Online Appendix for Cyclical Attention to Saving

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A Proofs

A.1 Proposition 1

Here I show that for b_t below a certain threshold, the household first order conditions are sufficient for utility maximization in the simple model (Section I), and in the quantitative model (Section IV).

First, write the household problem as an unconstrained maximization by substituting out for consumption using the budget constraint:

(A.1)
$$\max_{b_t, i_t^e, X_t} U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u \left(\frac{b_{t-1}}{\Pi_t} (1 + i_{t-1}^e) + y_t(X_t) - b_t \right) - \mu \mathcal{I}(\mathbb{E}_s i_t^e) + v(X_t) \right)$$

Here I have summarized all choice variables other than saving b_t and the effective interest rate i_t^e in the vector X_t . In the simple model there are no other choice variables, so X_t is empty and non-asset income y_t is exogenous. In the quantitative model X_t includes wage setting, investment, and capital utilization. Inflation erodes real bond holdings as in the quantitative model. Note that this proof corresponds to the simple model case if Π_t is set to 1 for all t.

I begin by defining H_s as the Hessian matrix of second-order partial derivatives of this utility function with respect to each choice variable that would result if there was no information friction, and so i_t^e was not a choice variable. The Hessian matrix for the full

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problem is then:

(A.2)
$$H = \begin{bmatrix} H_{S} & \vdots \\ 0 & \frac{\partial^{2}U}{\partial b_{t}\partial i_{t}^{e}} \\ 0 & \dots & 0 & \frac{\partial^{2}U}{\partial b_{t}\partial i_{t}^{e}} & \frac{\partial^{2}U}{\partial i_{s}^{e}^{2}} \end{bmatrix}$$

Here I have used the fact that the only choice variable that i_t^e interacts with in the utility function is b_t . For all other choice variables X_t , $\frac{\partial^2 U}{\partial X_t \partial i_t^e} = 0$. The first order conditions are sufficient for utility maximization if U is weakly concave, which is true if for any vector x:

(A.3)
$$xHx' = x_s H_s x_s' + 2yz \frac{\partial^2 U}{\partial b_t \partial i_t^e} + z^2 \frac{\partial^2 U}{\partial i_t^{e^2}} \le 0$$

Where $x_s = [x_1, ..., y]$ and $x = [x_s, z]$. If households cannot influence effective interest rates the utility function is concave, as then this is a standard household maximization problem (identical to that in Harrison and Oomen (2010) in the quantitative model). This implies that $x_s H_s x_s' < 0$.

Assuming a diminishing marginal utility of consumption we have that:

(A.4)
$$\frac{\partial^2 U}{\partial b_t^2} = u''(c_t) + \beta \mathbb{E}_t \frac{u''(c_{t+1})(1 + i_t^e)^2}{\Pi_{t+1}^2} < 0$$

It is therefore sufficient for the concavity of U to show that for any y, z:

(A.5)
$$y^2 \frac{\partial^2 U}{\partial b_t^2} + 2yz \frac{\partial^2 U}{\partial b_t \partial i_t^e} + z^2 \frac{\partial^2 U}{\partial i_t^{e^2}} \le 0$$

Using the definition of U this condition becomes:

(A.6)
$$y^2 u''(c_t) + y^2 \beta \mathbb{E}_t u''(c_{t+1}) \frac{(1+i_t^e)^2}{\Pi_{t+1}^2} + 2yz\beta \mathbb{E}_t u''(c_{t+1}) \frac{(1+i_t^e)b_t}{\Pi_{t+1}^2} + 2yz\beta \mathbb{E}_t u''(c_{t+1}) \frac{1}{\Pi_{t+1}} - z^2 \mu \mathcal{I}''(\mathbb{E}_s i_t^e) + z^2 \beta \mathbb{E}_t u''(c_{t+1}) \frac{b_t^2}{\Pi_{t+1}^2} \le 0$$

The two terms that don't depend on c_{t+1} are both negative. The remaining terms can

be written as:

$$(A.7) \quad \beta \mathbb{E}_{t} \frac{u''(c_{t+1})b_{t}^{2}}{\Pi_{t+1}^{2}} \left(\frac{y^{2}(1+i_{t}^{e})^{2}}{b_{t}^{2}} + z^{2} + 2yz \left(\frac{1+i_{t}^{e}}{b_{t}} + \frac{\Pi_{t+1}u'(c_{t+1})}{u''(c_{t+1})} \right) \right)$$

$$= -\beta y^{2} \mathbb{E}_{t} \frac{u''(c_{t+1})b_{t}^{2}}{\Pi_{t+1}^{2}} \left(\frac{2(1+i_{t}^{e})\Pi_{t+1}u'(c_{t+1})}{b_{t}u''(c_{t+1})} + \frac{\Pi_{t+1}^{2}(u'(c_{t+1}))^{2}}{(u''(c_{t+1}))^{2}} \right)$$

$$+ \beta \mathbb{E}_{t} \frac{u''(c_{t+1})b_{t}^{2}}{\Pi_{t+1}^{2}} \left(z + y \left(\frac{1+i_{t}^{e}}{b_{t}} + \frac{\Pi_{t+1}u'(c_{t+1})}{u''(c_{t+1})} \right) \right)^{2}$$

Since $u''(c_{t+1}) < 0$, the second term in this expression is negative. A sufficient condition for U to be concave is therefore:

(A.8)
$$\mathbb{E}_{t} \frac{u''(c_{t+1})}{\Pi_{t+1}^{2}} \left(\frac{2(1+i_{t}^{e})\Pi_{t+1}}{b_{t}\Gamma_{t+1}} - \frac{\Pi_{t+1}^{2}}{\Gamma_{t+1}^{2}} \right) \leq 0$$

where:

(A.9)
$$\Gamma_{t+1} = -\frac{u''(c_{t+1})}{u'(c_{t+1})} > 0$$

is the coefficient of absolute risk aversion.

Rearranging, a sufficient condition for U to be concave is:

(A.10)
$$b_t \le 2 \cdot \frac{\mathbb{E}_t \frac{u''(c_{t+1})(1+i_t^e)}{\Gamma_{t+1}\Pi_{t+1}}}{\mathbb{E}_t \frac{u''(c_{t+1})}{\Gamma_{t+1}^2}} \equiv \bar{b}_t$$

Therefore the first order conditions are sufficient for utility maximization as long as the amount saved is sufficiently low relative to the coefficient of absolute risk aversion.

The qualitative results in Section I hold as long as this condition is satisfied. In the quantitative model this is easily the case for plausible parameters. There $b_t = 1$, and with CRRA utility we have:

(A.11)
$$c_{t+1}\Gamma_{t+1} = \frac{1}{\sigma^c}$$

where σ^c is the intertemporal elasticity of substitution, estimated to be substantially below 1.

Condition A.10 can therefore be written as:

(A.12)
$$\mathbb{E}_t u''(c_{t+1})c_{t+1} \left[\frac{\sigma^c}{2} c_{t+1} - \frac{(1+i_t^e)}{\Pi_{t+1}} \right] \ge 0$$

In steady state, $\frac{(1+i_t^e)}{\Pi_{t+1}} = \beta^{-1} > 1$. Since steady state consumption is 0.662, in the region of the steady state the term inside the square brackets is negative, which along with $u''(c_{t+1}) < 0$ means that this condition is comfortably satisfied.

A.2 Inverse Relationship between \mathcal{I}_t and λ_t

The only way λ_t and \mathcal{I}_t can be related in equation 10 is through choice probabilities, which holding i_t^n constant are entirely summarized by the effective interest rate. Using the chain rule we therefore have:

(A.13)
$$\frac{\partial \lambda_t}{\partial \mathcal{I}_t} = \frac{\partial \lambda_t}{\partial \mathbb{E}_s i_t^e} \frac{\partial \mathbb{E}_s i_t^e}{\partial \mathcal{I}_t}$$

In Appendix A.3 I show that $\frac{\partial \lambda_t}{\partial \mathbb{E}_s i_t^e} < 0$ and $\frac{\partial \mathcal{I}_t}{\partial \mathbb{E}_s i_t^e} = \lambda_t^{-1} > 0$, implying that:

$$\frac{\partial \lambda_t}{\partial \mathcal{I}_t} < 0$$

A.3 Proposition 2

Substituting the optimal choice probabilities into the information constraint 10 gives (dropping time subscripts to simplify notation, as everything here is defined within the same period):

(A.15)
$$\mathcal{I} = \frac{\mathbb{E}_s i^e}{\lambda} - \sum_{s=1}^{S} \Pr(s) \log d_s$$

Where:

(A.16)
$$d_s = \sum_{k=1}^{N} \mathcal{P}_k \exp\left(\frac{i^k(s)}{\lambda}\right)$$

and $\{1, 2, ..., S\}$ is the set of all possible states of the world.

Differentiate this with respect to $\mathbb{E}_s i^e$, holding the offered interest rates $i^n(s)$ constant as individuals take them as given:

(A.17)
$$\frac{\partial \mathcal{I}}{\partial \mathbb{E}_{s} i^{e}} = \frac{1}{\lambda} - \frac{\mathbb{E}_{s} i^{e}}{\lambda^{2}} \frac{\partial \lambda}{\partial \mathbb{E}_{s} i^{e}} - \sum_{s=1}^{S} \frac{\Pr(s)}{d_{s}} \frac{\partial d_{s}}{\partial \mathbb{E}_{s} i^{e}}$$

Each term inside the sum is:

(A.18)

$$\frac{\Pr(s)}{d_s} \frac{\partial d_s}{\partial \mathbb{E}_s i^e} = \frac{\Pr(s)}{d_s} \frac{\partial \lambda}{\partial \mathbb{E}_s i^e} \left[\left(\sum_{k=1}^N \exp(\frac{i^k(s)}{\lambda}) \frac{\partial \mathcal{P}_k}{\partial \lambda} \right) - \frac{1}{\lambda^2} \left(\sum_{k=1}^N i^k(s) \mathcal{P}_k \exp(\frac{i^k(s)}{\lambda}) \right) \right] \\
= \frac{\partial \lambda}{\partial \mathbb{E}_s i^e} \Pr(s) \left(\sum_{k=1}^N \frac{\Pr(k|s)}{\mathcal{P}_k} \frac{\partial \mathcal{P}_k}{\partial \lambda} \right) - \frac{\mathbb{E}_s i^e}{\lambda^2}$$

Substituting this back into equation A.17 gives:

(A.19)
$$\frac{\partial \mathcal{I}}{\partial \mathbb{E}_{s} i^{e}} = \frac{1}{\lambda} - \frac{\partial \lambda}{\partial \mathbb{E}_{s} i^{e}} \sum_{s=1}^{S} \sum_{k=1}^{N} \frac{\Pr(s) \Pr(k|s)}{\mathcal{P}_{k}} \frac{\partial \mathcal{P}_{k}}{\partial \lambda}$$

Recall that \mathcal{P}_k is defined as the unconditional probability of choosing bank k, so it can be written as $\sum_{s=1}^{S} \Pr(k|s) \Pr(s)$. Using this, equation A.19 becomes:

(A.20)
$$\frac{\partial \mathcal{I}}{\partial \mathbb{E}_{s} i^{e}} = \frac{1}{\lambda} - \frac{\partial \lambda}{\partial \mathbb{E}_{s} i^{e}} \sum_{k=1}^{N} \frac{\partial \mathcal{P}_{k}}{\partial \lambda}$$

Since the sum of \mathcal{P}_k over banks is always equal to 1, the sum of the derivatives of \mathcal{P}_k must equal zero. We therefore have that:

(A.21)
$$\frac{\partial \mathcal{I}}{\partial \mathbb{E}_{s} i^{e}} = \frac{1}{\lambda}$$

Since λ is the Lagrange multiplier on the information constraint in the individual's problem, it is always strictly positive and $\mathcal{I}'(\mathbb{E}_s i^e) = \frac{\partial \mathcal{I}}{\partial \mathbb{E}_s i^e} > 0$.

Differentiating again with respect to $\mathbb{E}_s i^e$ we have:

(A.22)
$$\frac{\partial^2 \mathcal{I}}{\partial (\mathbb{E}_s i^e)^2} = -\frac{1}{\lambda^2} \frac{\partial \lambda}{\partial \mathbb{E}_s i^e}$$

 $\mathcal{I}''(\mathbb{E}_s i^e)$ is therefore positive if $\frac{\partial \lambda}{\partial \mathbb{E}_s i^e} < 0$.

Differentiating the definition of i^e (12) with respect to i^e we have:

(A.23)
$$\frac{d\lambda}{di^e} = \frac{\lambda^2 \left(\sum_n \mathcal{P}_n \exp(\frac{i^n}{\lambda})\right)^2}{\left(\sum_n i^n \mathcal{P}_n \exp(\frac{i^n}{\lambda})\right)^2 - \left(\sum_n i^{n^2} \mathcal{P}_n \exp(\frac{i^n}{\lambda})\right) \left(\sum_m \mathcal{P}_m \exp(\frac{i^m}{\lambda})\right)}$$

The numerator is always positive, so $\frac{d\lambda}{di^e}$ has the same sign as the denominator. After

expanding the terms in brackets the denominator is:

$$(A.24)$$

$$\sum_{n} i^{n^{2}} \mathcal{P}_{n}^{2} \exp(\frac{2i^{n}}{\lambda}) + \sum_{m \neq n} i^{n} i^{m} \mathcal{P}_{n} \mathcal{P}_{m} \exp(\frac{i^{n} + i^{m}}{\lambda}) - \sum_{n} i^{n^{2}} \mathcal{P}_{n}^{2} \exp(\frac{2i^{n}}{\lambda}) - \sum_{m \neq n} i^{n^{2}} \mathcal{P}_{n} \mathcal{P}_{m} \exp(\frac{i^{n} + i^{m}}{\lambda})$$

$$= -\sum_{m \neq n} (i^{n^{2}} - i^{n} i^{m}) \mathcal{P}_{n} \mathcal{P}_{m} \exp(\frac{i^{n} + i^{m}}{\lambda})$$

Inside the sum, each pair of banks $\{j,k\}$ appear twice: when m=k, n=j and when m=j, n=k. For each distinct pair of banks $\{j,k\}$, the terms inside the sum are equal to:

(A.25)
$$\mathcal{P}_{j}\mathcal{P}_{k}\exp(\frac{i^{j}+i^{k}}{\lambda})(i^{j^{2}}-i^{j}i^{k}+i^{k^{2}}-i^{k}i^{j}) = \mathcal{P}_{j}\mathcal{P}_{k}\exp(\frac{i^{j}+i^{k}}{\lambda})(i^{j}-i^{k})^{2} > 0$$

Each pair of terms inside the sum in equation A.24 is therefore positive, and so $\frac{d\lambda}{di^e}$ is negative in each state of the world. That therefore implies that $\mathcal{I}''(\mathbb{E}_s i^e) > 0$.

A.4 Corollary 1

With uninformative priors we can write the probability of choosing bank n in state s as:

(A.26)
$$\Pr(n|s) = \frac{1}{1 + \sum_{j \neq n}^{N} \exp(\frac{i^{j} - i^{n}}{\lambda})}$$

Now consider a mean-preserving spread of interest rates, so replace each i^n with $\tilde{i}^n = ki^n - \bar{i}(k-1)$, where \bar{i} is the unconditional mean of the pre-spread interest rates.

If choice probabilities are unchanged, and so attention \mathcal{I} is unchanged, then it must be that for all n:

(A.27)
$$\sum_{j\neq n}^{N} \exp(\frac{i^{j} - i^{n}}{\lambda}) = \sum_{j\neq n}^{N} \exp(\frac{\tilde{i}^{j} - \tilde{i}^{n}}{\tilde{\lambda}}) = \sum_{j\neq n}^{N} \exp(\frac{k(i^{j} - i^{n})}{\tilde{\lambda}})$$

This is satisfied when $\tilde{\lambda} = k\lambda$. If k > 1 the mean-preserving spread increases the dispersion of interest rates, and correspondingly λ rises. Since $\mathcal{I}'(i^e) = \lambda^{-1}$, this reduces $\mathcal{I}'(i^e)$.

A.5 Lemma 1

First, partially differentiate the first order condition for bank n (19) with respect to λ_t , denoting $S_n = \frac{\exp(i_t^n/\lambda_t)}{\sum_k = 1^N \exp(i_t^k/\lambda_t)}$ as the market share of bank n in period t, and $d_n = 1$

 $i_t^{CB} - i_t^n - \chi_t^n$ as the profit bank n makes per bond sold. Time subscripts are dropped for these variables to save notation, as all relevant variables occur within the same period.

(A.28)
$$-d_n \frac{\partial \mathcal{S}_n}{\partial \lambda_t} - (1 - \mathcal{S}_n) \frac{\partial i_t^n}{\partial \lambda_t} = 1$$

Using the definition of S_n (equation 15):

(A.29)
$$\frac{\partial \mathcal{S}_n}{\partial \lambda_t} = \frac{\mathcal{S}_n(1 - \mathcal{S}_n)}{\lambda_t} \frac{\partial i_t^n}{\partial \lambda_t} - \frac{\mathcal{S}_n(1 - \mathcal{S}_n)i_t^n}{\lambda_t^2} + \mathcal{S}_n \left(\sum_{j \neq n} \frac{\mathcal{S}_j}{\lambda_t^2} (i_t^j - \lambda_t \frac{\partial i_t^j}{\partial \lambda_t}) \right)$$

Substituting this in to equation A.28 and rearranging we obtain:

$$(A.30) \frac{\partial i_t^n}{\partial \lambda_t} = \frac{1}{\lambda_t (1 - \mathcal{S}_n)(\lambda_t + d_n \mathcal{S}_n)} \left[i_t^n d_n \mathcal{S}_n (1 - \mathcal{S}_n) - \lambda_t^2 - d_n \mathcal{S}_n \left(\sum_{j \neq n} \mathcal{S}_j (i_t^j - \lambda_t \frac{\partial i_t^j}{\partial \lambda_t}) \right) \right]$$

From equation 19 we have $d_n = \lambda_t (1 - \mathcal{S}_n)^{-1}$. Separately, we can write $\sum_{j \neq n} \mathcal{S}_j i_t^j = i_t^e - \mathcal{S}_n i_t^n$. Using these we obtain:

(A.31)
$$\frac{\partial i_t^n}{\partial \lambda_t} = \frac{S_n}{\lambda_t (1 - S_n)} (i_t^n - i_t^e + \lambda_t \sum_{j \neq n} S_j \frac{\partial i_t^j}{\partial \lambda_t}) - 1$$

We now proceed with a guess-and-verify approach. Suppose that $\frac{\partial i_t^n}{\partial \lambda_t} < 0$ for all banks n, so every bank increases their interest rate when attention rises (λ falls). In that case we have that:

(A.32)
$$\frac{\partial i_t^n}{\partial \lambda_t} < \frac{S_n}{\lambda_t (1 - S_n)} (i_t^n - i_t^e) - 1$$

A sufficient condition for $\frac{\partial i_t^n}{\partial \lambda_t} < 0$ is therefore:

(A.33)
$$i_t^n < i_t^e + \frac{\lambda_t (1 - \mathcal{S}_n)}{\mathcal{S}_n}$$

This is clearly true for all banks whose interest rate is below the effective interest rate. I now show that it is true for all banks provided λ_t is above a threshold $\underline{\lambda}$.

Recall that with uninformative priors the effective interest rate rises monotonically with attention and falls monotonically with λ (see Appendix A.3), so $i_t^e \geq \bar{i}_t$, where \bar{i}_t is the unweighted mean interest rate on offer in period t. Condition A.33 is therefore satisfied if:

$$(A.34) i_t^n < \bar{i}_t + \frac{\lambda_t (1 - \mathcal{S}_n)}{\mathcal{S}_n}$$

Substituting out for i_t^n and \bar{i}_t using the bank first order conditions, this becomes:

(A.35)
$$\lambda_t \left(\frac{1 - \mathcal{S}_n + \mathcal{S}_n^2}{\mathcal{S}_n (1 - \mathcal{S}_n)} - \frac{1}{N} \sum_{j=1}^N \frac{1}{1 - \mathcal{S}_j} \right) > \bar{\chi} - \chi_t^n$$

where $\bar{\chi}$ is the unweighted mean transaction cost, which is time-independent. Consider the two fractions inside the brackets. The first is minimized at $S_n = \frac{1}{2}$, at which point:

(A.36)
$$\min_{\mathcal{S}_n} \frac{1 - \mathcal{S}_n + \mathcal{S}_n^2}{\mathcal{S}_n (1 - \mathcal{S}_n)} = 3$$

The second is minimized when $S_j = N^{-1}$ for all j, at which point:

(A.37)
$$\min_{\mathcal{S}_j} \left(-\frac{1}{N} \sum_{j=1}^N \frac{1}{1 - \mathcal{S}_j} \right) = -\frac{N}{N - 1}$$

We therefore have:

(A.38)
$$\lambda_t \left(\frac{1 - \mathcal{S}_n + \mathcal{S}_n^2}{\mathcal{S}_n (1 - \mathcal{S}_n)} - \frac{1}{N} \sum_{j=1}^N \frac{1}{1 - \mathcal{S}_j} \right) > \lambda_t \left(\frac{2N - 3}{N - 1} \right)$$

A sufficient condition for all banks to increase interest rates when λ_t falls is therefore:

(A.39)
$$\lambda_t > \frac{N-1}{2N-3}(\bar{\chi} - \chi^{min}) = \underline{\lambda}$$

Where χ^{min} is the lowest cost experienced by any bank, which again is time-independent as the χ^n_t distribution is assumed to be constant.

Condition A.39 is sufficient rather than necessary, and may in fact be substantially more restrictive than necessary. In particular, it ignores the fact that interest rates are strategic complements $(\frac{\partial i_t^j}{\partial \lambda_t})$ enters equation A.31 with a positive coefficient), so low-rate banks increasing their interest rates when λ falls will incentivize higher-rate banks to do the same. We can see this difference when N=2, in which case the system of equations given by A.31 has a straightforward analytic solution:

(A.40)
$$\frac{\partial i_t^n}{\partial \lambda_t} = \frac{1}{1 - \mathcal{S}_n + \mathcal{S}_n^2} \left[\frac{\mathcal{S}_n^2}{\lambda_t} (i_t^n - i_t^{-n}) - 1 - \mathcal{S}_n \right]$$

This is negative as long as (substituting out for i_t^n and i_t^{-n} using the bank first order

condition):

(A.41)
$$\lambda_t > \frac{2S_n^2(1 - S_n)}{1 - 2S_n + 2S_n^2 + S_n^3 - S_n^4} (\bar{\chi} - \chi^{min})$$

This is substantially less restrictive than condition A.39. The right hand side of condition A.41 is maximized at $S_n = 0.589$, at which point the condition becomes $\lambda_t > 0.475(\bar{\chi} - \chi^{min})$, while condition A.39 in the two-bank case is $\lambda_t > (\bar{\chi} - \chi^{min})$. In the estimated quantitative model condition A.39 is easily satisfied in the region of steady state, so all interest rates rise with attention in the region of the steady state.

In this case with two banks, we can also show that interest rate dispersion always falls when attention rises (λ_t falls). Using equation A.40, we have that $\frac{\partial i_t^1}{\partial \lambda_t} > \frac{\partial i_t^2}{\partial \lambda_t}$ if:

(A.42)
$$i_t^1 - i_t^2 > \frac{\lambda_t(2S_1 - 1)}{2S_1^2 - 2S_1 + 1}$$

Substituting out for i_t^1 and i_t^2 using the two bank first order conditions we obtain:

$$(A.43) \quad \chi_t^2 - \chi_t^1 > \lambda_t \left[\frac{2S_1 - 1}{2S_1^2 - 2S_1 + 1} - \frac{1}{S_1(1 - S_1)} \right] = -\lambda_t \frac{(2S_1^3 - S_1^2 - S_1 + 1)}{S_1(1 - S_1)(2S_1^2 - 2S_1 + 1)}$$

The fraction on the right hand side is positive for all $S^1 \in (0,1)$. We therefore have that in response to an attention rise, bank 1 raises interest rates by less than bank 2 $(\frac{\partial i_t^1}{\partial \lambda_t} > \frac{\partial i_t^2}{\partial \lambda_t})$ whenever bank 2 has higher costs - so whenever bank 2 offers lower rates. That gives us that dispersion falls when attention rises.

