Appendix:
For online publication

The Effects of Sin Taxes and Advertising Restrictions in a Dynamic Equilibrium

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A Purchase Data

In Table A.1, we report the set of cola products over which we model demand and supply. A product is defined as a firm-brand-pack combination. For each product, we present its share of total cola expenditure and its average price per liter. We model consumer demand over this set of products and two outside options: other (non-cola) drinks, categorized as either sugar-sweetened or non-sugar-sweetened.

Table A.2 details the 12 demographic groups over which we allow all consumer preference parameters to vary. These groups are based on the interaction of household type and income quantile. Household types include: (i) working-age households without children, (ii) pensioner households without children, and (iii) households with children. A working-age household is one with at least one member aged 18–65, while a household with children has at least one member aged 18 or younger. Income quartiles are based on equivalized income, calculated as household income divided by the OECD equivalence scale. The table reports the number of households and transactions (including cola and outside option purchases) for each household type.

Table A.1: *Firms and brands*

Firm	Brand	Pack	Expenditure share	Average price (£ per liter)
Coca Cola Enterprises	Regular Coke	Bottle(s): 1.25l: Single	0.6%	0.83
		Bottle(s): 1.5l: Single	0.3%	0.72
		Bottle(s): 1.75l: Single	0.5%	0.83
		Bottle(s): 1.75l: Multiple	2.7%	0.63
		Cans: 10x330ml: Single	0.9%	0.99
		Cans: 12x330ml: Single	2.5%	0.96
		Cans: 15x330ml: Single	0.6%	0.88
		Cans: 24x330ml: Single	2.1%	0.84
		Bottle(s): 2l: Single	0.9%	0.83
		Bottle(s): 2l: Multiple	4.7%	0.61
		Cans: 30x330ml: Single	1.1%	0.76
		Bottle(s): 3l: Single	1.0%	0.61
		Bottle(s): 4x1.5l: Single	0.4%	0.65
		Cans: 6x330ml: Single	1.4%	1.10
		Cans: 8x330ml: Single	6.1%	0.99
	Diet Coke	Bottle(s): 1.25l: Single	0.5%	0.84
		Bottle(s): 1.5l: Single	0.3%	0.73
		Bottle(s): 1.75l: Single	0.4%	0.85
		Bottle(s): 1.75l: Multiple	3.1%	0.62
		Cans: 10x330ml: Single	1.5%	1.02
		Cans: 12x330ml: Single	4.6%	0.97
		Cans: 15x330ml: Single	1.0%	0.88
		Cans: 24x330ml: Single	2.8%	0.83
		Bottle(s): 2l: Single	0.9%	0.80
		Bottle(s): 2l: Multiple	5.4%	0.62
		Cans: 30x330ml: Single	1.3%	0.76
		Bottle(s): 3l: Single	0.6%	0.61
		Bottle(s): 4x1.5l: Single	0.4%	0.65
		Cans: 6x330ml: Single	1.8%	1.00
		Cans: 8x330ml: Single	10.3%	0.99
Pepsico	Regular Pepsi	Bottle(s): 2l: Single	5.1%	0.52
		Cans: 6x330ml: Single	0.4%	0.82
		Cans: 8x330ml: Single	2.1%	0.82
	Diet Pepsi	Bottle(s): 1.5l: Single	0.2%	0.63
		Cans: 12x330ml: Single	0.6%	0.82
		Bottle(s): 2l: Single	15.0%	0.52
		Cans: 6x330ml: Single	0.9%	0.84
		Cans: 8x330ml: Single	9.2%	0.83
Store brands	Regular store	Bottle(s): 2l: Single	2.1%	0.18
		Bottle(s): 4x2l: Single	0.2%	0.24
	Diet store	Bottle(s): 2l: Single	3.0%	0.19
		Bottle(s): 4x2l: Single	0.5%	0.24
All			100.0%	0.74

Notes: Authors' calculations using data from Kantar Take Home Purchase Panel for 2010-2016. Diet Coke includes Coke Zero and Diet Pepsi includes Pepsi Max.

Table A.2: *Households' demographic groups*

		Number of: households	Number of: transactions
Working age	Bottom income quartile	1660	184536
	2nd income quartile	1718	192576
	3rd income quartile	1398	163288
	Top income quartile	2550	257582
Pensioner	Bottom income quartile	1455	177450
	2nd income quartile	1154	134867
	3rd income quartile	568	71455
	Top income quartile	411	46172
Household with children	Bottom income quartile	3015	385244
	2nd income quartile	3447	448110
	3rd income quartile	1950	242701
	Top income quartile	2384	281669

Notes: Numbers are for our analysis sample from the Kantar FMCG At-Home Purchase Panel for 2010-2016.

B Advertising Market and Data

B.1 The UK TV Market

The UK TV market is heavily regulated. Four large public service broadcasters—BBC, ITV1, Channel 4 (C4), and Channel 5 (C5)—face constraints on advertising. The BBC, funded by an annual license fee, is not permitted to air adverts. ITV1, C4 and C5, which do not receive license fee income, are allowed to show adverts but face some restrictions regarding programming, and total time dedicated advertising. These public broadcasters have relatively large audience shares: BBC1 accounts for approximately 20%, ITV for 16%, BBC2 and C4 for 7% each and C5 for 5%. They compete for viewers by offering programs designed for broad audience appeal (see Crawford et al. (2017) for a detailed discussion of the UK television advertising market).

In addition to these public service broadcasters, there are numerous commercial channels that do not face specific programming restrictions.[1] Access to these channels depends on the household's TV subscription type. Households can watch TV in four ways: free-to-air, Freeview, satellite, or cable. All households with a TV must pay the BBC license fee. Free-to-air provides access only to public service broadcasters without additional cost. Freeview

[1]The BBC also operates additional channels (e.g., BBC3, BBC4, BBC News, BBC Parliament) with low viewership, which are legally prohibited from advertising.

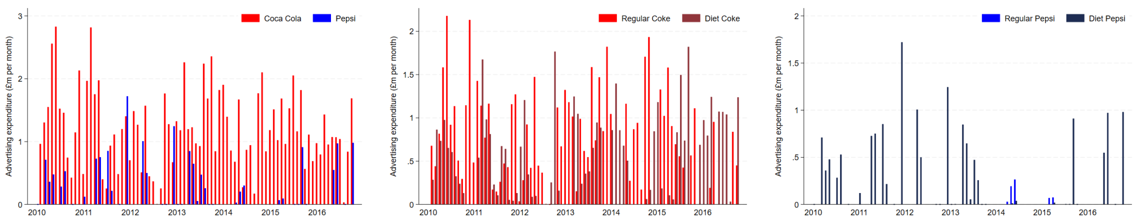
requires purchasing a compatible TV or set-top box but involves no further fees and offers a limited selection of additional channels. Satellite and cable subscriptions provide access to a broader range of mainly commercial channels while also including all free-to-air and Freeview channels.

B.2 Advertising Expenditure

Figure B.1 presents advertising spending over time, separately for Coca Cola Enterprises (Coca Cola) and Pepsico (Pepsi), and further disaggregated by Regular and Diet brands within each firm. The figure highlights fluctuations in spending and reveals distinct advertising strategies: Coca Cola Enterprises allocates more to advertising its Regular brand than its Diet brand, with the former accounting for 57% of its total spend. In contrast, Pepsico advertises almost exclusively its Diet brand. Our analysis focuses on Coca Cola's advertising decisions for its Regular and Diet brands and Pepsico's decision for its Diet brand.

Figure B.1: *Advertising Expenditure*

(a) by firm (b) Coca Cola, by brand (c) Pepsi, by brand

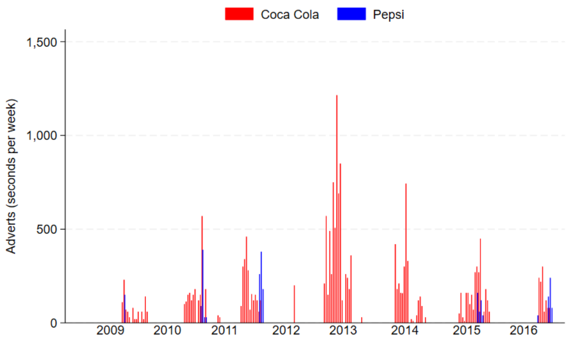


Notes: Authors' calculations using data from AC Nielsen Advertising data for 2010-2016.

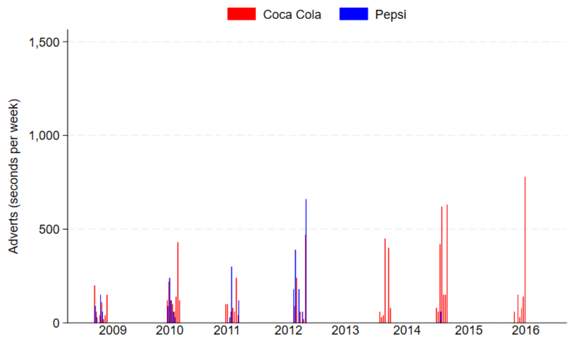
Figure B.2 illustrates weekly variation in advertising (measured in seconds) for Coca Cola and Pepsico brands during two prime-time talent shows, The X Factor and Britain's Got Talent. Both shows air on ITV but at different times of the year—one in spring and the other in autumn. According to TV viewing data, 46% of households regularly watch Britain's Got Talent (25% of whom do not regularly watch The X Factor), while 39% regularly watch The X Factor (12% of whom do not regularly watch Britain's Got Talent). Advertisements from both Coca Cola and Pepsico appear during each show, but the distribution differs: Pepsico accounts for only 11% of cola advertising time during The X Factor (2009–2016), whereas its share rises to 27% during Britain's Got Talent. As a result, households' exposure to advertising from each firm varies depending on whether they watch neither, one, or both shows.

Figure B.2: *Within genre advertising variation*

(a) The X Factor



(b) Britain's Got Talent



Notes: Authors' calculations using data from AC Nielsen Advertising data for 2010-2016. Figures show number of seconds of adverts shown during the indicated show per week week.

Table B.1 lists the advertising agencies in our dataset for 2016, covering all food and drink advertising. It shows that Coca Cola works with Mediacom, representing 29% of the agency's total food and drinks advertising, while Pepsico works with OMD, accounting for 3% OMD's food and drinks advertising.

Table B.1: *Advertising agencies in 2016*

	Total agency advertising spend (£m) on		
	All food & drink	Coca Cola	Pepsi
Omd	94.75	-	2.52
Zenith	77.35	-	-
Carat	57.04	-	-
Mediacom	37.93	10.87	-
Um	27.49	-	-
Blue 449	24.68	-	-
Mec	20.42	-	-
Mindshare Media Uk Ltd	16.80	-	-
Rocket	15.86	-	-
Initiative Media London	8.79	-	-
Arena Media	7.59	-	-
M/six	7.51	-	-
Phd	5.65	-	-
Maxus	4.13	-	-
The7stars	4.07	-	-
Starcom	3.85	-	-
Mnc	3.69	-	-
Spirit Media Scotland Ltd	1.17	-	-
Spark Foundry	0.92	-	-
Goodstuff Communications	0.77	-	-
Direct (In House) Advertising	0.64	-	-
Specialist Works Ltd	0.62	-	-
Ams Media Group Ltd	0.43	-	-
The Lane Agency	0.36	-	-
Nick Stewart Media Consultancy	0.22	-	-
Overseas Agency - Ireland	0.21	-	-
Bray Leino	0.19	-	-
Anderson Spratt Group	0.14	-	-
Not Allocated	0.11	-	-
We Are Boutique	0.10	0.01	-
Republic Of Media	0.09	-	-
Genesis Advertising Ltd	0.05	-	-
Rla Group	0.02	-	-
Morvah	0.02	-	-
John Ayling & Associates Ltd	0.01	-	-
Juice Media Uk Ltd	0.01	-	-
Hello Starling	0.01	-	-
Di5 Ltd	0.01	-	-
Walker Communications	0.01	-	-
Tcs Media Ltd	0.00	-	-

Notes: Authors' calculations using data from AC Nielsen Advertising data for 2016.

