + b^{OS}_h OS_{t-1} + \epsilon_{ft+h}\\ TFPQ_{t+h} - TFPQ_{t-1} &= b^{MS}_0 + b^{MS}_h MS_{t-1} + \epsilon_{ft+h}\end{aligned}$$

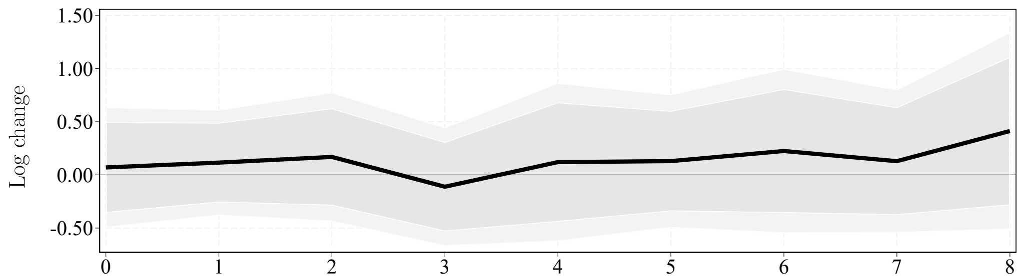
for $h = 0, ..., 8$ quarters. As in the paper, oil shocks (OS) are measured as in Känzig (2021) (unexpected movements in oil price futures the day after OPEC meetings) and money shocks (MS) are constructed as in Gürkaynak, Sack and Swanson (2005) (log change in the price of overnight index swaps within a narrow window around ECB monetary policy announcements).

Figure OA.1 plots the empirical IRF coefficients over different horizons. In Panel A, we plot the response to oil shocks. We normalize the oil shock so that b^{OS}_1 represents the effect on impact of a one-standard deviation shock to oil prices (a 15.7 percent increase in Brent crude oil price). In Panel B, we plot the response

[18]The firm age is from FPS Economy (2022).



(a) Response of aggregated TFP to oil shocks



(b) Response of aggregated TFP to money shocks

FIGURE OA.1. DYNAMIC EFFECTS OF OIL AND MONEY SHOCKS

Note: The dark (light) gray shaded areas are 90 (95) percent confidence bands obtained from Newey-West standard errors with four quarters of correlation.

to monetary shocks. The coefficient represents the effect of an annualized shock of 100 basis points. Reassuringly, both shocks have no explanatory power on our aggregate "purified" TFPQ measure at any horizon.

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