Online Appendix: On Taking a Skewed Risk More Than Once

Sebastian Ebert Mats Köster*

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Abstract

This online appendix (OA) contains additional results to the paper "On Taking a Skewed Risk More Than Once." Section OA.1 provides analytical results on prospect theory and penny-picking, and Section OA.2 discusses threshold gambling strategies.

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*Ebert: Heidelberg University, Department of Economics, Bergheimer Str. 58, 69115 Heidelberg, Germany. Email: sebastian.ebert@awi.uni-heidelberg.de. Köster: CEU, Department of Business and Economics, Quellenstraße 51, 1100 Vienna, Austria. Email: kosterm@ceu.edu. This project received funding from the European Research Council under the European Union's Horizon 2020 research and innovation programme (grant agreement No 803380).

OA.1 A cumulative prospect theory analysis of repeatedly gambling skewed risks

In this appendix, we provide an analytical result that shows that those cumulative prospect theory (CPT, Tversky and Kahneman 1992) decision-makers (DMs) who avert one or few left-skewed risks are the ones who most willingly gamble them using trailing stop-loss (as shown numerically in Figure 5 in the main text). To this end, we offer a closed-form expression for the CPT utility from gambling a moment-parametrized basic gamble trailing stop-loss (and penny-picking in particular). This closed-form expression is for the tractable CPT specification with neo-additive probability weighting and piecewise-linear utility, and allows for additional comparative statics results.

The neo-additive weighting function of Chateauneuf et al. (2007) is given by:

$$w_{\text{NA}}(p) = \begin{cases} 0 & p = 0 \\ mp + a & p \in (0,1) \\ 1 & p = 1. \end{cases} \tag{OA.1}$$

where $m > 0$, $a \geq 0$, and $m + a \leq 1$. These conditions ensure that w is indeed a weighting function (i.e., strictly increasing, non-negative, and bounded from above by one). Inverse-S-shape follows for $a > 0$ and $m < 1$. Wakker (2010, p. 210) remarks that "the neo-additive weighting functions are among the most promising candidates regarding the optimal tradeoff of parsimony and fit" and that "the interpretation of its parameters is clearer and more convincing than with other families." Ebert and Karehnke (2025) provide formal results on CPT with neo-additive probability weighting, which show that $a > 0$ is necessary and sufficient for skewness-seeking.[1] The tractability of the neo-additive function, as well as the restriction to piecewise-linear gain-loss utility, allows for an analytical expression for the CPT utility from gambling trailing stop-loss. The proof of the result is given at the end

[1]Note that $a > 0$ captures skewness-seeking regardless of the number of outcomes of a prospect. In particular, when $n \geq 3$, for any middle outcome x_i ($i = 2, \ldots, n-1$) the CPT decision weight is given by $\pi_i = mp_i$ and thus underweighted regardless of whether the objective probability p_i is large or small.

of this section.

Proposition OA.1 (Piecewise-linear, neo-additive CPT utility of TSL)**.** *For CPT with the neo-additive probability weighting function from equation* (OA.1) *and piecewise-linear utility ($\alpha = 1$):*

$$CPT(S_{\tau_{\mathrm{TSL}}\wedge T}) = \sqrt{\frac{\mathbb{V}[X]}{p(1-p)}}\Big(a(T(1-p)-\lambda p) - (\lambda-1)m(1-p)(T-k+1)p^{T-k+1}\Big), \tag{OA.2}$$

where $k = \max\left(1, \left\lfloor T+1-\frac{p}{1-p}\right\rfloor\right)$ and $\lfloor\cdot\rfloor$ denotes the largest integer smaller or equal than $\cdot$.

The result reduces in complexity by noting that $k = T \iff p \leq 0.5$. That is, for right-skewed basic gambles, $S_{\tau_{\mathrm{TSL}}\wedge T}$ features T gains and one loss. Then, equation (OA.2) simplifies to:

$$CPT(S_{\tau_{\mathrm{TSL}}\wedge T}) = \sqrt{\frac{\mathbb{V}[X]}{p(1-p)}}\Big(a(T(1-p)-\lambda p) - (\lambda-1)m(1-p)p\Big).$$

It is easily observed (also for the more general expression (OA.2)) that larger loss aversion λ unambiguously decreases utility. In the absence of skewness-seeking ($a = 0$), CPT utility is negative if $\lambda \geq 1$ so that loss aversion prevents the DM from gambling any gamble.

