Online Appendix of the paper

Exploiting Rivals' Strengths

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C Results

The quadratic example with linear prices. In the quadratic model (9), the demand functions are:

$$q_1 = \frac{(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2}, \quad q_2 = \frac{(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}. \tag{C1}$$

Remember that the marginal costs are $c_1 = 0$ and $c_2 = c$.

Let us begin with the case in which the dominant firm sets a simple linear price without referencing the rival's output, and acts as the price leader. Given p_1, firm 2 has two options. It may respond with its own linear price, in which case its best response is:

$$p_2 = \frac{1}{2}\left[1 + c - \gamma(1-p_1)\right], \tag{C2}$$

or it may offer an exclusive-dealing contract at a price slightly below p_1.

Anticipating firm 2's response, the dominant firm must ensure that the exclusive-dealing option is never profitable for its rival. To do so, it sets a price equal to the lower of two values: the unconstrained profit-maximizing price:

$$p_1 = \frac{1}{2} - \frac{\gamma(1-c)}{2\left(2-\gamma^2\right)} \tag{C3}$$

and the minimum price that deters firm 2 from deviating to exclusive dealing, given by:

$$\text{(C4)} \qquad p_1 = \frac{2(1-c)\sqrt{\gamma}(1-\gamma^2)}{\sqrt{1+\gamma}}$$

.

The unconstrained solution applies when $c \geq \overline{c}(\gamma)$, where $\overline{c}(\gamma)$ is the value of c for which the two prices above coincide. The constrained solution applies when this inequality is reversed. The resulting equilibrium profit for the dominant firm is:

$$\Pi_1^{NR} = \begin{cases} \dfrac{\left[2-(1-c)\gamma-\gamma^2\right]^2}{8(2-3\gamma^2+\gamma^4)} & \text{if } c \geq \overline{c}(\gamma) \\ \frac{(1-c)\left[\gamma^{5/2}-2\sqrt{\gamma}+\Gamma(\gamma^2+\gamma-2)\right]\left[(2c+1)\Gamma\gamma^2-2(c+1)\Gamma-(1-c)(2\gamma^{5/2}-\Gamma\gamma-2\sqrt{\gamma})\right]}{\left[\Gamma(4-3\gamma^2)\right]^2} & \text{if } \quad c \leq \overline{c}(\gamma) \end{cases}$$

where $\Gamma = \sqrt{\gamma+1}$.

When firms compete for exclusivity, firm 2 will price at cost, setting $p_2 = c$. This yields a reservation payoff for the buyer of $S^R = \frac{(1-c)^2}{2}$. The dominant firm then maximizes its profit under the buyer's participation constraint, $S \geq S^R$. The constraint may be binding or slack. (It may be slack because, with linear prices, even a monopolist must leave some rent to the buyer). When it is slack, the dominant firm can charge the monopoly price, $p^M = \frac{1}{2}$, and obtain the monopoly profit, $\Pi^M = \frac{1}{4}$. When instead the constraint is binding, the dominant firm must slightly undercut the rival to meet the constraint, pricing at $p_1 = c$ (minus a small discount). The former case applies when $c \geq \frac{1}{2}$, the latter when $c \leq \frac{1}{2}$. As a result, under exclusive dealing, the dominant firm obtains:

$$\text{(C5)} \qquad \Pi_1^E = \begin{cases} c(1-c) & \text{if } c \leq \frac{1}{2} \\ \\ \frac{1}{4} & \text{if } c \geq \frac{1}{2} \end{cases}$$

Finally, consider the Ramsey-Boiteux profit, which is what the dominant firm can obtain with MSR contracts. Like in the case of exclusive dealing, the buyer's participation constraint $S \geq S^R$ may be binding or slack. The optimization problem is then similar to that arising under exclusive dealing, with the difference that there is one extra degree of freedom as the multi-product monopolist sets two prices rather than one. Since the formulas for the equilibrium prices are not particularly informative, here we report only the Ramsey-Boiteux profit:

$$\Pi^{\mathrm{RB}}(S^R) = \begin{cases} (1-c)\left[1-c+\sqrt{\dfrac{2(1-c)(1-\gamma)+c}{1-\gamma^2}}\right] & \text{if } c \leq \dfrac{3+\gamma(1-4\gamma)+\sqrt{3(1-\gamma^2)}}{3-4\gamma^2} \\ \\ \dfrac{2(1-c)(1-\gamma)+c}{4\left(1-\gamma^2\right)} & \text{if } c \geq \dfrac{3+\gamma(1-4\gamma)+\sqrt{3(1-\gamma^2)}}{3-4\gamma^2} \end{cases}$$