In general, search models based on Burdett and Judd (1983) have price dispersion initially rising in search effort, and then falling with search effort once effort is above some threshold. The reason for the difference with the inattention model is that Burdett-Judd models feature a reservation price, above which consumers do not buy. If we impose that interest rates cannot fall below some lower bound, then as attention approaches zero interest rates again converge on this lower bound, just as prices converge on the reservation price in Burdett and Judd (1983). In that case interest rate dispersion initially rises with attention as banks move away from the lower bound, then falls as found above, just as in Burdett-Judd models. Since there are two banks and no interest rate lower bound in the quantitative model in Section IV, this model behaves in a qualitatively similar way to a Burdett-Judd model in the region where more search effort reduces price dispersion.

A.6 Proposition 3

The first line of equation 25 follows directly from differentiating equation 24, and applying the chain rule and the product rule. The second term inside the square brackets is negative as a consequence of Lemma 1. It only remains therefore to show that:

(A.44)
$$\sum_{n=1}^{N} \frac{\partial \Pr(n|s_t)}{\partial \lambda_t} i_t^n = -\frac{1}{\lambda_t^2} Var^e(i_t^n) < 0$$

First, from equation 15, holding all i_t^n constant, we obtain:

$$\frac{\partial \Pr(n|s_t)}{\partial \lambda_t} = -\frac{\Pr(n|s_t)(1 - \Pr(n|s_t))i_t^n}{\lambda_t^2} + \frac{\Pr(n|s_t)}{\lambda_t^2} \sum_{j \neq n} \Pr(j|s_t)i_t^j$$
(A.45)
$$= -\frac{\Pr(n|s_t)(1 - \Pr(n|s_t))i_t^n}{\lambda_t^2} + \frac{\Pr(n|s_t)}{\lambda_t^2}(i_t^e - \Pr(n|s_t)i_t^n)$$

$$= \frac{\Pr(n|s_t)}{\lambda_t^2}(i_t^e - i_t^n)$$

Substituting this into the left hand side of equation A.44:

(A.46)
$$\sum_{n=1}^{N} \frac{\partial \Pr(n|s_t)}{\partial \lambda_t} i_t^n = \frac{1}{\lambda_t^2} \sum_{n=1}^{N} \Pr(n|s_t) i_t^n (i_t^e - i_t^n)$$

$$= \frac{1}{\lambda_t^2} \left((i_t^e)^2 - \sum_{n=1}^{N} \Pr(n|s_t) (i_t^n)^2 \right)$$

$$= -\frac{1}{\lambda_t^2} Var^e (i_t^n)$$

Since variances are positive by definition, this term is strictly negative.

A.7 Relationship between Attention and φ_t

A.7.1 N=2 Banks, Uninformative Priors

As in Section IV, define p_t^g as the probability an individual chooses the high interest rate bank in period t:

(A.47)
$$p_t^g = \frac{\exp(\frac{i_t^g}{\lambda_t})}{\exp(\frac{i_t^g}{\lambda_t}) + \exp(\frac{i_t^b}{\lambda_t})}$$

Individuals paying no attention to bank choice choose bank n with probability $\mathcal{P}_n = 0.5$, so the benchmark no-attention rate in the model is the unweighted mean of the available

interest rates:

(A.48)
$$i_t^b = \mathcal{P}_1 i_t^1 + (1 - \mathcal{P}_1) i_t^2 = 0.5 (i_t^1 + i_t^2)$$

With two banks and uninformative priors, the attention constraint 10 becomes:

(A.49)
$$\mathcal{I}_t = \log(2) + p_t^g \log p_t^g + (1 - p_t^g) \log(1 - p_t^g)$$

Attention is therefore a monotonically increasing function of p_t^g (as $p_t^g \ge 0.5$). The empirical statistic φ_t is:

(A.50)
$$\varphi_t = \frac{p_t^g i_t^g + (1 - p_t^g) i_t^b - \frac{1}{2} (i_t^g + i_t^b)}{\frac{1}{2} (i_t^g - i_t^b)}$$

This simplifies to:

(A.51)
$$\varphi_t = \frac{p_t^g(i_t^g - i_t^b) - \frac{1}{2}(i_t^g - i_t^b)}{\frac{1}{2}(i_t^g - i_t^b)} = 2p_t^g - 1$$

In this case φ_t is therefore a linear function of the probability an individual successfully chooses the higher interest rate bank, which itself is an increasing concave function of attention. This case also highlights the importance of normalizing the spread $i_t^e - i_t^b$ by the standard deviation of interest rates to obtain φ_t : without that, φ_t would be increasing in $i_t^g - i_t^b$, even if p_t^g and so attention are held constant. The normalization therefore prevents changes in rate dispersion from mechanically affecting φ_t .

The normalization only exactly removes all dependence on the shape of the rate distribution in this case of N=2 and uninformative priors, but still helps mitigate the dependence of $i_t^e - i_t^b$ on the spread of interest rates more generally. In particular, it ensures that φ_t is homogeneous of degree 0 in interest rates, so a mean-preserving spread of the interest rate distribution (as studied in Appendix A.4) leaves φ_t unchanged unless attention, and so choice probabilities, change.

A.7.2 N>2 Banks

Since all variables here are defined within the same period I drop all time subscripts to simplify notation. Denoting the unweighted mean interest rate (which is again the model's no-attention rate) as \bar{i} , and the standard deviation of interest rates as $\sigma(i)$, the

model-implied φ is:

(A.52)
$$\varphi = \frac{\sum_{n} i^{n} \Pr(\text{choose } n) - \overline{i}}{\sigma(i)} = \frac{\frac{\sum_{n} i^{n} \exp(\frac{i^{n}}{\lambda})}{\sum_{m} \exp(\frac{i^{m}}{\lambda})} - \overline{i}}{\sigma(i)}$$

First, note that as \mathcal{I} approaches 0, λ approaches infinity, and so $\varphi = 0$ when attention is 0:

(A.53)
$$\lim_{\lambda \to \infty} \varphi = \frac{\frac{1}{N} \sum_{n} i^{n} - \overline{i}}{\sigma(i)} = 0$$

If attention \mathcal{I} reaches $\log(N)$, then each individual can perfectly identify the highest interest rate bank with probability 1, so denoting this as bank 1 (without loss of generality) we have $\varphi > 0$:

(A.54)
$$\varphi(\mathcal{I} = \log(N)) = \frac{i^1 - \overline{i}}{\sigma(i)} = \frac{\frac{1}{N} \sum_n (i^1 - i^n)}{\sigma(i)} > 0$$

Since φ is continuous in attention for $\mathcal{I} \in (0, \log(N))$, the statements above guarantee that \mathcal{I} and φ are positively related at least in some portions of this range.

To make further progress, I now consider how φ changes in the model assuming that interest rates are held fixed. We use the chain rule to write:

(A.55)
$$\frac{\partial \varphi}{\partial \mathcal{I}} = \frac{\partial \varphi}{\partial \lambda} \frac{\partial \lambda}{\partial \mathcal{I}}$$

From Appendix A.2, we have that $\partial \lambda / \partial \mathcal{I} < 0$.

Now consider $\frac{\partial \varphi}{\partial \lambda}$. Since $\frac{\partial \lambda}{\partial i^e} < 0$ we have:

(A.56)
$$\frac{\partial \varphi}{\partial \lambda} = \frac{1}{\sigma(i)} \frac{\partial i^e}{\partial \lambda} < 0$$

This implies that $\frac{\partial \varphi}{\partial \mathcal{I}} > 0$. Holding the distribution of interest rates constant, φ monotonically increases with attention.

This, however, is only the direct effect of a change in attention on φ . As shown in Appendix A.5, a change in attention also implies a change in the interest rate distribution, which when N > 2 will have an indirect effect on φ . Numerically, these indirect effects are small, such that attention and φ are positively related as long as attention is not extremely high.

If attention is very high, then φ can fall as attention increases, because an increase in attention causes the highest rate bank to lower their rates, or only raise them a small amount (see Appendix A.4). Since attention is very high, individuals choose this bank

with a very high probability, and so their effective interest rate only increases a small amount with attention. The increase in attention does, however, cause lower-rate banks to increase their interest rates, and so the benchmark rate increases more strongly than i^e . With N=2 this is counteracted in φ by the normalization by $\sigma(i)$, but with a larger number of banks this adjustment is incomplete because the N-1 lowest rate banks do not converge on each other at the same rate as they converge on the best bank. This breakdown of the link between φ and \mathcal{I} , however, only occurs at extreme levels of attention outside of plausible parameter ranges.

If χ^n are spaced equally on $[0, \chi_0^b]$, where χ_0^b is the highest bank cost in the steady state of the quantitative model, and i^{CB} is at the steady state value from that model, then with N=3 the peak of φ occurs when attention is such that individuals choose the highest rate bank with probability 0.87. As N rises the $\Pr(1|1)$ associated with the threshold level of attention does fall, but only gradually. With N=20, φ is increasing in \mathcal{I} as long as $\Pr(1|1) < 0.85$.

B Persistent Bank Costs

B.1 Modeling Persistent Bank Costs

Here I show how persistent bank costs affect equilibrium attention, interest rates, and individual choice probabilities. For simplicity, I keep to the case of N=2 banks.

Suppose that, as in Section IV, each period one bank is 'good' (cost χ^g) and the other is 'bad' (cost $\chi^b > \chi^g$). There are two possible states of the world: in state 1 bank 1 is good and bank 2 is bad, and in state 2 the ordering is reversed. Unlike in Section IV, assume that there is persistence in the state. Specifically, the state of the world, denoted s_t , follows a two-state Markov process, in which $\Pr(s_{t+1} = s | s_t = s) = g$, where $g \ge 0.5$.

B.1.1 Savers

Assume that savers know the previous state of the world: they observe whether they chose correctly or not when the interest rate payouts occur. Their choice problem in period t therefore remains a static problem. The persistence in s_t shows up as a prior belief biased towards the previous period's realized state, which I assume without loss of generality to be state 1. Savers know the bank policy functions, and so they know what interest rate each bank will set in each state of the world. They therefore face the payoff

¹An exploration of this kind of problem without the assumption that individuals know the history of states (but with exogenous payoffs) can be found in Steiner et al. (2017).

matrix, where again I have dropped time subscripts since the saver problem is static (the same will also be true of the bank problem):

Table 4: Payoff matrix, observed previous state

Here a_n indicates choosing bank n, and $i^{n,s}$ is the interest rate offered by bank n if state s is realized. This matrix is not, in general, symmetric, because bank policy functions depend on both their costs (i.e. the state of the world) and saver predispositions, so bank 1 will set different interest rates in state 1 than bank 2 would in state 2 if $q \neq 0.5$.

With a marginal cost of information of λ , the probability a saver chooses bank n in state s is as in equation 11:

(B.1)
$$P(n|i^{n,s}, i^{-n,s}, s) = \frac{\mathcal{P}_n \exp(\frac{i^{n,s}}{\lambda})}{\mathcal{P}_n \exp(\frac{i^{n,s}}{\lambda}) + (1 - \mathcal{P}_n) \exp(\frac{i^{-n,s}}{\lambda})}$$

The unconditional choice probabilities (predispositions) are found as the solution to two normalization conditions (following Matějka and McKay, 2015):

(B.2)
$$\frac{\exp(\frac{i^{1,1}}{\lambda})g}{\mathcal{P}_1 \exp(\frac{i^{1,1}}{\lambda}) + (1 - \mathcal{P}_1) \exp(\frac{i^{2,1}}{\lambda})} + \frac{\exp(\frac{i^{1,2}}{\lambda})(1 - g)}{\mathcal{P}_1 \exp(\frac{i^{1,2}}{\lambda}) + (1 - \mathcal{P}_1) \exp(\frac{i^{2,2}}{\lambda})} = 1$$

(B.3)
$$\frac{\exp(\frac{i^{2,1}}{\lambda})g}{\mathcal{P}_1 \exp(\frac{i^{1,1}}{\lambda}) + (1 - \mathcal{P}_1) \exp(\frac{i^{2,1}}{\lambda})} + \frac{\exp(\frac{i^{2,2}}{\lambda})(1 - g)}{\mathcal{P}_1 \exp(\frac{i^{1,2}}{\lambda}) + (1 - \mathcal{P}_1) \exp(\frac{i^{2,2}}{\lambda})} = 1$$

The \mathcal{P}_1 that satisfies these conditions is:

(B.4)
$$\mathcal{P}_{1} = \frac{e^{\frac{i^{21}}{\lambda}}e^{\frac{i^{22}}{\lambda}} - (1-g)e^{\frac{i^{21}}{\lambda}}e^{\frac{i^{12}}{\lambda}} - ge^{\frac{i^{11}}{\lambda}}e^{\frac{i^{22}}{\lambda}}}{e^{\frac{i^{11}}{\lambda}}e^{\frac{i^{12}}{\lambda}} - e^{\frac{i^{21}}{\lambda}}e^{\frac{i^{22}}{\lambda}} - e^{\frac{i^{21}}{\lambda}}e^{\frac{i^{22}}{\lambda}} + e^{\frac{i^{21}}{\lambda}}e^{\frac{i^{22}}{\lambda}}}$$

B.1.2 Banks

Since savers observe past states of the world, their priors are entirely determined by the true previous state and the transition probabilities, neither of which the banks can influence. The bank problem therefore remains static: banks choose interest rates to maximize their instantaneous expected profit, giving the same first order condition as in Section I.A (again dropping time subscripts):

(B.5)
$$\frac{d}{di^n}P(n|s)\cdot(i^{CB}-i^n-\chi^n)=P(n|s)$$

I assume that banks take saver predispositions as given when deciding their interest rates. Intuitively, predispositions reflect household knowledge of the exogenous law of motion for the state of the world, and of bank policy functions. If households learn about how banks respond to different costs over time, then a bank changing its policy will not have any effect on predispositions until households learn about the change over many periods. The assumption can therefore be seen as assuming that banks are myopic, and don't take into account the future benefits of manipulating predispositions. While predispositions must be consistent with interest rate policies in the long run, banks do not take this into account in their decisions. This is similar to the assumptions in the deep habits model of Ravn et al. (2006), in which consumption habits evolve very slowly over time, so firms have limited ability to influence them in the short run. This assumption avoids counter-intuitive equilibria in which a fall in attention implies fierce competition for predispositions as households lean more heavily on these in their decisions.

The bank first order condition is then as in Section I:

(B.6)
$$(1 - P(n|s)) \cdot (i^{CB} - i^n - \chi^n) = \lambda$$

The only difference is that Pr(n|s) here includes the predisposition, which comes from the prior beliefs, which are in turn driven by the persistence of bank costs.

B.1.3 Equilibrium

Given exogenous values for g, λ , χ^n , and i^{CB} , an equilibrium consists of values for $\{P(n|s), \mathcal{P}_1, i^n\}$ such that:

- 1. Individuals maximize their expected interest rate subject to the marginal cost of information λ , yielding a predisposition to bank 1 as in equation B.4, and choice probabilities for each bank n in each state s as in equation B.1.
- 2. Banks maximize profits, setting i^n according to equation B.6.

Since this equilibrium allows \mathcal{P}_1 to vary in response to interest rate strategies (equation B.4), this equilibrium can be taken as the steady state of the system after predispositions have had time to adjust.

B.1.4 Simulation Results

I solve this system numerically for an example calibration, and study how the resulting equilibrium varies with λ and g. The qualitative results are robust to a wide variety of calibrations.

All of the results from the static cost model still hold: as attention rises interest rate dispersion falls and average rates rise. The highest rate in the market rises as λ falls as long as λ is above some threshold level. Figure 5 shows this result for an example calibration.

In addition, we have two new results. First, increasing the persistence of bank costs reduces saver attention, as priors become more informative. This causes bank 1 (which is increasingly likely to be low cost) to offer lower interest rates, as savers will come to them with a high probability anyway. Conversely, bank 2 offers higher rates to try and maintain their market share.

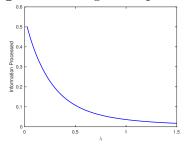
Second, the effective interest rate averaged over individuals depends on the state of the world. Bank 1 is more likely to be the low cost bank, so savers are predisposed to choose them. Bank 1 responds to this predisposition by offering lower interest rates. This only partially offsets the prior belief effect, so savers have $\mathcal{P}_1 > 0.5$ in equilibrium. This means that if the state stays at s_1 (bank 1 is low cost), savers are more likely to correctly identify the low cost bank than they are if the state changes to s_2 . This increases the effective interest rate in s_1 . At the same time, interest rates at the low cost bank are lower if that low cost bank is bank 1, as they are reacting to savers predispositions. Average interest rates are therefore higher in s_2 , which increases the effective interest rate in s_2 relative to s_1 . Which effect dominates depends on the calibration, but in either case there are two possible effective interest rates each period, and whenever there is a transition from one state to the other the effective interest rate will change even if all other variables are at steady state.

State transitions therefore produce shocks to the household effective interest rate, with $i^{e,1}$ realized with probability g and $i^{e,2}$ realized with probability 1-g. These shocks are the key qualitative difference between this model and the static cost model in Sections I and IV.

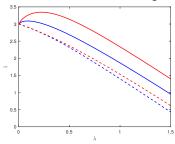
B.2 Empirical Persistence of Interest Rate Rankings

In Sections I and IV I assume that the ranking of a bank in the interest rate distribution has no persistence. Table 5 shows the bank transition probabilities between quintiles of the interest rate distribution of the products studied in Section III over a month and a year. The annual transition probabilities are relevant because these products have a term

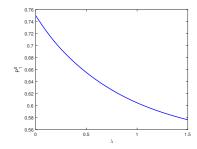
Figure 5: Long run equilibrium varies with λ in the model with persistent bank costs.



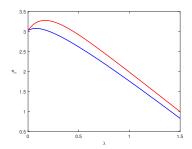
(a) Amount of information processed



(b) Equilibrium interest rates for bank 1 (blue) and bank 2 (red) when they are low (solid) and high (dashed) cost.



(c) Predisposition to choosing bank 1



(d) Effective interest rate if the low cost bank is bank 1 (blue) and bank 2 (red).

Note: Panels show results from simulations of the persistent bank cost model detailed in Appendix B.1, using the calibration: $g = 0.75, \chi^g = 0, \chi^b = 2, i^{CB} = 5, \lambda \in [0.024, 1.5]$. Quantity of information processed in panel (a) is defined as in equation 10. The effective interest rate in panel (d) is defined as $i^e(s) = \Pr(1|s)i^{1,s} + \Pr(2|s)i^{2,s}$.

of one year, so individual savers buying these products have to revisit their decision a year later (or exit the market).

Without persistence, every transition probability would equal 0.2. The values on the diagonal of the transition matrices are all greater than this, so there is some persistence in the data. However, the persistence is limited, even in the top and bottom quintiles where it is strongest. If a saver chose a bank in the top quintile of the interest rate distribution in a given period, then a year later when their product matures there is only a 36% probability of that bank still being in the top quintile. This explains why adding bank fixed effects do not account for much of the dispersion of interest rates, as discussed in Section III.A.

Table 5: Bank quintile transition matrices.

	1	2	3	4	5			
1	0.59	0.16	0.10	0.07	0.07			
2	0.19	0.51	0.19	0.07	0.04			
3				0.20				
4	0.01	0.08	0.30	0.41	0.20			
5				0.22				
(a) Monthly								

	1	2	3	4	5			
1	0.36	0.23	0.15	0.13	0.13			
2	0.25	0.30	0.22	0.13	0.09			
3	0.15	0.25	0.25	0.21	0.14			
4	0.09	0.19	0.21	0.28	0.23			
5	0.06	0.15	0.19	0.25	0.36			
(b) Annual								

Note: In each table the cell (n, m) indicates the probability of transitioning from the nth quintile to the mth quintile in the following period. Sample period: 1996-2009. Source: Moneyfacts Group (2009).

I test if these transition matrices are significantly different from a matrix where every element is 0.2 (the no-persistence case) with a likelihood ratio test:

(B.7)
$$-2\ln\left(\frac{\prod_{n=1}^{5}\prod_{m=1}^{5}p_{n,m}}{\prod_{n=1}^{5}\prod_{m=1}^{5}0.2}\right) \sim \chi_{19}^{2}$$

The critical value of the test statistic for 5% significance is 30.1. The monthly and annual transition matrices give test statistics of 25.9 and 4.3 respectively. We therefore cannot reject the hypothesis of no persistence at either an annual or a monthly frequency.

C Simple Model Extensions

C.1 Alternative Models

Here I show that the main mechanism of the inattention model of Section I is also present in a broad class of models in which households can pay a cost to increase the interest rate they face. This includes a model with frictional search for savings products, as in McKay (2013). To maintain simplicity here, I assume an exogenously fixed distribution of interest rates. I show in Appendix A.5 that attention affects the equilibrium interest rate distribution in the model of Section I in qualitatively the same way as search effort affects the equilibrium price distribution in Burdett and Judd (1983).

Consider an infinitely lived household who chooses consumption and saving each period to maximize expected lifetime utility subject to a standard budget constraint, where income comes from an endowment y_t and asset income. Households can choose in period t to pay a cost to increase the interest rate they face i_t^e . That is, to achieve i_t^e they must pay a cost $C(i_t^e)$, where C is an increasing convex function. I will consider two specifications for this cost, one in which the cost is an additively separable cost in the utility function, and another in which it is a monetary cost entering the budget constraint. The utility cost specification could be thought of as time or effort spent searching for

products, while the monetary cost would be paying an advisor or intermediary to search on their behalf. The specification in use is determined by the binary variable ϕ : when $\phi = 0$ the cost is a utility cost, when $\phi = 1$ we are studying the monetary cost specification.

(C.1)
$$\max_{c_t, b_t, i_t^e} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - (1 - \phi) C(i_t^e) \right]$$

subject to

(C.2)
$$c_t + b_t + \phi C(i_t^e) = y_t + b_{t-1}(1 + i_{t-1}^e)$$

We obtain a familiar consumption Euler equation, and a first order condition on i_t^e :

(C.3)
$$u'(c_t) = \beta(1 + i_t^e) \mathbb{E}_t u'(c_{t+1})$$

(C.4)
$$\beta b_t \mathbb{E}_t u'(c_{t+1}) = (1 - \phi) C'(i_t^e) + \phi u'(c_t) C'(i_t^e)$$

The household problem in Section I is a special case of this problem. The household equates the marginal utility of higher asset income with the marginal cost of achieving such a rise in interest rates. With a diminishing marginal utility of consumption, when expected future consumption falls the marginal utility of higher interest rates rises. If $\phi = 0$ households will respond by paying to increase their interest rate, since C is convex. If $\phi = 1$, households will only pay to increase i_t^e (and so $C'(i_t^e)$) if expected future consumption has fallen relative to current consumption, as increasing future asset income is achieved by sacrificing current consumption.

After a persistent contractionary shock, expected future consumption will fall, so households will pay to increase their interest rate, which will cause current consumption to fall further through the consumption Euler equation, amplifying the shock. This is true in both the utility cost and monetary cost specifications, as long as future consumption is expected to fall by more than current consumption, as is the case in most quantitative models (including that in Section IV) that feature hump-shaped impulse responses. This amplification is the mechanism explored in Section I: the rational inattention problem is a tractable way to motivate and model the cost $C(i_t^e)$ as a utility cost (in which case hump-shaped IRFs are not required), and allows for the distribution of available interest rates to be endogenized as a bank pricing equilibrium. It is not, however, the only way to do this. I now show that a model with frictional search for banks also fits into this class of models.

Suppose that the household is made up of many individuals. Many banks offer savings products, with interest rates that are distributed according to some CDF F(i). Individuals can only choose a bank for their saving if they have observed its interest rate. All individuals observe one bank drawn at random from F, then with probability ψ they observe a second bank (again drawn at random) before choosing where to place their savings. The meeting rate ψ is an increasing function of the search effort of the individual, denoted e, which is decided by the household.