B.3 Estimating Advertising Impact Probability

For one year, 2015, we have data on advertising impacts—the industry-standard measure of viewership—collected by the Broadcasters Audience Research Board (BARB).[2]

Impacts are measured based on Ratecard Weighted TV Ratings (TVRs), also known as Gross Rating Points (GRPs). TVRs are calculated as the number of impacts divided by the total target audience. Broadcasters use ratecard-weighted TVRs to sell advertising slots, applying weights to adjust for differences in slot length. While one impact typically represents a single viewer watching a 30 second ad, a pair of 15 second slots may hold greater value for advertisers than a single 30 second slot. Ratecard weighting accounts for these differences, enabling revenue comparisons—e.g., a slot generating 50 ratecard-weighted impacts produces half as much advertising revenue as a slot generating 100 ratecard-weighted impacts.

Table B.2 presents descriptive statistics on the match between our purchase data (which includes information on households' TV viewing habits by show, station, and time slot) and our advertising data. While we undertake this matching for all years in our dataset, we focus on 2015 in Table B.2 since it is the only year where we observe impacts.

In 2015, there were 35,481 Coca Cola and Pepsico adverts for which we could match at the show level, meaning we observe whether households watched the show during which the advert aired. Since the purchase data only record the most popular shows watched by households, some adverts in the advertising data could not be matched at the show level. In these cases, we matched based on station and time slot, covering an additional 77,083 adverts. A small number of adverts aired on minor stations for which household viewing behavior is not recorded in the purchase data; in these cases, we could only match on time slot. However, as shown in Table B.2, these adverts account for a small fraction of advertising spending and have very low measured impacts.

[2] BARB collects these data as follows: A sample of households is provided with a remote control featuring a button for each household member (and an additional button for guests). Each individual must press their button whenever they enter or leave the room while the television is on. Each household's TV is fitted with a meter that records 15 seconds of audio from TV adverts and matches this to a reference library. (See https://www.barb.co.uk/about-us/how-we-do-what-we-do/)

Table B.2: *Match in 2015 between Kantar media data and AC Nielsen advert data*

Matched on	Total agency advertising spend (£m) on No. adverts	Mean impacts (TVR)	Total expenditure (£m)
Show	35481	0.0534	7.58
Station & Time slot	77083	0.0170	8.10
Time slot only	62270	0.0007	0.83

Consumer advertising exposure (equation (1)) depends on whether a household has seen an advert during slot k, denoted as w_{ik}. These weights correspond to the Q possible values of ordinal survey responses:

$$w_{ik} = \sum\nolimits_{q=1,..,Q} w_r 1_{\{v_{ik}=q\}}$$

where $v_{ik} = q$ if household i reported response q to the survey question related to slot k (e.g., "how regularly do you watch show X?" if they show aired during that slot).

Households' answers to these questions are qualitative and categorized as: "never", "hardly ever", "sometimes", and "regularly". Since these responses are not directly quantitative, we leverage data from 2015—when we observe advertising impacts—to estimate the probabilities associated with each response category.

Let $q = \{1, 2, 3\}$ correspond to the three nonzero responses {"hardly ever", "sometimes", and "regularly"}, with v_{ik} denoting household i's response for slot k and w_q representing the probability of watching corresponding to answer q.

We estimate w_q using constrained nonlinear least squares:

$$TVR_k = \sum\nolimits_q w_q \left(\frac{1}{N} \sum\nolimits_i 1_{\{v_{ik}=q\}} \right) + e_k$$

subject to

$$0 \leq w_1 \leq w_2 \leq w_3 \leq 1$$

We estimate this separately for slots matched based on the show and for slots matched based on station and time slot. Table B.3 presents the estimates, where we find that the constraint that $w_1 \leq w_2$ binds.

Table B.3: *Estimates of* w_q *(*$q = 1, 2, 3$*)*

	TVR	
	show	station slot
w_1	0.0352	0.0274
	(0.0223)	(0.0040)
w_2	0.0352	0.0274
	(0.0223)	(0.0040)
w_3	0.4975	0.4454
	(0.1153)	(0.0159)
N	88	1208

Note that if total viewership were unavailable, we could estimate w_q directly within the demand model. To see this, we we can rewrite individual advertising exposure as

$$\begin{aligned} a_{ibt} &= \sum\nolimits_{q=1}^{Q} w_q \sum\nolimits_{\{k|t(k)=t\}} 1_{\{v_{ik}=q\}} \omega(T_{bk}) \\ &= \sum\nolimits_{q=1}^{Q} w_q a^q_{ibt} \end{aligned}$$

where $a^q_{ibt} = \sum_{\{k|t(k)=t\}} 1_{\{v_{ik}=q\}} \omega(T_{bk})$. This formulation implies that we could estimate w_q as part of the demand model instead of relying on estimates derived from the TV survey and viewership data.

The main advantage of estimating w_q within the demand model is that it would allow for additional heterogeneity, for example, through demographic-specific w_q. However, given that we already allow for substantial heterogeneity in how advertising exposure affects random utility—including demographic-specific effects—this approach would add little benefit while significantly increasing the number of advertising controls in the demand model.

B.4 Advertising Exposure and Stock

We specify the consumer's exposure stock to brand b advertising at the beginning of week t as the discounted sum of past advertising exposure:

$$A_{ibt} = \sum\nolimits_{s=0}^{t-1} \delta^{t-1-s} a_{ibs} = \delta A_{ibt-1} + a_{ibt-1}.$$

To initialize exposure stocks, we use data on advertising and household TV viewing behavior from a pre-sample year (2009), as advertising exposure older than 52 weeks has a negligible impact on stocks.

We set $\delta = 0.9$. To support this choice we use the regression (equation (2)) in Section I.D as the basis for conducting non-nested hypothesis test. We evaluate this equation with $\delta = 0.9$ against alternative values $\delta = 0.1, 0.15, 0.2, ..., 0.95$. We use the test proposed by MacKinnon (1983).

The idea behind this test is to obtain the fitted values of equation (2) for two competing models and then to re-estimate one model additionally including the fitted values from the alternative model as an extra regressor. The test itself is a t-test on the significance of the fitted value, if they are significant, it suggests that the alternative model has additional explanatory power. We conduct two sets of tests. First, we examine whether models with $\delta \neq 0.9$ provide additional expanatory power compared to the baseline model with $\delta = 0.9$ (reported in the Table B.4). Second, we test whether including advertising with $\delta = 0.9$ improves explanatory power in models where $\delta \neq 0.9$ (reported in the Table B.5). The results from the first table indicate we almost always reject the additional explanatory power of models with $\delta \neq 0.9$, compared to the $\delta = 0.9$ model. Conversely, the results in the second table show that for models with $\delta \neq 0.9$, we almost always cannot reject the additional explanatory power of including advertising with $\delta = 0.9$.

Table B.4: *MacKinnon tests*

	Coke Reg			Coke Diet			Pepsi Diet		
x	α	s.e.	P-value	α	s.e.	P-value	α	s.e.	P-value
10	0.078	0.068	0.255	0.031	0.075	0.684	0.038	0.087	0.658
15	0.092	0.070	0.188	0.017	0.077	0.826	0.045	0.086	0.599
20	0.104	0.071	0.145	0.008	0.078	0.920	0.040	0.088	0.653
25	0.114	0.073	0.120	0.001	0.080	0.988	0.034	0.090	0.704
30	0.120	0.075	0.108	-0.005	0.081	0.953	0.028	0.092	0.761
35	0.125	0.077	0.103	-0.011	0.083	0.896	0.021	0.095	0.826
40	0.129	0.079	0.103	-0.018	0.085	0.835	0.014	0.099	0.888
45	0.133	0.082	0.104	-0.026	0.087	0.764	0.011	0.105	0.919
50	0.137	0.085	0.109	-0.037	0.090	0.678	0.014	0.111	0.897
55	0.139	0.089	0.119	-0.052	0.093	0.578	0.026	0.120	0.826
60	0.140	0.094	0.136	-0.071	0.098	0.468	0.044	0.130	0.737
65	0.143	0.101	0.156	-0.098	0.107	0.356	0.053	0.145	0.717
70	0.155	0.112	0.166	-0.138	0.121	0.254	0.028	0.172	0.870
75	0.187	0.131	0.153	-0.201	0.146	0.169	-0.087	0.223	0.697
80	0.256	0.172	0.138	-0.315	0.194	0.106	-0.344	0.307	0.262
85	0.407	0.296	0.169	-0.622	0.338	0.066	-0.897	0.491	0.068
90	0.000	0.000		0.000	0.000		0.000	0.000	
95	-0.085	0.187	0.651	0.362	0.221	0.100	0.647	0.219	0.003

Notes: $p < 0.05$ we reject the null; i.e., $\delta = x$ matters (in addition to $\delta = 0.9$)

Table B.5: *MacKinnon, Inverse*

	Coke Reg			Coke Diet			Pepsi Diet		
x	α	s.e.	P-value	α	s.e.	P-value	α	s.e.	P-value
10	0.922	0.068	0.000	0.969	0.075	0.000	0.962	0.087	0.000
15	0.908	0.070	0.000	0.983	0.077	0.000	0.955	0.086	0.000
20	0.896	0.071	0.000	0.992	0.078	0.000	0.960	0.088	0.000
25	0.886	0.073	0.000	0.999	0.080	0.000	0.966	0.090	0.000
30	0.880	0.075	0.000	1.005	0.081	0.000	0.972	0.092	0.000
35	0.875	0.077	0.000	1.011	0.083	0.000	0.979	0.095	0.000
40	0.871	0.079	0.000	1.018	0.085	0.000	0.986	0.099	0.000
45	0.867	0.082	0.000	1.026	0.087	0.000	0.989	0.105	0.000
50	0.863	0.085	0.000	1.037	0.090	0.000	0.986	0.111	0.000
55	0.861	0.089	0.000	1.052	0.093	0.000	0.974	0.120	0.000
60	0.860	0.094	0.000	1.071	0.098	0.000	0.956	0.130	0.000
65	0.857	0.101	0.000	1.098	0.107	0.000	0.947	0.145	0.000
70	0.845	0.112	0.000	1.138	0.121	0.000	0.972	0.172	0.000
75	0.813	0.131	0.000	1.201	0.146	0.000	1.087	0.223	0.000
80	0.744	0.172	0.000	1.315	0.194	0.000	1.344	0.307	0.000
85	0.593	0.296	0.045	1.622	0.338	0.000	1.897	0.491	0.000
90	0.000	0.000		0.000	0.000		0.000	0.000	
95	1.085	0.187	0.000	0.638	0.221	0.004	0.353	0.219	0.108

Notes: $p < 0.05$ *we reject the null; i.e.,* $\delta = 0.9$ *matters (in addition to* $\delta = x$*)*

Table B.6 summarizes the variation in brand advertising flows and stocks using the within-group standard deviation (measured over time and individuals). Figures B.3 and B.4 present heatmaps illustrating the weekly variation in the distribution of advertising flow and stock for each brand, pooled across demographic groups.