The moral hazard model. We first derive the equilibrium prices under exclusive dealing. When the buyer can purchase only product i, the demand for it is given by $q_i^E = 1+\theta-p_i$, and the firm's expected profit is:

$$\Pi_i^E = (p_i - c_i)(1-p_i) + F_i. \tag{C6}$$

The corresponding expected buyer surplus is $E\left[V(U(q_i^E,0) - p_i q_i^E - F_i)\right]$. Expressing the surplus in terms of a certainty equivalent, defined as the solution to:

$$V(S^{CE}) = E\left[V(U(q_i^E,0) - p_i q_i^E - F_i)\right], \tag{C7}$$

it is easy to verify that:

$$S_i^{CE}(p_i, F_i) = \frac{1}{2\eta}\log\left(1+\eta\sigma^2\right) + \frac{(1-p_i)^2}{2(1+\eta\sigma^2)} - F_i. \tag{C8}$$

As a first step in deriving the equilibrium, consider the profit-maximization problem of firm i, under the constraint that the buyer receives at least a payoff of S. It is easy to see that the optimal marginal price is:

$$p_i^E = c_i + \frac{(1-c_i)\eta\sigma^2}{1+2\eta\sigma^2}, \tag{C9}$$

and the corresponding fixed fee is:

$$F_i^E(S) = \frac{(1-c_i)^2\left(1+\eta\sigma^2\right)}{\left(1+2\eta\sigma^2\right)^2} + \frac{\log\left(1+\eta\sigma^2\right)}{2\eta} - S. \tag{C10}$$

As soon as $\sigma^2 > 0$ and $\eta > 0$, the marginal price is distorted upward. The extent of the distortion is independent of the buyer's payoff S; any change in S is fully absorbed by an equal and opposite adjustment in the fixed fee, leaving the marginal price unchanged.

Next, consider the equilibrium behavior of the two firms. As usual, firm 2—being foreclosed—must maximize the buyer's surplus subject to a break-even constraint. This is the dual of the profit-maximization problem described above, so the marginal price remains the same. By setting the fixed fee such that $\Pi_2^E = 0$, firm 2 can ensure the buyer a certainty equivalent surplus of:

$$S_{CE}^R = \frac{(1-c)^2\left(1+\eta\sigma^2\right)}{2(1+2\eta\sigma^2)} + \frac{1}{2\eta}\log\left(1+\eta\sigma^2\right). \tag{C11}$$

Being more efficient, firm 1 can match this payoff while still earning a positive profit. The equilibrium two-part tariff is given by the above formulas with $S = S_{CE}^R$. Simple algebra then yields:

$$\Pi_1^E = \frac{c(2-c)\left(1+\eta\sigma^2\right)}{2(1+2\eta\sigma^2)}. \tag{C12}$$

Next, let us analyze the Ramsey-Boiteux pricing structure. The expression for the certainty equivalent surplus now becomes:

$$S(p_1,p_2,F)=\frac{1}{2\eta}\log\left(\frac{2\eta\sigma^2}{\gamma+1}+1\right)+\frac{(p_1+p_2-2)^2}{4(1+\gamma+2\eta\sigma^2)}+\frac{(p_1-p_2)^2}{4(1-\gamma)}-F. \tag{C13}$$

The Ramsey-Boiteux solution maximizes $p_1q_1+(p_2-c)q_2+F$, where the demand functions are:

$$q_1=\frac{(1-\gamma)(1+\theta)-p_1+\gamma p_2}{1-\gamma^2} \quad \text{and} \quad q_2=\frac{(1-\gamma)(1+\theta)-p_2+\gamma p_1}{1-\gamma^2}, \tag{C14}$$

under the buyer's participation constraint. Again, the profit-maximizing marginal prices:

$$p_1^{\mathrm{RB}}=\sigma^2\frac{(2-c)\eta}{1+\gamma+4\eta\sigma^2} \quad \text{and} \quad p_2^{\mathrm{RB}}=c+\sigma^2\frac{(2-c)\eta}{1+\gamma+4\eta\sigma^2}, \tag{C15}$$