If an individual observes the interest rates of two banks, they choose the bank offering the higher interest rate, so the interest rate chosen has distribution $(F(i))^2$. The expected interest rate for an individual before we know how many banks they will observe, that is the effective interest rate faced by the household overall, is therefore:

(C.5)
$$i_t^e = (1 - \psi(e_t)) \int if(i)di + 2\psi(e_t) \int if(i)F(i)di$$

This is increasing in the probability of seeing a second bank $\psi(e_t)$, as the expected maximum of two draws from a distribution must be (weakly) greater than the expectation of a single draw. We can rearrange this to express search effort in terms of the interest rate the household ends up facing:

(C.6)
$$e_t = \psi^{-1} \left(\frac{i_t^e - \int if(i)di}{2 \int if(i)F(i)di - \int if(i)di} \right)$$

The fraction inside the inverse ψ function increases linearly in i_t^e . If there are diminishing returns to effort (ψ is concave) then effort will be a convex function of the desired interest rate. If effort is costly, then the costs of increasing i_t^e will be a direct cost in the household utility function. As long as there are weakly diminishing returns to effort, and the cost of effort is weakly convex in effort, and at least one of those two curvatures is strict, then we obtain the first specification discussed above: there is a direct cost in utility which is convex in the desired (chosen) level of the interest rate. Formally, if the cost of effort in the utility function is $C_e(e)$, then we have:

(C.7)
$$C(i_t^e) = C_e \left(\psi^{-1} \left(\frac{i_t^e - \int if(i)di}{2 \int if(i)F(i)di - \int if(i)di} \right) \right)$$

(C.8)
$$C''(i_t^e) > 0$$
 if $C''_e(i_t^e) \ge 0$ and $\psi''(e_t) \le 0$, one inequality strict

C.2 Attention to Borrowing: Details

C.2.1 Model

A finite number $N^d \geq 2$ of lending banks choose their interest rate i_t^{nd} to maximize:

(C.9)
$$i_t^n = \arg\max_{\hat{i}_t^{nd}} \Pr(n|\hat{i}_t^{nd}, i_t^{-nd}) \cdot (i_t^{nd} - i_t^{CB} - \chi_t^{nd})$$

where $\Pr(n|i_t^{nd}, i_t^{-nd})$ is the probability an individual chooses bank n for their borrowing with a given interest rate distribution, and χ_t^{nd} is the transaction cost per unit lending of bank n. This implies the first order condition:

(C.10)
$$\frac{d}{di_t^{nd}} \Pr(n|i_t^{nd}, i_t^{-nd}) \cdot (i_t^{nd} - i_t^{CB} - \chi_t^{nd}) = -\Pr(n|i_t^{nd}, i_t^{-nd})$$

As in Section I.E, I assume for simplicity that the distribution of χ_t^{nd} is constant over time, so the household's effective interest rate on debt is not affected by the realizations of χ_t^{nd} . Each bank is also equally likely to draw each χ_t^{nd} , so individuals will have uninformative priors.

The household problem with debt is:

(C.11)
$$\max_{c_t, b_t, i_t^e, i_t^{eb}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \mu \mathcal{I}(i_t^e) - \mu \mathcal{I}^d(i_t^{ed}) \right)$$

subject to

(C.12)
$$c_t + b_t - d = b_{t-1}(1 + i_{t-1}^e) - d(1 + i_{t-1}^{ed}) + y_t$$

(C.13)
$$\mathcal{I}'(i_t^e) > 0, \ \mathcal{I}''(i_t^e) > 0, \ \mathcal{I}^{d\prime}(i_t^{ed}) < 0, \ \mathcal{I}^{d\prime\prime}(i_t^{ed}) > 0$$

where $\mathcal{I}^d(i_t^{ed})$ is the information processing required to achieve an effective debt interest rate of i_t^{ed} . The signs of $\mathcal{I}^{d'}(i_t^{ed}), \mathcal{I}^{d''}(i_t^{ed})$ are assumed here, and are verified below. The household first order conditions are:

(C.14)
$$u'(c_t) = \beta \mathbb{E}_t (1 + i_t^e) u'(c_{t+1})$$

(C.15)
$$\beta b_t \mathbb{E}_t u'(c_{t+1}) = \mu \mathcal{I}'(i_t^e)$$

(C.16)
$$\beta d\mathbb{E}_t u'(c_{t+1}) = -\mu \mathcal{I}'(i_t^{ed})$$

As with savings, each individual chooses which bank to use for their portion of the household's borrowing by solving a discrete-choice rational inattention problem. The only difference to the problem in Section I.C is that for borrowing products, household indirect utility is decreasing in i_t^{ed} . Individuals therefore aim to choose the bank with the lowest interest rate on borrowing. The quantity of information processed is therefore defined as in equation 10, with uninformative priors:

(C.17)
$$\mathcal{I}^d(i_t^{ed}) = \log(N^d) + \sum_{n=1}^{N^d} \Pr(n|s_t^d) \log(\Pr(n|s_t^d))$$

where to reduce notation s_t^d summarizes the state of the world in the lending market: that is, the interest rate at each lending bank.

Minimizing i_t^{ed} subject to this information constraint gives the probability of an individual choosing bank n, and the corresponding effective interest rate:

(C.18)
$$\Pr(n|s_t^d) = \frac{\exp(-\frac{i_t^{nd}}{\lambda_t^d})}{\sum_{k=1}^{N^d} \exp(-\frac{i_t^{kd}}{\lambda_t^d})}$$

(C.19)
$$i_t^{ed} = \sum_{n=1}^{N^d} \Pr(n|s_t^d) i_t^{nd} = \frac{\sum_{n=1}^{N^d} i_t^{nd} \exp(-\frac{i_t^{nd}}{\lambda_t^d})}{\sum_{n=1}^{N^d} \exp(-\frac{i_t^{nd}}{\lambda_t^d})}$$

Equation C.18 is the same as equation 28 in the main text.

Substituting equation C.18 into equation C.17, we obtain:

(C.20)
$$\mathcal{I}^d(i_t^{ed}) = -\frac{i_t^{ed}}{\lambda_t^d} - \log\left(\sum_{n=1}^{N^d} \frac{1}{N^d} \exp\left(-\frac{i_t^{nd}}{\lambda_t^d}\right)\right)$$

Comparing this with equation A.15 in the case with uninformative priors and no variation across cost states S, we have that information processed about debt can be expressed using the same function defining information processing about saving:

(C.21)
$$\mathcal{I}^{d}(i_{t}^{ed}, \{i_{t}^{nd}\}_{n=1}^{N^{d}}, \lambda_{t}^{d}) = \mathcal{I}(-i_{t}^{ed}, \{-i_{t}^{nd}\}_{n=1}^{N^{d}}, \lambda_{t})$$

As a result, we can employ Proposition 2 to obtain:

(C.22)
$$\mathcal{I}^{d'}(i_t^{ed}) = -(\lambda_t^d)^{-1} < 0, \ \mathcal{I}^{d''}(i_t^{ed}) > 0$$

which when combined with Proposition 2 verifies condition C.13.

Combining equations C.16 and C.22 yields equation 27. Differentiating equation C.18 with respect to i_t^{nd} and substituting into equation C.10 yields equation 29.

Differentiating equation C.19 with respect to an arbitrary shock z_t , we obtain:

(C.23)
$$\frac{\partial i_t^{ed}}{\partial z_t} = \left[\sum_{n=1}^{N^d} \frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d} i_t^{nd} + \sum_{n=1}^{N^d} \Pr(n|s_t^d) \frac{\partial i_t^{nd}}{\partial \lambda_t^d} \right] \frac{\partial \lambda_t^d}{\partial z_t}$$

Holding all i_t^{nd} constant, we have that:

(C.24)
$$\frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d} = \frac{\Pr(n|s_t^d)(1 - \Pr(n|s_t^d))i_t^{nd}}{(\lambda_t^d)^2} - \frac{\Pr(n|s_t^d)}{(\lambda_t^d)^2} \sum_{k \neq n} \Pr(k|s_t^d)i_t^{kd} \\
= -\frac{\Pr(n|s_t^d)}{(\lambda_t^d)^2} (i_t^{ed} - i_t^{nd})$$

This in turn implies that:

(C.25)
$$\sum_{n=1}^{N^d} \frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d} i_t^{nd} = -\frac{1}{(\lambda_t^d)^2} \sum_{n=1}^{N^d} \Pr(n|s_t^d) i_t^{nd} (i_t^{ed} - i_t^{nd})$$
$$= \frac{1}{(\lambda_t^d)^2} Var^e(i_t^{nd})$$

where $Var^e(i_t^{nd})$ is defined as in equation 31. Substituting this into equation C.23, we obtain equation 30.

C.2.2 Proof of Proposition 4

First, combine equations 22 and 27 to obtain:

(C.26)
$$\lambda_t^d = \frac{b_t}{d} \lambda_t$$

where λ_t is determined as in equation 22, and is therefore independent of d. Differentiating both sides with respect to z_t implies equation 32.

Second, without loss of generality, denote i_t^{1d} as the lowest interest rate on offer in the borrowing market: $i_t^{1d} < i_t^{nd}$ for all n > 1. As a consequence, $i_t^{ed} \ge i_t^{1d}$, from which we

have:

(C.27)
$$\frac{1}{(\lambda_t^d)^2} Var^e(i_t^{nd}) = \sum_{n=1}^{N^d} \Pr(n|s_t^d) \cdot \left(\frac{i_t^{nd} - i_t^{ed}}{\lambda_t^d}\right)^2$$

(C.28)
$$\leq \sum_{n=1}^{N^d} \Pr(n|s_t^d) \cdot \left(\frac{i_t^{nd} - i_t^{1d}}{\lambda_t^d}\right)^2$$

To make further progress, it is useful to derive an expression linking interest rates and choice probabilities. Manipulating equation 28, we can rewrite the probability of an individual choosing bank n as:

(C.29)
$$\Pr(n|s_t^d) = \frac{\exp(\frac{i_t^{md} - i_t^{nd}}{\lambda_t^d})}{\sum_{k=1}^{N^d} \exp(\frac{i_t^{md} - i_t^{kd}}{\lambda_t^d})}$$

for any $m \in \{1, ..., N^d\}$. Setting n = m, this becomes

(C.30)
$$\Pr(m|s_t^d) = \frac{1}{\sum_{k=1}^{N^d} \exp(\frac{i_t^{md} - i_t^{kd}}{\lambda_t^d})}$$

Combining these:

(C.31)
$$\Pr(n|s_t^d) = \exp(\frac{i_t^{md} - i_t^{nd}}{\lambda_t^d}) \Pr(m|s_t^d)$$

(C.32)
$$\implies \frac{i_t^{md} - i_t^{nd}}{\lambda_t^d} = \log\left(\frac{\Pr(n|s_t^d)}{\Pr(m|s_t^d)}\right)$$

Substituting this in to equation C.28 with m = 1, we have:

$$(C.33) \qquad \frac{1}{(\lambda_t^d)^2} Var^e(i_t^{nd}) \le \sum_{n=1}^{N^d} \Pr(n|s_t^d) \cdot \left(\log\left(\frac{\Pr(n|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2$$

We now proceed by taking the limit of the right hand side as $d \to \infty$. From equation 27, this is equivalent to $\lambda_t^d \to 0$. From equation 28, this implies $\Pr(1|s_t^d) \to 1$, $\Pr(n \neq 1|s_t^d) \to 0$. That is, as d becomes large, individuals pay large amounts of attention, and in the limit they identify the lowest interest rate in the market with certainty. It is convenient to work directly with the limits as $\Pr(1|s_t^d) \to 1$, and by extension as $\Pr(n \neq 1|s_t^d) \to 0$, noting that in this partial-equilibrium setting this is equivalent to

 $d \to \infty$, as the stock of debt has no effect on $u'(c_{t+1})$ or i_t^{CB} . (C.34)

$$\lim_{\Pr(1|s_t^d) \rightarrow 1} \sum_{n=1}^{N^d} \Pr(n|s_t^d) \cdot \left(\log\left(\frac{\Pr(n|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2 = \sum_{n=1}^{N^d} \lim_{\Pr(1|s_t^d) \rightarrow 1} \Pr(n|s_t^d) \cdot \left(\log\left(\frac{\Pr(n|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2$$

Expanding the limit inside the summation gives:

(C.35)
$$\lim_{\Pr(1|s_t^d) \to 1} \Pr(n|s_t^d) \cdot \left(\log\left(\frac{\Pr(n|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2 = \lim_{\Pr(1|s_t^d) \to 1} \Pr(n|s_t^d) \left(\log(\Pr(1|s_t^d))\right)^2 + \lim_{\Pr(1|s_t^d) \to 1} \Pr(n|s_t^d) \left(\log\Pr(n|s_t^d)\right)^2 - 2 \lim_{\Pr(1|s_t^d) \to 1} \Pr(n|s_t^d) \log(\Pr(n|s_t^d)) \log(\Pr(1|s_t^d))$$

The first limit in this expanded expression is trivially equal to 0. Applying l'Hôpital's rule, we further find that:

(C.36)
$$\lim_{\Pr(1|s_t^d)\to 1} \Pr(n|s_t^d) \left(\log \Pr(n|s_t^d)\right)^2 = 0$$

(C.37)
$$\lim_{\Pr(1|s_t^d)\to 1} \Pr(n|s_t^d) \log(\Pr(n|s_t^d)) \log(\Pr(1|s_t^d)) = 0$$

Combining these results, we therefore have:

(C.38)
$$\lim_{\Pr(1|s_t^d)\to 1} \sum_{n=1}^{N^d} \Pr(n|s_t^d) \cdot \left(\log\left(\frac{\Pr(n|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2 = 0$$

The definition of $Var^e(i_t^{ed})$ (equation 31) implies $Var^e(i_t^{ed}) \geq 0$. Combining this and equation C.33, by the squeeze theorem we therefore have:

(C.39)
$$\lim_{d \to \infty} \frac{1}{(\lambda_t^d)^2} Var^e(i_t^{nd}) = \lim_{\Pr(1|s_t^d) \to 1} \frac{1}{(\lambda_t^d)^2} Var^e(i_t^{nd}) = 0$$

This completes the proof of equation 33.

Finally, we turn to equation 34. Differentiate bank n's first order condition (29) to obtain:

(C.40)
$$\frac{\partial i^{nd}}{\partial \lambda_t^d} = \frac{1}{1 - \Pr(n|s_t^d)} + \frac{(i_t^{nd} - i_t^{CB} - \chi_t^{nd})}{1 - \Pr(n|s_t^d)} \frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d}$$

(C.41)
$$= \frac{1}{1 - \Pr(n|s_t^d)} + \frac{\lambda_t^d}{(1 - \Pr(n|s_t^d))^2} \frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d}$$

where the second equality follows from substituting $i_t^{nd} - i_t^{CB} - \chi_t^{nd}$ using equation 29.

Differentiating equation 28 with respect to λ_t^d , we obtain:

(C.42)
$$\frac{\partial \Pr(n|s_t^d)}{\partial \lambda_t^d} = -\frac{\Pr(n|s_t^d)}{\lambda_t^d} \left[\frac{\partial i_t^{nd}}{\partial \lambda_t^d} - \sum_{k=1}^{N^d} \Pr(k|s_t^d) \frac{\partial i_t^{kd}}{\partial \lambda_t^d} - \frac{i_t^{nd} - i_t^{ed}}{\lambda_t^d} \right]$$

Combining equations C.41 and C.42 and rearranging we obtain:

$$(C.43) \quad \frac{\partial i_t^{nd}}{\partial \lambda_t^d} = \frac{1 - \Pr(n|s_t^d)}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} + \frac{\Pr(n|s_t^d)}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} \left(\frac{i_t^{nd} - i_t^{ed}}{\lambda_t^d} + \sum_{k=1}^{N^d} \Pr(k|s_t^d) \frac{\partial i_t^{kd}}{\partial \lambda_t^d}\right)$$

It will be useful now to note that, from the definition of i_t^{ed} (C.19):

(C.44)
$$\frac{i_t^{nd} - i_t^{ed}}{\lambda_t^d} = \frac{i_t^{nd} - \sum_{k=1}^{N^d} \Pr(k|s_t^d) i_t^{kd}}{\lambda_t^d} = \sum_{k=1}^{N^d} \Pr(k|s_t^d) \frac{i_t^{nd} - i_t^{kd}}{\lambda_t^d}$$

Using equation C.32, this can further be rewritten as:

(C.45)
$$\frac{i_t^{nd} - i_t^{ed}}{\lambda_t^d} = \sum_{k=1}^{N^d} \Pr(k|s_t^d) \log \left(\frac{\Pr(k|s_t^d)}{\Pr(n|s_t^d)}\right)$$

Substituting this into equation C.43, we have:

$$(C.46) \quad \frac{\partial i_t^{nd}}{\partial \lambda_t^d} = \frac{1 - \Pr(n|s_t^d)}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2}$$

$$+ \frac{\Pr(n|s_t^d)}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} \left(\sum_{k=1}^{N^d} \Pr(k|s_t^d) \log \left(\frac{\Pr(k|s_t^d)}{\Pr(n|s_t^d)} \right) + \sum_{k=1}^{N^d} \Pr(k|s_t^d) \frac{\partial i_t^{kd}}{\partial \lambda_t^d} \right)$$

We now proceed with a guess-and-verify approach. Suppose at sufficiently high d, $\sum_{k=1}^{N} \Pr(k|s_t^d) \frac{\partial i_t^{kd}}{\partial \lambda_t^d} < 0$. In that case we have that:

$$(\text{C.47}) \quad \Pr(n|s_{t}^{d}) \frac{\partial i_{t}^{nd}}{\partial \lambda_{t}^{d}} < \frac{1 - \Pr(n|s_{t}^{d})}{1 - \Pr(n|s_{t}^{d}) + (\Pr(n|s_{t}^{d}))^{2}} \\ + \frac{\Pr(n|s_{t}^{d})}{1 - \Pr(n|s_{t}^{d}) + (\Pr(n|s_{t}^{d}))^{2}} \left(\sum_{k=1}^{N^{d}} \Pr(k|s_{t}^{d}) \log \left(\frac{\Pr(k|s_{t}^{d})}{\Pr(n|s_{t}^{d})} \right) \right)$$

Taking limits:

$$(C.48) \quad \lim_{\Pr(1|s_t^d) \to 1} \left(\Pr(n|s_t^d) \frac{\partial i_t^{nd}}{\partial \lambda_t^d} \right) < \lim_{\Pr(1|s_t^d) \to 1} \left(\frac{\Pr(n|s_t^d)(1 - \Pr(n|s_t^d))}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} \right)$$

$$+ \lim_{\Pr(1|s_t^d) \to 1} \left(\frac{(\Pr(n|s_t^d))^2}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} \sum_{k=1}^{N^d} \Pr(k|s_t^d) \log \left(\Pr(k|s_t^d)\right) \right)$$

$$- \lim_{\Pr(1|s_t^d) \to 1} \left(\frac{(\Pr(n|s_t^d))^2}{1 - \Pr(n|s_t^d) + (\Pr(n|s_t^d))^2} \log(\Pr(n|s_t^d)) \right)$$

For all banks, whether they have $\Pr(n|s_t^d) \to 1$ (i.e. if n=1) or $\Pr(n|s_t^d) \to 0$ $(n \neq 1)$, all three of the limits on the right hand side are 0. Summing up across banks n we therefore have:

$$(C.49) \qquad \lim_{\Pr(1|s_t^d)\to 1} \left(\sum_{n=1}^{N^d} \Pr(n|s_t^d) \frac{\partial i_t^{nd}}{\partial \lambda_t^d} \right) = \sum_{n=1}^{N^d} \lim_{\Pr(1|s_t^d)\to 1} \left(\Pr(n|s_t^d) \frac{\partial i_t^{nd}}{\partial \lambda_t^d} \right) < 0$$

By the same argument used in deriving equation 33, the limit as $\Pr(1|s_t^d) \to 1$ is equivalent to the limit as $d \to \infty$. This verifies our guess, and completes the proof.

C.2.3 Proof of Corollary 2

With $N^d=2$, there are several results that will prove helpful in simplifying $\partial i_t^{ed}/\partial \lambda_t^d$. First, using the fact that $\Pr(2|s_t^d)=1-\Pr(1|s_t^d)$:

(C.50)
$$1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2 = 1 - \Pr(2|s_t^d) + (\Pr(2|s_t^d))^2$$

Next, from equation C.45:

(C.51)
$$\frac{i_t^{1d} - i_t^{ed}}{\lambda_t^d} = (1 - \Pr(1|s_t^d)) \log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right)$$

(C.52)
$$\frac{i_t^{2d} - i_t^{ed}}{\lambda_t^d} = -\Pr(1|s_t^d)\log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right)$$

Substituting these into equation C.23, we can write $\partial i_t^{ed}/\partial \lambda_t^d$ in terms of $\Pr(1|s_t^d)$ only:

(C.53)
$$\frac{\partial i_t^{ed}}{\partial \lambda_t^d} = 1 + (2\Pr(1|s_t^d) - 1)\log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right) + \Pr(1|s_t^d)(1 - \Pr(1|s_t^d))\left(\log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2$$

Differentiating with respect to $Pr(1|s_t^d)$, we find:

(C.54)
$$\frac{d}{d\Pr(1|s_t^d)} \left(\frac{\partial i_t^{ed}}{\partial \lambda_t^d}\right) = \frac{1 - 2\Pr(1|s_t^d)}{\Pr(1|s_t^d)(1 - \Pr(1|s_t^d))} \cdot \left[1 + \Pr(1|s_t^d)(1 - \Pr(1|s_t^d)) \left(\log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right)\right)^2\right]$$

The term inside the square brackets is strictly positive, so the expression takes on the sign of $1 - 2\Pr(1|s_t^d)$ (i.e. it is positive for $\Pr(1|s_t^d) < 0.5$, 0 for $\Pr(1|s_t^d) = 0.5$, and negative for $\Pr(1|s_t^d) > 0.5$).

Next, I turn to how $Pr(1|s_t^d)$ changes with λ_t^d . Applying the same substitutions (C.50-C.52) to equation C.42 and simplifying, we obtain:

$$(C.55) \quad \frac{\partial \Pr(1|s_t^d)}{\partial \lambda_t^d} = \frac{\Pr(1|s_t^d)(1 - \Pr(1|s_t^d))}{\lambda_t^d} \left[\log \left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)} \right) - \left(\frac{\partial i_t^{1d}}{\partial \lambda_t^d} - \frac{\partial i_t^{2d}}{\partial \lambda_t^d} \right) \right]$$

To evaluate this, we therefore need to evaluate $\partial i_t^{nd}/\partial \lambda_t^d$. Solving the system of equations implied by equation C.41 for $n = \{1, 2\}$, we obtain:

$$\begin{split} &(\mathrm{C}.56) \\ &\frac{\partial i_t^{1d}}{\partial \lambda_t^d} = \frac{1 + \Pr(1|s_t^d)}{1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2} + \frac{(\Pr(1|s_t^d))^2}{1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2} \log \left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right) \\ &(\mathrm{C}.57) \\ &\frac{\partial i_t^{2d}}{\partial \lambda_t^d} = \frac{2 - \Pr(1|s_t^d)}{1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2} - \frac{(1 - \Pr(1|s_t^d))^2}{1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2} \log \left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right) \end{split}$$

Substituting these in to equation C.55 and simplifying, we obtain:

(C.58)
$$\frac{\partial \Pr(1|s_t^d)}{\partial \lambda_t^d} = \frac{\Pr(1|s_t^d)(1 - \Pr(1|s_t^d))(1 - 2\Pr(1|s_t^d))}{\lambda_t^d(1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2)} + \frac{(\Pr(1|s_t^d))^2(1 - \Pr(1|s_t^d))^2}{\lambda_t^d(1 - \Pr(1|s_t^d) + (\Pr(1|s_t^d))^2)} \log\left(\frac{1 - \Pr(1|s_t^d)}{\Pr(1|s_t^d)}\right)$$

Both terms are positive for $\Pr(1|s_t^d) < 0.5, \ 0$ for $\Pr(1|s_t^d) = 0.5$, and negative for

 $\Pr(1|s_t^d) > 0.5$. From this and the sign of equation C.54 we obtain that:

(C.59)
$$\frac{d}{d\lambda_t^d} \left(\frac{di_t^{ed}}{d\lambda_t^d} \right) = \frac{\partial \Pr(1|s_t^d)}{\partial \lambda_t^d} \cdot \frac{d}{d\Pr(1|s_t^d)} \left(\frac{di_t^{ed}}{d\lambda_t^d} \right) > 0 \text{ if } \Pr(1|s_t^d) \neq 0.5$$

Finally, we show that the restriction $\Pr(1|s_t^d) \neq 0.5$ is never binding. From equation C.32 with n=1, m=2, we have that $\Pr(1|s_t^d)=0.5$ if $i_t^{1d}=i_t^{2d}$ (as $\lambda_t^d>0$). Using the bank first order conditions (29) we have:

(C.60)
$$i_t^{1d} - i_t^{2d} = \lambda_t^d \frac{2\Pr(1|s_t^d) - 1}{\Pr(1|s_t^d)(1 - \Pr(1|s_t^d))} + \chi_t^{1d} - \chi_t^{2d}$$

Since $\chi_t^{1d} < \chi_t^{2d}$, this implies $\Pr(1|s_t^d) = 0.5, i_t^{1d} = i_t^{2d}$ can never be an equilibrium. With non-zero attention $(\lambda_t^d > 0)$, individuals always improve on their priors.