Table B.6: *Advertising exposure*

Demographic	Income quartile	Flow s.d. Coke Reg	Flow s.d. Coke Diet	Flow s.d. Pepsi Diet	Stock s.d. Coke Reg	Stock s.d. Coke Diet	Stock s.d. Pepsi Diet
Working age	1	32.8	29.2	24.0	168.6	138.6	67.3
	2	32.6	28.7	23.7	165.3	135.1	66.5
	3	32.2	28.3	23.2	162.3	132.6	65.3
	4	31.3	27.5	22.8	157.9	128.6	63.9
Pensioner	1	30.6	26.4	22.5	152.3	123.9	62.8
	2	29.5	24.9	21.3	147.4	116.9	59.7
	3	31.9	26.3	22.7	157.9	123.7	63.5
	4	28.6	24.2	20.4	141.4	114.1	57.3
Household with children	1	31.4	28.2	23.2	161.9	132.6	65.7
	2	32.0	28.6	23.7	164.1	133.6	67.1
	3	31.7	27.8	23.7	158.2	127.4	63.7
	4	30.0	26.4	21.7	152.6	122.6	61.5

Figure B.3: *Advertising flow*

(a) Regular Coke (b) Diet Coke (c) Diet Pepsi

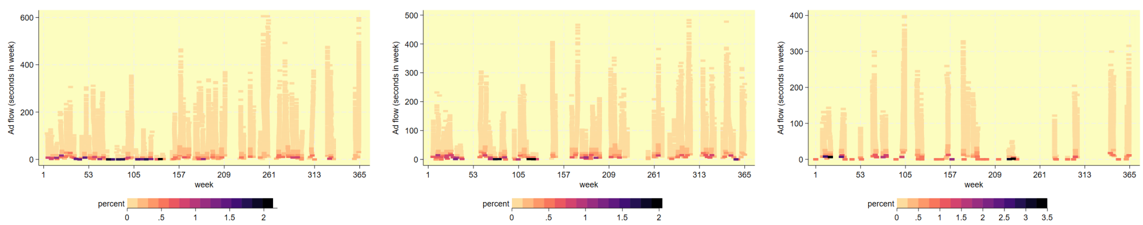
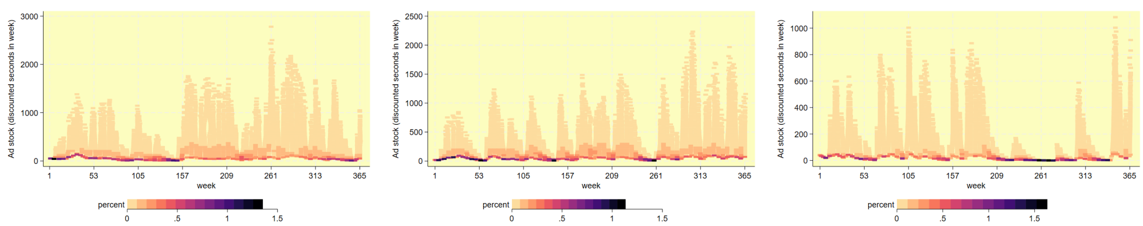


Figure B.4: *Advertising stock*

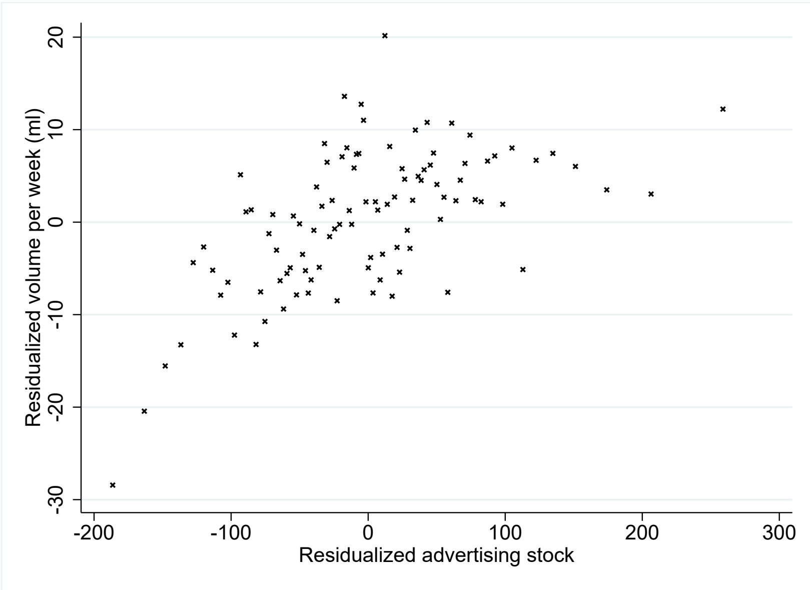
(a) Regular Coke (b) Diet Coke (c) Diet Pepsi



B.5 Non-parametric Evidence of Advertising Effects

In Section I.D we provide evidence based on within-household variation on the relationship between purchase volume and advertising exposure stocks. Here, we estimate a more flexible version of equation (2) by first residualizing vol_{ibt}, and A_{ibt} using regression on τ_t, $\iota_{d,q(t)}$, $\kappa_{r,q(t)}$ and η_i. In Figure B.5 we plot the non-parametric conditional expectation of vol_{ibt} given A_{ibt} for Regular Coke, the most heavily advertised brand. It provides data-driven support for a concave relationship between purchases and advertising stocks.

Figure B.5: *Non-parametric relationship between residualized Regular Coke volume and advertising*



Notes: For both Regular Coke volumes and advertising stocks, we regress the variable on week, demographic-quarter, region-quarter and household fixed effects, and obtain the residuals. The graphs plots the relationship between residualized Regular Coke volume and percentiles of the distribution of residualized Regular Coke volume.

C Equilibrium Delegation

To simplify notation and without loss of generality, we assume each firm sells a single product. A firm that directly chooses its advertising slots (rather than delegating decisions to an advertising agency) and its price solves the following problem:

$$\max_{\{p_{jt}\}_{\forall t},\{T_{jkt}\}_{\forall k,t}} \sum_{t=0}^{\infty} \beta^t \pi_{jt}(p_{1t}, .., p_{Jt}, (T_{11\tau}, ..., T_{JK\tau})_{\tau\leq t}) \tag{C.1}$$

where

$$\pi_{jt}(p_{1t}, .., p_{Jt}, (T_{11\tau}, ..., T_{JK\tau})_{\tau\leq t}) \equiv (p_{jt} - c_{jt})q_{jt}(p_{1t}, .., p_{Jt}, (T_{11\tau}, ..., T_{JK\tau})_{\tau\leq t}) - \sum_k \rho_{kt}T_{jkt}$$

and ρ_{kt} represents the price of advertising on channel k (Here, k indexes both channels and time slots, but for simplicity, we refer to it as a channel.) The firm's profits depend on the decisions of other firms. We seek a Markov Perfect Equilibrium.

If the firm delegates advertising decisions to an advertising agency, its problem becomes:

$$\max_{\{p_{jt},e_{jt}\}\forall t} \sum\nolimits_{t=0}^{\infty} \beta^t \pi_{jt}(p_{1t}, .., p_{Jt}, (T^*_{11t}(e_{1t}), ..., T^*_{JKt}(e_{jt}))_{\tau \leq t}), \tag{C.2}$$

where

$$T^*_{jk}(e_{jt}) = \arg\max \omega(T_{j1t}, .., T_{jKt})$$
$$\text{s.t.} \ \sum\nolimits_k \rho_k T_{jkt} \leq e_{jt}$$

This represents the optimal choice of an advertising agency, which aims to maximize aggregate impact $\omega(T_{j1t}, .., T_{jKt})$ subject to the budget e_{jt}:

A firm can either:

1. Directly set prices and advertising to maximize its discounted sum of profits, or
2. Delegate advertising choices to an agency, which maximizes impacts subject to a budget.

We first analyze a game where the delegation decision is made in a static equilibrium, and then extend the analysis to a dynamic equilibrium.

C.1 Endogenous Delegation in Static Equilibrium

Price and advertising competition without delegation Denote the profit of firm j, whose product is sold at price p_j and advertised for duration T_{jk} on slot k as:

$$\pi_j(p_j, T_j, p_{-j}, T_{-j}) = (p_j - c_j) q_j(p_j, T_j, p_{-j}, T_{-j}) - \sum\nolimits_k \rho_k T_{jk}$$

where T_j is the vector of $(T_{jk})_{k=1,..,K}$ and ρ_k is the price of advertising on channel k (where k indexes both channels and time slots, but for simplicity, we refer to it as a channe).

Let * denote the Nash equilibrium when firms do not delegate advertising. A Nash equilibrium $(p^*_j, T^*_j, p^*_{-j}, T^*_{-j})$ will be solution of:

$$\max_{p_j, T_j} \pi_j(p_j, T_j, p^*_{-j}, T^*_{-j}) \equiv \pi^*_j$$

with a symmetric condition holding for firm $-j$.

Price and advertising competition with delegation When the firm delegates advertising decisions to an agency, it provides an impact function $\omega(T_{j1}, .., T_{jK})$ to be maximized This function is independent of prices and the competing firm's choices. The firm's problem then reduced to choosing prices and an advertising budget to solve:

$$\max_{p_j, e_j} \pi_j(p_j, \tilde{T}_j(e_j), p^{**}_{-j}, \tilde{T}_{-j}(e^{**}_{-j}))) \equiv \pi^{**}_j$$

subject to the advertising agency's optimal allocation of advertising across slots:

$$\begin{aligned} \tilde{T}_j(e_j) &= \arg\max \omega(T_{j1}, .., T_{jK}) \\ &\quad \text{s.t. } \sum\nolimits_k \rho_k T_{jk} \leq e_j \end{aligned}$$

and given the optimal choices of competing firms, p^{**}_{-j} and e^{**}_{-j}. The Nash Equilibrium $(p^{**}_j, T^{**}_j, p^{**}_{-j}, T^{**}_{-j})$ consists of solutions of the above problem, where the equilibrium advertising allocation satisfies $T^{**}_j \equiv \tilde{T}_j(e^{**}_{-j})$.

Depending on the own and cross-demand effects of advertising, the firm's profit under delegation may be higher or lower than when it controls advertising directly:

$$\pi^*_j \leq \pi^{**}_j \text{ or that } \pi^*_j \geq \pi^{**}_j$$

Choice of delegation of advertising Now, suppose each firm can choose whether or not to delegate its advertising decisions. Each firm incurs an additional fixed cost κ_j if it chooses to manage both price and advertising decisions in-house. However, this cost is not incurred if the firm delegates slot selection to an advertising agency while retaining control over prices and the overall advertising budget.[3]

If $\kappa_j = 0$ for both firms, the unique equilibrium outcome is that neither firm delegates its advertising decisions. This is because, in the absence of delegation costs, each firm finds it optimal to control both price and advertising in order to maximize profit, given the competitor's choices. This remains true even if delegation would lead to higher profits $\pi^{**}_j \geq \pi^*_j$. If firms are free to choose delegation, the equilibrium outcome will always be non-delegation, as each firm has an incentive to compete more aggressively on both price

[3] We do not explicitly model the cost that firms may incur when engaging advertising agencies, such as markups charged by agencies. The fixed cost κ_j represents the additional burden of in-house management, which may arise due to efficiency gains from delegation, specialized marketing expertise, or agencies' superior knowledge of television advertising markets.

and advertising when its rival delegates. In equilibrium, this leads all firms to retain direct control over advertising.

However, when $\kappa_j > 0$, delegation can emerge as a Nash equilibrium. If both firms delegate their advertising decisions,[4] they may achieve higher profits than under direct competition. This is because the structure of demand can be such that delegation softens competition in advertising, mitigating the intense rivalry that would otherwise arise in a business-stealing environment.

To see this in more detail, define the following:

- $p_j^*(p_{-j}, T_{-j})$ and $T_j^*(p_{-j}, T_{-j})$ as firm j's price and advertising best responses to the competing price and advertising choice when the firm does not delegate to an agency.
- $p_j^{**}(p_{-j}, T_{-j})$ and $T_j^{**}(p_{-j}, T_{-j})$ as firm j's price and advertising best responses when it does delegate, where $T_j^{**}(p_{-j}, T_{-j}) \equiv \tilde{T}_j(e_j^{**}(p_{-j}, T_{-j}))$ and $e_j^{**}(p_{-j}, T_{-j})$ is firm j's advertising choice, solving: $\max_{p_j, e_j}(p_j - c_j)q_j(p_j, \tilde{T}_j(e_j), p_{-j}, T_{-j}) - \sum_k \rho_k \tilde{T}_{jk}(e_j)$

Next, we denote firm j's profit under its best response without delegation as $\pi_j^*(p_{-j}, T_{-j})$ and with delegation as $\pi_j^{**}(p_{-j}, T_{-j})$, given by:

$$\pi_j^*(p_{-j}, T_{-j}) \equiv (p_j^*(p_{-j}, T_{-j}) - c_j)q_j(p_j^*(p_{-j}, T_{-j}), T_j^*(p_{-j}, T_{-j})), p_{-j}, T_{-j}) - \sum_k \rho_k T_{jk}^*(p_{-j}, T_{-j})$$

and

$$\pi_j^{**}(p_{-j}, T_{-j}) \equiv (p_j^{**}(p_{-j}, T_{-j}) - c_j)q_j(p_j^{**}(p_{-j}, T_{-j}), T_j^{**}(p_{-j}, T_{-j})), p_{-j}, T_{-j}) - \sum_k \rho_k T_{jk}^{**}(p_{-j}, T_{-j})$$

By construction, we always have $\pi_j^{**}(p_{-j}, T_{-j}) \leq \pi_j^*(p_{-j}, T_{-j})$ for any given (p_{-j}, T_{-j}), meaning that delegating cannot be a Nash Equilibrium if there is no delegation cost ($\kappa_j = 0$). However, delegation can be an equilibrium if the delegation costs κ_j and κ_{-j} satisfy:

$$\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^*(p_{-j}^{**}, T_{-j}^{**}) - \kappa_j \quad \text{and} \quad \pi_{-j}^{**}(p_j^{**}, T_j^{**}) \geq \pi_{-j}^*(p_j^{**}, T_j^{**}) - \kappa_{-j}$$

That is, delegation becomes an equilibrium when firms find it optimal to avoid the fixed cost of managing advertising in-house.