are independent of the buyer's reservation payoff. The fixed fee is then determined by the buyer's participation constraint and is:

$$\begin{aligned}F^{\mathrm{RB}}=\frac{1}{4}\Bigg[&\frac{c^2}{1-\gamma}+\frac{(2-c)^2\left(1+\gamma+2\eta\sigma^2\right)}{\left(\gamma+4\eta\sigma^2+1\right)^2}+\\&-\frac{2(1-c)^2\left(\eta\sigma^2+1\right)}{2\eta\sigma^2+1}+\frac{2\log\left(\eta\sigma^2+1\right)}{\eta}+\frac{\log\left(\frac{2\eta\sigma^2}{\gamma+1}+1\right)}{\eta}\Bigg].\end{aligned} \tag{C16}$$

Consequently, the Ramsey-Boiteux profit is:

$$\Pi^{\mathrm{RB}}(S^R_{CE})=\frac{c^2}{4(1-\gamma)}+\frac{(2-c)^2\left(\gamma+2\eta\sigma^2+1\right)}{4(\gamma+1)\left(\gamma+4\eta\sigma^2+1\right)}-\frac{(1-c)^2\left(\eta\sigma^2+1\right)}{4\eta\sigma^2+2}+\frac{\log\left[\frac{\gamma+2\eta\sigma^2+1}{(\gamma+1)(\eta\sigma^2+1)}\right]}{2\eta}.$$

Before calculating the profit that the dominant firm can achieve without referencing the rival's output, let us complement the informal discussion in the main text regarding why the dominant firm cannot attain the Ramsey-Boiteux profit without referencing the rival's volume, by providing more rigorous arguments. First, observe that the demand functions (C14) are bijective. Therefore, in order to replicate the Ramsey-Boiteux outcome, marginal prices must be set at the Ramsey-Boiteux levels. However, these price levels cannot be sustained in a non-cooperative equilibrium.

To understand why, consider any two-part tariff $p_1 q_1 + F_1$ that the dominant firm may offer when it cannot reference the rival. Given this pricing schedule, the buyer will choose to purchase from firm 2 only if doing so provides at least the same payoff as accepting firm 1's offer alone. Accordingly, for any given p_2, firm 2 will set the fixed fee F_2 so that the buyer is indifferent between purchasing from both firms or exclusively from firm 1. (As discussed later, firm 2 may consider offering an exclusive-dealing contract. However, it is never optimal for the dominant firm to provoke such a response from its rival, so we can safely disregard this possibility for now—it will be addressed in detail below.) Using certainty equivalents, this indifference condition can be written as:

$$\frac{\log\left(1+\eta\sigma^2\right)}{\eta} + \frac{(1-p_1)^2}{2(1+\eta\sigma^2)} = \frac{\log\left(\frac{1+2\eta\sigma^2}{\gamma+1}\right)}{2\eta} + \frac{(2-p_1-p_2)^2}{4(1+\gamma+2\eta\sigma^2)} + \frac{(p_1-p_2)^2}{4(1-\gamma)} - F_2.$$

Solving for F_2, substituting into the profit function, and optimizing with respect to p_2 yields:

$$p_2 = \frac{c\left(\gamma+2\eta\sigma^2+1\right) - (1-\gamma)\eta(p_1-2)\sigma^2}{(3-\gamma)\eta\sigma^2+\gamma+1},$$

$$\text{(C17)} \qquad F_2 = \frac{\log\left(\frac{2\eta}{\gamma+1}+\frac{1}{\sigma^2}\right) - \log\left(\eta+\frac{1}{\sigma^2}\right)}{2\eta} + \\ - \frac{\left(\gamma+2\eta\sigma^2+1\right)\left(\eta\sigma^2(c-\gamma+(\gamma-2)p_1+1)+c+\gamma-\gamma p_1-1\right)^2}{2(\gamma-1)\left(\eta\sigma^2+1\right)\left(-(\gamma-3)\eta\sigma^2+\gamma+1\right)^2}.$$