From C.26 we have that λ_t^d is strictly decreasing in d. This, combined with equation C.59, implies equation 35.

C.2.4 Google trends data

Section I.F discusses evidence that mortgages are on average large relative to interest-bearing assets among those who hold them, and that interest rate dispersion is indeed lower in mortgage markets as predicted by the model. Attention is, however, difficult to measure directly, as the method in Section III is not appropriate for mortgages (see discussion in Section II.A). Here I provide supplementary evidence using data from Google trends (Google, 2023). Note that this is not a perfect measure of attention, as some people searching for mortgage-related terms could be exploring (for example) whether they want to buy a house or not. They may not be actively engaged in choosing between different mortgages. Similarly, those searching for saving-related terms may not actually be choosing between saving products at that time.

In Table 6 I report summary statistics for the comparison between search intensity for savings accounts and mortgages. Specifically, for each panel, I construct monthly series of relative search intensity, where for each month I divide the intensity of searches related to saving products by the intensity of searches related to mortgages. This is done from January 2004 to June 2023, for the UK, US, and for global searches. Numbers below 1 indicate there are more searches for information on mortgages than saving products.

For the left panel, I use search intensity on Google-generated search topics, which cover a variety of search terms. I take the ratio of searches on the topic "saving accounts" to those on the topic "mortgages". As this may include searches not related to product comparisons, in the left panel I instead use search intensity on the specific searches "saving comparison" and "mortgage comparison". With both measures, the relative

search intensity is below 1 in every month, with the majority of months seeing more than 4 times the search intensity for mortgages than saving products.

Table 6: Summary statistics for relative search intensity of savings to mortgage products.

	Panel A: search type topic				Panel B: search type comparisons			
Region	mean	p25	p50	p75	mean	p25	p50	p75
UK	0.22	0.17	0.21	0.25	0.17	0.11	0.14	0.20
US	0.12	0.08	0.11	0.14	0.14	0.07	0.13	0.20
World	0.15	0.11	0.14	0.19	0.18	0.13	0.17	0.21

Note: Each panel presents summary statistics of the time series of $s_t(\text{saving})/s_t(\text{mortgage})$, where $s_t(\cdot)$ is the Google search intensity for that product for that region in month t. In panel A $s(\cdot)$ is the search intensity for the topics "saving accounts" and "mortgages" respectively, in panel B $s_t(\cdot)$ is the search intensity for the terms "saving comparison" and "mortgage comparison". In each case, $s_t(\cdot)$ is normalized so that the greatest search intensity across the two topics or terms is given a value of 100. The columns p25, p50, p75 give the 25th, 50th, and 75th percentiles of each series. The sample runs from January 2004 to June 2023. Source: Google (2023).

D Further Results for Section III

D.1 Relationship between Bank Positions in Different Market Segments

To calculate the Quoted Household Interest Rate used to construct φ_t in Section III, the Bank of England computes a weighted average of the interest rates in the set of products detailed in Section II.A. The weights are the quantities of new deposits per bank across a broader set of products than those from which the interest rates are taken. Here I show that a bank's position in the distribution of interest rates qualifying for inclusion in the Quoted Household Interest Rate is closely related to their position in the other market segments included when the weights are calculated. As argued in Section II.A, this implies that the cyclical patterns in φ_t found in Section III.C reflect a systematic shift towards banks at the top of all of these market segments when unemployment is high and interest rates are low. φ_t is therefore informative about the position of household choices within the distribution of available rates despite this data limitation.

The weights for the Quoted Household Interest Rate are constructed using new deposits in all fixed interest rate bonds with terms up to one year. The products qualifying for inclusion in the Quoted Household Interest Rate make up 30% of the products in this broader set. Taking all products in the broader set from the Moneyfacts data, I divide them into market segments based on their term, investment size, and interest payment frequencies. The set of such characteristics is given in Table 7.

Taking all combinations of these characteristics yields 72 market segments. Many products are included in multiple segments because an investment of £10000, for example,

Table 7: Bank product characteristics used for subdividing the fixed rate bond market

Characteristic	Division
Term length (months)	{1-3, 4-6, 7-9, 10-12}
Investment size (£000s)	$\{1, 2.5, 5, 10, 25, 50\}$
Interest payment frequency	{Monthly, Quarterly, On maturity}

is often eligible for products with lower minimum investments.

For each segment each month, I rank the banks that compete in that segment-month by their interest rate in that segment-month. If a bank has multiple products that qualify for the Quoted Household Interest Rate, I follow the construction of the Quoted Household Interest Rate series and only consider the one with the highest interest rate. I similarly rank the set of products included in the Quoted Household Interest Rate (the Q segment). I then compute the correlation between these ranks each month, then finally for each market segment I take the mean of these rank correlations over the months, weighting by the number of banks competing in both that segment and the Q segment that month (i.e. weighting by the number of observations used to construct that month's correlation). This gives an average interest rate rank correlation between the Q segment and every other market segment used in constructing the Quoted Household Interest Rate weights.

For 30 of the market segments, there are either no products with that set of characteristics, or there are no occasions where more than one bank simultaneously competes in that segment and the Q segment. This leaves 42 segments for which the rank correlation with the Q segment can be computed.

In these remaining market segments, bank rankings are highly correlated with the rankings in the Q segment. The mean rank correlation across the segments is 0.70, and this is distorted by a small number of market segments which very few banks ever compete in. Of the six segments with rank correlations with the Q segment below 0.5, four are 7-9 month bonds with a monthly payment frequency, which contain less than 1 product per month on average. The other two are also very small segments, with an average of 1.02 and 1.17 banks competing simultaneously in them and the Q segment each month. These correlations are therefore based off very few observations, and the small number of banks competing there each month suggests that they are not large market segments, making them unlikely to play a big role in the weights used to calculate the Quoted Household Interest Rate.

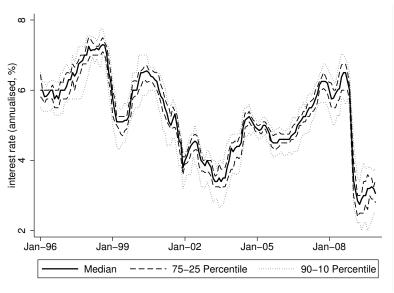
Other market segments are larger. In the ten largest market segments, the average number of banks competing in those segments and the Q segment each month is greater than 11. For the largest five segments, it exceeds 25.

The mean rank correlation across the segments rises to 0.84 when segments are weighted by this mean number of banks competing there and in the Q segment each month. If we take the number of banks competing in a segment as indicative of the size of that market segment, this shows that bank positions within the interest rate distribution analyzed in Section III (in the Q segment), are strongly correlated with bank positions in the other substantial market segments that are included in the weights behind the Quoted Household Interest Rate data.

D.2 Time Series Behavior of Interest Rates and φ_t

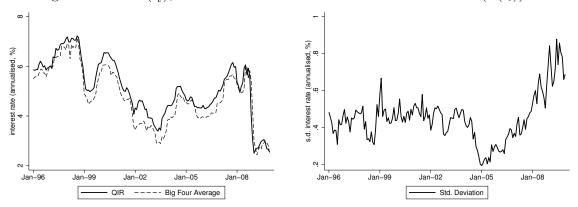
Figure 6 plots the median interest rate each month, alongside the 10th, 25th, 75th, and 90th percentiles of the interest rate distribution. Figure 7 then plots the time series of the three components used to construct φ_t in equation 36, whose summary statistics are reported in Table 2. Although, as discussed above, the spread $\mathbb{E}_h i_t - i_t^b$ is substantially more volatile than $\sigma(i_t)$, the standard deviation of interest rates is still important in determining φ_t . As an example, during 2004 interest rates became substantially less dispersed. If choice probabilities remained constant, we would therefore observe a large fall in $\mathbb{E}_h i_t - i_t^b$. However, no such convergence is observed, suggesting savers became less successful in selecting the highest-rate products in this period. This highlights the importance of normalizing by $\sigma(i_t)$ in the construction of φ_t .

Figure 6: Time series of the 10th, 25th, 50th, 75th, and 90th percentiles of the within-month interest rate distribution.



Note: Percentiles are computed using the products listed in Moneyfacts magazine that qualify for inclusion in the Quoted Household Interest Rate (defined in Section II.A). Source: Moneyfacts Group (2009).

Figure 7: Time series of Quoted Household Interest Rate $(\mathbb{E}_h i_t)$, average interest rate at the 'big four' banks (i_t^b) , and the standard deviation of interest rates $(\sigma(i_t))$.



Note: The big four banks are Barclays, HSBC, Lloyds, and RBS. The within-month standard deviation is computed using the products listed in Moneyfacts magazine that qualify for inclusion in the Quoted Household Interest Rate (defined in Section II.A). Source: Moneyfacts Group (2009), Bank of England (nda).

The raw series for φ_t constructed from these components may in principle be driven by movements in the position of the big four banks in the distribution, as this is an imperfect proxy for the benchmark interest rate obtained with no information processing. To combat this concern, I estimate the following regression equation using OLS:

$$(D.1) \varphi_t = \alpha_0 + \alpha_1 pos_t + v_t$$

where pos_t is measure of the position of the big four banks within the rate distribution each month, constructed similarly to φ_t and defined in equation 37. The results of this regression are shown in Table 8.

The coefficient on pos_t is positive and significant, indicating that raw φ_t is indeed higher when the big four are lower down in the interest rate distribution. However, this mechanical effect is small, as the R^2 of the regression is low.

Table 8: Regression of φ_t on the position of the big four in the interest rate distribution.

	$arphi_t$
pos_t	0.466
	(0.0466)
Constant	0.396
	(0.0614)
R-squared	0.227
Observations	165

Note: Table shows estimated coefficients α_0, α_1 from OLS estimation of the regression $\varphi_t = \alpha_0 + \alpha_1 pos_t + v_t$. φ_t is defined in equation 36, and pos_t is defined in equation 37. Robust standard errors in parentheses. Sample period: 1996-2009. Source: Moneyfacts Group (2009), Bank of England (nda).

D.3 Alternative Measures of φ_t

Here I present two alternatives to the household choice statistic φ_t , which corroborate the evidence in Section III.C that households move up through the distribution of interest rates when unemployment is high and the level of average rates is low.

First, I define a new variable $\varphi_{best,t}$ in a similar way to φ_t , but rather than comparing the average rate achieved by households each month with the rate at the big four banks, I compare it with the highest interest rate available in the market. Intuitively, rather than comparing choices to a 'no attention' benchmark, this compares choices to a full information benchmark.

(D.2)
$$\varphi_{best,t} = \frac{\mathbb{E}_h i_t - i_t^{best}}{\sigma(i_t)}$$

Second, I define $\varphi_{pct,t}$ to be the percentile of the interest rate distribution at which the average interest rate achieved by the household sits. This takes no stance on the appropriate benchmark for choices. As with the previous two statistics, it is homogeneous of degree 0. The downside is that it does not consider the shape of the interest rate distribution either side of the average rate achieved by households.

(D.3)
$$\varphi_{pct,t} = \Pr(i_t^n < \mathbb{E}_h i_t)$$

When households are more successful at choosing the higher interest rate products in the market, $\varphi_{best,t}$ is low and $\varphi_{pct,t}$ is high. The pairwise correlations between the baseline (residualized) φ_t measure, the raw (unresidualized) version, these two alternative statistics ($\varphi_{best,t}$, $\varphi_{pct,t}$), unemployment, and mean interest rates are shown in Table 9. As in Section III, all correlations are between the cyclical components of each variable, extracted with a HP filter.

When unemployment is high and interest rates are low, $\varphi_{pct,t}$ and φ_t (raw and residualized) are high, while $\varphi_{best,t}$ is low. All correlations are strongly significant. The alternative measures of household choice success therefore deliver the same qualitative implications as those found in Section III: in contractions households move up within the distribution of interest rates, away from the low rate offered by the big four banks and towards the highest rate in the market.

D.4 Market Composition and Selection

In this appendix I show that the composition of households holding fixed term savings bonds does not vary significantly through the Great Recession.

Table 9: Pairwise contemporaneous correlations of attention proxies, the unemployment rate, and within-month mean interest rates.

	φ_t (residual)	φ_t (raw)	$\varphi_{best,t}$	$\varphi_{pct,t}$	U_t	$ar{i}_t$
φ_t (residual)	1					
φ_t (raw)	0.895	1				
	(0.000)					
$\varphi_{best,t}$	-0.627	-0.430	1			
	(0.000)	(0.000)				
$arphi_{pct,t}$	0.712	0.416	-0.548	1		
	(0.000)	(0.000)	(0.000)			
U_t	0.273	0.157	-0.548	0.340	1	
	(0.000)	(0.045)	(0.000)	(0.000)		
\overline{i}_t	-0.277	-0.156	0.458	-0.367	-0.792	1
	(0.000)	(0.045)	(0.000)	(0.000)	(0.000)	

Note: All alternative φ_t statistics are computed as detailed in Appendix D.3. U refers to the unemployment rate (ONS series MGSX), and \bar{i} refers to the unweighted mean interest rate on products listed that month in Moneyfacts qualifying for inclusion in the Quoted Household Interest Rate (details in Appendix II.A). All variables are HP-filtered before computing pairwise correlations. P-values in parentheses. Sample period: 1996-2009. Source: Moneyfacts Group (2009), Bank of England (nda), Office for National Statistics (2020e).

Drechsler et al. (2017) show that when the Federal Funds Rate rises in the US, retail banks increase their deposit spreads, and deposits flow out of the retail market. In principle, this kind of switching could drive the countercyclicality in Figure 2. If households differ in their propensity to pay attention to savings, then it could be that when the level of interest rates rises the high-attention households switch out of the retail deposit market. The savers that remain buying fixed-rate savings bonds from banks are the low-attention households, and so the average attention of households in the market falls without any individual household changing their attention.

To explore if this compositional change is occurring, I study waves 1-3 (2006, 2008, 2010) of the Wealth and Assets Survey (WAS) (Office for National Statistics, 2019). This survey asks a large number of households about their assets, including whether they hold fixed term savings bonds (note that this is a broader set of products that those used to construct Figure 2). As the three waves span the Great Recession, if a composition effect is driving the cyclicality of φ_t we should find that characteristics associated with being more attentive to financial decisions become relatively more common over the recession, among the people who hold fixed-term bonds.

Iscenko (2018) and Bhutta et al. (2020) find that households are more likely to be attentive to mortgage decisions if they have high incomes and high levels of education. Iscenko (2018) also finds a non-linear association with age. I therefore explore compositional changes among fixed-term bond-holders along these lines. Specifically, I consider household income by decile of the overall income distribution, indicators for any educational qualifications

and for degree-level qualifications, and an indicator for whether the household is aged 45-54, the age identified by Finke et al. (2017) as corresponding to peak financial knowledge. Income deciles are computed from self-reported labor income plus self-employed income within each survey wave. Table 10 reports the results of regressing each of these variables on indicators for the wave in which the person was surveyed, using the subset of households who hold a fixed-term bond.

Table 10: Regressions on variables related to financial literacy.

	(1)	(2)	(3)	(4)
	Income decile	Some qualification	Degree qualification	Aged 45-54
wave=2	-0.126	0.00423	0.0169	-0.0138
	(0.106)	(0.0133)	(0.0167)	(0.0124)
wave=3	-0.396	-0.0228	0.00258	-0.00820
	(0.105)	(0.0136)	(0.0164)	(0.0127)
Constant	4.991	0.836	0.304	0.137
	(0.0730)	(0.00945)	(0.0116)	(0.00877)
Observations	6138	6138	6138	6138

Note: Table shows estimated coefficients $\alpha_0, \alpha_2, \alpha_3$ from OLS estimation of the regression $X_{it} = \alpha_0 + \alpha_2 \mathbb{1}(t=2) + \alpha_3 \mathbb{1}(t=3) + v_{it}$, for a range of dependent variables X_{it} defined in the text. The baseline is wave t=1 (2006). Waves t=2 and t=3 took place in 2008 and 2010. All regressions are weighted using the survey weights in the Wealth and Assets Survey. Robust standard errors in parentheses. Source: Wealth and Assets Survey, waves 1-3 (Office for National Statistics, 2019).

The only composition change that is significantly different from zero is that the income of fixed term bond-holders declined slightly relative to the overall income distribution between waves 1 and 3. This is the opposite direction to the compositional change that would be required to explain the cyclical patterns of φ_t . All other compositional changes are not significantly different from zero. It is therefore unlikely that compositional changes explain the cyclicality of φ_t .

E Quantitative Model: Further Details

E.1 Quantitative Model Equations

Table 11 lists the (endogenous and exogenous) variables of the quantitative model, and Table 12 lists the log-linearized model equations. For a complete derivation see Appendix F. In the tables below, \overline{X} denotes the steady state of the variable X_t , and e_{Xt} is an i.i.d. normal innovation. To reduce notation, it is convenient below to work with gross interest rates. For any interest rate i_t^x , I use $r_t^x \equiv 1 + i_t^x$ in the tables below. To further reduce notation, I do not use \hat{X}_t to denote the log-deviation of X_t from \bar{X} : rather, in Table 12

any reference to X_t denotes that variable's log-deviation.

Notice that along with the monetary policy and labor disutility shocks, the attention shock ζ^{μ} is assumed to be i.i.d. This is because the estimation finds the shock has a negligible effect on all of the observables, so cannot identify the shock's persistence. As the shock is so small, calibrating the persistence to any other value [0,1) makes no difference to the results.

Table 11: Description of variables in the quantitative model

Variable	Description	Variable	Description
nfa	Net foreign assets	π^m	Inflation: imports
c	Consumption: total	π^w	Inflation: wage
c^h	Consumption: domestic goods	π^{xvf}	Inflation: producer price of exports
c^m	Consumption: imports	q	Real exchange rate
h	Hours	r	Rental rate on capital
inv	Investment	r^b	Gross bad bank interest rate
k	Capital	r^{CB}	Gross policy interest rate
λ	Shadow value of information	r^e	Gross effective interest rate
p^g	Probability of choosing the good bank	r^g	Gross good bank interest rate
p^h	Relative price of domestic final goods	u_c	Marginal utility of c
p^{hv}	Relative price of domestic intermediate goods	w	Real wage
p^m	Relative price of imported goods	x	Exports
p^x	Relative price of exported goods	y^h	Output: used domestically
p^{xv}	Relative producer price of exported final goods	y^v	Output: total
π	Inflation: total	z	Capital utilization
π^{hv}	Inflation: domestic intermediates		
c^f	Foreign demand	$\hat{\zeta}^{\chi}$	Bank interest rate level shock
g	Government spending	$\hat{\zeta}^{\chi b}$	Bank interest rate dispersion shock
π^f	Foreign inflation	ζ^{hb}	Markup shock
p^{xf}	Foreign relative export prices	ζ^k	Capital adjustment cost shock
r^f	Foreign interest rate	$\zeta^{\kappa h}$	Labor disutility shock
tfp	$\overline{\mathrm{TFP}}$	ζ^{rcb}	Monetary policy shock
ζ^{c}	Risk premium shock	ζ^{μ}	Attention shock