Delegation can also arise as an equilibrium if:

$$\pi_j^*(p_{-j}^*, T_{-j}^*) - \kappa_j \geq \pi_j^{**}(p_{-j}^*, T_{-j}^*) \quad \text{and} \quad \pi_{-j}^*(p_j^*, T_j^*) - \kappa_{-j} \geq \pi_{-j}^{**}(p_j^*, T_j^*)$$

[4] A mixed strategy where only one firm delegates can also be an equilibrium, but we do not explore this case here.

Thus, firms may endogenously choose to delegate advertising to an agency and obtain higher profits if there is some fixed cost associated with directly managing advertising slot choices.

In the absence of this cost ($\kappa_j = 0$), delegation cannot be an equilibrium in this one-period static game. However, in a dynamic setting, where firms decide on delegation for the long term, the outcome can differ.

C.2 Endogenous Delegation in Dynamic Equilibrium

For simplicity, we consider the case where advertising has no dynamic effect on demand (i.e., consumers are memoryless).

Consider the repeated game where firms maximize their intertemporal sum of profits with discount factor $\beta \in (0, 1)$. In this setting, delegating to an agency can be a Subgame perfect Nash Equilibrium even if $\kappa_j = \kappa_{-j} = 0$, provided firms are sufficiently patient (i.e., β large enough). The standard trigger strategy can sustain delegation as an equilibrium: firms delegate as long as their competitor does, but if one firm deviates by not delegating, both switch permanently to the no-delegation equilibrium. For this strategy to work, we need β to be large enough to satisfy (assuming stationary, where demand and profit function are time invariant)

$$\frac{1}{1-\beta} \underbrace{\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})}_{\substack{\text{Profit of } j \text{ with delegation} \\ \text{given}(p_{-j}^{**}, T_{-j}^{**})}} \geq \underbrace{\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**})}_{\substack{\text{Profit of } j \text{ without delegation} \\ \text{given}(p_{-j}^{**}, T_{-j}^{**})}} + \frac{\beta}{1-\beta} \underbrace{\pi_j^{*}(p_{-j}^{*}, T_{-j}^{*})}_{\substack{\text{profit of } j \\ \text{under no delegation equilibrium}}}$$

and symmetrically for firm $-j$:

$$\frac{1}{1-\beta} \pi_{-j}^{**}(p_j^{**}, T_j^{**}) \geq \pi_{-j}^{*}(p_j^{**}, T_j^{**}) + \frac{\beta}{1-\beta} \pi_{-j}^{*}(p_j^{*}, T_j^{*})$$

Since we know that $\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})$ and given that $\frac{1}{1-\beta} > 1$ while $\frac{\beta}{1-\beta} < \frac{1}{1-\beta}$, the condition holds whenever

$$\beta \geq \frac{\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**})}{\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{*}(p_{-j}^{*}, T_{-j}^{*})}.$$

This condition is always satisfied if: $\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{*}(p_{-j}^{*}, T_{-j}^{*}) < 0$. However, delegation cannot be sustained as an equilibrium if:

$$\pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \geq \pi_j^{*}(p_{-j}^{**}, T_{-j}^{**}) - \pi_j^{*}(p_{-j}^{*}, T_{-j}^{*})$$

that is $\pi_j^{**}(p_{-j}^{**}, T_{-j}^{**}) \leq \pi_j^{*}(p_{-j}^{*}, T_{-j}^{*})$ meaning that delegation is only an equilibrium of the dynamic game if the per-period profit under mutual delegation is higher than under mutual non-delegation. If this condition holds, there exists a discount factor $\beta^* < 1$ such that for all $\beta \geq \beta^*$, delegation is a Subgame Perfect Nash Equilibrium.

This model provides a rationale for why firms delegate advertising to agencies in equilibrium. It shows that delegation can be a more profitable strategy than direct competition in advertising, particularly in a dynamic setting where firms use strategies that sustain tacit coordination on delegation.

D Monopoly Advertising Response to Tax

In the case of a static single-product monopolist, we illustrate how tax policy affects the profit-maximizing advertising choice. This highlights two key mechanisms that shape a firm's incentives to adjust advertising in response to the introduction or modification of a tax.

The monopolist chooses its price p and its level of advertising A to maximize profits, facing the demand function $Q(p, A)$ (where $Q_p < 0$ and $Q_A > 0$), a constant marginal cost of production c, a specific tax τ, and a constant marginal cost of advertising k. The firm's problem is: $(p^*, A^*) = \arg\max_{p,A}(p - c - \tau)Q(p, A) - kA$. We assume that the profit function in concave in (p, A). Denote optimal output by $Q^* \equiv Q(p^*, A^*)$, optimal price-cost margin by $\mu^* \equiv p^* - \tau - c$ and pass-through of a marginal tax increase (holding advertising fixed) on the tax-exclusive price $(p^* - \tau)$, relative to the tax-inclusive price, by $\rho^* \equiv \left(\frac{dp^*}{d\tau}\big|_{A^*} - 1\right) / \frac{dp}{d\tau}\big|_{A^*}$. Note $\rho^* > 0$ ($\rho^* < 0$) implies that a marginal tax increase is over-shifted (under-shifted) to prices—i.e., the monopolist increases (decreases) its margin in response, holding advertising fixed. The impact of a marginal tax increase on optimal advertising is determined by:[5]

$$\text{sign}\left\{\frac{dA^*}{d\tau}\right\} = \text{sign}\left\{\mu^* Q_{Ap}^* + \rho^* Q_A^*\right\}.$$

To interpret this condition, first consider the case where the monopolist sets an exogenous fixed margin, meaning $\frac{dp^*}{d\tau} = 1$ and $\rho^* = 0$. In this case, whether the tax increases advertising depends on the cross-derivative of demand, Q_{Ap}^*. A tax rise increases the (tax-inclusive) price, pushing the firm further up its demand curve. If, at this new higher price level, consumers

[5]The condition stated in terms of demand primitives is: $\text{sign}\left\{\frac{dA^*}{d\tau}\right\} = \text{sign}\left\{ -\frac{Q^*}{Q_p^*} Q_{Ap}^* + \left(-1 + \frac{Q^* Q_{pp}^*}{(Q_p^*)^2}\right) Q_A^* \right\}$.

are more (less) responsive to advertising, the firm has an incentivize to increase (decrease) its advertising.

When the firm can adjust its margin—so price is also a choice variable—an additional effect comes into play. If the firm raises its margin in response to the tax (so $\rho^* > 0$), this increases the profitability of acquiring the marginal consumer, incentivizing greater advertising. Conversely, if the firm lowers its margin $\rho^* < 0$, advertising incentives weaken.

Thus, in the monopoly case, advertising responses to taxes depend on two factors:

1. Variation in demand responsiveness to advertising along the demand curve: If consumer sensitivity to advertising changes with price, this shapes the firm's advertising incentives.
2. Pass-through of the tax: Whether the tax is under- or over-shifted depends on the tax structure and demand curvature, influencing the firm's optimal margin and, consequently, its advertising decisions.

Moreover, the ability to adjust advertising introduces a feedback effect on price-setting. This creates both direct and indirect effects on consumption.[6]

In reality, most firms sell multiple products, tax liabilities varies across products, firms engage in competition, and advertising has persistent effects on consumer choice meaning that competition is dynamic in nature. Our model incorporates the additional factors influencing advertising decisions, while also capturing the two forces highlighted in this simplified example.

E Solution to Advertising Agency Problem

The optimal advertising length during slot k satisfies equation (8), which we repeat here

$$T^*_{bk} = \omega'^{-1}\left(\frac{\rho_k}{\sum_{i\in\Omega_b} w_{ik}}\frac{1}{\lambda^*_{bt}}\right).$$

[6] In particular, tax pass-through depends on advertising adjustments. Specifically, $\frac{d(p-\tau)^*}{d\tau} > 0$ if and only if $\left(-1 + \frac{Q^* Q^*_{pp}}{(Q^*_p)^2}\right) > \frac{1}{(-Q_p)(-Q_A)}\left(-Q^2_{Ap}\frac{Q}{-Q_P} - Q_A Q_{Ap}\right)$. In contrast, with fixed advertising, $\frac{d(p-\tau)^*}{d\tau} > 0$ if and only if $\left(-1 + \frac{Q^* Q^*_{pp}}{(Q^*_p)^2}\right) > 0$.

We specify the power function, $\omega(T) = T^{\gamma}$, hence $(\omega')^{-1}(x) = (\frac{x}{\gamma})^{\frac{1}{\gamma-1}}$, and therefore:

$$T^*_{bk} = \left(\frac{1}{\gamma}\frac{\rho_k}{\sum_{i\in\Omega_b} w_{ik}}\frac{1}{\lambda^*_{bt}}\right)^{\frac{1}{\gamma-1}}.$$

Note, total brand advertising expenditure is

$$e_{bt} = \sum_{\{k|t(k)=t\}} \rho_k T^*_{bk} = \sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\gamma\sum_{i\in\Omega_b} w_{ik}}\right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\lambda^*_{bt}}\right)^{\frac{1}{\gamma-1}}$$

Hence, combining the last two equations, we obtain:

$$T^*_{bk} = \left(\frac{\rho_k}{\sum_{i\in\Omega_b} w_{ik}}\right)^{\frac{1}{\gamma-1}} \left(\sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\sum_{i\in\Omega_b} w_{ik}}\right)^{\frac{1}{\gamma-1}}\right)^{-1} e_{bt} \tag{E.1}$$

Allowing for a multiplicative error in the measurement of ρ_k, this implies

$$\begin{aligned}\log\left(\frac{\rho_k}{\sum_{i\in\Omega_b} w_{ik}}\right) =& \tau_{t(k)} - (1-\gamma)\log(T^*_{bk}/e_{bt(k)}) + \omega_k \\ =& \tau_{t(kb)} - (1-\gamma)\log(T^*_{bk}) + \omega_k \end{aligned} \tag{E.2}$$

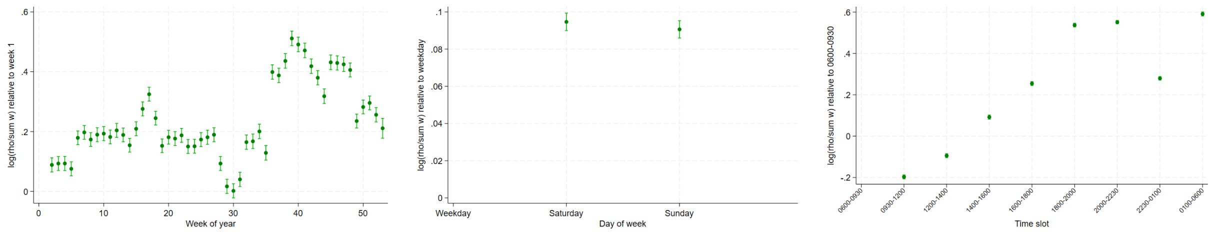
where $\tau_{t(kb)}$ is a slot-brand fixed effect.

We estimate equation (E.2) using 2015 television advertising data for all food and drink brands. We aggregate the data slightly to the level of brand-station-week-slot type level, where slot type is defined by the interaction of weekday/Saturday/Sunday with thwo following time intervals: 1am-6am, 6am-9.30am, 9.30am-12pm, 12pm-2pm, 2pm-4pm, 4pm-6pm, 6pm-10pm, 10pm-10.30pm and 10.30pm-1.00am. We measure price per view, $\frac{\rho_k}{\sum_i w_{ik}}$, as the advertising spend for brand-station-week-slot type divided by rate card-weighted television rating among adult viewers.

Figure E.1 illustrates variation in these prices, plotting mean differences across weeks, days of the week, and time slots. These patterns align with intuition—for instance, advertising tends to be more expensive (and impactful) during Easter and Christmas, on weekends, and in the evening.

Figure E.1: *Variation in* $\log(\frac{\rho_k}{\sum_i w_{ik}})$

(a) by week (b) by day of week (c) by time slot



We measure advertising length, T^*_{bk}, as advertising duration in seconds. We report estimates in Table E.1. These correspond to the $\hat{\gamma} = 0.64$ (with p-value is smaller than 0.0001) reported in the paper.