These expressions represent firm 2's best response to any possible strategy of the dominant firm. Now, to re-produce the Ramsey-Boiteux marginal prices, firm 2's best response should yield $p_2 = p_2^{\mathrm{RB}}$ when $p_1 = p_1^{\mathrm{RB}}$. However, it is easy to verify that firm 2's best response to $p_1 = p_1^{\mathrm{RB}}$ is:

$$p_2 = p_2^{\mathrm{RB}} + \sigma^2 \frac{(2-c)\gamma\eta\,(1+\gamma+2\eta\sigma^2)}{(1+\gamma+4\eta\sigma^2)\,[(3-\gamma)\eta\sigma^2+\gamma+1]} \tag{C18}$$

As soon as $\sigma^2 > 0$, this is different from p_2^{RB}. In other words, the Ramsey-Boiteux prices cannot represent an equilibrium of the pricing game between the two firms when firms cannot use contracts that reference rivals' volumes. This implies that the dominant firm must obtain strictly less than the Ramsey-Boiteux profit as soon as $\sigma^2 > 0$.

Finally, let us compute the equilibrium profits when the dominant firm cannot reference the rival's output and is restricted to offering a simple two-part tariff of the form $p_1 q_1 + F_1$. Since firm 1 acts as the price leader and firm 2 as the follower, we solve for the equilibrium using backward induction.

Given firm 1's tariff, firm 2 has two options: it can either offer a standard (unconditional) two-part tariff or propose an exclusive-dealing contract. If it chooses the former, its best response has already been derived above. Denote the profit associated with this response as Π_2^{CR}, where CR stands for common representation. If, instead, firm 2 opts for an exclusive-dealing contract, its two-part tariff would be:

$$\text{(C19)} \qquad \begin{aligned} p_2^E &= \frac{c+(1-c)\eta\sigma^2}{1+2\eta\sigma^2}, \\ F_2^E &= F_1 + \frac{1}{4}\left[\frac{(1-c)^2}{(1+2\eta\sigma^2)^2} + \frac{(1-c)^2}{1+2\eta\sigma^2} - \frac{2(1-p_1)^2}{1+\eta\sigma^2}\right] \end{aligned}$$

resulting in a profit of Π_2^E.

The dominant firm must then set its fixed fee, F_1, high enough to deter a potential deviation by firm 2 to an exclusive-dealing contract. In other words, F_1 must satisfy the condition $\Pi_2^{CR} \geq \Pi_2^E$. Solving this constraint for F_1, substituting the result into firm 1's profit function, and optimizing yields the dominant firm's optimal two-part tariff:

$$p_1 = \frac{(2-c)\left(1-\gamma^2\right)\eta\sigma^2}{2\left(2-\gamma^2\right)\eta\sigma^2+\gamma+1},$$

$$F_1 = \frac{1}{8}\left\{-4\frac{\log\left[\eta+\frac{1}{\sigma^2}-\log\left(\frac{2\eta}{\gamma+1}+\frac{1}{\sigma^2}\right)\right]}{\eta}+\right.$$

$$\left.+\frac{(c+\kappa-2)^2\left[2(\gamma-1)\eta\sigma^2-2(\gamma+1)\right]}{(\gamma+1)\left[(\gamma-3)\eta\sigma^2-\gamma-1\right]}+\frac{2(c-\kappa)^2}{1-\gamma}\right\}-\frac{(c-1)^2\left(\eta\sigma^2+1\right)}{4\eta\sigma^2+2},$$

where

$$\kappa = \frac{(2-c)\left(1-\gamma^2\right)\eta\sigma^2}{1+\gamma-2\left(\gamma^2-2\right)\eta\sigma^2}.$$

Given these equilibrium tariffs, firm 1's profit can be readily calculated. Specifically, it can be expressed as:

$$\Pi_1 = \Pi^{\mathrm{RB}}(S^R_{CE}) - \sigma^4 \frac{c^2\gamma^2\eta^3\left(1+\gamma+2\eta\sigma^2\right)}{\left(1+\gamma+4\eta\sigma^2\right)\left[1+\gamma+(3-\gamma)\eta\sigma^2\right]\left[1+\gamma+2\left(2-\gamma^2\right)\eta\sigma^2\right]}.$$

This expression clarifies that MSR contracts are strictly profitable as soon as $\sigma^2 > 0$.