The impact of skewness-seeking ($a > 0$) can be inferred from:

$$\frac{\partial CPT(S_{\tau_{\mathrm{TSL}}\wedge T})}{\partial a} = \sqrt{\frac{\mathbb{V}[X]}{p(1-p)}}(T(1-p)-\lambda p),$$

which is strictly positive if:

$$\lambda < T\frac{1-p}{p}. \tag{OA.3}$$

That is, greater skewness-seeking a increases risk-taking if enough time (and thus strategy skewness) is available and/or if the basic risk is right-skewed enough to begin with. Moreover, equation (OA.3) formalizes the message from Figure 5 in the main text that those DMs who avert one or few left-skewed risks are the ones who most willingly gamble them trailing stop-loss. Specifically, when T is small and p is large (i.e., $(1-p)/p$ is small), then the CPT utility of risk-taking decreases in skewness-seeking a. As T increases, eventually, the utility of taking this same risk increases in a. Moreover, from equation (OA.2) it is evident that, for every skewness-seeker ($a > 0$), a sufficiently left-skewed risk is disliked once but liked if offered sufficiently many times for gambling trailing stop-loss.[2]

A strong implication from Proposition OA.1 is that, as $T \to \infty$, CPT utility not only turns positive but approaches infinity, regardless of the value of p (no matter how left-skewed) and regardless of the degree of loss aversion λ. The extreme result is a consequence of the neo-additive weighting implying extreme skewness-seeking already when a is close to zero but strictly positive. In particular, under neo-additive weighting, the best payoff x_1 receives a decision weight of at least a, regardless of how tiny its probability. This observation thus implies that the DM is not only willing to gamble any gamble trailing stop-loss (if sufficient time is available), but also that his willingness to pay for doing so approaches infinity—a St. Petersburg paradox-like result. In the next paragraph we show, however, that for large (but not infinite) T this limit result on extreme risk-taking is not observed even for DMs that are not loss-averse.

If the DM is skewness-seeking but not loss-averse, ($a > 0$ and $\lambda = 1$), then equation (OA.2) simplifies to:

$$CPT(S_{\tau_{\text{TSL}} \wedge T}) > 0 \iff T(1-p) - p > 0 \iff p < \frac{T}{T+1}. \qquad \text{(OA.4)}$$

The latter inequality means that with a horizon of $T = 99$ periods, the DM is willing to gamble any gamble trailing stop-loss with $p < \frac{99}{100}$. With loss aversion, this threshold value for repeated risk-taking decreases further. Recall that Figure 5 in the main text showed that, with the CPT parametrization of Tversky and Kahneman (1992), it depends on parameters whether a very left-skewed risk with $p = 0.99$

[2]Note that the continuous weighting function underlying Figure 5 allows for a more refined result regarding this effect. Rather than "skewness-seeking yes/no" being the necessary and sufficient condition for taking any risk eventually (if the horizon is long enough), the result depends on the strength of skewness-seeking δ (as well as other CPT parameters).

is taken. Unlike with neo-additive weighting, therefore, not every gamble will be taken if sufficient gambling time is available.

As a final remark, note that trailing stop-loss gambling for one period corresponds to gambling buy-and-hold. Therefore, the myopic CPT DM considered in Section 5 gambles buy-and-hold if and only if the basic gamble is right-skewed ($p < \frac{1}{1+1} = 0.5$) and abstains from gambling otherwise. The non-myopic DM, taking into account strategy skewness, gambles strictly more gambles whenever $T < 1$.

Proof of Proposition OA.1. First note that for the neo-additive weighting function and when the best outcome is a gain and the worst outcome is a loss, for a general distribution X with outcomes $x_1, \ldots, x_{T+1}$ and corresponding probabilities $p_1, \ldots, p_{T+1}$ it holds that:

$$\pi_1 = w(p_1) = mp_1 + a \text{ and } \pi_{T+1} = w(p_{T+1}) = mp_{T+1} + a.$$

For an intermediate outcome x_i, $i = 2, \ldots, T$ it holds that:

$$\pi_i = m(p_1 + \ldots + p_i) + a - (m(p_1 + \ldots + p_{i-1}) + a) = mp_i$$

and for an intermediate outcome x_i, $i = 2, \ldots, T$ that is a loss it also holds that:

$$\pi_i = m(p_{T+1} + \ldots + p_i) + a - (m(p_1 + \ldots + p_{i+1}) + a) = mp_i.$$