Table 12: Log-linearized quantitative model equations

Name	Equation
Wage inflation definition	$\pi_t^w = w_t - w_{t-1} + \pi_t$
Wage Phillips Curve	$(1+\beta\epsilon^w)\pi_t^w - \epsilon^w\pi_{t-1}^w$
	$= \beta \mathbb{E}_t \pi_{t+1}^w + \frac{\psi^w (1 - \beta (1 - \psi^w))}{1 - \psi^w} \left(\frac{\sigma^h}{\sigma^h + \sigma^w} \right) \left(\frac{1}{\sigma^h} h_t - u_{ct} - w_t + \zeta_t^{\kappa h} \right)$
Marginal Utility of c	$u_{ct} = -\frac{1}{\sigma^c}c_t + \psi^{hab}(\frac{1}{\sigma^c} - 1)c_{t-1}$
Consumption Euler equation	$u_{ct} = \mathbb{E}_{t}(u_{ct+1} + r_{t}^{e} - \pi_{t+1}) + \zeta_{t}^{c}$
k first order condition	$p_t^h + \chi^k(k_t - k_{t-1} - \epsilon^k(k_{t-1} - k_{t-2})) + \hat{\zeta}_t^k + r_t^e - \mathbb{E}_t \pi_{t+1}$
z first order condition	$= \beta \mathbb{E}_t(\chi^k(k_{t+1} - k_t - \epsilon^k(k_t - k_{t-1})) + \chi^z r_{t+1} + (1 - \delta) p_{t+1}^h + \hat{\zeta}_{t+1}^h)$ $r_t = \sigma^z z_t + p_t^h$
Relative import demand	$c_t^m = -\sigma^m p_t^m + c_t$
Relative home good demand	$c_t^m = -\sigma^m p_t^m + c_t \ c_t^h = -\sigma^m p_t^h + c_t$
Consumption basket	$c_t = \frac{\overline{c}^h}{\overline{c}}(p_t^h + c_t^h) + \frac{\overline{c}^m}{\overline{c}}(p_t^m + c_t^m)$
Attention first order condition	$\mathbb{E}_t u_{ct+1} - \mathbb{E}_t \pi_{t+1} = -\lambda_t + \zeta_{\mu t}$
Optimal bank choice probability	$p_t^g = \frac{1-\overline{p}^g}{\overline{\lambda}} (\overline{r}^g r_t^g - \overline{r}^b r_t^b - (\overline{r}^g - \overline{r}^b) \lambda_t)$
Effective rate definition	$rac{1}{eta}r_t^e = \overline{p}^g(\overline{r}^g - \overline{r}^b)p_t^g + \overline{p}^g\overline{r}^gr_t^g + (1 - \overline{p}^g)\overline{r}^br_t^b$
Production function	$y_t^v = tfp_t + \frac{(1-\alpha)\overline{h}\frac{\sigma^y - 1}{\sigma^y}}{(1-\alpha)\overline{h}\frac{\sigma^y - 1}{\sigma^y} + \alpha\overline{k}\frac{\sigma^y - 1}{\sigma^y}} h_t + \frac{\alpha\overline{k}\frac{\sigma^y - 1}{\sigma^y}}{(1-\alpha)\overline{h}\frac{\sigma^y - 1}{\sigma^y} + \alpha\overline{k}\frac{\sigma^y - 1}{\sigma^y}} (k_{t-1} + z_t)$
Domestically consumed inflation definition	$\pi_t^{-} = p_t^{-} - p_{t-1}^{-} + \pi_t^{-}$
Export inflation definition	$\pi_t^{xvf} = p_t^{xv} - p_{t-1}^{xv} + \pi_t^f + q_t - q_{t-1} $ $(1 + \beta \epsilon^{hv}) \pi_t^{hv} - \epsilon^{hv} \pi_{t-1}^{hv}$
Domestic good Phillips Curve	
	$= \beta \mathbb{E}_t \pi_{t+1}^{hv} - \frac{\sigma^{hb} - 1}{\chi^{hv}} (p_t^{hv} + \frac{1}{\sigma^y} (y_t^v - h_t) - w_t + \frac{\sigma^y - 1}{\sigma^y} t f p_t) + \zeta_t^{hb}$
Export good Phillips Curve	$(1+eta\epsilon^{xv})\pi_t^{xvf} \stackrel{\chi}{-} \epsilon^{xv}\pi_{t-1}^{xvf}$
	$= \beta \mathbb{E}_{t} \pi_{t+1}^{xvf} - \frac{\sigma^{xb} - 1}{\chi^{xv}} (p_{t}^{xv} + \frac{1}{\sigma^{y}} (y_{t}^{v} - h_{t}) - w_{t} + \frac{\sigma^{y} - 1}{\sigma^{y}} t f p_{t}) + \zeta_{t}^{hb}$
Optimal k - h ratio	$z_t + k_{t-1} - h_t = \sigma^y(w_t - r_t)$
Good bank profit maximization	$\lambda_t = rac{1}{\overline{r}^{CB} - \overline{r}^g - \chi_0^g} (\overline{r}^{CB} r_t^{CB} - \overline{r}^g r_t^g - \hat{\zeta}_t^\chi) - rac{\overline{p}^g}{1 - \overline{p}^g} p_t^g$
Bad bank profit maximization	$\lambda_t = \frac{1}{\bar{r}^{CB} - \bar{r}^b - \chi_0^b} (\bar{r}^{CB} (1 - \chi_1) r_t^{CB} - \bar{r}^b r_t^b - \hat{\zeta}_t^{\chi} - \hat{\zeta}_t^{\chi b}) + p_t^g$
Taylor rule	$r_t^{CB} = \zeta_t^{rcb} + \theta^{rcb} r_{t-1}^{CB} + (1 - \theta^{rcb}) (\theta^p \pi_t + \theta^y (y_t^v - tf p_t))$
Export demand	$x_t = c_t^f - \sigma^x (q_t + p_t^x - p_t^{xf})$
Import inflation definition	$\pi_t^m = p_t^m - p_{t-1}^m + \pi_t$
Import Phillips Curve	$(1 + \beta \epsilon^{m}) \pi_{t}^{m} - \epsilon^{m} \pi_{t-1}^{m}$ $= \beta \mathbb{E}_{t} \pi_{t+1}^{m} + \frac{\psi^{m} (1 - \beta (1 - \psi^{m}))}{1 - \psi^{m}} (p_{t}^{xf} - q_{t} - p_{t}^{m})$
	$=\beta \mathbb{E}_t \pi_{t+1}^m + \frac{\varphi^{-(1-\varphi^{-1})}}{1-\psi^m} (p_t^{x_j} - q_t - p_t^m)$
Price of domestic consumption basket	$p_t^h = \kappa^{hv} p_t^{hv} + (1 - \kappa^{\tilde{h}v}) p_t^m$ $p_t^x = \kappa^{xv} p_t^{xv} + (1 - \kappa^{xv}) p_t^m$
Price of export consumption basket k law of motion	$ \begin{aligned} p_t - \kappa & p_t + (1 - \kappa^z)p_t \\ \delta inv_t &= k_t - (1 - \delta)k_{t-1} + \chi^z z_t \end{aligned} $
Goods market clearing	$y_t^v = \kappa^{hv}(\overline{c}^h c_t^h + \overline{invinv_t} + \overline{g}g_t) + \kappa^{xv}\overline{x}x_t$
Domestic goods market clearing	$y_t^h = rac{\overline{c}^h}{\overline{\eta}^h} c_t^h + rac{i \overline{n} v}{\overline{\eta}^h} i n v_t + rac{\overline{g}}{\overline{\eta}^h} g_t$
nfa law of motion	$nfa_{t} = (1 + \overline{i}^{CB})(nfa_{t-1} + \overline{nfa}(r_{t-1}^{f} - \pi_{t}^{f} - q_{t} + q_{t-1})) + \overline{x}(p_{t}^{x} + x_{t}) - \overline{c}^{m}c_{t}^{m}$
•	$-(1-\kappa^{hv})\overline{y}^h y_t^h - (1-\kappa^{xv})\overline{x}x_t - (\overline{c}^m + (1-\kappa^{hv})\overline{y}^h + (1-\kappa^{xv})\overline{x})p_t^m$
Real UIP	$\mathbb{E}_{t}q_{t+1} - q_{t} + \chi^{nfa} nfa_{t} = r_{t}^{f} - r_{t}^{CB} - \mathbb{E}_{t}(\pi_{t+1}^{f} - \pi_{t+1})$
TFP	$tfp_t = \rho_{tfp}tfp_{t-1} + e_{tfpt}$
Government spending	$g_t = \rho_g g_{t-1} + e_{gt}$
Risk premium shock	$\zeta_c^c = \rho_{\zeta^c} \zeta_{t-1}^c + (1 - \rho_{\zeta^c}^2)^{\frac{1}{2}} e_{\zeta^c t}$
Markup shock	$\zeta_t^{hb} = \rho_{\zeta^{hb}} \zeta_{t-1}^{hb} + (1 - \rho_{\zeta^{hb}_{t-1}}^2)^{\frac{1}{2}} e_{\zeta^{hb}t}$
Capital adjustment cost shock	$\hat{\zeta}_{t}^{k} = \rho_{\zeta^{k}} \hat{\zeta}_{t-1}^{k} + (1 - \rho_{\zeta^{k}}^{2})^{\frac{1}{2}} e_{\zeta^{k} t}$
Labor disutility shock	$\zeta_t^{\kappa h} = e_{\zeta^{\kappa h}t}$
Monetary policy shock	$\zeta_t^{rcb} = e_{\zeta^{cb}t}$
Bank rate level shock	$\hat{\zeta}_{t}^{\chi} = \rho_{\zeta^{\chi}} \hat{\zeta}_{t-1}^{\chi} + (1 - \rho_{\zeta^{\chi}}^{2})^{\frac{1}{2}} e_{\zeta^{\chi} t}$
Bank rate dispersion shock	$\hat{\zeta}_t^{\chi b} = \rho_{\zeta \chi b} \hat{\zeta}_{t-1}^{\chi b} + (1 - \rho_{\zeta \chi b}^2)^{\frac{1}{2}} e_{\zeta \chi b_t}$
Attention shock	$\zeta_t^\mu = e_{\zeta^\mu t}$
Foreign variables	VAR(4) in Appendix E.2.2

E.2 Estimation Details

E.2.1 Data Sources and Treatment

There are 11 standard observable variables: domestic (UK) GDP, consumption, inflation, the 3-month treasury bill rate, investment, real wages, hours worked, and foreign inflation, industrial production, interest rates, and relative export prices. The foreign variables are trade-weighted averages of the other G7 countries. On top of these I add 3 observables from the Moneyfacts data described in Section II: the mean and standard deviation of deposit rates, and φ_t as calculated in Section III.B. I use data from 1993-2009.

I follow Harrison and Oomen (2010) to source the standard observables. See their paper and the replication package associated with this paper for details of the data series. The only exception to the Harrison-Oomen method is that I use industrial production for all foreign countries, where they use a mix of industrial production and GDP. The data are obtained from Board of Governors of the Federal Reserve System (2020), International Monetary Fund (2020), and Office for National Statistics (2020a,b,c,d,e,f).

I take log first differences of all domestic real variables, and transform inflation and interest rates into quarterly gross rates before taking logs and de-meaning. For the foreign real variables, I take logs and then extract the cyclical components using a one-sided HP filter. For the average and standard deviation of interest rates in Moneyfacts I follow the same procedure used for the treasury bill rate, averaging across months within each quarter before taking logs, and leaving a quarter as missing when a month of data is missing. I include φ_t in levels to avoid losing more observations after the quarters with missing months through first-differencing. I therefore use a one-sided HP filter to extract the cyclical component of φ_t . I do not take logs of φ_t as on several occasions it is close to zero. This is therefore a measure of linearized, not log-linearized, φ_t . The observation equation is adjusted accordingly. I choose χ_0^g and χ_0^b to match two moments: the average gap between the highest and the (unweighted) mean interest rate available in the Moneyfacts data, and the average gap between the unweighted mean interest rate in Moneyfacts and the policy rate.

Using N=2 banks in the quantitative model keeps the equations simple, but it also means that the model-implied φ_t is always in the range [0,1]. The observed data has larger numbers of banks, so to map that into suitable data for the model I measure the maximum possible φ_t in the data each period, that would be achieved if the Quoted Household Interest Rate was equal to the highest rate available that month. I divide the observed φ_t by the mean of these values (2.993) before HP-filtering to give an approximate mapping into the $\varphi_t \in [0,1]$ range seen in the model.

E.2.2 Foreign VAR

Foreign variables are assumed to follow a VAR(4) process estimated outside of the model, as in Adolfson et al. (2007). Denoting the vector of foreign variables as Y_t , the structural VAR process is:

(E.1)
$$F_0Y_t = F_1Y_{t-1} + F_2Y_{t-2} + F_3Y_{t-3} + F_4Y_{t-4} + u_t$$

To identify the parameters, I start with the Adolfson et al. (2007) restrictions: output and inflation are assumed to be unaffected by contemporaneous shocks to anything other than themselves, but interest rates respond to both. As I have an extra variable not in Adolfson et al. (2007) (relative export prices), I add that inflation and output also do not respond contemporaneously to shocks to relative export prices. Furthermore, I assume that the foreign interest rate does not respond contemporaneously to shocks to relative export prices, but that relative export prices can respond contemporaneously to all variables. Intuitively, this reflects the notion that the exchange rate can vary rapidly in response to shocks, and that this will affect the relative export price. This gives:

(E.2)
$$F_0 = \begin{bmatrix} 1, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0 \\ -\gamma_{\pi}, & -\gamma_{y}, & 1, & 0 \\ -\gamma_{\pi}^{p}, & -\gamma_{y}^{p}, & \gamma_{r}^{p}, & 1 \end{bmatrix}$$

Where the order of variables in Y_t is inflation, output, interest rates, relative export prices. The model is over-identified. We cannot reject the over-identifying restrictions (p-value 0.87).

E.2.3 Calibration, Priors, and Estimation Results

Table 13 gives descriptions of each calibrated parameter and its calibrated value. Table 14 gives descriptions of each estimated parameter and its prior. See Harrison and Oomen (2010) for the sources of each calibrated value and prior except those specific to the attention block, which are discussed in Section IV.C.

Tables 15 and 16 show the estimation results for the baseline model and the full information model in Section IV.E respectively. Impulse response functions of consumption to all shocks listed in Table 3, in both the baseline model and the fixed-attention alternative, are shown in Figure 8.

Table 13: Calibrated parameters

Parameter	Description	Value
α	Capital income share	0.3
β	Discount factor	0.99
δ	Depreciation rate	0.025
χ^{nfa}	Net foreign asset adjustment cost	0.01
χ^z	Capital utilization cost	$\beta^{-1} - 1 + \delta$
κ^{hv}	Share of domestic value added in home goods	0.935
κ^{xv}	Share of domestic value added in export goods	0.748
ψ^m	Expenditure weight of imports in consumption	0.248
ψ^{pm}	Imports Calvo parameter	0.4
σ^{hb}	Elasticity of substitution: goods varieties	9.668
σ^m	Elasticity of substitution: home vs. foreign goods	1.77
σ^w	Elasticity of substitution: labor varieties	8.3
σ^x	Elasticity of substitution: exports	1.5
σ^{xb}	Elasticity of substitution: export varieties	9.668
σ^y	Elasticity of substitution: labor vs. capital in production	0.5
$ar{g}$	Steady state government spending share of output	0.19
\overline{inv}	Steady state investment share of output	0.138
$\bar{\sigma}(i^n)$	Steady state standard deviation of interest rates	0.002*
$\bar{i}^n - \bar{i}^{CB}$	Steady state saving interest rate - policy rate spread	0.001*

^{*}The steady state bank costs χ_0^g, χ_0^b are the parameters that adjust to ensure these targets are met.

Table 14: Description and priors of estimated parameters

Parameter	Description	Prior Distribution
σ^c	Intertemporal elasticity of substitution	N(0.66, 0.198)
ψ^{hab}	Consumption habit parameter	Beta(0.69, 0.05)
σ^h	Labor supply elasticity	N(0.43, 0.108)
χ^k	Capital adjustment cost constant	N(201, 60.3)
ϵ^k	Indexation to past capital adjustment in capital adjustment cost	Beta(0.5, 0.25)
σ^z	Capital utilization cost elasticity	N(0.56, 0.168)
χ^{hv}	Domestic goods price adjustment cost	N(326, 97.8)
ϵ^{hv}	Domestic goods inflation indexation	Beta(0.26, 0.1)
χ^{xv}	Export goods price adjustment cost	N(43, 12.5)
ϵ^{xv}	Export goods inflation indexation	Beta(0.14, 0.05)
ψ^{pm}	Imported goods Calvo parameter	Beta(0.40, 0.15)
ϵ^m	Imported goods inflation indexation	Beta(0.17, 0.05)
ψ^w	Wage Calvo parameter	Beta(0.21, 0.05)
ϵ^w	Wage inflation indexation	Beta(0.58, 0.145)
$ heta^p$	Taylor Rule inflation weight	N(1.87, 0.131)
θ^y	Taylor Rule output weight	N(0.11, 0.028)
$ heta^{rcb}$	Taylor Rule persistence	Beta(0.87, 0.05)
μ	Marginal cost of information	InvGamma(0.005, 0.5)
χ_1	Elasticity of inefficient bank costs to the policy rate	N(0, 0.25)
$ ho_{tfp}$	Persistence of TFP shock	Beta(0.89, 0.05)
σ_{tfp}	s.d. TFP shock	InvGamma(0.006, 2)
$ ho_g$	Persistence of government spending shock	Beta(0.96, 0.025)
σ_g	s.d. government spending shock	InvGamma(0.009, 2)
$ ho_x$	Persistence of shock x	U(0.5, 0.289)
$\sigma_{\zeta^{\kappa h}}$	s.d. labor disutility shock	InvGamma(0.01, 2)
σ_{ζ^c}	s.d. monetary policy shock	InvGamma(0.025, 2)
$\sigma_{\zeta^h b}$	s.d. price markup shock	InvGamma(0.006, 2)
σ_{ζ^k}	s.d. capital adjustment cost shock	InvGamma(0.06, 2)
σ_y	s.d. shock y	InvGamma(0.001, 2)
$\sigma_{ u z}$	s.d. measurement error on z	InvGamma(0.01, 2)

 $x = \zeta^c, \zeta^{hb}, \zeta^k, \zeta^\mu, \zeta^\chi, \zeta^{\chi b}$ refers to the shock to the risk premium, price markups, capital adjustment costs, information costs, interest rate level and dispersion. All other shocks are assumed i.i.d. $y = \zeta^{rg}, \zeta^\mu, \zeta^\chi, \zeta^{\chi b}$. z contains the mean and standard deviation of bank deposit rates, and φ_t .

Table 15: Estimated posteriors in baseline model

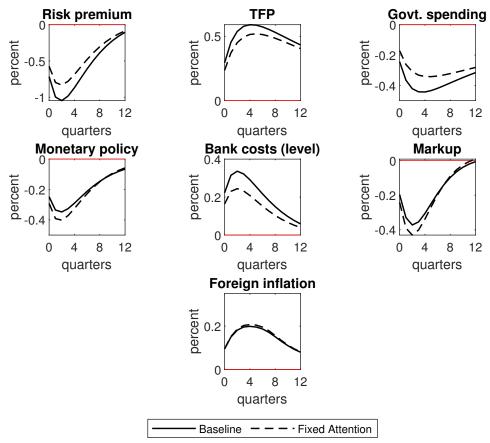
Parameter	Mean	5%	95%	Parameter	Mean	5%	95%
σ^c	0.237	0.172	0.308	$ ho_{\zeta^{hb}}$	0.248	0.003	0.443
ψ^{hab}	0.740	0.675	0.820	$ ho_{\zeta^k}$	0.376	0.000	0.832
σ^h	0.464	0.271	0.631	$\rho_{\zeta} \chi$	0.924	0.860	0.983
χ^k	152.448	58.233	241.595	$ ho_{\zeta\chi b}$	0.785	0.690	0.875
ϵ^k	0.475	0.010	0.822	μ	0.037	0.026	0.047
σ^z	0.564	0.315	0.841	χ_1	-0.280	-0.494	-0.063
χ^{hv}	422.303	274.872	554.110	σ_g	0.033	0.028	0.038
ϵ^{hv}	0.223	0.078	0.363	$\sigma_{\zeta^{\kappa h}}$	1.550	0.676	2.386
χ^{xv}	37.092	13.306	60.056	$\sigma_{_{\mathcal{C}^{rcb}}}$	0.001	0.001	0.002
ϵ^{xv}	0.135	0.058	0.217	σ_{tfp}	0.007	0.006	0.008
ψ^{pm}	0.632	0.371	0.894	σ_{ζ^c}	0.009	0.006	0.012
ϵ^m	0.165	0.079	0.244	$\sigma_{\zeta^{hb}}$	0.007	0.005	0.008
ψ^w	0.267	0.202	0.328	σ_{ζ^k}	0.140	0.051	0.221
ϵ^w	0.335	0.184	0.474	$\sigma_{\zeta^{\mu}}$	0.000	0.000	0.001
$ heta^p$	1.813	1.598	2.024	σ_{ζ}^{\prime}	0.003	0.002	0.004
θ^y	0.144	0.102	0.187	$\sigma_{\zeta\chi b}$	0.003	0.002	0.004
$ heta^{rcb}$	0.912	0.891	0.933	$\sigma_{ u \varphi}$	0.093	0.078	0.108
$ ho_{tfp}$	0.957	0.934	0.981	$\sigma_{ u s}$	0.009	0.002	0.019
$ ho_g$	0.954	0.921	0.983	$\sigma_{ u m}$	0.002	0.001	0.002
$ ho_{\zeta^c}$	0.892	0.831	0.947				

Table 16: Estimated posteriors in full information model

Parameter	Mean	5%	95%	Parameter	Mean	5%	95%
σ^c	0.187	0.108	0.265	$\rho_{\zeta hb}$	0.266	0.044	0.474
ψ^{hab}	0.723	0.652	0.796	ρ_{ζ^k}	0.753	0.578	0.952
σ^h	0.443	0.280	0.582	$\rho_{\zeta \chi}$	NA	NA	NA
χ^k	162.775	66.424	252.815	$\rho_{\zeta\chi b}$	NA	NA	NA
ϵ^k	0.143	0.001	0.295	μ	NA	NA	NA
σ^z	0.534	0.267	0.826	χ_1	NA	NA	NA
χ^{hv}	412.895	270.223	548.295	σ_g	0.033	0.028	0.038
ϵ^{hv}	0.215	0.073	0.365	$\sigma_{\zeta^{\kappa h}}$	2.074	0.852	3.303
χ^{xv}	33.019	4.005	54.746	σ_{crcb}	0.001	0.001	0.002
ϵ^{xv}	0.138	0.057	0.216	σ_{tfp}	0.007	0.006	0.008
ψ^{pm}	0.652	0.401	0.893	σ_{ζ^c}	0.012	0.007	0.018
ϵ^m	0.165	0.087	0.241	$\sigma_{\zeta^{hb}}$	0.007	0.005	0.008
ψ^w	0.239	0.173	0.298	σ_{ζ^k}	0.213	0.070	0.364
ϵ^w	0.320	0.172	0.473	$\sigma_{\zeta\mu}$	NA	NA	NA
$ heta^p$	1.851	1.646	2.058	$\sigma_{\zeta^{\chi}}$	NA	NA	NA
θ^y	0.146	0.103	0.186	$\sigma_{\zeta\chi b}$	NA	NA	NA
$ heta^{rcb}$	0.912	0.893	0.935	$\sigma_{\nu\varphi}$	NA	NA	NA
$ ho_{tfp}$	0.962	0.939	0.987	$\sigma_{ u s}$	NA	NA	NA
$ ho_g$	0.953	0.919	0.990	$\sigma_{ u m}$	NA	NA	NA
ρ_{ζ^c}	0.895	0.837	0.950	<u> </u>	.1:		

Note: full information implies zero interest rate dispersion, so this can no longer discipline parameters of bank cost functions as in the baseline model. I therefore set $\chi_0^b = \chi_1 = 0$. This implies $r_t^g = r_t^b = r_t^{CB} - \xi_t^{\chi} - \xi_t^{\chi b}$. In the log-linearized model, bank cost shocks are then isomorphic to risk premium shocks, so are excluded without loss of generality.

Figure 8: Impulse Response Functions of c_t in response to various 1 standard deviation shocks.



Note: Solid lines are simulations of a 1 standard deviation shock in the estimated model described in Section IV.A. Estimation details and estimated parameters are listed in Appendix E.2. Dashed lines are simulations from the same model, with the same parameters, but where p_t^g has been held at steady state in all periods, so households are no longer on their first order condition for attention (equation 39) in each period.

E.3 Attention to Borrowing in the Quantitative Model

E.3.1 Model

I introduce borrowing to the model by assuming that a fraction q_d of households are less patient than others, as in Iacoviello (2005), Eggertsson and Krugman (2012), and many others. These households accumulate debt up to an exogenous constraint, so I refer to them as debtors, and index their idiosyncratic variables and parameters by d.

Lending banks. The banks engaged in lending are set up as in Section I.F and Appendix C.2. There are $N^d=2$ lending banks, who borrow from the government at the policy rate i_t^{CB} and lend this money out to individuals in debtor households. As with the deposit-taking banks in Section IV.A.2, each period one lender draws a low cost, χ_t^{gd} . The other draws a high cost χ_t^{bd} . There is no persistence in these cost rankings.

The first order conditions for the good (low cost) and bad (high cost) lending bank are (following equation 29):

(E.3)
$$(1 - p_t^{gd})(i_t^{gd} - i_t^{CB} - \chi_t^{gd}) = \lambda_t^d$$

(E.4)
$$p_t^{gd}(i_t^{bd} - i_t^{CB} - \chi_t^{bd}) = \lambda_t^d$$

where p_t^{gd} is the probability a borrower chooses the good lender in period t.

Log-linearizing around steady state as described in Section IV.B, these become:

(E.5)
$$\frac{1}{\bar{i}^{gd} - \bar{i}^{CB} - \bar{\chi}^{gd}} (\bar{i}^{gd} \hat{i}^{gd}_t - \bar{i}^{CB} \hat{i}^{CB}_t - \bar{\chi}^{gd} \hat{\chi}^{gd}_t) - \frac{\bar{p}^{gd}}{1 - \bar{p}^{gd}} \hat{p}^{gd}_t = \hat{\lambda}^d_t$$

(E.6)
$$\frac{1}{\bar{i}^{bd} - \bar{i}^{CB} - \bar{\chi}^{bd}} (\bar{i}^{bd} \hat{i}^{bd}_t - \bar{i}^{CB} \hat{i}^{CB}_t - \bar{\chi}^{bd} \hat{\chi}^{bd}_t) + \hat{p}^{gd}_t = \hat{\lambda}^d_t$$

where \bar{x} indicates the steady state of x_t , and \hat{x}_t indicates the log-deviation of x_t from \bar{x} .

Households. A fraction q_d of households are debtors, while the remaining $1 - q_d$ are savers. The savers are identical to the households in the model presented in Section IV, except for a detail in the budget constraint discussed below. The debtors have the same preferences, except that their discount factor is $\beta^d < \beta$. Denote the consumption of debtors as c_t^d .