The adverse selection model. Let us begin by considering the Ramsey-Boiteux solution, which corresponds to the problem of maximizing the profit of a multi-product monopolist, subject to the constraint that the buyer receives a reservation payoff of $S^R(\theta)$. For analytical convenience, however, we initially consider a more general version of the problem, in which the reservation payoff is given by an arbitrary function $\widetilde{S}(\theta)$.

Applying the Revelation Principle, the monopolistic screening problem can be formulated as the choice of a direct mechanism $\{q_1(\theta), q_2(\theta)\}$, subject to the relevant incentive compatibility and participation constraints. By standard arguments, this problem can be recast as an optimal control problem, with $q_1(\theta)$ and $q_2(\theta)$ as control variables and the buyer's surplus $S(\theta)$ as the state variable:

$$
\text{(C20)} \qquad \begin{aligned}
\max_{\{q_1(\theta),q_2(\theta)\}} \Pi\left[q_1(\theta), q_2(\theta)\right] &= \int\limits_{\Delta}^{\Delta} \left[U(q_1(\theta), q_2(\theta), \theta) - cq_2(\theta) - S(\theta)\right] f(\theta) d\theta \\
\text{s.t.} \qquad \frac{dS(\theta)}{d\theta} &= q_1(\theta) + q_2(\theta) \quad \text{(IC)} \\
S(\theta) &\geq \widetilde{S}(\theta) \quad \text{(IR)}
\end{aligned}
$$

where $F(\theta) = \frac{\theta+\Delta}{2\Delta}$ is the cumulative distribution function of the uniform distribution, and $f(\theta) = \frac{1}{2\Delta}$ is the corresponding density.

An additional constraint is the monotonicity condition:

$$
\text{(C21)} \qquad \sum_{i=1,2} U_{\theta q_i} \frac{dq_i(\theta)}{d\theta} = \frac{dq_1(\theta)}{d\theta} + \frac{dq_2(\theta)}{d\theta} \geq 0,
$$

which ensures that the allocation $\{q_1(\theta), q_2(\theta)\}$ is implementable. As is standard in the literature, we initially omit this constraint and verify ex-post that the resulting solution satisfies it.

Assume that the participation constraint (IR) in (C20) binds only for the lowest type $\theta = -\Delta$. ((It is straightforward to verify that this condition holds when $\widetilde{S}(\theta) = S^R(\theta)$ and Δ is sufficiently small, as ensured by condition (12) in the main text). Under this assumption, the problem reduces to the pointwise maximization of the virtual surplus function—i.e., the Hamiltonian associated with problem (C20)):

$$
\text{(C22)} \qquad \begin{aligned}
H(q_1, q_2, \theta) &= U(q_1, q_2, \theta) - cq_2 - \frac{1 - F(\theta)}{f(\theta)} U_\theta(q_1, q_2, \theta) \\
&= (1 + 2\theta - \Delta)(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2\right) - \gamma q_1 q_2 - cq_2.
\end{aligned}
$$

The first-order conditions are:

$$(C23) \qquad (1+2\theta-\Delta) - q_1 - \gamma q_2 = 0$$
$$(1+2\theta-\Delta) - q_2 - \gamma q_1 - c = 0.$$

Therefore, considering the concavity of the Hamiltonian, the optimal quantities are:

$$(C24) \qquad q_1^{\mathrm{RB}}(\theta) = \frac{(1+2\theta-\Delta)(1-\gamma)+\gamma c}{1-\gamma^2}$$

$$(C25) \qquad q_2^{\mathrm{RB}}(\theta) = \frac{(1+2\theta-\Delta)(1-\gamma)-c}{1-\gamma^2}.$$

Condition (12) in the main text ensures that the optimal quantities of both products are positive for any θ. Furthermore, it is immediate to verify that both quantities are increasing in θ, so the monotonicity condition (C21) is satisfied.

