Changing Business Cycles: The Role of Women's Employment
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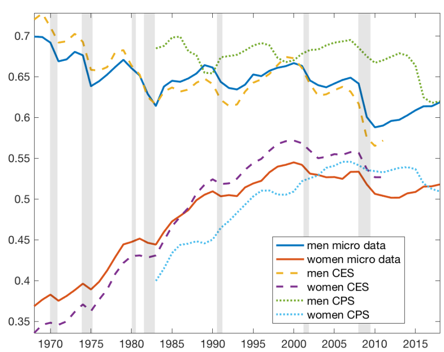
# Supplemental Appendix

# (I) Data Comparisons

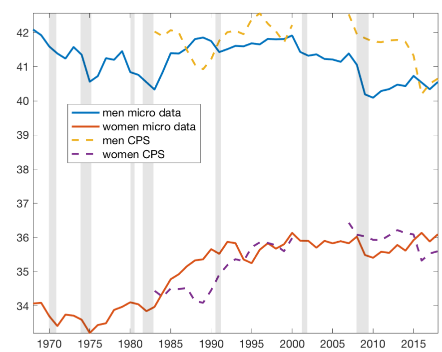
I compare the employment, hours and wage series by gender constructed from microdata with other series by gender directly available from the Bureau of Labor Statistics starting in later periods. The results are displayed in figure 24. Panel (A) reports the employment to population ratio by gender I constructed with the micro data with the value of this variable directly available from the CPS. I also include the corresponding value from the CES, obtained by dividing the number non-farm employees by the population. To derive the number of men employees, I take the difference between the total number of employees and the number of women employees. The three series match quite closely for both genders. Panel (B) reports hour estimate of hours worked per week by gender to the values reported by the CPS, which start in 1976. Again, for the periods in which I have both measures, they track each other very closely. Panel (C) reports my estimate for hourly wages by gender and the corresponding series provided by the CPS, which starts in 1990. Here, the time pattern is also very similar for the overlapping period. For this analysis, the most important series is the female/male wage ratio. I report three different values of it on panel (D). The solid line corresponds to the value I estimate from micro data, the dashed line to the value obtained from the CPS hourly wage series by gender also described in panel (C), while the dotted line corresponds to the ratio of usual weekly earnings by gender, which are available directly from the CPS starting in 1979. The three series have a very similar pattern for the overlapping periods. The ratios coming from CPS hourly wages and weekly earnings are somewhat higher, which may reflect smaller gender wage gaps for wage and salary workers than in the overall sample.

I also compare the aggregate values of hours and wages implied by our micro based estimates with commonly used aggregate series in the literature. Specifically, I consider the following series for aggregate hours:

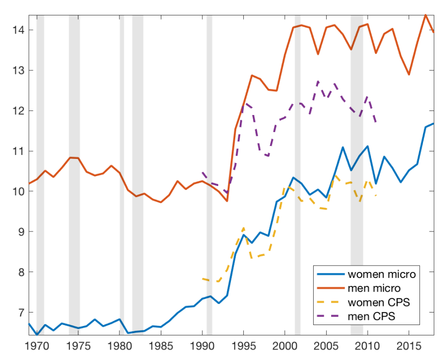
- Hours of Wage and Salary Workers on Nonfarm Payrolls: Total (TOTLA). Source: U.S. Bureau of Labor Statistics.



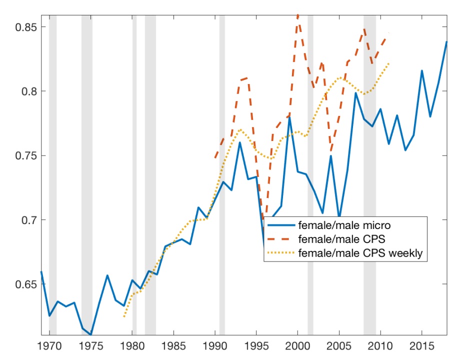

(A) $E/P$




(B) Hours worked per week




(C) Hourly wages




(D) Female/male wages

FIGURE 24: $E/P$, average number of hours worked per Iek, hourly wages and female/male wage ratios, comparison between micro data and aggregate series. Source: Author's calculation based on CPS March Supplement and CES.