The debtor household budget constraint is:²

(E.7)
$$PC_t c_t^d - D_t + (1 + i_{t-1}^{ed})D_{t-1} = W_t h_t - PC_t \tau_t + PC_t \tau_0$$

where D_t is nominal debt taken out in period t, i_t^{ed} is the effective interest rate on that debt, and τ_0 is a steady state transfer. Since debtors have lower discount factors than savers, they reduce their asset holdings in all assets until they hit the relevant constraints. This is why there is no capital income or firm profit in equation E.7: debtor households reduce their capital and equity holdings to zero. It is also the reason that these households choose to hold debt D_t rather than savings.

The constraint on debt takes the simple form $D_t/PC_t \leq d$. That is, real debt holdings cannot exceed the constant level d. β^d will be set sufficiently low that this constraint

²Note I assume here that debtor households supply labor to the same union as the savers, and that the labor of the two types of households are perfect substitutes, so wages and hours worked are the same for all households. The wage Phillips curve is modified accordingly. See the online appendix for details.

always binds for debtors, so in real terms the debtor budget constraint is:

(E.8)
$$c_t^d - d + \frac{1 + i_{t-1}^{ed}}{\pi_t} d = w_t h_t - \tau_t + \tau_0$$

As in Section I.F, the first order condition on attention takes a very similar form to that of savers:

(E.9)
$$\beta^{d} d\mathbb{E}_{t} \frac{u_{ct+1}^{d}}{\pi_{t+1}} = \mu e^{\zeta_{t}^{\mu}} (\lambda_{t}^{d})^{-1}$$

where

(E.10)
$$u_{ct+1}^d = \frac{1}{(\tilde{c}_t)^{\psi^{hab}}} \left(\frac{c_{t+1}^d}{(\tilde{c}_t)^{\psi^{hab}}} \right)^{-\frac{1}{\sigma_c}}$$

where habits are dependent on aggregate consumption across both household types $\tilde{c}_t \equiv (1 - q_d)c_t + q_dc_t^d$. The same is true for savers. Note that the marginal cost of information $\mu e^{\zeta_t^{\mu}}$ is part of preferences, and so is assumed to be common to savers and debtors.

Within debtor households, individuals choose banks as in Appendix C.2, and this implies a bank choice probability given by

(E.11)
$$p_t^{gd} = \frac{\exp(-\frac{i_t^{gd}}{\lambda_t^d})}{\exp(-\frac{i_t^{gd}}{\lambda_t^d}) + \exp(-\frac{i_t^{bd}}{\lambda_t^d})}$$

Finally, the effective interest rate on debt is defined as:

(E.12)
$$i_t^{ed} = p_t^{gd} i_t^{gd} + (1 - p_t^{gd}) i_t^{bd}$$

Log-linearizing in the fashion described in Section IV.B, these become:

(E.13)
$$\bar{c}^d \hat{c}_t^d = \bar{w} \bar{h} (\hat{w}_t + \hat{h}_t) - \bar{\tau} \hat{\tau}_t - d\bar{i}^{ed} \hat{i}_{t-1}^{ed} + d(1 + \bar{i}^{ed}) \hat{\pi}_t$$

$$(E.14) \qquad -\frac{1}{\sigma_c} \mathbb{E}_t \hat{c}_{t+1}^d + \psi^{hab} \left(\frac{1}{\sigma_c} - 1 \right) \hat{c}_t^d - \mathbb{E}_t \hat{\pi}_{t+1} = -\hat{\lambda}_t^d + \zeta_t^{\mu}$$

(E.15)
$$\hat{p}_t^{g,d} = \left(\frac{1 - \bar{p}^{g,d}}{\bar{\lambda}^d}\right) \left[\bar{i}^{bd}\hat{i}_t^{bd} - \bar{i}^{gd}\hat{i}_t^{gd} + (\bar{i}^{gd} - \bar{i}^{bd})\hat{\lambda}_t^d\right]$$

(E.16)
$$\bar{i}^{ed}\hat{i}^{ed}_t = \bar{p}^{g,d}(\bar{i}^{gd} - \bar{i}^{bd})\hat{p}^{g,d}_t + \bar{p}^{g,d}\bar{i}^{g,d}\hat{i}^{g,d}_t + (1 - \bar{p}^{g,d})\bar{i}^{b,d}\hat{i}^{b,d}_t$$

The transfer τ_0 in the debtor household budget constraint (equation E.7) is funded by a lump sum tax on savers, equal to $\tau_0 \cdot q_d/(1-q_d)$. This has no effect on the loglinearized first order conditions for the savers, but allows me to control the steady state level of consumption inequality, which is important for aggregate dynamics in two-agent New Keynesian models such as this (Debortoli and Galí, 2018). Since the majority of the equilibrium conditions are unchanged from the representative-agent model, I leave the full derivation of this extended model to the online appendix.

E.3.2 Quantification

The calibrations and priors of all parameters present in the representative-agent model are kept the same for this extended model. There are 4 new parameters for this model: β^d , d, q_d , τ_0 . In addition, there are two new shock processes: χ_t^{gd} , χ_t^{bd} .

First, I calibrate β^d to 0.98. This is sufficiently low that the debt constraint binds in steady state.³ Since the stock of saving among saver households has been normalized to 1, I set d to match the ratio of median mortgage debt among mortgage-holders to median gross financial assets among non-mortgage-holders with positive savings in the UK in the first wave of the Wealth and Assets Survey (Office for National Statistics, 2019), conducted towards the end of the sample period used for estimating the baseline model (2006-2008). This implies a calibration of d = 10. I focus on mortgage-holders since the data on interest rate dispersion used to specify lender bank costs concerns mortgages. Furthermore, Cloyne et al. (2020) find that mortgagors account for a large majority of the liquidity-constrained households in the UK.

I set the proportion of debtors to $q_d = 0.21$, following Debortoli and Galí (2018) who note this is in the middle of the range of calibrations common in the two-agent New Keynesian literature. This is also close to the proportion of UK households estimated to be liquidity constrained and hold a mortgage by Cloyne et al. (2020). Finally, I follow Galí et al. (2007) and set τ_0 such that steady state consumption is equal across savers and debtors. This does not, however, imply that consumption is equal in all periods, as shocks will affect the two household types in different ways.

Next, I turn to the cost processes χ_t^{gd} and χ_t^{bd} . As with the deposit-taking banks, I assume that the costs of each lending bank consist of a constant component and a

³Note that $\beta^d < \beta$ is necessary but not sufficient for this, as debtors face different effective interest rates from savers.

time-varying component:

$$\chi_t^{gd} = \chi_0^{gd} + \tilde{\chi}_t^{gd}$$

(E.18)
$$\chi_t^{bd} = \chi_0^{bd} + \tilde{\chi}_t^{bd}$$

where $\tilde{\chi}_t^{gd}, \tilde{\chi}_t^{bd}$ are mean-zero stationary processes.

For the constants χ_0^{gd} and χ_0^{bd} , I follow the procedure for the deposit-taking banks and calibrate them to target two empirical moments, once concerning the average spread between mortgage interest rates and the policy rate, and another concerning the dispersion of mortgage interest rates. For the first of these, I compute the average spread between the Quoted Household Interest Rate series for 5-year fixed-rate mortgages (Bank of England, ndb) and the 3-month T-bill rate over the period 1996-2009. This is the data counterpart of $\bar{i}^{ed} - \bar{i}^{CB}$ in the model. Targeting this pins down the average of the constant components of lending bank costs.

To calibrate the dispersion of the constant components of lending bank costs, I first use the data from Moneyfacts described in Section II to compute the spread between the highest and lowest-yield saving products offered in December 2000 in the sample used to construct φ_t . This spread is 250 basis points. This is useful, because Cook et al. (2002) measure the equivalent spread for comparable 5-year fixed-rate mortgages available in the same month. They measure this spread as 33 basis points. I set the constant cost dispersion $\chi_0^{bd} - \chi_0^{gd}$ to match this ratio, i.e. so that in steady state the spread between maximum and minimum debt interest rates is 33/250 times the equivalent spread for saving.

Finally, I set the dynamic components of lending bank costs to be equal to the equivalent processes for deposit-taking banks. That is, the total costs at each lending bank are given by:

(E.19)
$$\chi_t^{gd} = \chi_0^{gd} + \zeta_t^{\chi}$$

(E.20)
$$\chi_t^{bd} = \chi_0^{bd} + \chi_1(i_t^{CB} - \bar{i}^{CB}) + \zeta_t^{\chi} + \zeta_t^{\chi b}$$

This means that the dynamics of bank costs are the same in borrowing as they are for saving. Economically, this is consistent with a banking environment in which cost shocks are common across the retail finance sector, and are not specific to product types. This assumption is particularly helpful here because identification of lending-bank cost dynamics that are separate from those in saving banks would be very weak in the absence of time-series data on mortgage rate dispersion (see discussion in Section II.A).

 $^{^4}$ Quoted Household Interest Rate is constructed in the same way as the equivalent for saving products, described in Section II.

I estimate this model in the same way as the representative agent model, as detailed in Appendix E.2.

E.3.3 Results

Posterior distributions for all estimated parameters are given in Table 17.

Table 17: Estimated posteriors in two-agent model

Parameter	Mean	5%	95%	Parameter	Mean	5%	95%
σ^c	0.790	0.570	1.015	$ ho_{\zeta^{hb}}$	0.232	0.005	0.426
ψ^{hab}	0.694	0.617	0.772	$ ho_{\zeta k}$	0.279	0.000	0.747
σ^h	0.496	0.340	0.658	$\rho_{\zeta \chi}$	0.942	0.898	0.990
χ^k	135.946	45.724	225.798	$ ho_{\zeta \chi b}$	0.782	0.692	0.877
ϵ^k	0.503	0.035	0.793	μ	0.026	0.019	0.035
σ^z	0.489	0.186	0.761	χ_1	-0.322	-0.545	-0.115
χ^{hv}	419.759	279.800	558.862	σ_g	0.033	0.028	0.037
ϵ^{hv}	0.215	0.085	0.358	$\sigma_{\zeta^{\kappa h}}$	1.221	0.582	1.791
χ^{xv}	33.269	4.597	55.553	$\sigma_{\zeta^{rcb}}$	0.001	0.001	0.002
ϵ^{xv}	0.135	0.063	0.212	σ_{tfp}	0.007	0.006	0.008
ψ^{pm}	0.686	0.478	0.899	σ_{ζ^c}	0.016	0.009	0.024
ϵ^m	0.168	0.089	0.249	$\sigma_{\zeta^{hb}}$	0.007	0.005	0.008
ψ^w	0.287	0.224	0.349	σ_{ζ^k}	0.111	0.038	0.182
ϵ^w	0.379	0.221	0.529	$\sigma_{\zeta^{\mu}}$	0.001	0.000	0.002
θ^p	1.848	1.631	2.048	$\sigma_{\zeta \chi}$	0.003	0.002	0.004
θ^y	0.153	0.112	0.192	$\sigma_{\zeta\chi b}$	0.003	0.002	0.004
$ heta^{rcb}$	0.914	0.893	0.935	$\sigma_{ u arphi}$	0.093	0.079	0.110
$ ho_{tfp}$	0.954	0.925	0.983	$\sigma_{ u s}$	0.009	0.003	0.015
$ ho_g$	0.938	0.896	0.981	$\sigma_{ u m}$	0.002	0.001	0.002
$ ho_{\zeta^c}$	0.738	0.606	0.875				

To see the effects of cyclical attention to saving and borrowing, Table 18 repeats the exercise of Table 3 for the estimated two-agent model. Specifically, I compute the cumulative 4-quarter response of aggregate consumption to a range of shocks in the baseline estimated model, and then in two alternatives. In the first alternative ('fixed attention') all parameters are as in the baseline, but attention of both savers and borrowers are held at their respective steady states. In the second ('saver attention'), saver attention is allowed to vary optimally, but borrower attention is held at its steady state.

In the first column of Table 18, I compute the aggregate consumption response to each shock in the fixed attention model relative to the response in the saver attention model. This therefore shows the extent of consumption amplification due to variable attention to saving. As in Table 3, for the most important shocks, the ratio is less than 1, implying that cyclical attention to saving amplifies aggregate consumption. In the second column, I compute the aggregate consumption responses in the fixed attention model relative to the full estimated model. This shows the effect of cyclical attention to saving and borrowing combined. The amplification is slightly smaller for the main shocks (i.e. risk premium, TFP, and government spending): cyclical attention to borrowing dampens fluctuations,

but the effect is small.

Table 18: Cumulative aggregate consumption response to shocks relative to variable attention baseline.

Shock	Saver vs Fixed	Full vs Fixed
Risk premium	0.879	0.884
TFP	0.905	0.917
Govt. spending	0.861	0.863
Monetary policy	1.137	1.018
Bank costs (level)	0.594	0.553
$Markup^a$	0.881	0.857
Foreign inflation	1.085	1.115

^a Markup shock ratios are calculated the same as all other shocks, except I use the impact response of aggregate consumption rather than the cumulative response over a year, because aggregate consumption rises on impact then falls below zero, so the cumulative response over 4 quarters is very close to zero in all models.

Note: For each shock, the reported statistics are calculated by taking the 12-month cumulative response of consumption to the shock in the estimated quantitative model, assuming that attention is held fixed at its steady state value, then dividing that by the equivalent cumulative consumption responses in the saver attention model (column 1) and the full model with variable attention of both savers and borrowers (column 2).

Overall, cyclical attention to saving alone amplifies the variance of aggregate consumption (relative to the fixed attention model) by 15.7%. Allowing attention to borrowing to vary as well, the variance of consumption is still 12.3% larger than in the fixed attention model. Cyclical attention to saving therefore remains the dominant way in which attention affects aggregate consumption, consistent with the findings in Section I.F.

Interestingly, cyclical attention to saving actually has a greater amplification effect on aggregate consumption in this model than it does in the representative agent model. This is why the overall amplification from cyclical attention of all households is comparable to that in the representative agent model. This occurs even though cyclical attention to saving directly affects only a subset of the population of households.

The reason, as outlined in Section IV.D, is that amplification of saver consumption has second-round effects on debtors, through labor income. To show this, I compute a decomposition similar to that in Kaplan et al. (2018), in which I split the response of risk premium shocks and monetary policy shocks into Euler-equation effects and indirect (general-equilibrium) effects.

Specifically, collect all income from labor, capital, and profits, minus investment and taxes, into a single variable m_t , so that the flow budget constraint of saver households is

(E.21)
$$c_t + b_t = \frac{1 + i_{t-1}^e}{\pi_t} b_{t-1} + m_t$$

and the corresponding present-value budget constraint is

(E.22)
$$\sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^{t-1} \frac{1+i_k^e}{\pi_{k+1}}} c_t = \sum_{t=0}^{\infty} \frac{1}{\prod_{k=0}^{t-1} \frac{1+i_k^e}{\pi_{k+1}}} m_t + \frac{1+i_{t-1}^e}{\pi_t} b_{t-1}$$

Log-linearizing, and imposing that all variables are at steady state in period -1, this becomes

(E.23)
$$\sum_{t=0}^{\infty} \beta^{t} (\hat{c}_{t} - \sum_{k=0}^{t-1} \hat{r}_{k}^{e}) = \frac{\bar{m}}{\bar{c}} \sum_{t=0}^{\infty} \beta^{t} (\hat{m}_{t} - \sum_{k=0}^{t-1} \hat{r}_{k}^{e})$$

where \bar{c} , \bar{m} are the steady states of c_t and m_t respectively. Hatted variables are logdeviations from steady state, and \hat{r}_t^e denotes the log-deviation of the real effective interest rate $(1+i_t^e)/\pi_{t+1}$ from its steady state.

Next, log-linearize the Euler equation (as in the representative-agent model, this is given by equation 38)

(E.24)
$$\hat{u}_{ct} = \zeta_t^c + \hat{r}_t^e + \hat{u}_{ct+1}$$

Substituting forwards T times, and using the definition of marginal utility, this becomes

(E.25)
$$-\frac{1}{\sigma^c}\hat{c}_t + \psi^{hab}(\frac{1}{\sigma^c} - 1)((1 - q_d)\hat{c}_{t-1} + q_d\hat{c}_{t-1}^d)$$

$$= \sum_{k=t}^{T-1} (\zeta_k^c + \hat{r}_k^e) - \frac{1}{\sigma^c}\hat{c}_T + \psi^{hab}(\frac{1}{\sigma^c} - 1)((1 - q_d)\hat{c}_{T-1} + q_d\hat{c}_{T-1}^d)$$

With repeated substitutions of this into equation E.23, saver consumption in period 0 can be written as

(E.26)
$$\hat{c}_0 = (1 - \beta)(1 - \beta\Omega) \sum_{t=1}^{\infty} \beta^t \left[\frac{b}{\beta \bar{c}} - \frac{\sigma^c}{1 - \beta} \Omega^{t-1} \right] \sum_{k=0}^{t-1} \hat{r}_k^e$$

$$- \frac{\sigma^c \beta}{1 - \beta \rho \zeta^c} \zeta_0^c + (1 - \beta)(1 - \beta\Omega) \left(1 - \frac{b}{\beta \bar{c}} \right) \sum_{t=0}^{\infty} \hat{m}_t$$

where

(E.27)
$$\Omega \equiv \psi^{hab} (1 - \sigma^c) (1 - q_d)$$

As in Kaplan et al. (2018), I use this to compute the effects of shocks that operate specifically through intertemporal substitution in the Euler equation of savers. To do this,

I assume that a one standard-deviation shock hits the economy in period 0. I then feed into equation E.26 the paths of \hat{r}_t^e and ζ_t^c from the relevant impulse responses computed from the estimated model, but I hold \hat{m}_t constant at 0 (steady state). The share of that shock's transmission on impact through Euler equation effects is given by dividing the Euler equation effect by the true impact of the shock on aggregate consumption, which incorporates all changes in \hat{m}_t .

(E.28)
$$\hat{c}_0^{\text{euler}} \equiv \frac{(1 - q_d)\hat{c}_0|\hat{m}_t = 0}{(1 - q_d)\hat{c}_0 + q_d\hat{c}_0^d}$$

Note that here I am including the response of attention and interest rates to the shock in the Euler-equation effects, so this is not the same as a partial vs. general equilibrium effect decomposition. It rather gives the share of transmission that occurs through intertemporal substitution in the Euler equation of savers. Table 19 shows the results of this decomposition for both risk premium shocks and monetary policy shocks, both of which have no direct effect on borrowers, but can only affect them through indirect income effects. For both shocks, I compute the decomposition for the fixed attention model, and for the saver attention model.

Table 19: Share of shock transmission due to direct Euler-equation effects on savers.

Shock	Fixed Attention	Saver Attention
Risk premium	1.110	0.999
Monetary policy	0.050	0.042

Note: The share of shock transmission due to direct Euler-equation effects is calculated as defined in Equation E.28. The 'Fixed Attention' and 'Saver Attention' models are as defined in the note accompanying Table 18.

In the fixed attention model, more than 100% of the impact of risk premium shocks is through the Euler equation of savers. Indirect effects actually dampen the shock a little, principally because profits rise after a contractionary risk premium shock, and this prevents saver consumption falling too far. In the saver attention model, recall that variable attention amplifies the effects of the shock on saver consumption. Mechanically, this would increase the share of transmission through Euler-equation effects. However, this is more than outweighed by the second-round effects on borrower incomes, such that the Euler-equation share actually falls. Therefore although cyclical attention to saving only directly influences the consumption of savers, debtor consumption is also affected, because labor income is affected and their consumption is very sensitive to income.

With monetary policy shocks, the first thing to note is that this model confirms the results in Kaplan et al. (2018), Bilbiie (2019) and others that the majority of monetary transmission occurs through indirect effects. Variable attention to saving further increases the share due to indirect effects, because this is one of the rare shocks in which attention

choices dampen the direct saver response to the shock.

F Quantitative model mathematical details

This appendix lays out the full mathematical details of the quantitative model in Section IV. Aside from the attention problem and banking sector, the model closely follows that of Harrison and Oomen (2010) (HO), which is in turn based on Smets and Wouters (2007), extended to an open economy as in Adolfson et al. (2007).

Households

Households maximize expected discounted utility $E_t \sum_{s=0}^{\infty} \beta^s U_{t+s}$, where instantaneous utility is given by:

(F.1)
$$U_{t} = \frac{1}{1 - \frac{1}{\sigma^{c}}} \left(\frac{c_{t}}{\bar{c}_{t-1}^{\psi^{hab}}} \right)^{1 - \frac{1}{\sigma^{c}}} - (\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t}^{\kappa h}} \frac{1}{1 + \frac{1}{\sigma^{h}}} (h_{t})^{1 + \frac{1}{\sigma^{h}}} - \mu e^{\zeta_{t}^{\mu}} \mathcal{I}_{t}(i_{t}^{e})$$

where $\beta \in (0, 1)$ is the discount factor, $\sigma^c > 0$ is the elasticity of intertemporal substitution, $\sigma^h > 0$ is the elasticity of labor supply, and $\kappa^h > 0$ gives the weight of labor supply in utility. \bar{c}_{t-1} is lagged aggregate consumption, taken as given by households, and so the parameter ψ^{hab} gives the degree of external habit formation. c_t is household consumption, h_t is labor supply, and $\zeta_t^{\kappa h}$ is an exogenous shock to the disutility of labor. In equilibrium $\bar{c}_t = c_t$ as all households are identical, but the households do not take this into account when making choices. Finally, $\mu > 0$ is the marginal cost of information, ζ_t^{μ} is an exogenous shock to this cost, i_t^e is the effective nominal interest experienced by the household, and $\mathcal{I}_t(i_t^e)$ is the information processing required to achieve that effective interest rate, as formalized in Section I of the paper.

The budget constraint is:

$$(F.2) PC_t c_t + PI_t inv_t + B_t - (1 + i_{t-1}^e) B_{t-1} = W_t h_t + R_t k_t^s + \Pi_t^v + \Pi_t^b - PC_t \tau_t$$

 PC_t is the consumer price index. As well as consumption, household spending consists of investment inv_t at price PI_t , and asset accumulation. One-period domestic bonds B_t are subject to the attention problem studied in Section I, and thus carry an effective nominal interest rate of i_t^e . Income comes from supplying labor at nominal wage W_t , supplying capital services k_t^s at rental rate R_t , profits from firms Π_t^v , and a transfer from the banking system Π_t^b that includes both bank profits and transaction costs. There is a lump sum tax of $PC_t\tau_t$ from the government.

Consumption indices

The equations above consider composite consumption c_t , with the composite price index PC_t . c_t is a CES combination of domestically-produced goods c_t^h and foreign-produced goods (imports) c_t^m :

(F.3)
$$c_t \equiv \kappa^c \left((1 - \psi^m) (c_t^h)^{1 - \frac{1}{\sigma^m}} + \psi^m (c_t^m)^{1 - \frac{1}{\sigma^m}} \right)^{\frac{\sigma^m}{\sigma^{m-1}}}$$

where $\kappa^c > 0$ is a parameter, $\psi^m \in (0,1)$ is the expenditure weight of imported consumption goods in aggregate consumption, and $\sigma^m > 1$ is the elasticity of substitution between domestic and foreign consumption.

The associated price index is:

(F.4)
$$PC_{t} \equiv \frac{1}{\kappa^{c}} \left[(1 - \psi^{m})^{\sigma^{m}} (PH_{t})^{1 - \sigma^{m}} + (\psi^{m})^{\sigma^{m}} (PM_{t})^{1 - \sigma^{m}} \right]^{\frac{1}{1 - \sigma^{m}}}$$

where PH_t is the price of goods produced at home and PM_t is the price of imported consumption goods. It will be convenient to express these relative to PC_t :

(F.5)
$$1 = \frac{1}{\kappa^c} \left[(1 - \psi^m)^{\sigma^m} \left(p_t^h \right)^{1 - \sigma^m} + (\psi^m)^{\sigma^m} \left(p_t^m \right)^{1 - \sigma^m} \right]^{\frac{1}{1 - \sigma^m}}$$

where $p_t^h = PH_t/PC_t$, $p_t^m = PM_t/PC_t$.