Therefore,

$$\pi_i = \begin{cases} mp_i + a & \text{if } i = 1 \text{ or } i = T+1 \text{ and} \\ mp_i & \text{if } i = 2, \ldots T. \end{cases} \tag{OA.5}$$

After an index change, equation (A.7) can be expressed as:

$$\mathbb{P}\left(S_{\tau_{\text{TSL}} \wedge T} = T\sqrt{\mathbb{V}[X]\frac{1-p}{p}}\right) = p^{T-t+1} \text{ and}$$

$$\mathbb{P}\left(S_{\tau_{\text{TSL}} \wedge T} = (T-t+1)\sqrt{\mathbb{V}[X]\frac{1-p}{p}} - \sqrt{\mathbb{V}[X]\frac{p}{1-p}}\right) = p^{T-t+1}(1-p) \text{ for } t = 2, \ldots, T+1.$$

Here, $T - t + 1$ corresponds to the number of upward movements and the indices

$t = 1, \dots, T + 1$ enumerate the payoffs x_t from best to worst (so that the standard CPT formula is easily applied). There are $T + 1$ possible payoffs, and the best state x_1 is the one with T upward movements. Clearly, x_1 is a gain and x_{T+1} is a loss. A middle state x_t $(t = 2, \dots, T)$ is a gain state, if and only if,

$$(T - t + 1)\sqrt{\mathbb{V}[X]\frac{1-p}{p}} - \sqrt{\mathbb{V}[X]\frac{p}{1-p}} \geq 0 \Longleftrightarrow t \leq T + 1 - \frac{p}{1-p}.$$

Therefore, the index of the smallest gain, k, is given by $k = \max\left(1, \left\lfloor T + 1 - \frac{p}{1-p} \right\rfloor\right)$.[3] Therefore, from the definition of CPT and equation (OA.5),

$$\begin{aligned}
CPT(S_{\tau_{\mathrm{TSL}} \wedge T}) = {} & (mp^T + a)\left(T\sqrt{\mathbb{V}[X]\frac{1-p}{p}}\right) \\
& + \sum_{t=2}^{k} mp^{T-t+1}(1-p)\left((T-t+1)\sqrt{\mathbb{V}[X]\frac{1-p}{p}} - \sqrt{\mathbb{V}[X]\frac{p}{1-p}}\right) \\
& - \lambda \sum_{t=k+1}^{T} mp^{T-t+1}(1-p)\left(-\left((T-t+1)\sqrt{\mathbb{V}[X]\frac{1-p}{p}} - \sqrt{\mathbb{V}[X]\frac{p}{1-p}}\right)\right) \\
& - \lambda(m(1-p)+a)\sqrt{\mathbb{V}[X]\frac{p}{1-p}},
\end{aligned}$$

which simplifies to:

$$\begin{aligned}
CPT(S_{\tau_{\mathrm{TSL}} \wedge T}) = {} & \sqrt{\frac{\mathbb{V}[X]}{p(1-p)}}\Bigg((mp^T + a)T(1-p) + m(1-p)\sum_{t=2}^{k} p^{T-t+1}\left((T-t+1)(1-p) - p\right) \\
& + \lambda m(1-p)\sum_{t=k+1}^{T} p^{T-t+1}\left((T-t+1)(1-p) - p\right) - \lambda(m(1-p)+a)p\Bigg).
\end{aligned}$$

[3]Intuitively, when $p \leq 0.5$, then $t \leq T$ so that all but the worst payoffs are gains. When the basic gamble is left-skewed ($p > 0.5$), however, several wins are necessary to make an overall profit with trailing stop-loss, because the loss from a lost gamble is larger than the gain from a won gamble.