The price schedule that supports these quantity levels is given by:

$$P^{\mathrm{RB}}(q_1,q_2) = \frac{1+\Delta(1-\gamma)-(1-c)\gamma}{2(1-\gamma)}(q_1+q_2) - \frac{c(1-3\gamma)}{2(1-\gamma)}q_2 + \\ - \frac{1}{4}(1+\gamma)(q_1^2+q_2^2) - \widetilde{S}(-\Delta).$$

Note that the reservation payoff affects only the fixed component of the price schedule—namely, the fixed fee—and does not influence marginal prices. As a result, a one-dollar increase in the reservation payoff of the lowest type leads to a one-dollar reduction in the Ramsey-Boiteux profit:

$$\Pi^{\mathrm{RB}}(\widetilde{S}(\theta)) = \frac{2(1-\gamma)\left[3c(2\gamma+1)(1-\Delta)-3\gamma(1-\Delta)^2+4\Delta^2\right]-3\left(1-2\gamma^2\right)c^2}{6\left(1-\gamma^2\right)} + \\ - \widetilde{S}(-\Delta).$$

This property also holds when the profit-maximization problem is further constrained by the condition $q_1(\theta)=0$—which corresponds to the maximum profit firm

2 could obtain by offering an exclusive-dealing contract. Consequently, the same reasoning as in footnote 22 of the main text implies that the dominant firm cannot earn more than the Ramsey-Boiteux profit $\Pi^{\mathrm{RB}}(S^R(\theta))$. This profit level can be determined using the formula derived above, noting that when $\widetilde{S} = S^R(\theta)$, it follows that $\widetilde{S}(-\Delta) = \frac{(1-\Delta-c)^2}{2}$.

For completeness, it is useful to explicitly describe the MSR contracts which, according to Proposition 1, allow the dominant firm to attain a profit of $\Pi^{\mathrm{RB}}(S^R(\theta))$. First, the latent contract is an exclusive-dealing agreement that guarantees each buyer type the same payoff they would receive under the price schedule $P^{\mathrm{RB}}(q_1, q_2)$. This payoff is:

$$(\text{C26}) \quad S^{\mathrm{RB}}(\theta) = \frac{c^2 + \gamma(c + \Delta - 1)^2 + \Delta(6 - 7\Delta) + 1}{2(\gamma + 1)} - \frac{[c + 2(1 + \Delta)]\theta}{\gamma + 1} + \frac{2\theta^2}{\gamma + 1}$$

To reproduce this payoff, the latent, exclusive-dealing price schedule must be:

$$(\text{C27}) \quad \begin{aligned} P^E(q_1) = {} & \frac{4[c(1 + \Delta) + 2\Delta(4\Delta - 1)] - 3c^2 - 4\gamma(c + \Delta - 1)^2}{8(\gamma + 1)} + \\ & + \frac{1}{4}[c + 2(1 + \Delta)]q_1 + \frac{1}{8}(\gamma - 3)q_1^2. \end{aligned}$$

Second, the dominant firm must offer the MSR schedule that the buyer is expected to accept. As shown in the main text, this corresponds to equation (12), which—under the quadratic specification—takes the following form:

$$\begin{aligned} P_1(q_1) = {} & \frac{1}{8}\left[\frac{(c + 6\Delta - 2)^2}{1 + \gamma} - 4(1 - c - \Delta)^2 - \frac{2 - 3(1 - \gamma)}{(1 - \gamma)^2}c^2\right] + \\ & + \frac{(1 - \gamma)(1 + \Delta) - c[1 - \sigma_1(1 + \gamma)]}{2(1 - \gamma)\sigma_1}q_1 - \frac{(1 + \gamma)[1 + 2(\sigma_1 - 1)\sigma_1]}{4\sigma_1^2}q_1^2. \end{aligned}$$

It is easy to verify that if the dominant firm offers the above schedule, and the rival prices at cost (as it must do in equilibrium), the buyer chooses precisely the quantities (C24) and (C25).

Next, consider the case where both firms compete by offering only exclusive-dealing schedules.[33] Firm 2, which is being foreclosed, must price at cost: $P_2^E(q_2) = cq_2$, guaranteeing to the buyer the reservation payoff $S^R(\theta)$. The dominant firm must then maximize its profit under the constraint that the buyer obtains at least $S^R(\theta)$ for each possible state of demand.