Table E.1: *Estimation of* γ

	$\log\left(\frac{\rho_k}{\sum_i w_{ik}}\right)$
$-(1-\gamma)$	-0.358
	0.001
Constant	10.268
	0.005
Brand-week fixed effects	yes
R-Square	0.08
N	2,503,591

F Additional Estimation Results

Our purchase data covers 21,710 households and 2,585,650 choice occasions (i.e., weeks in which a drink is purchased). To estimate our demand model, we randomly select up to 1,000 households from each of the 12 demographic groups and up to 25 choice occasions per household. This results in 267,677 choice occasions, which we use for estimation. We estimate the model separately for each demographic group, allowing all parameters to vary across groups. We use simulated maximum likelihood, approximating each random coefficient integral using 50 Modified Latin Hypercube draws per observation (see Hess et al. 2006) and allowing for correlated draws for the price and advertising coefficients. Table F.1 reports the parameter estimates, omitting product and time-effects for brevity.

In Table F.2 we report selected mean product-level price elasticities. In Table F.3 we report product-level mean marginal cost and markups.

Table F.1: *Coefficient estimates*

	No kids				Pensioner			
Inc. qrt	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Price	0.173	0.174	0.050	-0.087	0.017	0.086	-0.130	0.012
	(0.040)	(0.033)	(0.034)	(0.037)	(0.039)	(0.040)	(0.054)	(0.058)
Adv	-1.074	-1.591	-2.217	-1.415	-1.637	-1.215	-0.981	-0.981
	(0.147)	(0.215)	(0.279)	(0.191)	(0.304)	(0.200)	(0.188)	(0.272)
Price (σ^2)	0.180	0.129	0.164	0.151	0.147	0.172	0.340	0.198
	(0.017)	(0.012)	(0.016)	(0.016)	(0.014)	(0.019)	(0.045)	(0.029)
Adv (σ^2)	0.475	0.597	1.766	0.642	0.559	0.517	0.426	0.383
	(0.088)	(0.104)	(0.281)	(0.151)	(0.186)	(0.137)	(0.091)	(0.185)
Price-Adv (COV)	0.283	0.276	0.463	0.311	0.079	0.293	0.348	0.207
	(0.031)	(0.027)	(0.040)	(0.041)	(0.021)	(0.044)	(0.049)	(0.057)
Coke (σ^2)	2.390	2.062	1.921	2.385	2.640	1.563	2.354	1.834
	(0.192)	(0.148)	(0.139)	(0.171)	(0.215)	(0.134)	(0.209)	(0.221)
Pepsi (σ^2)	3.834	3.943	3.556	5.882	5.451	3.831	4.448	2.941
	(0.240)	(0.260)	(0.248)	(0.358)	(0.385)	(0.302)	(0.359)	(0.390)
Sugary (σ^2)	1.731	2.029	1.898	2.702	2.150	2.079	2.358	2.254
	(0.088)	(0.099)	(0.098)	(0.130)	(0.104)	(0.105)	(0.153)	(0.161)
Adv within firm	0.126	0.076	0.142	0.066	0.234	0.299	0.118	0.364
	(0.062)	(0.057)	(0.053)	(0.056)	(0.065)	(0.065)	(0.081)	(0.097)
Adv across firm	0.190	-0.028	0.096	0.107	0.440	0.303	0.093	-0.292
	(0.061)	(0.060)	(0.057)	(0.062)	(0.070)	(0.071)	(0.089)	(0.108)
Entertainment× Coke	1.156	-0.858	0.234	-1.477	0.393	1.418	-0.997	1.765
	(0.454)	(0.440)	(0.353)	(0.500)	(0.515)	(0.544)	(0.564)	(0.720)
Shows× Coke	-0.101	-0.130	-0.505	0.023	0.479	-1.428	1.306	0.680
	(0.335)	(0.299)	(0.225)	(0.271)	(0.297)	(0.371)	(0.354)	(0.570)
Factual× Coke	0.797	0.699	-0.498	0.705	0.114	-0.106	-0.298	-0.484
	(0.314)	(0.289)	(0.279)	(0.297)	(0.271)	(0.320)	(0.451)	(0.492)
Drama× Coke	-1.260	-0.031	0.326	-0.936	-0.272	-0.088	1.318	-1.430
	(0.361)	(0.315)	(0.374)	(0.323)	(0.324)	(0.308)	(0.378)	(0.504)
Reality× Coke	-1.157	1.698	0.810	-0.862	0.533	-1.309	1.034	2.575
	(0.434)	(0.456)	(0.437)	(0.461)	(0.536)	(0.604)	(0.716)	(0.946)
Sports× Coke	1.057	0.602	-0.031	-0.197	-1.221	-0.273	-0.513	0.025
	(0.175)	(0.186)	(0.169)	(0.167)	(0.182)	(0.159)	(0.193)	(0.270)
Entertainment× Pepsi	-0.909	0.380	0.056	0.558	-2.768	1.830	-2.161	-2.044
	(0.463)	(0.517)	(0.447)	(0.521)	(0.624)	(0.585)	(0.731)	(0.924)
Shows× Pepsi	0.865	-0.880	-1.200	-1.648	-0.199	-2.538	0.806	3.575
	(0.297)	(0.362)	(0.420)	(0.394)	(0.399)	(0.403)	(0.445)	(0.448)
Factual× Pepsi	-1.052	-1.120	1.006	1.785	0.679	0.612	-0.597	-2.840
	(0.340)	(0.347)	(0.405)	(0.514)	(0.442)	(0.397)	(0.501)	(0.703)
Drama× Pepsi	-0.498	0.791	-0.057	0.642	-0.365	-0.293	1.336	2.083
	(0.387)	(0.369)	(0.476)	(0.476)	(0.368)	(0.365)	(0.489)	(0.604)
Reality× Pepsi	1.210	3.152	2.082	0.588	1.341	3.091	2.704	0.546
	(0.450)	(0.662)	(0.727)	(0.602)	(0.604)	(0.590)	(0.787)	(1.267)
Sports× Pepsi	0.628	0.728	-0.042	-0.226	-1.301	0.754	0.356	-0.262
	(0.177)	(0.217)	(0.235)	(0.197)	(0.226)	(0.204)	(0.253)	(0.326)
ITV× Coke	0.480	-0.237	0.126	0.188	-0.180	0.216	-0.376	-0.600
	(0.169)	(0.118)	(0.097)	(0.114)	(0.110)	(0.128)	(0.129)	(0.183)
C4× Coke	-0.105	0.007	0.192	-0.222	-0.388	-0.428	0.015	-0.515
	(0.123)	(0.126)	(0.102)	(0.105)	(0.124)	(0.109)	(0.178)	(0.196)
C5× Coke	-0.166	-0.635	-0.219	-0.191	-0.239	-0.024	-0.239	0.132
	(0.123)	(0.130)	(0.110)	(0.108)	(0.120)	(0.106)	(0.160)	(0.180)
Cable× Coke	0.984	0.380	0.331	0.633	-0.141	0.273	0.202	-0.082
	(0.138)	(0.119)	(0.112)	(0.111)	(0.121)	(0.116)	(0.130)	(0.181)
ITV× Pepsi	-0.257	-0.681	-0.335	0.327	0.097	-0.087	-0.200	-0.262
	(0.153)	(0.141)	(0.118)	(0.176)	(0.143)	(0.161)	(0.201)	(0.266)
C4× Pepsi	0.035	0.020	0.233	0.516	-0.348	-0.571	0.144	0.441
	(0.118)	(0.138)	(0.134)	(0.152)	(0.143)	(0.154)	(0.227)	(0.327)
C5× Pepsi	0.089	0.243	-0.312	-0.926	0.044	0.120	-1.001	-0.031
	(0.124)	(0.132)	(0.202)	(0.169)	(0.138)	(0.148)	(0.186)	(0.314)
Cable× Pepsi	-0.102	0.157	0.097	1.079	0.806	0.073	-0.097	0.694
	(0.134)	(0.133)	(0.144)	(0.151)	(0.144)	(0.158)	(0.149)	(0.220)
Wkend-prime× Coke	0.289	-0.152	-0.054	-0.369	-0.781	-1.306	0.818	-0.244
	(0.222)	(0.170)	(0.140)	(0.168)	(0.229)	(0.238)	(0.311)	(0.307)
Wkend-non prime× Coke	-0.337	-0.394	-0.513	0.505	-0.155	0.777	0.490	-0.298
	(0.168)	(0.127)	(0.113)	(0.134)	(0.170)	(0.162)	(0.211)	(0.252)
Wkday-prime× Coke	-0.368	0.380	0.403	-0.169	0.140	0.326	0.007	-0.479
	(0.277)	(0.203)	(0.183)	(0.168)	(0.281)	(0.300)	(0.267)	(0.313)
Wkday-non prime× Coke	-0.500	0.145	0.278	-0.106	-0.066	-0.390	0.379	-0.198
	(0.168)	(0.144)	(0.105)	(0.117)	(0.181)	(0.187)	(0.194)	(0.183)
Wkend-prime× Pepsi	-0.092	-0.496	-0.173	-0.607	0.290	-0.239	0.595	0.604
	(0.206)	(0.209)	(0.216)	(0.207)	(0.357)	(0.293)	(0.352)	(0.504)
Wkend-non prime× Pepsi	0.065	0.383	0.533	-0.226	-0.372	0.821	-0.569	0.544
	(0.162)	(0.175)	(0.152)	(0.187)	(0.241)	(0.219)	(0.220)	(0.284)
Wkday-prime× Pepsi	0.517	0.570	-0.208	-1.041	1.133	0.511	0.428	-0.548
	(0.220)	(0.281)	(0.231)	(0.281)	(0.422)	(0.383)	(0.341)	(0.406)
Wkday-non prime× Pepsi	0.233	0.062	-0.236	-0.183	-0.844	-0.360	0.295	-0.031
	(0.150)	(0.161)	(0.152)	(0.155)	(0.241)	(0.215)	(0.211)	(0.277)
Viewing hours× Coke	-0.125	0.007	-0.060	-0.043	-0.389	-0.048	-0.105	0.072
	(0.087)	(0.077)	(0.072)	(0.063)	(0.087)	(0.087)	(0.112)	(0.079)
Viewing hours× Pepsi	-0.262	-0.188	-0.141	0.238	-0.600	-0.170	-0.219	-0.039
	(0.064)	(0.075)	(0.074)	(0.103)	(0.107)	(0.117)	(0.148)	(0.158)

Coefficient estimates cont.