Expenditure allocation

Given c_t , the allocation of expenditure between home and foreign is:

(F.6)
$$c_t^h = (1 - \psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} c_t$$

(F.7)
$$c_t^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} c_t$$

Total consumption expenditure is:

$$(F.8) c_t = p_t^h c_t^h + p_t^m c_t^m$$

Capital accumulation

Capital accumulates according to the law of motion:

(F.9)
$$k_{t} = inv_{t} + (1 - \delta)k_{t-1} - \Delta_{t}^{k} - \Delta_{t}^{z}$$

where k_t is capital purchased in period t, which will be available for production in period t+1. $\delta > 0$ is the depreciation rate. Δ_t^k is a quadratic cost associated with changing the capital stock:

(F.10)
$$\Delta_{t}^{k} \equiv \frac{\chi^{k}}{2\bar{k}_{t-1}} \left[k_{t} - \left(\frac{\bar{k}_{t-1}}{\bar{k}_{t-2}} \right)^{\epsilon^{k}} k_{t-1} + \frac{\bar{k}_{t-1}}{\chi^{k}} \zeta_{t}^{k} \right]^{2}$$

Note that these costs arise if a household's own capital accumulation deviates from the aggregate rate of capital accumulation in the previous period, as \bar{k}_t denotes aggregate capital that the household takes as given. This cost is controlled by the parameters χ^k , $\epsilon^k > 0$. ζ_t^k is an exogenous shock to the capital adjustment cost.

 Δ_t^z is an additional depreciation which is increasing in capital utilization. Households rent capital services to firms, which depend on previously installed capital and utilization z_t :

$$(F.11) k_t^s = z_t k_{t-1}$$

Choosing a higher z_t increases the capital services the household can supply, but implies a faster depreciation of the capital stock, through Δ_t^z :

(F.12)
$$\Delta_t^z \equiv \frac{\chi^z}{1+\sigma^z} \left[(z_t)^{1+\sigma^z} - 1 \right] k_{t-1}$$

where $\chi^z, \sigma^z > 0$ control the magnitude and slope of utilization-related depreciation.

First Order Conditions

The household chooses c_t , i_t^e , k_t , inv_t , B_t , z_t to maximize the present discounted sum of the utility in equation (F.1) subject to equations (F.2), (F.9), (F.11), and the convex costs of increasing i_t^e . The first order conditions are:

(F.13)
$$\frac{1}{\overline{c}_{t-1}^{\psi^{hab}}} \left(\frac{c_t}{\overline{c}_{t-1}^{\psi^{hab}}} \right)^{-\frac{1}{\sigma^c}} = PC_t \Lambda_t$$

(F.14)
$$\beta E_t \Lambda_{t+1} B_t = \mu e^{\zeta_t^{\mu}} \mathcal{I}'_t(i_t^e)$$

$$(F.15) \qquad \Theta_t \left(1 + \frac{\partial \Delta_t^k}{\partial k_t} \right) = \beta E_t \left[\Lambda_{t+1} R_{t+1} z_{t+1} + \Theta_{t+1} \left(1 - \delta - \frac{\partial \Delta_{t+1}^k}{\partial k_t} - \frac{\partial \Delta_{t+1}^z}{\partial k_t} \right) \right]$$

(F.16)
$$\Lambda_t PI_t = \Theta_t$$

$$(F.17) \qquad \Lambda_t = \beta E_t (1 + i_t^e) \Lambda_{t+1}$$

$$(F.18) \qquad \Lambda_t R_t k_{t-1} = \Theta_t \chi^z z_t^{\sigma^z} k_{t-1}$$

where Λ_t, Θ_t are the Lagrange multipliers on (F.2) and (F.9), and:

(F.19)
$$\frac{\partial \Delta_t^k}{\partial k_t} = \chi^k \frac{k_t - \left(\frac{\bar{k}_{t-1}}{\bar{k}_{t-2}}\right)^{\epsilon^k} k_{t-1} + \frac{\bar{k}_{t-1}}{\chi^k} \zeta_t^k}{\bar{k}_{t-1}}$$

$$(F.20) \qquad \frac{\partial \Delta_{t+1}^k}{\partial k_t} = -\chi^k \left(\frac{\bar{k}_t}{\bar{k}_{t-1}}\right)^{\epsilon^k} \frac{k_{t+1} - \left(\frac{\bar{k}_t}{\bar{k}_{t-1}}\right)^{\epsilon^k} k_t + \frac{\bar{k}_t}{\chi^k} \zeta_{t+1}^k}{\bar{k}_t}$$

(F.21)
$$\frac{\partial \Delta_{t+1}^z}{\partial k_t} = \chi^z \frac{z_{t+1}^{1+\sigma^z} - 1}{1 + \sigma^z}$$

Combining equations (F.13) and (F.17) gives the consumption Euler equation,⁵ while combining equations (F.13) and (F.14) (and transforming to be in terms of real bonds $b_t = B_t/PC_t$) gives the attention first order condition (equation 39 in the main paper).

Labor unions

Households supply labor to a continuum of unions, who in turn set wages. Rather than choosing labor supply directly, households agree to supply all labor demanded at the wage set by the union. Unions supply differentiated labor varieties $h_t(i)$ to a perfectly competitive labor packer, who combines varieties with a CES aggregator to an aggregate labor supply h_t :

(F.22)
$$h_t \equiv \left[\int_0^1 h_t(i)^{\frac{\sigma^w - 1}{\sigma^w}} di \right]^{\frac{\sigma^w}{\sigma^w - 1}}$$

where $\sigma^w > 1$ is the elasticity of substitution between labor varieties.

Cost minimization implies the demand for each variety is:

(F.23)
$$h_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\sigma^w} h_t$$

where $W_t(i)$ is the nominal wage set by the union i, and W_t is the aggregate nominal wage:

(F.24)
$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\sigma^w} di \right]^{\frac{1}{1-\sigma^w}}$$

Unions set wages to maximize expected discounted utility of their members, subject to a cost of wage adjustment and this demand function. The adjustment cost is quadratic

⁵See equation F.93 below for how the risk premium shock is incorporated into this equation.

in deviations from a target wage inflation rate Ξ_t^w . Their problem is therefore:

$$\max_{W_{t}(i)} E_{t} \sum_{s=0}^{\infty} \beta^{s} \left\{ \Lambda_{t+s} W_{t+s}(i) h_{t+s}(i) - (\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t+s}^{\kappa h}} \frac{1}{1 + \frac{1}{\sigma^{h}}} (h_{t+s}(i))^{1 + \frac{1}{\sigma^{h}}} - \Lambda_{t+s} \frac{\chi^{w}}{2} \left(\frac{W_{t+s}(i)}{W_{t+s-1}(i) \Xi_{t+s}^{w}} - 1 \right)^{2} W_{t+s} \right\}$$
(F.25)

subject to (F.23), and:

where $\chi^w > 0$ controls the strength of wage adjustment costs, and $\epsilon^w \geq 0$ controls the degree of indexation to past wage inflation. If $\epsilon^w = 0$, then the cost is the standard Rotemberg-style cost, with no indexation to past wage changes. If $\epsilon^w > 0$, wages are instead partially indexed to past wage growth. Notice that while wages received and wage adjustment costs are discounted by Λ_{t+s} , the disutility of labor is not, as it is a utility cost rather than a monetary cost.

Taking the first order condition and then imposing symmetry among unions $(W_t(i) = W_t, h_t(i) = h_t)$ yields:

$$(F.27) \quad (1 - \sigma^{w})h_{t} + (\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t}^{\kappa h}} \sigma^{w} \frac{h_{t}^{1 + \frac{1}{\sigma^{h}}}}{W_{t} \Lambda_{t}} - \frac{\chi^{w} W_{t}}{W_{t-1} \Xi_{t}^{w}} \left(\frac{W_{t}}{W_{t-1} \Xi_{t}^{w}} - 1 \right) + E_{t} \frac{\beta \chi^{w} W_{t+1}^{2} \Lambda_{t+1}}{W_{t}^{2} \Xi_{t+1}^{w} \Lambda_{t}} \left(\frac{W_{t+1}}{W_{t} \Xi_{t+1}^{w}} - 1 \right) = 0$$

Now rewrite in terms of real wages $w_t \equiv W_t/PC_t$:

$$(1 - \sigma^{w})h_{t} = -(\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t}^{\kappa h}} \sigma^{w} \frac{h_{t}^{1 + \frac{1}{\sigma^{h}}}}{w_{t}u_{ct}} + \frac{\chi^{w}w_{t}\pi_{t}}{w_{t-1}\Xi_{t}^{w}} \left(\frac{w_{t}}{w_{t-1}\Xi_{t}^{w}}\pi_{t} - 1\right) - E_{t} \frac{\beta\chi^{w}w_{t+1}^{2}\pi_{t+1}u_{ct+1}}{w_{t}^{2}\Xi_{t+1}^{w}u_{ct}} \left(\frac{w_{t+1}}{w_{t}\Xi_{t+1}^{w}}\pi_{t+1} - 1\right)$$

where $u_{ct} \equiv \Lambda_t PC_t$ is the marginal utility of consumption:

(F.29)
$$u_{ct} = \frac{1}{\bar{c}_{t-1}^{\psi^{hab}}} \left(\frac{c_t}{\bar{c}_{t-1}^{\psi^{hab}}}\right)^{-\frac{1}{\sigma^c}}$$

and we note that Ξ_t^w can be written in real terms as:

(F.30)
$$\Xi_t^w = \left(\frac{w_{t-1}}{w_{t-2}}\pi_{t-1}\right)^{\epsilon^w}$$

Note that in HO, they use Calvo staggered wage setting, rather than this Rotembergstyle setup. I use the quadratic adjustment cost setup to keep the exposition of the model brief. Since we consider a steady state with no trend inflation, the steady steady state and log-linearized wage Phillips curve are identical in these two setups. To map from the parameters here to the wage-resetting probability in HO, replace χ^w with:

(F.31)
$$\chi^w \equiv \frac{(\sigma^w - 1)(1 - \psi^w)\bar{h}}{\psi^w(1 - \beta(1 - \psi^w))} \left(1 + \frac{\sigma^w}{\sigma^h}\right)$$

where ψ^w is the probability a union can reset W_t each period. With this substitution, equation (F.28) implies exactly the same log-linearized wage Phillips curve as HO. See Born and Pfeifer (2020) for details of how this χ^w expression is derived.

Firms

Domestic producers

There is a continuum of monopolistically competitive intermediate goods producers, who produce output for production of domestic goods y_t^{hv} and for production of export goods y_t^{xv} . Their total output, $y_t^v = y_t^{hv} + y_t^{xv}$, is given by a CES production function over labor and capital services:

$$(F.32) y_t^v = tfp_t \left[(1 - \alpha) \left(h_t \right)^{\frac{\sigma^y - 1}{\sigma^y}} + \alpha \left(k_t^s \right)^{\frac{\sigma^y - 1}{\sigma^y}} \right]^{\frac{\sigma^y}{\sigma^y - 1}}$$

where tfp_t is aggregate productivity, $\alpha > 0$ is the capital share, and $\sigma^y > 0$ is the elasticity of substitution between factors of production. Letting $r_t \equiv R_t/PC_t$ be the real rental rate, real total costs are $w_t h_t + r_t k_t^s$. Minimizing this cost for a given y_t^v gives:

(F.33)
$$\frac{w_t}{r_t} = \frac{1 - \alpha}{\alpha} \left[\frac{k_t^s}{h_t} \right]^{\frac{1}{\sigma^y}}$$

(F.34)
$$h_t = \left(\frac{w_t}{1-\alpha}\right)^{-\sigma^y} \left[(1-\alpha)^{\sigma^y} w_t^{1-\sigma^y} + \alpha^{\sigma^y} r_t^{1-\sigma^y} \right]^{\frac{\sigma^y}{1-\sigma^y}} \frac{y_t^v}{t f p_t}$$

$$(F.35) k_t^s = \left(\frac{r_t}{\alpha}\right)^{-\sigma^y} \left[(1-\alpha)^{\sigma^y} w_t^{1-\sigma^y} + \alpha^{\sigma^y} r_t^{1-\sigma^y} \right]^{\frac{\sigma^y}{1-\sigma^y}} \frac{y_t^v}{t f p_t}$$

Marginal costs are then given by:

(F.36)
$$mc_{t} = \left[(1 - \alpha)^{\sigma^{y}} w_{t}^{1 - \sigma^{y}} + \alpha^{\sigma^{y}} r_{t}^{1 - \sigma^{y}} \right]^{\frac{1}{1 - \sigma^{y}}} \frac{1}{t f p_{t}}$$

Perfectly competitive final goods producers combine intermediate goods varieties from domestic and foreign firms using a Leontief technology:

$$(F.37) y_t^h = \min\{\frac{y_t^{hv}}{\kappa^{hv}}, \frac{mi_t^h}{1 - \kappa^{hv}}\}$$

(F.38)
$$y_t^x = \min\{\frac{y_t^{xv}}{\kappa^{xv}}, \frac{mi_t^x}{1 - \kappa^{xv}}\}$$

where κ^{hv} , κ^{xv} are parameters, y_t^h , y_t^x denote final goods production for the domestic and export markets respectively, and mi_t^h , mi_t^x denote imported intermediate inputs used for each final good. The indices y_t^{hv} and y_t^{xv} are CES aggregates of intermediate varieties:

$$(F.39) y_t^{hv} \equiv \left[\int_0^1 y_t^{hv}(i)^{\frac{\sigma^{hb}}{-}1} \sigma^{hb} di \right]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}}$$

(F.40)
$$y_t^{xv} \equiv \left[\int_0^1 y_t^{xv}(i)^{\frac{\sigma^{xb}-1}{\sigma^{xb}}} di \right]^{\frac{\sigma^{xb}}{\sigma^{xb}-1}}$$

(F.41)

where σ^{hb} , $\sigma^{xb} > 1$ are elasticities of substitution between varieties. Minimizing final good producer costs yields:

(F.42)
$$\frac{\kappa^{hv}}{1 - \kappa^{hv}} = \frac{y_t^{hv}}{mi_t^h}$$

(F.43)
$$\frac{\kappa^{xv}}{1 - \kappa^{xv}} = \frac{y_t^{xv}}{mi_t^x}$$

$$(F.44) y_t^{hv}(i) = \left(\frac{p_t^{hv}(i)}{p_t^{hv}}\right)^{-\sigma^{hb}} y_t^{hv}$$

(F.45)
$$y_t^{xv}(i) = \left(\frac{p_t^{xv}(i)}{p_t^{xv}}\right)^{-\sigma^{xb}} y_t^{xv}$$

(F.46)
$$mc_t^h = \kappa^{hv} p_t^{hv} + (1 - \kappa^{hv}) p_t^m$$

(F.47)
$$mc_t^x = \kappa^{xv} p_t^{xv} + (1 - \kappa^{xv}) p_t^m$$

where mc_t^h, mc_t^x denote the final good producer's (real) marginal costs in the domestic and export sectors. Since final goods producers are perfectly competitive, the prices of domestic and export goods, again expressed relative to PC_t , are equal to their respective

marginal costs: $p_t^h = mc_t^h$ and $p_t^x = mc_t^x$. In the expressions for individual variety demands and marginal costs, the price indices for intermediate inputs for domestic and export production (p_t^{hv}) and p_t^{xv} are defined as:

(F.48)
$$p_t^{hv} \equiv \left[\int_0^1 (p_t^{hv}(i))^{1-\sigma^{hb}} di \right]^{\frac{1}{1-\sigma^{hb}}}$$

(F.49)
$$p_t^{xv} = \left[\int_0^1 (p_t^{xv}(i))^{1-\sigma^{xb}} di \right]^{\frac{1}{1-\sigma^{xb}}}$$

where $y_t^{hv}(i)$ and $y_t^{xv}(i)$ are the quantities of intermediate goods demanded from producer i for each type of production, and $p_t^{hv}(i)$ and $p_t^{xv}(i)$ are the prices set by that producer. p_t^{xv} is specifically the relative export price expressed in domestic currency, defined as:

$$(F.50) p_t^{xv} \equiv \frac{PXVF_t}{PC_tER_t}$$

where $PXVF_t$ is the price of intermediate goods in the export sector in foreign currency terms, and ER_t is the nominal exchange rate.

Price setting

Intermediate goods produces can set different prices for goods used in the production of final goods for domestic consumption and for export: i.e. different prices for $y_t^{hv}(i)$ and $y_t^{xv}(i)$. In both cases, they set prices to maximize expected discounted profits, net of quadratic price adjustment costs. Their optimization problem is therefore:

$$\max_{p_t^{hv}(i), p_t^{xv}(i)} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \left\{ p_{t+s}^{hv}(i) y_{t+s}^{hv}(i) + p_{t+s}^{xv} y_{t+s}^{xv}(i) - w_{t+s} h_{t+s}(i) - r_{t+s} k_{t+s}^s(i) \right. \\
\left. \left. - \frac{\chi^{hv}}{2} \left(\frac{p_{t+s}^{hv}(i)}{p_{t+s-1}^{hv}(i)} \frac{\pi_{t+s}}{\Xi_{t+s}^{hv}} - 1 \right)^2 p_{t+s}^{hv} y_{t+s}^{hv} - \frac{\chi^{xv}}{2} \left(\frac{p_{t+s}^{xv}(i) q_{t+s}}{p_{t+s-1}^{xv}(i) q_{t+s-1}} \frac{\pi_{t+s}^f}{\Xi_{t+s}^{xv}} - 1 \right)^2 p_{t+s}^{xv} y_{t+s}^{xv} \right\}$$

subject to the production function (F.32), and demand for domestic and export varieties (F.44) and (F.45). Note the objective function is specified here in real terms, with nominal profits and costs divided through by PC_t . This is also why inflation π_t appears in the adjustment cost for domestic-use goods: if the original costs are in the growth of prices PHV_{t+s}/PHV_{t+s-1} , expressing each price relative to PC_{t+s} transforms that ratio into $\pi_{t+s}p_{t+s}^{hv}/p_{t+s-1}^{hv}$. The same logic generates the adjustment costs for export goods, which

depend on the change in those prices in foreign currency:

(F.52)
$$\frac{PXVF_{t+s}}{PXVF_{t+s-1}} = \frac{p_{t+s}^{xv}PC_{t+s}ER_{t+s}}{p_{t+s-1}^{xv}PC_{t+s-1}ER_{t+s-1}} = \frac{p_{t+s}^{xv}q_{t+s}\pi_{t+s}^{f}}{p_{t+s-1}^{xv}q_{t+s-1}}$$

where q_t is the real exchange rate, defined as:

$$(F.53) q_t = \frac{PC_t E R_t}{PC F_t}$$

in which PCF_t is the foreign price level. $\pi_t^f \equiv PCF_t/PCF_{t-1}$ is foreign inflation.

As with wage setting, there is partial indexation of domestic and export prices to past inflation in those prices through Ξ_t^{hv}, Ξ_t^{xv} :

(F.54)
$$\Xi_t^{hv} = \left(\frac{p_{t-1}^{hv}}{p_{t-2}^{hv}} \pi_{t-1}\right)^{\epsilon^{hv}}$$

(F.55)
$$\Xi_t^{xv} = \left(\frac{p_{t-1}^{xv}q_{t-1}}{p_{t-2}^{xv}q_{t-2}}\pi_{t-1}^f\right)^{\epsilon^{xv}}$$

 $\chi^{hv}, \chi^{xv} > 0$ control the degree of price stickiness, and $\epsilon^{hv}, \epsilon^{xv} > 0$ control the degree of price indexation.

Taking the first order conditions, and then imposing cost minimization (F.33)-(F.36) and that all intermediate goods firms are symmetric, so set the same prices in equilibrium, we obtain Phillips curves for each good:

$$(F.56) 1 - \sigma^{hb} + \frac{\sigma^{hb}mc_t}{p_t^{hv}} - \chi^{hv} \frac{p_t^{hv}\pi_t}{p_{t-1}^{hv}\Xi_t^{hv}} \left(\frac{p_t^{hv}}{p_{t-1}^{hv}} \frac{\pi_t}{\Xi_t^{hv}} - 1\right)$$

$$+ \beta \chi^{hv} E_t \frac{\Lambda_{t+1} y_{t+1}^{hv}}{\Lambda_t y_t^{hv}} \frac{(p_{t+1}^{hv})^2 \pi_{t+1}}{(p_t^{hv})^2 \Xi_{t+1}^{hv}} \left(\frac{p_{t+1}^{hv}}{p_t^{hv}} \frac{\pi_{t+1}}{\Xi_{t+1}^{hv}} - 1\right) = 0$$

$$1 - \sigma^{xb} + \frac{\sigma^{xb}mc_t}{p_t^{xv}} - \chi^{xv} \frac{p_t^{xv} q_t \pi_t^f}{p_{t-1}^{xv} q_{t-1} \Xi_t^{xv}} \left(\frac{p_t^{xv} q_t}{p_{t-1}^{xv} q_{t-1}} \frac{\pi_t^f}{\Xi_t^{xv}} - 1\right)$$

$$+ \beta \chi^{xv} E_t \frac{\Lambda_{t+1} y_{t+1}^{xv}}{\Lambda_t y_t^{xv}} \frac{(p_{t+1}^{xv})^2 q_{t+1} \pi_{t+1}^f}{(p_t^{xv})^2 q_t \Xi_{t+1}^{xv}} \left(\frac{p_{t+1}^{xv} q_{t+1}}{p_t^{xv} q_t} \frac{\pi_{t+1}^f}{\Xi_{t+1}^{xv}} - 1\right) = 0$$

$$(F.57)$$

Like consumption, investment goods are a CES aggregate of many investment good varieties:

(F.58)
$$inv_t \equiv \left[\int_0^1 inv_t(i)^{\frac{\sigma^{hb}-1}{\sigma^{hb}}} di \right]^{\frac{\sigma^{hb}}{\sigma^{hb}-1}}$$

All investment goods are assumed to be produced domestically, and are produced

with the same technology as the domestic consumption good c_t^h . Since the elasticity of substitution between varieties is also the same (σ^{hb}) , we have that the price indices will be identical: $PI_t = PH_t$. The demand for an individual investment good variety is given by:

(F.59)
$$inv_t(i) = \left(\frac{p_t^h(i)}{p_t^h}\right)^{-\sigma^{hb}} inv_t$$

Banks

These are described in detail in Section IV.A.2 of the paper. Equations 41, 42, 44, and 45, reproduced as (F.60)-(F.63) here, define the probability of choosing the good bank p_t^g , the effective interest rate i_t^e , and the first order conditions of good and bad banks.

(F.60)
$$p_t^g = \frac{\exp(\frac{i_t^g}{\lambda_t})}{\exp(\frac{i_t^g}{\lambda_t}) + \exp(\frac{i_t^b}{\lambda_t})}$$

(F.61)
$$i_t^e = p_t^g i_t^g + (1 - p_t^g) i_t^b$$

(F.62)
$$(1 - p_t^g) \cdot (i_t^{CB} - i_t^g - \chi_0^g - \zeta_t^{\chi}) = \lambda_t$$

(F.63)
$$p_t^g \cdot (i_t^{CB}(1 - \chi_1) - i_t^b - (\chi_0^b - \chi_1 \bar{i}^{CB}) - \zeta_t^{\chi} - \zeta_t^{\chi b}) = \lambda_t$$

where i_t^g, i_t^b are the nominal interest rates set by the good and bad bank respectively, λ_t is the shadow value of information, i_t^{CB} is the interest rate set by the central bank, $\chi_0^g, \chi_0^b, \chi_1$ are parameters setting the levels and responsiveness to i_t^{CB} of bank transaction costs, \bar{i}^{CB} is the steady state of i_t^{CB} , and $\zeta_t^\chi, \zeta_t^{\chi b}$ are exogenous shocks to the level and dispersion of bank costs. As in Section I of the paper, the shadow value of information is related to information processing:

$$\mathcal{I}_t'(i_t^e) = \lambda_t^{-1}$$

Government

The government budget constraint is:

(F.65)
$$PC_{t}\tau_{t} = PH_{t}g_{t} + i_{t-1}^{CB}B_{t}^{g}$$

where g_t is government spending, which is spent on home goods only. Contrary to HO, we assume that the government issues a positive supply of bonds B_t^g , so alongside g_t government expenditure includes interest payments on these bonds, paid at the central bank interest rate. The lump sum tax τ_t adjusts each period to satisfy this budget constraint.