Exercising the sum formulas from equations (A.3) and (A.4) yields:

$$
\begin{aligned}
CPT(S_{\tau_{\mathrm{TSL}} \wedge T}) = \sqrt{\frac{\mathbb{V}[X]}{p(1-p)}} \Big(& (mp^T + a)T(1-p) \\
& + m(1-p)[(T-k+1)p^{T-k+1} - Tp^T - \lambda((T-k+1)p^{T-k+1} - p)] \\
& - \lambda(m(1-p) + a)p \Big)
\end{aligned}
$$

and simplification yields the expression stated in the claim. ■

OA.2 Two-threshold strategies

Although a trailing stop-loss strategy is the guiding example of the paper, Proposition 1 on the decomposition of total into basic and strategy skewness applies to arbitrary strategies. In this section, we consider *two-threshold strategies,* that is, stopping once reaching one of two thresholds (one lower and one higher than the initial profit of zero). Similar to trailing stop-loss, these strategies are appealing from practical, theoretical, and experimental viewpoints alike. Compared to trailing stop-loss, the analysis is less tractable when the basic risk is skewed and/or the horizon is finite because the distribution of the stopped random walk is more complicated.[4] For the sake of simplicity, therefore, we assume a symmetric basic risk (a 50-50 risk to win or lose one), stopping thresholds $a < 0 < b$, with $a, b \in \mathbb{Z}$, and an infinite horizon $T = \infty$. We show that a and b can be chosen such that the skewness of $S_{\tau_{a,b}}$ can take any value; that is, symmetric binary risks can be gambled in such a way that total skewness takes any desired value.

Denoting the gambling strategy by $\tau_{a,b}$, the induced profit distribution $S_{\tau_{a,b}}$ has two outcomes that correspond to the two stopping thresholds a and b. One can thus apply the insights on the basic skewness used in the main text on total skewness in

[4] For example, suppose the basic risk gives -4 with 20% probability and $+1$ otherwise and suppose stopping occurs once wealth reaches -4 or $+8$. Either threshold my be hit exactly but also be overshot (e.g., an upward move followed by two downward moves results in -7, overshooting the lower threshold of -4 by three units). The resulting profit distribution is thus not binary. Similarly, even when the basic gamble is symmetric, the profit distribution has many outcomes when the horizon is finite.

this section. In particular, the following statements are equivalent (Ebert 2015):

(i) The lower threshold is closer than the upper threshold, $|a| < b$.

(ii) It is less-likely to reach the upper than the lower threshold, $P[S_{\tau_{a,b}} = b] < 0.5$.

(iii) It takes longer to reach the upper than the lower threshold, $\rho(\tau_{a,b}, S_{\tau_{a,b}}) > 0$.

(i) characterizes what is often referred to as a stop-loss strategy, emphasizing that it stops before significant losses (relative to the gain potential) can arise. (ii) means that total skewness of such a strategy is positive ($S_{\tau_{a,b}}$ yields a larger outcome with low probability and smaller outcome with high probability; Ebert 2015 shows that this property is equivalent to many others that indicate positive skewness), and (iii) means that strategy skewness is positive. The equivalence of (ii) and (iii) is in line with Proposition 1: if basic skewness is zero (as assumed here), total skewness equals strategy skewness. (i) thus says that the two-threshold strategy is right-skewed both in the sense that strategy skewness and total skewness are positive.

The observation above supports the terminology used in Heimer et al. (2025) and Dertwinkel-Kalt and Frey (2024), who refer to a two-threshold strategy as right-skewed, if and only if, it induces a right-skewed overall distribution (i.e, right-skewed $S_{\tau_{a,b}}$, which means positive total skewness). In the experiment of Heimer et al. (2025), subjects commit to thresholds for gambling a symmetric basic gamble that pays \$0.50 with equal probability for up to $T = 26$ rounds. Subjects' average thresholds are given by $a = -4.06\$$ and $b = +7.88\$$, which implies a total skewness of 0.27 (computed by simulation). Conditional on choosing a stop-loss strategy, the ratio between the gain loss limit is necessarily larger, and so is the induced total skewness.

The current paper goes beyond the case of symmetric basic gambles as well as beyond the case of threshold strategies in order to study phenomena such as penny-picking. The consideration of negatively skewed risks to this end is evidently essential; the focus on trailing-stop loss is merely for the sake of concreteness. The result that risks with arbitrary negative skewness can be gambled in such a way that total skewness is positive and that they become attractive to prospect theory individuals, holds for this strategy. The result is first and foremost an existence result—the trailing stop-loss strategy is a strategy that "does the job." Others may do as well, but analytical results are more difficult to establish. Also, the intuition for how other strategies work, and applying them in practice, may be more complex than "stopping after the first loss of a given size."

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