	Family			
Inc. qrt	Q1	Q2	Q3	Q4
Price	0.154	0.149	0.092	-0.036
	(0.031)	(0.032)	(0.033)	(0.033)
Adv	-2.754	-1.658	-2.210	-1.372
	(0.652)	(0.232)	(0.332)	(0.166)
Price (σ^2)	0.145	0.118	0.159	0.118
	(0.012)	(0.011)	(0.014)	(0.013)
Adv (σ^2)	0.777	0.659	0.889	0.451
	(0.424)	(0.194)	(0.257)	(0.082)
Price-Adv (COV)	-0.015	0.229	0.339	0.230
	(0.013)	(0.040)	(0.053)	(0.027)
Coke (σ^2)	2.448	2.401	2.059	1.983
	(0.172)	(0.174)	(0.156)	(0.136)
Pepsi (σ^2)	3.169	3.999	4.178	3.677
	(0.229)	(0.251)	(0.338)	(0.238)
Sugary (σ^2)	1.773	1.904	1.909	1.720
	(0.088)	(0.096)	(0.096)	(0.088)
Adv within firm	0.063	0.065	0.046	0.123
	(0.053)	(0.055)	(0.054)	(0.054)
Adv across firm	0.134	0.034	0.080	-0.124
	(0.057)	(0.058)	(0.057)	(0.058)
Entertainment× Coke	-0.283	0.325	-1.250	-0.065
	(0.331)	(0.375)	(0.392)	(0.402)
Shows× Coke	0.346	-0.789	0.825	-0.050
	(0.259)	(0.295)	(0.248)	(0.250)
Factual× Coke	0.391	0.297	-0.422	-0.842
	(0.279)	(0.261)	(0.256)	(0.252)
Drama× Coke	-1.472	0.862	-0.222	0.330
	(0.389)	(0.349)	(0.422)	(0.444)
Reality× Coke	1.619	-0.915	1.702	1.238
	(0.357)	(0.367)	(0.452)	(0.441)
Sports× Coke	-0.610	0.016	-0.819	0.434
	(0.154)	(0.177)	(0.210)	(0.153)
Entertainment× Pepsi	0.598	0.219	-0.825	0.230
	(0.372)	(0.489)	(0.403)	(0.500)
Shows× Pepsi	0.402	0.518	0.338	-1.426
	(0.254)	(0.353)	(0.303)	(0.309)
Factual× Pepsi	-0.759	-1.878	0.383	0.998
	(0.308)	(0.309)	(0.311)	(0.390)
Drama× Pepsi	-1.698	0.193	-0.452	0.691
	(0.370)	(0.486)	(0.401)	(0.852)
Reality× Pepsi	3.237	-0.486	-0.024	1.898
	(0.414)	(0.418)	(0.669)	(0.528)
Sports× Pepsi	-0.086	0.017	-0.173	0.152
	(0.196)	(0.210)	(0.212)	(0.192)
ITV× Coke	0.109	0.083	-0.105	-0.308
	(0.113)	(0.112)	(0.161)	(0.107)
C4× Coke	-0.493	0.452	0.001	-0.559
	(0.119)	(0.108)	(0.119)	(0.105)
C5× Coke	-0.358	-0.390	-0.090	-0.273
	(0.113)	(0.108)	(0.125)	(0.146)
Cable× Coke	0.188	0.134	0.339	-0.051
	(0.117)	(0.129)	(0.146)	(0.102)
ITV× Pepsi	0.103	0.002	-0.766	0.400
	(0.123)	(0.131)	(0.167)	(0.140)
C4× Pepsi	-0.635	0.472	0.393	-1.129
	(0.144)	(0.127)	(0.119)	(0.134)
C5× Pepsi	-0.160	0.223	0.427	0.135
	(0.137)	(0.122)	(0.153)	(0.145)
Cable× Pepsi	0.174	0.616	-0.031	0.568
	(0.131)	(0.125)	(0.141)	(0.150)
Wkend-prime× Coke	-0.167	0.234	-0.518	-0.038
	(0.157)	(0.163)	(0.198)	(0.141)
Wkend-non prime× Coke	0.069	-0.115	0.477	-0.023
	(0.122)	(0.128)	(0.146)	(0.123)
Wkday-prime× Coke	0.293	-0.073	0.327	0.082
	(0.171)	(0.213)	(0.193)	(0.149)
Wkday-non prime× Coke	-0.241	-0.059	0.190	0.402
	(0.113)	(0.113)	(0.130)	(0.104)
Wkend-prime× Pepsi	0.338	-0.182	0.608	-0.515
	(0.183)	(0.218)	(0.236)	(0.184)
Wkend-non prime× Pepsi	-0.280	0.216	-0.221	-0.076
	(0.128)	(0.135)	(0.216)	(0.188)
Wkday-prime× Pepsi	0.352	0.543	-0.080	0.478
	(0.192)	(0.226)	(0.203)	(0.203)
Wkday-non prime× Pepsi	0.213	-0.400	0.852	0.069
	(0.122)	(0.130)	(0.190)	(0.170)
Viewing hours× Coke	0.014	0.118	-0.103	0.059
	(0.087)	(0.087)	(0.079)	(0.056)
Viewing hours× Pepsi	-0.074	0.158	0.001	-0.031
	(0.104)	(0.078)	(0.074)	(0.080)

Table F.2: *Product level price elasticities*

	Reg Coke		Diet Coke		Reg Pepsi		Diet Pepsi	
	2l	10×330ml	2l	10×330ml	2l	8×330ml	2l	10×330ml
Regular Coke: 1.5l	0.047	0.041	0.024	0.034	0.037	0.012	0.062	0.024
Regular Coke: 2l	-1.915	0.044	0.024	0.040	0.039	0.013	0.061	0.024
Regular Coke: 10x330ml	0.023	-3.829	0.013	0.044	0.035	0.014	0.058	0.033
Regular Coke: 24x330ml	0.012	0.051	0.006	0.044	0.029	0.015	0.046	0.037
Diet Coke: 1.5l	0.024	0.021	0.049	0.059	0.018	0.006	0.099	0.038
Diet Coke: 2l	0.023	0.024	-1.793	0.069	0.020	0.006	0.097	0.038
Diet Coke: 10x330ml	0.012	0.026	0.021	-3.844	0.016	0.007	0.085	0.051
Diet Coke: 24x330ml	0.007	0.026	0.011	0.078	0.014	0.007	0.072	0.056
Reg Pepsi: 2l	0.008	0.013	0.004	0.011	-2.019	0.091	0.361	0.156
Regular Pepsi: 8x330ml	0.007	0.015	0.004	0.012	0.242	-2.890	0.332	0.171
Diet Pepsi: 1.5l	0.005	0.006	0.008	0.014	0.117	0.037	0.565	0.214
Diet Pepsi: 2l	0.005	0.007	0.008	0.019	0.119	0.041	-1.951	0.240
Diet Pepsi: 8x330ml	0.004	0.008	0.006	0.022	0.101	0.042	0.473	-3.302
Regular store: 2l	0.011	0.015	0.006	0.012	0.047	0.016	0.073	0.030
Diet store: 2l	0.006	0.008	0.011	0.022	0.024	0.008	0.116	0.048
Regular outside	0.011	0.012	0.007	0.011	0.039	0.012	0.068	0.026
Diet outside	0.007	0.007	0.012	0.019	0.021	0.007	0.108	0.040

Table F.3: *Product level markups*

Firm	Brand	Pack	Marginal cost (£/l)	Price-cost margin (£/l)	Lerner index
Coca Cola Enterprises	Regular Coke	Bottle(s): 1.25l: Single	0.07	0.77	0.92
		Bottle(s): 1.5l: Single	0.21	0.71	0.77
		Bottle(s): 1.75l: Single	0.12	0.78	0.87
		Bottle(s): 1.75l: Multiple	0.33	0.41	0.56
		Cans: 10x330ml: Single	0.60	0.42	0.41
		Cans: 12x330ml: Single	0.57	0.38	0.40
		Cans: 15x330ml: Single	0.58	0.39	0.40
		Cans: 24x330ml: Single	0.58	0.24	0.29
		Bottle(s): 2l: Single	0.17	0.70	0.80
		Bottle(s): 2l: Multiple	0.30	0.34	0.53
		Cans: 30x330ml: Single	0.56	0.24	0.30
		Bottle(s): 3l: Single	0.29	0.30	0.50
		Bottle(s): 4x1.5l: Single	0.41	0.31	0.43
		Cans: 6x330ml: Single	0.73	0.64	0.47
		Cans: 8x330ml: Single	0.57	0.42	0.42
	Diet Coke	Bottle(s): 1.25l: Single	0.03	0.82	0.96
		Bottle(s): 1.5l: Single	0.10	0.70	0.88
		Bottle(s): 1.75l: Single	0.09	0.79	0.90
		Bottle(s): 1.75l: Multiple	0.31	0.41	0.56
		Cans: 10x330ml: Single	0.59	0.42	0.42
		Cans: 12x330ml: Single	0.56	0.37	0.40
		Cans: 15x330ml: Single	0.50	0.39	0.44
		Cans: 24x330ml: Single	0.58	0.25	0.30
		Bottle(s): 2l: Single	0.03	0.67	0.96
		Bottle(s): 2l: Multiple	0.26	0.33	0.56
		Cans: 30x330ml: Single	0.56	0.24	0.30
		Bottle(s): 3l: Single	0.30	0.28	0.48
		Bottle(s): 4x1.5l: Single	0.44	0.32	0.42
		Cans: 6x330ml: Single	0.69	0.55	0.44
		Cans: 8x330ml: Single	0.58	0.41	0.42
Pepsico	Regular Pepsi	Bottle(s): 2l: Single	0.14	0.38	0.74
		Cans: 6x330ml: Single	0.27	0.59	0.68
		Cans: 8x330ml: Single	0.36	0.47	0.56
	Diet Pepsi	Bottle(s): 1.5l: Single	-0.03	0.66	1.04
		Cans: 12x330ml: Single	0.49	0.48	0.49
		Bottle(s): 2l: Single	0.16	0.37	0.70
		Cans: 6x330ml: Single	0.28	0.59	0.68
		Cans: 8x330ml: Single	0.44	0.41	0.48

G Transition Function

We posit that firms track a summary statistic of the brand-specific consumer exposure distribution and present evidence that doing so results in negligible prediction error. Specifically, we assume that the state space consists of the expected value of the exposure stock distribution for each brand, denoted as $(\mathbb{A}_{1t}, \ldots, \mathbb{A}_{Bt})$, where $\mathbb{A}_{bt} = \frac{1}{I}\sum_i \mathcal{A}_{ibt} = \delta\mathbb{A}_{bt-1} + \mathbb{a}_{bt-1}$, and where $\mathbb{a}_{bt} = \frac{1}{I}\sum_i a_{ibt}$ is the average flow exposure. This sum is taken over the set of soft drinks consumers, consistent with them being the targeted population in the agency problem (equation (7)). Alternatively, firms could track exposure stocks among a subset of this population. We experimented with the possibility that firms track exposure stocks for specific demographic groups. However, since average stocks across groups tend to co-move, this results in qualitatively similar outcomes in the dynamic game.

By tracking the mean of the distribution, firms make a prediction error in their demands, equal to $s_{jt}(\mathbf{p}_t, \mathbb{A}_{1t}, \ldots, \mathbb{A}_{Bt}) - E_{\mathcal{A}_t}[s_{jt}(\mathbf{p}_t, \mathcal{A}_{i1t}, \ldots, \mathcal{A}_{iBt})]$. In practice, this error is small, with the average absolute error (across products) being 2% of product level demands. This occurs because errors are upward for consumers who are more exposed than the mean and downward for those less exposed than the mean, and thus those errors tend to compensate each other on average.

Combining the consumer-level advertising exposure (equation (1)) with our estimate of the optimal condition for the choice of advertising slots (captured by our estimate of the curvature parameter for $\omega(\cdot)$ in equation (8), γ), the evolution of the brand b state variable can be rewritten as $\mathbb{A}_{bt} = \delta\mathbb{A}_{bt-1} + \lambda_{t-1}e^{\gamma}_{bt-1}$, where λ_{t-1} is a period specific rate of transformation of advertising expenses into additional brand-level advertising exposure, and depends on advertising slot prices (see below).

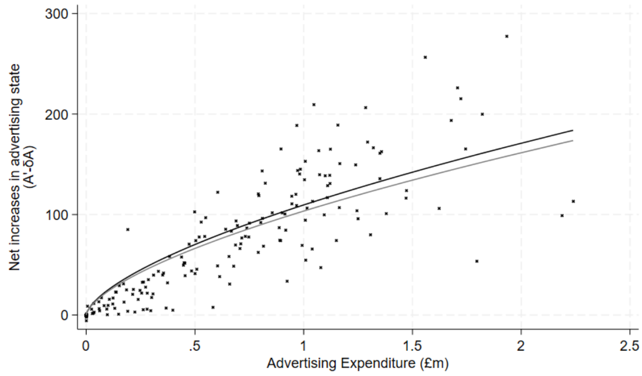
Firms do not observe the realization of λ_{t-1} when making decisions about their advertising budgets e_{bt-1} (since slot advertising prices are not yet known). Therefore, at this point in time, λ_{t-1} is a random variable. We assume that firms form expectations about changes in the advertising state conditional on expenditure, which implies the stock satisfies:

$$\mathbb{A}_{bt} - \delta\mathbb{A}_{bt-1} = \lambda e^{\gamma}_{bt-1} + \upsilon_{bt-1}, \tag{G.1}$$

where $\upsilon_{bt-1} = (\lambda_{t-1} - \lambda)e^{\gamma}_{bt-1}$. We estimate this equation with linear methods (as γ is already known).

Table G.1: *Advertising state law of motion*

	$\mathbb{A}_{bt} - \delta\mathbb{A}_{bt-1}$ (1)	$\mathbb{A}_{bt} - \delta\mathbb{A}_{bt-1}$ (2)
e_{bt-1} $(\hat{\lambda})$	0.0153	0.0145
	(0.0004)	(0.0006)
$\text{var}(v_{bt-1})$	776	800
r^2	0.8751	0.8728
N	249	246
Instrument	No	Yes



Notes: Table shows estimates of equation (G.1). Column (1) are OLS estimates, column (2) are IV estimates instrumenting e^{λ}_{bt-1} with $\mathbb{A}_{bt-2}$. The figure shows a scatter plot of monthly advertising expenditure, e_{bt-1}, and net changes in the advertising state, $\mathbb{A}_{bt} - \delta\mathbb{A}_{bt-1}$ (across brands and year-months). The black line is based on the OLS estimate and the grey line on the IV estimate (in both cases with $\gamma = 0.64$).