The problem faced by the dominant firm can be formulated as an optimal control problem with $q_1(\theta)$ as the control variable and the buyer's surplus $S(\theta)$ as the state variable:

$$
\text{(C28)} \qquad
\begin{aligned}
&\max_{q_1(\theta)} \Pi_1 = \int_{-\Delta}^{\Delta} \left[U(q_1(\theta), 0, \theta) - S(\theta)\right] f(\theta) d\theta \\
&\text{s.t.} \quad \frac{dS(\theta)}{d\theta} = q_1(\theta) && \text{(IC)} \\
&\qquad S(\theta) \geq \tfrac{1}{2}(1 + \theta - c)^2 && \text{(IR)}
\end{aligned}
$$

Using standard techniques, it is easy to verify that when condition (12) holds, the solution to this problem is $q_1^E(\theta) = 1 + 2\theta - \Delta$. The price schedule that implements this quantity and meets the buyer's participation constraint is:

$$
\text{(C29)} \qquad P_1^E(q_1) = \frac{1}{4}\left[4(1-\Delta)c + (7\Delta - 2)\Delta - 1 - 2c^2\right] + \frac{1+\Delta}{2} q_1 - \frac{1}{4} q_1^2.
$$

Consequently, the dominant firm's exclusive-dealing profit is:

$$
\text{(C30)} \qquad \Pi_1^E = c\left(1 - \frac{c}{2}\right) + \Delta\left(\frac{2\Delta}{3} - c\right).
$$

Finally, consider the case in which the dominant firm does not reference the rival's output. To obtain a closed-form solution, we restrict the dominant firm to offering quadratic price schedules.[34]

33. Note the difference with Calzolari and Denicolo (2015), where firms simultaneously offer both exclusive and non-exclusive schedules.

34. This restriction is justified by the observation that, in all previously analyzed cases, the equilibrium price schedules are indeed quadratic. Moreover, Martimort and Stole (2009) show that under simultaneous pricing—and assuming that neither firm can reference the rival's output—there exists an equilibrium in which both firms offer quadratic schedules.

Proceeding backward, we first derive firm 2's best response to the dominant firm's strategy. Thus, let us suppose that firm 1 offers a price schedule:

$$P_1(q_1) = \alpha_{01} + \alpha_{11}q_1 + \alpha_{21}q_1^2, \tag{C31}$$

and define the buyer's indirect payoff function as:

$$\begin{aligned} v^2(q_2,\theta) &= \max_{q_1} [U(q_1,q_2,\theta) - P_1(q_1)] \\ &= \frac{(1+\theta-\alpha_{11})^2}{2+4\alpha_{21}} - \alpha_{01} + \frac{(1+\theta)(1-\gamma+2\alpha_{21})+\gamma\alpha_{11}}{1+2\alpha_{21}}q_2 - \frac{1+2\alpha_{21}-\gamma^2}{2+4\alpha_{21}}q_2^2 \end{aligned}$$

This is, effectively, the buyer's payoff function when he contracts with firm 2. Given this payoff function, firm 2 solves a standard monopolistic screening problem. The solution is obtained by point-wise maximization of the virtual surplus function:

$$H^2(q_2,\theta) = v^2(q_2,\theta) - c_2 q_2 - \frac{1-F(\theta)}{f(\theta)} v_\theta^2(q_2,\theta), \tag{C32}$$

which gives:

$$q_2(\theta) = \frac{(\alpha_{11}-1)\gamma + 2\alpha_{21}(1-c_2-\Delta+2\theta) + (2\theta-\Delta)(1-\gamma) + 1 - c_2}{1-\gamma^2+2\alpha_{21}}. \tag{C33}$$

To implement this solution, firm 2 must offer a quadratic price schedule, the coefficients of which are:

$$\begin{aligned} \hat{\alpha}_{12}(\alpha_{11},\alpha_{21}) &= \frac{(1+\Delta+c_2)(1+2\alpha_{21}) - \gamma(1+\Delta-\alpha_{11})}{2(1+2\alpha_{21})} \\ \hat{\alpha}_{22}(\alpha_{11},\alpha_{21}) &= -\frac{1+2\alpha_{21}-\gamma^2}{4(1+2\alpha_{21})}. \end{aligned}$$

To complete the derivation of firm 2's best response, it remains to determine the fixed fee. This is given by the buyer's participation constraint, which now is:

$$\max_{q_1,q_2} [U(q_1,q_2,\theta) - P_1(q_1) - P_2(q_2)] \geq \max_{q_1} [U(q_1,0,\theta) - P_1(q_1)] \tag{C34}$$

and, once again, binds for the lowest type $\theta = -\Delta$. This yields:

$$\hat{\alpha}_{02}(\alpha_{11}, \alpha_{21}) = \frac{[\gamma(1-\alpha_{11}-3\Delta)-(1+2\alpha_{21})(1-c-3\Delta)]^2}{4(1+2\alpha_{21})\,(1+2\alpha_{21}-\gamma^2)}. \tag{C35}$$

Having derived firm 2's best response to any quadratic price schedule offered by firm 1, we can now determine the dominant firm's optimal schedule. As in the moral hazard setting, firm 1's fixed fee must be set to ensure that firm 2 has no incentive to deviate by inducing the buyer to deal exclusively with it.