- Nonfarm Business Sector: Hours of All Persons (HOANBS). Source: U.S. Bureau of Labor Statistics. Release: Productivity and Costs. Frequency: Quarterly, Seasonally Adjusted

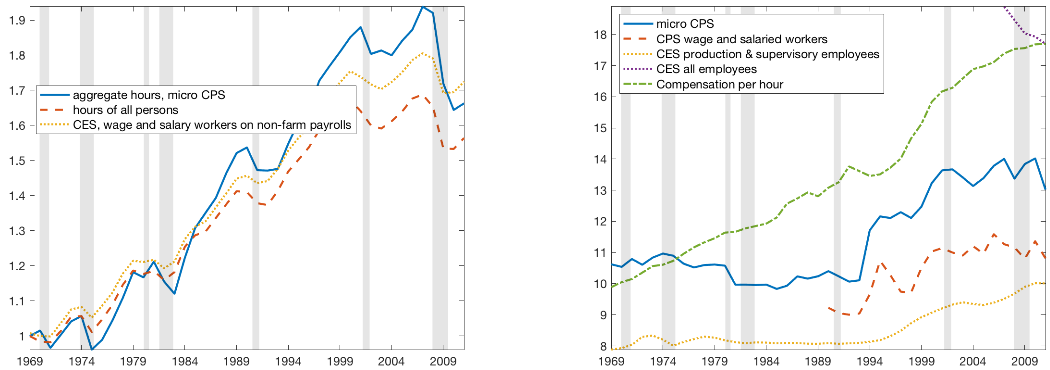
The series are obtained from the FRED Database https://fred.stlouisfed.org. For more information, see http://www.bls.gov/lpc/hoursdatainfo.htm.

For wages, I consider:

- Hourly wages for wage and salaried workers estimated from weekly earnings and usual weekly hours. Frequency: Monthly. Source: Current Population Survey.
- Hourly wages of production and supervisory employees. Frequency: Monthly. Source: Current Establishment Survey.
- Hourly wages of all employees. Frequency: Monthly. Initial availability: 2006 January. Source: Current Establishment Survey.
- Nonfarm Business Sector: Compensation Per Hour (COMPNFB). Source: U.S. Bureau of Labor Statistics. Release: Productivity and Costs. Frequency: Quarterly, Seasonally Adjusted. Obtained from the FRED Database.

All wage series are expressed in 1982-1984 dollars. The Compensation Per Hour is reported as an index and I rescale it to be the same as Nonfarm Payrolls Hourly Compensation for All Employees in 2011 (this series is only available since 2006).

The results are displayed in figure 25. The aggregate hours per capita series used in the analysis closely follows the ones available from the Current Employment Survey, though it grows at a slightly faster rate starting in the mid-1980s. This may reflect that I also include the self-employed and I consider total hours, which may include hours on a second job, which would not be considered by the CES. Our aggregate wage series is considerably higher than aggregate wages for wage and salaried workers from the CPS and wages for production and supervisory employees from the CES. However, for the available years it is lower than the CES measure of hourly wage for all workers. The growth in my aggregate hourly wage series reflects quite closely the growth rate of compensation per hour from the Productivity and Costs release.




(A) Hours worked per week

(B) Hourly wages

FIGURE 25: Aggregate hours per capita, index 1969=1, and hourly wages in 1982-1984 dollars, comparison between micro based estimates for economy wide averages and aggregate series. Source: Author's calculation based on CPS March Supplement and CES.

# (II) Model Derivations

## Steady State

The aggregate variables in the steady state for the rescaled version of the model are characterized by the following system of equations, using the normalization $L = 1$:

$$1 = \beta e^{\gamma} r^k + (1-\delta)\beta e^{\gamma} \tag{30}$$

$$r^k = \frac{e^{-\gamma}}{\beta} - (1-\delta) \tag{31}$$

$$k = \left(\frac{\alpha}{r^k}\right)^{\frac{1}{1-\alpha}}$$

$$y = k^{\alpha}$$

$$\bar{k} = k e^{\gamma}$$

$$i = \left[1 - (1-\delta)e^{-\gamma}\right]\bar{k}$$

$$c = \frac{y}{g} - i$$

$$\lambda = \frac{1}{c}\frac{e^{\gamma} - \beta\eta}{e^{\gamma} - \eta}.$$

## Log Linear Approximation

I can now derive the model's log-linear approximation. Log-linear deviations from steady state are defined as follows, for a generic variable $x_t$ with s.s. value $x$:

$$\hat{x}_t \equiv \log x_t - \log x,$$

except for $\hat{z}_t \equiv z_t - \gamma$. The set of state equations that will be used in the estimation comprise (32)-(44) derived below.

### Households

- Consumption

$$\begin{aligned}(e^\gamma - \eta\beta)(e^\gamma - \eta)\hat{\lambda}_t = \eta\beta e^\gamma E_t \hat{c}_{t+1} - \left(e^{2\gamma} + \eta^2\beta\right)\hat{c}_t + \eta e^\gamma \hat{c}_{t-1} \\ + \eta e^\gamma(\beta\rho_z - 1)\hat{z}_t + (e^\gamma - \eta\beta\rho_b)(e^\gamma - \eta)\hat{b}_t \end{aligned} \quad (32)$$

- Physical capital $\left(\bar{K}_t\right)$

$$\hat{\phi}_t = (1-\delta)\beta e^{-\gamma} E_t\left(\hat{\phi}_{t+1} - \hat{z}_{t+1}\right) + \left(1 - (1-\delta)\beta e^{-\gamma}\right) E_t\left[\hat{\lambda}_{t+1} - \hat{z}_{t+1} + \hat{r}^k_{t+1}\right] \quad (33)$$