Column (1) in Table G.1 shows estimates of λ and the variance of the error term under the assumption that $\mathbb{E}[v_{bt-1}|e_{bt-1}] = 0$ (which holds if $\mathbb{E}[\lambda_{t-1}|e_{bt-1}] = \lambda$). In column (2), we allow for this possibility that $\mathbb{E}[v_{bt-1}|e_{bt-1}] \neq 0$ by instrumenting e^{γ}_{bt-1} with the two period lagged mean advertising stock $\mathbb{A}_{bt-2}$. Since this variable is observed, it is included in firms' information sets when they choose advertising expenditure e_{bt-1}. Moreover, given the likely diminishing returns to investment in a brand's advertising stock, $\mathbb{A}_{bt-2}$ is likely to influence the firm's flow investment decision. We find that instrumenting with $\mathbb{A}_{bt-2}$ leads to a modest decline in $\hat{\lambda}$ relative to column (1). Additionally, we include a scatter plot of the underlying data and plot the relationship implied between the change in net stock and advertising investment, which shows that the implied relationship is very similar across both sets of estimates.

To solve for a Markov Perfect Equilibrium we discretize the state space. Specifically, for a set of evenly spaced discrete values $\{A_1, \dots, A_K\}$, where $A_1 = 0$, we use the state transition function:

$$P(\mathbb{A}_{bt} = \mathbb{A}_{k'}|\mathbb{A}_{bt-1} = \mathbb{A}_k, e_{bt-1}) = \int_{\mathbb{A}_{k'-1}}^{\mathbb{A}_{k'}} f_v(\mathbb{A}_{bt} - \delta\mathbb{A}_k - \lambda e^{\gamma}_{bt-1}) \frac{\mathbb{A}_{bt} - \mathbb{A}_{k-1}}{\mathbb{A}_{k'} - \mathbb{A}_{k'-1}} d\mathbb{A}_{bt} + \int_{\mathbb{A}_{k'}}^{\mathbb{A}_{k'+1}} f_v(\mathbb{A}_{bt} - \delta\mathbb{A}_k - \lambda e^{\gamma}_{bt-1}) \frac{\mathbb{A}_{k'+1} - \mathbb{A}_{bt}}{\mathbb{A}_{k'+1} - \mathbb{A}_{k'}} d\mathbb{A}_{bt}. \quad (G.2)$$

Since there are three advertising states—one for Regular Coke, Diet Coke and Diet Pepsi—the state grid $\{A_1, ..., A_K\}^3$ has dimension K^3. We set a value for A_K above the 99^{th} percentile of observed mean stocks in the data and check ex post that the maximum state has zero probability mass in the equilibrium ergodic distribution. We use an evenly spaced grid and set $K = 21$, meaning there are 9,261 points in the discretized state space.

G.1 State Transition Function

The mean exposure flow for brand b advertising is

$$\mathbb{a}_{bt} = \frac{1}{I} \sum\nolimits_i \sum\nolimits_{\{k|t(k)=t\}} w_{ik} \omega(T_{bk}^*),$$

and the mean exposure stock is

$$\mathbb{A}_{bt} = \sum_{s=0}^{t-1} \delta^{t-1-s} \mathbb{a}_{bs} = \delta \mathbb{A}_{bt-1} + \mathbb{a}_{bt-1}.$$

Given our power function specification for $\omega(.)$, $\omega(T_{bk}^*) = T_{bk}^{*\gamma}$, and the optimality condition for T_{bk}^* (equation (E.1)), this implies that

$$\begin{aligned}
\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1} =& \frac{1}{I} \sum\nolimits_i \sum\nolimits_{\{k|t(k)=t-1\}} w_{ik} T_{bk}^{*\gamma} \\
=& \frac{1}{I} \sum\nolimits_i \sum\nolimits_{\{k|t(k)=t-1\}} w_{ik} \left(\left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \left(\sum_{\{k|t(k)=t\}} \rho_k \left(\frac{\rho_k}{\sum_i w_{ik}} \right)^{\frac{1}{\gamma-1}} \right)^{-1} \right)^{\gamma} e_{bt-1}^{\gamma} \\
\equiv& \lambda_{t-1} e_{bt-1}^{\gamma}
\end{aligned}$$

Defining λ as $\mathbb{E}[\mathbb{A}_{bt} - \delta \mathbb{A}_{t-1}] = \lambda e_{bt-1}^{\gamma}$, we get

$$\mathbb{A}_{bt} - \delta \mathbb{A}_{bt-1} = \lambda e_{bt-1}^{\gamma} + \nu_{bt-1}$$

with $\nu_{bt-1} = (\lambda_{t-1} - \lambda) e_{bt-1}^{\gamma}$.

H Solution Algorithm

Our solution algorithm is similar in spirit to that of Pakes and McGuire (1994).

State space discritization. The state space consists of the expected value of the exposure stock for each of brand, $(\mathbb{A}_{1t}, \ldots, \mathbb{A}_{Bt})$. In our application $B = 3$, (corresponding to Regular Coke (RC), Diet Coke (DC) and Diet Pepsi (DP)). For each b, we discretize the state spaced into $K = 21$ evenly spaced values, $A_1, \ldots, A_K$. We set a value for A_K above the

99^{th} percentile of observed mean stocks in the data and check ex post that the maximum state has zero probability mass in the equilibrium ergodic distribution. The state space is of dimension $21^3 = 9,261$. Denote by $a_{\Bbbk}$ a single point in the state space grid, which corresponds to discrete advertising levels for each brand, i.e., $(A_{RC,k}, A_{DC,k'}, A_{DP,k''})$ where $k, k', k'' \in \{1, \ldots, 21\}$.

Profit function. In our application there are two firms, $f = \{C, P\}$, which correspond to Coca Cola Enterprises and Pepsico. Denote the state-specific gross profit function (i.e., prior to deducting any advertising expenditure) of firm f by $\pi_f(a_{\Bbbk})$. Note, $\pi_f(a_{\Bbbk})$ is evaluated at the state-specific equilibrium price vector $\mathbf{p}(a_{\Bbbk})$. We compute $\pi_f(a_{\Bbbk})$ for $f \in \{C, P\}$ in each of the 9,261 states. This entails, at each point in the state space grid, solving the price vector that satisfies the set of first-order conditions (equation (5)). In matrix notation, these conditions are:

$$\mathbf{p}(a_{\Bbbk}) = \mathbf{c} - \left[\mathbf{\Gamma} \circ \left(\frac{\partial \mathbf{q}(a_{\Bbbk}, \mathbf{p}(a_{\Bbbk}))}{\partial \mathbf{p}}\right)\right]^{-1} \mathbf{q}(a_{\Bbbk}, \mathbf{p}(a_{\Bbbk}))$$

where $\mathbf{\Gamma}$ is the product ownership matrix. Re-write this as $\mathbf{p}_{\Bbbk} = f_{\Bbbk}(\mathbf{p}_{\Bbbk})$. We start with an initial guess of $\mathbf{p}^r_{\Bbbk}$, compute $\mathbf{p}^{r+1}_{\Bbbk} = f_{\Bbbk}(\mathbf{p}^r_{\Bbbk})$ and continue updating until $||\mathbf{p}^{r+1}_{\Bbbk} - \mathbf{p}^r_{\Bbbk}|| = \max |\mathbf{p}^{r+1}_{\Bbbk} - \mathbf{p}^r_{\Bbbk}| < 10^{-4}$. Once we have obtained state-specific equilibrium prices we also compute the state-specific equilibrium quantity vector, $\mathbf{q}(a_{\Bbbk})$, and consumer surplus, $\mathrm{CS}(a_{\Bbbk})$.

Our counterfactual simulations entail the imposition of a specific and (separately) an ad valorem tax. In order to implement these counterfactuals we must repeat the computation of the state-specific profit functions with each tax in place.

Bellman equations. Let $a = (a_{RC}, a_{DC}, a_{DP})$ denote the current levels of the Regular Coke, Diet Coke and Diet Pepsi advertising states. The two firms value functions are joint solutions of:

$$\begin{aligned} V_C(a, e_{RC}, e_{DC}) = \pi_C(a) + \max_{e_{RC}, e_{DC} \in R^+} \Bigg\{ &- (\psi_{RC} e_{RC} + \psi_{DC} e_{DC}) + \beta \sum_{a'_{RC}, a'_{DC}} \\ &\bar{V}_C(a'_{RC}, a'_{DC}, e_{RC}, e_{DC}) p(a'_{RC}|a_{RC}, e_{RC}) p(a'_{DC}|a_{DC}, e_{DC}) \Bigg\} \end{aligned} \tag{H.1}$$

$$V_P(a, e_{DP}) = \pi_P(a) + \max_{e_{DP} \in R^+} \Bigg\{ - \psi_{DP} e_{DP} + \beta \sum_{a'_{DP}} \bar{V}_P(a'_{DP}, e_{DP}) p(a'_{DP}|a_{DP}, e_{DP}) \Bigg\}, \tag{H.2}$$

where

$$\bar{V}_C(a'_{RC}, a'_{DC}, e_{RC}, e_{DC}) = \sum_{a'_{DP}} V_C(a', e_{RC}, e_{DC}) p(a'_{DP}|a_{DP}, e_{DP})$$

$$\bar{V}_P(a'_{DP}, e_{DP}) = \sum_{a'_{RC}, a'_{DC}} V_P(a', e_{RC}, e_{DC}) p(a'_{RC}|a_{RC}, e_{RC}) p(a'_{DC}|a_{DC}, e_{DC}),$$

and the transition function, $p(a'_b|a_b, e_b)$, is given by equation (G.2).

Solving for the MPE. The solution algorithm is as follows:

1. Start with an initial guess of optimal advertising expenditures and value functions in each advertising state. When solving for the no tax equilibrium we use as starting values, for all $\mathbb{k}$:

$$e^l_{RC}(a_{\mathbb{k}}) = e^l_{DC}(a_{\mathbb{k}}) = 0.3e^6, \quad e^l_{DP}(a_{\mathbb{k}}) = 0.2e^6 \quad V^l_C(a_{\mathbb{k}}) = \frac{\pi_C}{1-\beta} \quad V^l_P(a_{\mathbb{k}}) = \frac{\pi_P}{1-\beta}$$

   When solving for the specific or ad valorem tax equilibrium we use the optimal values from the no tax equilibrium as starting values.

2. For each point in the state space, $\mathbb{k}$, use equations (H.1) and (H.2), evaluated at the initial guess of $(V^l_C(a_{\mathbb{k}}), V^l_R(a_{\mathbb{k}}), e^l_{CR}(a_{\mathbb{k}}), e^l_{CD}(a_{\mathbb{k}}), e^l_{PD}(a_{\mathbb{k}}))$ to solve for the optimal advertising expenditures $\tilde{e}^{l+1}_{CR}(a_{\mathbb{k}}), \tilde{e}^{l+1}_{CD}(a_{\mathbb{k}}), \tilde{e}^{l+1}_{PD}(a_{\mathbb{k}})$.

3. Use as the iteration $l+1$ advertising expenditures $e^{l+1}_b(a_{\mathbb{k}}) = (1-\lambda)e^l_b(a_{\mathbb{k}}) + \lambda\tilde{e}^{l+1}_b(a_{\mathbb{k}})$ with dampening parameter $\lambda = 0.5$.

4. Use these expenditures to evaluate the right hand side equations (H.1) and (H.2) and thereby update the value functions $(V^{l+1}_C(a_{\mathbb{k}}), V^{l+1}_P(a_{\mathbb{k}}))$.

5. Repeat steps 2-4 until the stopping criteria, for $f = \{C, P\}$:

$$\left|\left|\frac{V^{l+1}_f - V^l_f}{1+|V^l_f|}\right|\right| = \max_{\mathbb{k}} \left|\frac{V^{l+1}_f - V^l_f}{1+|V^l_f|}\right| < 10^{-6}$$

   is satisfied.