To that end, consider the most attractive offer firm 2 can make to the buyer type $\theta = -\Delta$, who is the most susceptible to being lured into an exclusive relationship. The optimal deviation for firm 2 involves marginal cost pricing—, i.e., $P_2^d(q_2) = \alpha_{02}^d + cq_2$—combined with a fixed fee α_{02}^d that makes the buyer $\theta = -\Delta$ just indifferent between purchasing from both firms or exclusively from firm 2:

$$\max_{q_1,q_2}\left[U(q_1,q_2,-\Delta)-P_1(q_1)-P_2(q_2)\right] = \max_{q_2}\left[U(0,q_2,-\Delta)-cq_2-\alpha_{02}^d\right]. \tag{C36}$$

Solving for α_{02}^d, we obtain the profit that firm 2 can make in this deviation:

$$\Pi_2^d(-\Delta) = \alpha_{02}^d = \alpha_{01} - \frac{(1-\alpha_{11}-\Delta)^2}{2(2\alpha_{21}+1)} + \frac{1}{2}(1-c-\Delta)^2. \tag{C37}$$

Firm 1 must then set its fixed fee α_{01} such that firm 2 does not find it profitable to deviate. When firm 2 does not deviate, its profit is:

$$\Pi_2^{CR} = \int_{-\Delta}^{\Delta} \frac{\hat{\alpha}_{02}(\alpha_{11},\alpha_{21}) + (\hat{\alpha}_{012}(\alpha_{11},\alpha_{21})-c)\hat{q}_2(\theta) + \hat{\alpha}_{22}(\alpha_{11},\alpha_{21})\hat{q}_2(\theta)^2}{2\Delta} d\theta,$$

where:

$$\begin{aligned}(\hat{q}_1(\theta),\hat{q}_2(\theta)) = \arg\max_{q_1,q_2}\Big[&U(q_1,q_2,\theta)-P_1(q_1)+\\ &-\hat{\alpha}_{02}(\alpha_{11},\alpha_{21})-\hat{\alpha}_{012}(\alpha_{11},\alpha_{21})q_2(\theta)-\hat{\alpha}_{22}(\alpha_{11},\alpha_{21})q_2(\theta)^2\Big]\end{aligned}$$

Equating Π_2^{CR} to Π_2^d, we obtain:

$$\hat{\alpha}_{01} = \frac{(1-\alpha_{11}-\Delta)^2}{2(2\alpha_{21}+1)} - \frac{1}{2}(1-c-\Delta)^2 + \hat{\alpha}_{02} + (\hat{\alpha}_{12}-c)q_2(-\Delta) + \hat{\alpha}_{22}\left[q_2(-\Delta)\right]^2.$$

Firm 1 then solves the following problem,

$$\text{(C38)} \qquad \max_{\alpha_{11},\alpha_{21}} \int_{-\Delta}^{\Delta} \left[\hat{\alpha}_{01} + \alpha_{11}\hat{q}_1(\theta) + \alpha_{21}\hat{q}_1(\theta)^2\right] \frac{1}{2\Delta} d\theta$$

The solution can be calculated explicitly. The derivation and the complete expression are available in this Mathematica file, which also verifies that the dominant firm's profit is always strictly lower than the Ramsey-Boiteux profit whenever $\Delta > 0$. The analysis in that file also explicitly confirms that, in all screening problems considered thus far, the participation constraint binds only for the lowest type, $\theta = -\Delta$. Additionally, it extends the analysis to cases in which condition (12) does not hold and the participation constraint binds over an interval of types. In all cases, it remains true that the dominant firm cannot attain the Ramsey-Boiteux profit without referencing the rival's output.