- Investment

$$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - e^{2\gamma}\zeta(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t) + \beta e^{2\gamma}\zeta E_t\left[\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{z}_{t+1}\right] \quad (34)$$

- Utilization

$$\hat{r}^k_t = \chi \hat{u}_t \quad (35)$$

- Definition of effective capital

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t \quad (36)$$

- Physical capital accumulation

$$\hat{\bar{k}}_t = (1-\delta)e^{-\gamma}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t\right) + \left(1 - (1-\delta)e^{-\gamma}\right)(\hat{\mu}_t + \hat{\imath}_t) \quad (37)$$

- Labor supply ($H^j_t$ for $j = f, m$)

$$\hat{w}^j_t = \hat{\varphi}^j_t + \nu^j \hat{H}^j_t - \hat{\lambda}_t \quad (38)$$

**Firms**

- Production function
$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{L}_t \tag{39}$$

- Labor input
$$\hat{L}_t = \omega^f \left(\hat{\tilde{a}}_t^f + \hat{H}_t^f\right) + \omega^m \hat{H}_t^m \tag{40}$$

- Return to capital
$$\hat{r}_t^k = (1-\alpha)\left(\hat{L}_t - \hat{k}_t\right) \tag{41}$$

- Female labor demand $\left(H_t^f\right)$

$$\hat{w}_t^f = \hat{y}_t + (\rho - 1)\hat{H}_t^f + \rho \hat{\tilde{a}}_t^f - \rho \hat{L}_t \tag{42}$$

- Male labor demand $(H_t^m)$

$$\hat{w}_t^m = \hat{y}_t + (\rho - 1)\hat{H}_t^m - \rho \hat{L}_t \tag{43}$$

**Resource Constraint**

- Resource constraint
$$\frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{r^k k}{y}\hat{u}_t = \frac{1}{g}\hat{y}_t - \frac{1}{g}\hat{g}_t \tag{44}$$

## Shocks

Following **?**, I normalize the intertemporal preference shock as:

$$\hat{b}_t^* = \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}\hat{b}_t$$

so as to make the coefficient on consumption in the Euler equation equal to one. The Euler equation pricing a real one-period bond with interest rate $r_t$, which I did not consider explicitly, reads:

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1} - \hat{z}_{t+1}\right] + \hat{r}_t$$

Table 9: Exogenous Shocks

| | |
|---|---|
| $\hat{b}_t^*$ | $= \rho_b \hat{b}_{t-1}^* + \varepsilon_{b,t}$ |
| $\hat{\mu}_t$ | $= \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t}$ |
| $\hat{z}_t$ | $= \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$ |
| $\hat{g}_t$ | $= \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}$ |
| $\hat{\tilde{a}}_t^f$ | $= \hat{\tilde{a}}_t^{fT} + \hat{\tilde{a}}_t^{fC}$ |
| $\hat{\tilde{a}}_t^{fT}$ | $= \rho_{\tilde{a}^{fT}} \hat{\tilde{a}}_{t-1}^{fT} + \varepsilon_{\tilde{a}^{fT},t}$ |
| $\hat{\tilde{a}}_t^{fC}$ | $= \rho_{\tilde{a}^{fC}} \hat{\tilde{a}}_{t-1}^{fC} + \varepsilon_{\tilde{a}^{fC},t}$ |
| $\hat{\tilde{\varphi}}_t^f$ | $= \hat{\tilde{\varphi}}_t^{fT} + \hat{\tilde{\varphi}}_t^{fC}$ |
| $\hat{\tilde{\varphi}}_t^{fT}$ | $= \rho_{\tilde{\varphi}^{fT}}^T \hat{\tilde{\varphi}}_{t-1}^{fT} + \varepsilon_{\tilde{\varphi}^{fT},t}^T$ |
| $\hat{\tilde{\varphi}}_t^{fC}$ | $= \rho_{\tilde{\varphi}^{fC}}^T \hat{\tilde{\varphi}}_{t-1}^{fC} + \varepsilon_{\tilde{\varphi}^{fC},t}$ |
| $\hat{\varphi}_t^m$ | $= \rho_{\varphi^m} \hat{\varphi}_{t-1}^m + \varepsilon_{\varphi^m,t}$ |

and substituting equation (32), I obtain:

$$\begin{aligned} \frac{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}{(e^\gamma - \eta\beta)(e^\gamma - \eta)} \hat{c}_t + (...) \quad &= \quad (...) + \frac{(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^\gamma - \eta\beta} \hat{b}_t \\ \hat{c}_t + (...) \quad &= (...) + \quad \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1-\rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma} \hat{b}_t. \end{aligned}$$

Using this normalization, the resulting set of exogenous shocks in the model is summarized in 9.