I Consumer Surplus Decomposition

Denote the advertising state-specific consumer surplus under regime $\chi \in \{\emptyset, \mathbb{s}, \mathbb{a}\}$ (corresponding to no-tax, specific tax and ad valorem tax), by $\text{cs}_\chi(\mathbb{A}, \mathbf{p}_\chi(\mathbb{A}))$, where $\mathbb{A} = \{\mathbb{A}\}_b$

denotes the value of the brand advertising state and $\mathbf{p}_\chi(\mathbb{A})$ the optimal price vector. Denote the equilibrium distribution over states in regime $\chi \in \{\emptyset, \mathbb{r}, \mathbb{s}, \mathbb{sr}, \mathbb{a}, \mathbb{ar}\}$ (where $\mathbb{r}$ corresponds to advertising restriction) by $g_\chi(\mathbb{A})$. Consider the change in equilibrium consumer surplus that results from the introduction of a specific tax (relative to when no tax is in place, and where advertising is unrestricted). This is given by:

$$\Delta \text{CS}_s = \int_{\mathbb{A}} \text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A}))g_s(\mathbb{A}) - \int_{\mathbb{A}} \text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A}))g_0(\mathbb{A}).$$

We decompose this into a static component, which reflects the change in the state-specific consumer surplus function, and a dynamic component, which reflects the change in the equilibrium distribution over states. In particular:

$$\Delta \text{CS}_s = \underbrace{\int_{\mathbb{A}} \left(\frac{1}{2}g_0(\mathbb{A}) + \frac{1}{2}g_s(\mathbb{A})\right)\left(\text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A})) - \text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A}))\right)}_{\text{static effect}} + \underbrace{\int_{\mathbb{A}} \left(\frac{1}{2}\text{cs}_s(\mathbb{A}, \mathbf{p}_s(\mathbb{A})) + \frac{1}{2}\text{cs}_0(\mathbb{A}, \mathbf{p}_0(\mathbb{A}))\right)\left(g_s(\mathbb{A}) - g_0(\mathbb{A})\right)}_{\text{dynamic effect}}.$$

We decompose the consumer surplus effects of the other policy interventions analogously. Notice that the advertising restriction only impacts the equilibrium distribution, so the impact of an advertising restriction (in the absence of any tax) engenders zero static effect.

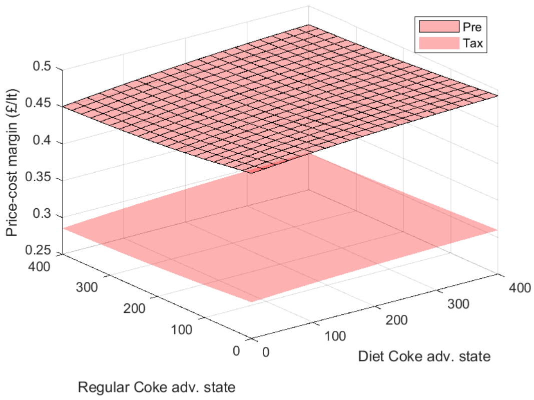
J Additional Counterfactual Results

In Figure J.1 we show the impact of the ad valorem tax on price-cost margins and the equilibrium distribution. The corresponding figure for a specific tax is reported in the main paper (Figure 6). Tables J.1 and J.2 shows aggregate effects by brand (providing a by-brand breakdown on Table 4). Table J.3 reports distributional effects, including the dynamic consumer surplus effect (Table 5 reports these results excluding the dynamic effect).

Figure J.1: *Impact of ad valorem tax and advertising restriction*

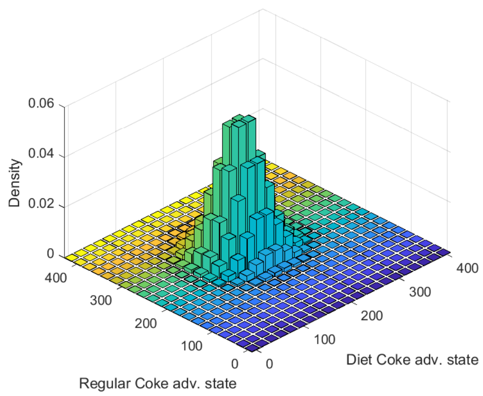
On static-specific optimal margins

(a) Average Regular Coke margins

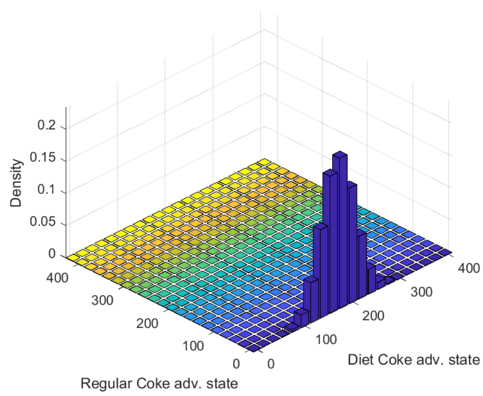


On equilibrium distribution

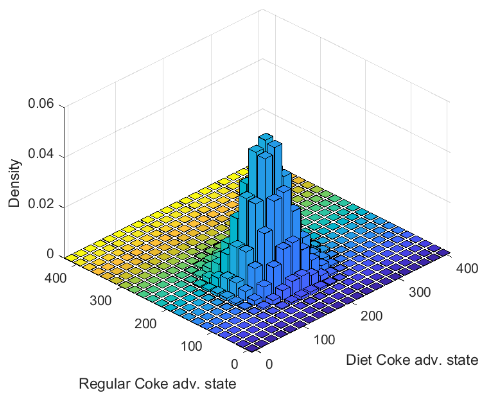
(b) Pre-policy



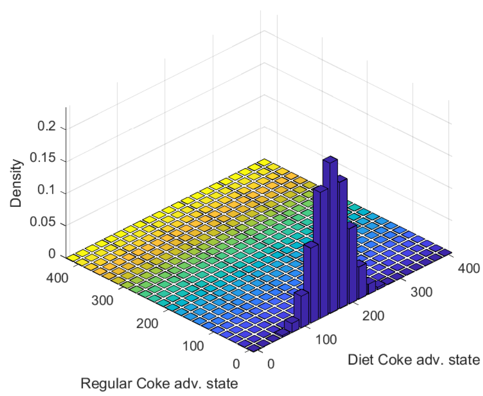
(c) Advertising restriction



(d) Tax



(e) Tax and advertising restriction



Notes: Panel (a) shows variation in the average price-cost margin for Regular Coke products. The hatched surface is pre-policy (and repeats Figure 3(a)) and the smooth surface corresponds to when an ad valorem tax is in place. In each case we hold fixed the Diet Pepsi advertising state at the highest probability state in the pre-policy equilibrium distribution. Panels (b)-(e) show the ergodic distribution, integrating over the Diet Pepsi advertising state space. Panel (b) repeats Figure 5(b).

Table J.1: *Aggregate impact of counterfactual policies, by brand*

	No tax	Specific tax			Ad valorem tax		
	Adv. restrict. (1)	Fixed adv. (2)	+ Eq. adv. response (3)	+ Adv. restrict. (4)	Fixed adv. (5)	+ Eq. adv. response (6)	+ Adv. restrict. (7)
Δ price							
Reg Coke	0.9%	28.2%	0.1%	0.6%	38.4%	0.1%	0.5%
Diet Coke	-1.3%	-1.6%	-0.1%	-0.8%	-1.6%	-0.2%	-0.7%
Reg Pepsi	-0.1%	34.2%	-0.0%	-0.1%	25.6%	-0.1%	-0.1%
Diet Pepsi	-0.0%	-0.6%	-0.0%	-0.0%	-0.2%	-0.0%	-0.0%
Δ margin							
Reg Coke	1.9%	5.0%	0.3%	1.3%	-34.6%	0.2%	0.7%
Diet Coke	-2.8%	-3.4%	-0.3%	-1.8%	-3.6%	-0.5%	-1.6%
Reg Pepsi	-0.1%	5.7%	-0.0%	-0.2%	-35.9%	-0.1%	-0.1%
Diet Pepsi	-0.0%	-0.9%	-0.0%	-0.0%	-0.3%	-0.1%	-0.0%
Δ advertising exp.							
Reg Coke	-100.0%	-	-33.1%	-100.0%	-	-47.3%	-100.0%
Diet Coke	-12.0%	-	-6.4%	-17.5%	-	-13.7%	-23.5%
Reg Pepsi	-	-	-	-	-	-	-
Diet Pepsi	0.1%	-	2.3%	1.6%	-	1.0%	0.3%
Δ quantity							
Reg Coke	-16.4%	-55.6%	-1.2%	-5.6%	-62.0%	-1.9%	-4.7%
Diet Coke	-6.0%	14.2%	-1.6%	-7.3%	15.5%	-2.9%	-6.7%
Reg Pepsi	-1.8%	-53.6%	-0.2%	-0.9%	-33.0%	-0.5%	-1.2%
Diet Pepsi	-1.6%	8.0%	-0.2%	-1.9%	5.7%	-0.5%	-1.7%
Reg Store	3.2%	7.9%	0.4%	2.0%	7.6%	0.7%	1.9%
Diet Store	2.8%	3.5%	0.4%	2.1%	3.4%	0.8%	1.9%
Reg Outside	3.1%	5.8%	0.4%	1.9%	5.4%	0.7%	1.7%
Diet Outside	2.6%	2.7%	0.4%	1.9%	2.5%	0.7%	1.7%

Notes: Numbers are expressed as a percentage of the pre-policy (i.e., pre tax and advertising restriction) level. Columns (1), (2) and (5) show changes relative to the pre-policy level. Column (3) (column (6)) shows the incremental change relative to column (2) (column (5)) and column (4) (column (7)) shows the incremental change relative to column (3) (column (6)). As stores brands prices, margins and advertising expenditures are held fixed we omit them from the table.

Table J.2: *Aggregate impact of counterfactual policies, by brand*

	No tax	Specific tax			Ad valorem tax		
	Adv. restrict.	Fixed adv.	+ Eq. adv. response	+ Adv. restrict.	Fixed adv.	+ Eq. adv. response	+ Adv. restrict.
Δ profits	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Reg Coke	-2.2%	-23.1%	1.1%	0.6%	-33.9%	1.9%	1.3%
Diet Coke	-3.4%	4.7%	-0.6%	-3.7%	5.1%	-1.0%	-3.3%
Reg Pepsi	-1.3%	-33.7%	-0.2%	-0.7%	-39.2%	-0.2%	-0.6%
Diet Pepsi	-1.0%	4.2%	-0.3%	-1.1%	3.3%	-0.4%	-1.0%

Notes: Numbers for price, margins, advertising expenditure and quantities are expressed as a percentage of the pre-policy (i.e., pre tax and advertising restriction) level; numbers for profits are expressed as a percentage of pre-policy total consumer expenditure. Columns (1), (2) and (5) show changes relative to the pre-policy level. Column (3) (column (6)) shows the incremental change relative to column (2) (column (5)) and column (4) (column (7)) shows the incremental change.

Table J.3: *Distributional impact of counterfactual policies (under "Total effect"' consumer surplus)*

	No tax	Specific tax		Ad valorem tax	
Income quartile	Adv. restrict. (1)	(2)	Adv. restrict. (3)	(4)	Adv. restrict. (5)
Change in sugar					
Bottom	-2.88%	-17.64%	-18.12%	-17.88%	-18.25%
2nd	-2.78%	-17.07%	-17.45%	-17.23%	-17.45%
3rd	-2.32%	-17.29%	-17.63%	-17.70%	-17.96%
Top	-2.83%	-12.22%	-12.73%	-12.56%	-12.83%
Change in consumer surplus					
Bottom	-6.20%	-9.11%	-13.50%	-9.78%	-13.72%
2nd	-3.87%	-7.13%	-9.73%	-7.52%	-9.85%
3rd	-4.10%	-7.81%	-10.73%	-8.38%	-11.03%
Top	-3.60%	-4.60%	-7.11%	-5.15%	-7.33%
Change in consumer surplus net of internalities					
Bottom	-4.98%	-1.66%	-5.84%	-2.22%	-6.01%
2nd	-2.86%	-0.96%	-3.43%	-1.29%	-3.55%
3rd	-3.40%	-2.54%	-5.36%	-2.99%	-5.56%
Top	-2.91%	-1.63%	-4.00%	-2.08%	-4.20%

Notes: Change in sugar is expressed as a percent of the income quartile specific pre-policy total drink sugar consumption. Change in consumer surplus (including net of internalities) is expressed as a percent of income quartile specific pre-policy total expenditure. The consumer surplus measure includes both the static impact of policy on the state-specific optimal prices and the impact of the changes in the equilibrium distribution over advertising state due to changes in optimal advertising expenditure.

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