The supply of bonds is such that the real supply is constant at $B_t^g/PC_t = b$.

Monetary policy

The central bank chooses the nominal policy rate i_t^{CB} according to a Taylor rule with interest-rate smoothing determined by parameter θ^{rcb} :

(F.66)
$$\frac{1 + i_t^{CB}}{1 + \bar{i}^{CB}} = \left(\frac{1 + i_{t-1}^{CB}}{1 + \bar{i}^{CB}}\right)^{\theta^{rcb}} \left\{ \pi_t^{\theta^p} \left(\frac{y_t^v}{\bar{y}^v t f p_t}\right)^{\theta^y} \right\}^{1 - \theta^{rcb}} e^{\zeta_t^{rcb}}$$

where \bar{y}^v is steady state output, and so $\bar{y}^v t f p_t$ is a measure of potential output. ζ_t^{rcb} is an exogenous monetary policy shock.

Market clearing

Market clearing of domestic goods and export goods requires

$$(F.67) y_t^h = c_t^h + inv_t + g_t$$

$$(F.68) y_t^x = x_t$$

where x_t is the quantity of exports demanded by foreign countries.

Total domestic output is equal to the total production of intermediate goods:

(F.69)
$$y_t^v = y_t^{hv} + y_t^{xv} = \kappa^{hv} y_t^h + \kappa^{xv} y_t^x$$

where the final equality uses final goods producer production functions ((F.37) and (F.38)) and cost minimization ((F.42) and (F.43)).

Factor market clearing requires that labor and capital services supplied by households equal labor and capital services demanded by intermediate firms. Domestic bond market clearing requires that real bonds demanded by households equal b, the constant supply of such bonds from the government.

Foreign variables

Demand for final export goods is given by:

(F.70)
$$x_t = \kappa^x \left(\frac{q_t p_t^x}{p_t^{xf}}\right)^{-\sigma^x} c_t^f$$

where $\kappa^x, \sigma^x > 0$ are parameters, p_t^{xf} is exogenous world export prices, expressed relative to PCF_t . c_t^f is exogenous export demand from foreign countries.

Imports prices are set in the domestic currency, and are assumed to be the same for all imports, no matter whether they are used directly for consumption, or in the domestic production of final goods for domestic use or export. Monopolistically competitive foreign firms face Rotemberg-style quadratic costs of price adjustment, partially indexed to past import good inflation. Domestic final goods producers aggregate imported goods as intermediate inputs using the CES aggregator:

(F.71)
$$y_t^m \equiv \left[\int_0^1 (y_t^m(i))^{\frac{\sigma^{mb}-1}{\sigma^{mb}}} di \right]^{\frac{\sigma^{mb}}{\sigma^{mb}-1}}$$

where $\sigma^{mb} > 1$ is the elasticity of substitution between varieties of imported goods, which is the same no matter whether the imports are for consumption or use as further intermediate inputs into production. The price index is therefore given by:

(F.72)
$$p_t^m \equiv \left[\int_0^1 (p_t^m(i))^{1-\sigma^{mb}} di \right]^{\frac{1}{1-\sigma^{mb}}}$$

Thus, the demand facing foreign exporter i is:

(F.73)
$$y_t^m(i) = \left(\frac{p_t^m(i)}{p_t^m}\right)^{-\sigma^{mb}} y_t^m$$

where $y_t^m(i) \equiv c_t^m(i) + mi_t^h(i) + mi_t^x(i)$ is the total demand for imports from firm i.

The problem of foreign exporter i is therefore:⁶

(F.74)
$$\max_{p_t^m(i)} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s}^f \left\{ p_{t+s}^m(i) y_{t+s}^m(i) - m c_{t+s}^f(i) y_{t+s}^m(i) - \frac{\chi^m}{2} \left(\frac{p_{t+s}^m \pi_{t+s}}{p_{t+s-1}^m \Xi_{t+s}^m} - 1 \right)^2 p_{t+s}^m y_{t+s}^m \right\}$$

subject to demand (F.73). Λ_t^f is the marginal utility of real income to the owners of the foreign firm, $mc_t^f(i)$ is foreign exporter *i*'s marginal cost, and Ξ_t^m captures the partial indexation to past import inflation:

(F.75)
$$\Xi_t^m = \left(\frac{p_{t-1}^m}{p_{t-2}^m} \pi_{t-1}\right)^{\epsilon^m}$$

To proceed, we assume that foreign producers purchase goods on world markets at the exogenous price p_t^{xf} , which implies (PC_t -deflated) marginal costs of foreign producers in domestic currency terms are:

$$mc_t^f = \frac{p_t^{xf}}{q_t}$$

Taking first order conditions and rearranging, we obtain a Phillips curve for imports:

$$1 - \sigma^{mb} + \frac{\sigma^{mb} p_t^{xf}}{q_t p_t^m} - \chi^m \frac{p_t^m \pi_t}{p_{t-1}^m \Xi_t^m} \left(\frac{p_t^m}{p_{t-1}^m} \frac{\pi_t}{\Xi_t^m} - 1 \right)$$

$$+ \beta \chi^m E_t \frac{\Lambda_{t+1}^f y_{t+1}^m}{\Lambda_t^f y_t^m} \frac{(p_{t+1}^m)^2 \pi_{t+1}}{(p_t^m)^2 \Xi_{t+1}^m} \left(\frac{p_{t+1}^m}{p_t^m} \frac{\pi_{t+1}}{\Xi_{t+1}^m} - 1 \right) = 0$$
(F.77)

Note that we do not need to specify a process for the foreign stochastic discount factor $(\Lambda_{t+1}^f/\Lambda_t^f)$. As long as it is assumed to be stationary, it cancels out in both the steady state, and when we log-linearize the model before solving.

Exchange rates and the balance of payments

Assume that foreign exchange market participants can trade in domestic and foreign bonds, but make up a negligible amount of the domestic market and so do not affect the government budget constraint. They can access domestic government bonds directly, so earn i_t^{CB} on them, not the household i_t^e . The nominal interest rate on foreign bonds is

⁶Note that domestic inflation π_t only features in this problem because the objective function has been normalized by PC_t . Adjustment costs are quadratic in deviations of PM_t/PM_{t-1} from the target rate Ξ_t^m . When we express that ratio in terms of relative import prices $p_t^m \equiv PM_t/PC_t$, it becomes $\pi_t \cdot p_t^m/p_{t-1}^m$.

 i_t^f . The real exchange rate q_t is then determined according to the UIP condition:

(F.78)
$$\mathbb{E}_{t} \frac{1 + i_{t}^{CB}}{\pi_{t+1}} \left(1 + \chi^{nfa} (nfa_{t} - \overline{nfa}) \right) = \mathbb{E}_{t} \frac{1 + i_{t}^{f}}{\pi_{t+1}^{f}} \frac{q_{t}}{q_{t+1}}$$

where nfa_t is the domestic country's real net foreign asset position, and \overline{nfa} is the steady state of nfa_t . If the parameter $\chi^{nfa}=0$, this reduces to the standard UIP condition. I instead calibrate χ^{nfa} to a small positive value, implying that movements in the domestic country's net foreign asset position will create a small wedge in UIP. This could come, for example, from quadratic costs in holding net foreign asset positions that deviate from the steady state position.⁷ The wedge is necessary to ensure that the steady state nfa_t is determinate, as discussed in Ghironi and Melitz (2005).

The net foreign asset position evolves to satisfy the balance of payments, i.e. so that changes in the financial account balance those in the current account:

(F.79)
$$nfa_t = nfa_{t-1} \frac{1 + i_{t-1}^f}{\pi_t^f} \frac{q_{t-1}}{q_t} + p_t^x x_t - p_t^m (c_t^m + mi_t^h + mi_t^x)$$

Additional definitions

Table 12 above lists the log-linearized model equations. To express the second (the wage Phillips curve) concisely, it is helpful to define wage inflation:

(F.80)
$$\pi_t^w \equiv \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \pi_t$$

Similarly, other inflation rates are defined:

(F.81)
$$\pi_t^{hv} \equiv \frac{PHV_t}{PHV_{t-1}} = \frac{p_t^{hv}}{p_{t-1}^{hv}} \pi_t$$

(F.82)
$$\pi_t^{xvf} \equiv \frac{PXVF_t}{PXVF_{t-1}} = \frac{p_t^{xv}}{p_t^{xv}} \frac{q_t}{q_{t-1}} \pi_t^f$$

(F.83)
$$\pi_t^m \equiv \frac{PM_t}{PM_{t-1}} = \frac{p_t^m}{p_{t-1}^m} \pi_t$$

(F.84)

In addition, it simplifies the log-linearization to use gross rather than nominal interest

⁷Harrison and Oomen (2010) indeed have such a microfoundation for the wedge, which they include in the household budget constraint. See the discussion of departures from their model below for further details.

rates:

$$(F.85) r_t^e \equiv 1 + i_t^e$$

$$(F.86) r_t^{CB} \equiv 1 + i_t^{CB}$$

$$(F.87) r_t^g \equiv 1 + i_t^g$$

$$(F.88) r_t^b \equiv 1 + i_t^b$$

$$(F.89) r_t^f \equiv 1 + i_t^f$$

In equations (F.63) and (F.62), the shocks ζ_t^{χ} and $\zeta_t^{\chi b}$ are introduced as mean-zero shocks. To avoid them dropping out in the log-linearization, we substitute out for them using:

$$\zeta_t^{\chi b} = e^{\hat{\zeta}_t^{\chi b}} - 1$$

where $\hat{\zeta}_t^{\chi}, \hat{\zeta}_t^{\chi b}$ are also mean-zero AR(1) shocks.

In the same way, we also replace the mean-zero capital adjustment cost shock ζ_t^k with:

$$\zeta_t^k = e^{\hat{\zeta}_t^k} - 1$$

Finally, we add two non-microfounded shocks which are common in the DSGE literature. The first is a risk premium shock ζ_t^c , which modifies the consumption Euler equation obtained by combining equation (F.17) with (F.13) and the definition of u_{ct} to:

(F.93)
$$u_{ct} = \beta e^{\zeta_t^c} \mathbb{E}_t \frac{(1 + i_t^e)}{\pi_{t+1}} u_{ct+1}$$

The second is a markup shock ζ_t^{hb} , which modifies the Phillips curve for domestic goods (F.56) to:

$$(F.94) 1 - \sigma^{hb} + \frac{\sigma^{hb}mc_{t}}{p_{t}^{hv}} - \chi^{hv} \frac{p_{t}^{hv}\pi_{t}}{p_{t-1}^{hv}\Xi_{t}^{hv}} \left(\frac{p_{t}^{hv}}{p_{t-1}^{hv}\Xi_{t}^{hv}} - 1\right)$$

$$+ \beta \chi^{hv} E_{t} \frac{\Lambda_{t+1}y_{t+1}^{hv}}{\Lambda_{t}y_{t}^{hv}} \frac{(p_{t+1}^{hv})^{2}\pi_{t+1}}{(p_{t}^{hv})^{2}\Xi_{t+1}^{hv}} \left(\frac{p_{t+1}^{hv}}{p_{t}^{hv}}\frac{\pi_{t+1}}{\Xi_{t+1}^{hv}} - 1\right) = \chi^{hv} (e^{\zeta_{t}^{hb}} - 1)$$

Model variables

The model can be reduced to a system of 27 endogenous variables:⁸ c_t , c_t^h , c_t^m , h_t , i_t^b , i_t^{CG} , i_t^e , i_t^g

$$(F.95) tf p_{t} = \rho_{tfp} tf p_{t-1} + e_{tfpt}$$

$$(F.96) g_{t} = \rho_{g} g_{t-1} + e_{gt}$$

$$(F.97) \zeta_{t}^{c} = \rho_{\zeta^{c}} \zeta_{t-1}^{c} + (1 - \rho_{\zeta^{c}}^{2})^{\frac{1}{2}} e_{\zeta^{c}t}$$

$$(F.98) \zeta_{t}^{hb} = \rho_{\zeta^{hb}} \zeta_{t-1}^{hb} + (1 - \rho_{\zeta^{hb}}^{2})^{\frac{1}{2}} e_{\zeta^{hb}t}$$

$$(F.99) \zeta_{t}^{k} = \rho_{\zeta^{k}} \hat{\zeta}_{t-1}^{k} + (1 - \rho_{\zeta^{k}}^{2})^{\frac{1}{2}} e_{\zeta^{k}t}$$

$$(F.100) \zeta_{t}^{\kappa h} = e_{\zeta^{\kappa h}t}$$

$$(F.101) \zeta_{t}^{\kappa cb} = e_{\zeta^{rcb}t}$$

$$(F.102) \hat{\zeta}_{t}^{\chi} = \rho_{\zeta^{\chi}} \hat{\zeta}_{t-1}^{\chi} + (1 - \rho_{\zeta^{\chi}}^{2})^{\frac{1}{2}} e_{\zeta^{\chi}t}$$

$$(F.103) \hat{\zeta}_{t}^{\chi b} = \rho_{\zeta^{\chi b}} \hat{\zeta}_{t-1}^{\chi b} + (1 - \rho_{\zeta^{\chi b}}^{2})^{\frac{1}{2}} e_{\zeta^{\chi b}t}$$

$$(F.104) \zeta_{t}^{\mu} = e_{\zeta^{\mu}t}$$

with $e_{xt} \sim i.i.d.N(0, \sigma_x^2)$ for $x \in \{tfp, g, \zeta^c, \zeta^{hb}, \zeta^k, \zeta^{\kappa h}, \zeta^{rcb}, \zeta^{\chi}, \zeta^{\chi b}, \zeta^{\mu}\}$. $\pi_t^f, c_t^f, r_t^f, p_t^{xf}$ then follow the VAR(4) process detailed in Appendix E.2.2.

Steady state

For this section, \bar{x} refers to the steady state of the associated variable x_t . I consider a steady state in which inflation is zero in all goods. That is, $\bar{\pi} = 1$, $\bar{\pi}^f = 1$. As a direct result, the indexation variables $\bar{\Xi}^{hv}$, $\bar{\Xi}^{xv}$, $\bar{\Xi}^{w}$ are all also equal to 1.

Relative prices Without loss of generality, I impose that $\bar{p}^h = \bar{p}^m$. In equation F.5 in steady state, this implies

(F.105)
$$\bar{p}^h = \kappa^c \left[(1 - \psi^m)^{\sigma^m} + (\psi^m)^{\sigma^m} \right]^{\frac{1}{\sigma^m - 1}}$$

⁸All variables excluded from this list are simple functions of the included variables. For example, taxes τ_t are a function of ph_t , g_t , and i_{t-1}^{CB} through the government budget constraint (F.65).

I set the parameter κ^c to

(F.106)
$$\kappa^{c} = \left[(1 - \psi^{m})^{\sigma^{m}} + (\psi^{m})^{\sigma^{m}} \right]^{\frac{1}{1 - \sigma^{m}}}$$

which is a normalization that ensures $\bar{p}^h = \bar{p}^m = 1$.

Using this, and the fact that final goods producers price at marginal cost, equation F.46 in steady state is

$$(F.107) 1 = \kappa^{hv} \bar{p}^{hv} + 1 - \kappa^{hv}$$

which implies $\bar{p}^{hv} = 1$.

From this, equation F.56 implies

$$\overline{mc} = \frac{\sigma^{hb} - 1}{\sigma^{hb}}$$

which can be substituted into equation F.57 to give

(F.109)
$$\bar{p}^{xv} = \left(\frac{\sigma^{xb}}{\sigma^{xb} - 1}\right) \left(\frac{\sigma^{hb} - 1}{\sigma^{hb}}\right)$$

Equation F.47 in steady state, again using the fact that final goods producers price at marginal cost, gives

(F.110)
$$\bar{p}^x = 1 + \kappa^{xv} \left(\frac{\sigma^{hb} - \sigma^{xb}}{\sigma^{hb}(\sigma^{xb} - 1)} \right)$$

In all quantitative exercises, I assume as in HO that elasticities of substitution are equal across export and domestic markets (i.e. $\sigma^{xb} = \sigma^{hb}$). In these equations, the assumption implies that $\bar{p}^{xv} = \bar{p}^x = 1$.

Finally, from equation F.77 in steady state

$$\bar{p}^{xf} = \frac{\sigma^{mb} - 1}{\sigma^{mb}} \bar{q}$$

where the steady state real exchange rate \bar{q} is derived below, and as with the other elasticities of substitution I set $\sigma^{mb} = \sigma^{hb}$ in all quantitative exercises.

Firms Without loss of generality, I fix the steady state of output at $\bar{y}^v = 1$. This aids the calibration of steady state government spending and investment as the relative contributions of those objects to UK GDP in the data. Given these, I then use the equations of the firm problem back out the steady state TFP required $\bar{y}^v = 1$ to hold.

Specifically, I calibrate χ^z such that steady state capital utilization $\bar{z}=1$. Equation F.11 then implies that in steady state $\bar{k}^s=\bar{k}$. I fix steady state investment to \overline{inv} , which is calibrated to match the average share of investment in output in UK national accounts. Equation F.9 then implies that:

$$\bar{k} = \frac{\bar{i}nv}{\delta}$$

Rearranging the firm first order condition on capital services (equation F.35) then gives the steady state of TFP:

(F.113)
$$\overline{tfp} = \bar{k}^{\frac{1}{\sigma^y - 1}} \left(\frac{\bar{r}}{\alpha}\right)^{\frac{\sigma^y}{\sigma^y - 1}} (\overline{mc})^{-\frac{\sigma^y}{\sigma^y - 1}}$$

From equations F.16 and F.18 we have

$$\bar{R} = \chi^z \overline{PI}$$

Using the definition $R_t = r_t \cdot PC_t$, and the result above that $PI_t = PH_t$, this rearranges to

$$\bar{r} = \bar{p}_t^h \chi^z = \chi^z$$

where the final equality uses the fact that relative prices are 1 in steady state.

Using all of these results, and equation F.108, equation F.113 becomes:

(F.116)
$$\overline{tfp} = \left(\frac{\chi^z \sigma^{hb}}{\alpha(\sigma^{hb} - 1)}\right)^{\frac{\sigma^y - 1}{\sigma^y}} \bar{k}^{\frac{1}{\sigma^y - 1}}$$

Next, I find the steady state of hours \bar{h} . Take the production function of intermediate goods producers (equation F.32), substitute out for $\bar{y}^v = 1$, and rearrange to obtain:

(F.117)
$$\bar{h} = \left[\frac{1}{1 - \alpha} \left(\frac{1}{\overline{tfp}} \right)^{\frac{\sigma^{y} - 1}{\sigma^{y}}} - \frac{\alpha}{1 - \alpha} \bar{k}^{\frac{\sigma^{y} - 1}{\sigma^{y}}} \right]^{\frac{\sigma^{y} - 1}{\sigma^{y} - 1}}$$

Using equation F.33, and substituting in that $\bar{r} = \chi^z$ and $\bar{k}^s = \bar{k}$, we obtain steady state real wages

(F.118)
$$\bar{w} = \frac{1 - \alpha}{\alpha} \chi^z \left(\frac{\bar{k}}{\bar{h}}\right)^{\frac{1}{\sigma^y}}$$

Exhange rates and balance of payments Without loss of generality, I normalize \bar{c}^f to 1. Given this, and the previously derived $\bar{p}^x = 1$, equation F.70 implies

(F.119)
$$\bar{x} = \kappa^x \left(\frac{\bar{q}}{\bar{p}^{xf}}\right)^{-\sigma^x}$$

I set the parameter κ^x to $\bar{x}(\bar{p}^{xf})^{-\sigma^x}$, which implies $\bar{q}=1$.

From UIP (equation F.78) we obtain:

$$(F.120) \bar{i}^f = \bar{i}^{CB}$$

Next, I turn to the steady state current account balance. From market clearing (equation F.69), and cost minimization (equations F.42 and F.43), we have:

(F.121)
$$\bar{y}^v = \bar{y}^{hv} + \bar{y}^{xv} = \frac{\kappa^{hv}}{1 - \kappa^{hv}} \overline{mi}^h + \frac{\kappa^{xv}}{1 - \kappa^{xv}} \overline{mi}^x$$

which rearranges to:

(F.122)
$$\bar{y}^v = \frac{1}{1 - \kappa^{hv}} \overline{mi}^h + \frac{1}{1 - \kappa^{xv}} \overline{mi}^x - \overline{mi}^h - \overline{mi}^x$$

Substituting out for the first two terms using the production functions for final goods for home and export consumption (equations F.37 and F.38):

$$\bar{y}^v = \bar{y}^h + \bar{y}^x - \overline{mi}^h - \overline{mi}^x$$

Rearranging this, and using $\bar{y}^x = \bar{x}$ from equation F.68, we obtain:

(F.124)
$$\bar{x} - \overline{mi}^h - \overline{mi}^x = \bar{y}^v - \bar{y}^h = 1 - \bar{y}^h$$

Substituting out for \bar{y}^h using equation F.67, this becomes

$$(F.125) \bar{x} - \overline{mi}^h - \overline{mi}^x = 1 - \bar{c}^h - \overline{inv} - \bar{g}$$

Subtracting consumption of imports from both sides, and noting from equation F.8 that $\bar{c}^h + \bar{c}^m$ adds up to total consumption \bar{c} , we obtain an expression for the current account balance $\bar{c}a$:

(F.126)
$$\overline{ca} \equiv \bar{x} - \bar{c}^m - \overline{mi}^h - \overline{mi}^x = 1 - \bar{c} - \overline{inv} - \bar{g}$$

Steady state investment and government spending are calibrated externally (see Firm

section above). I therefore fix \bar{c} to match the average current account balance as a percentage of GDP over the period considered (since total output \bar{y}^v is normalized to 1). Steady state exports adjust to ensure that this \bar{c} is consistent with market clearing, as derived below.

With these results in hand, rearrange the law of motion for net foreign assets (equation F.79) in steady state to:

$$-\bar{i}^f \overline{nfa} = \bar{x} - \bar{c}^m - \overline{mi}^h - \overline{mi}^x$$

where I have used that relative prices \bar{p}^m and \bar{p}^x are both 1 in steady state. The right hand side of this equation is equal to $\bar{c}a$, so using equations F.126 and F.120, this becomes

(F.128)
$$\overline{nfa} = -\left(\frac{1 - \bar{c} - i\bar{n}v - \bar{g}}{\bar{i}^{CB}}\right)$$

Households From equation F.17, we have $\bar{i}^e = \beta^{-1} - 1$.

Given the steady state consumption calibration described above, equations F.6 and F.7 then give the consumption of domestic and imported goods

(F.129)
$$\bar{c}^h = (1 - \psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} \bar{c}$$

(F.130)
$$\bar{c}^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} \bar{c}$$

In addition, the steady state marginal utility of consumption comes from equation F.13

(F.131)
$$\bar{u}_c = \bar{c}^{-\frac{1}{\sigma^c} + \psi^{hab}(\frac{1}{\sigma^c} - 1)}$$

With the results derived here we also obtain, from equations F.67 and F.37 respectively

$$(F.132) \bar{y}^h = \bar{c}^h + \overline{inv} + \bar{g}$$

(F.133)
$$\overline{mi}^h = (1 - \kappa^{hv})\overline{y}^h$$

Information and banks To find the steady state parameters in the attention block of the model, I first define two new steady state objects, which are calibration targets. First, \overline{mn} is the spread between the policy rate and the unconditional mean interest rate available on savings:

$$\overline{mn} \equiv \overline{i}^{CB} - \frac{\overline{i}^g + \overline{i}^b}{2}$$

Second, \overline{sd} as the standard deviation of available interest rates:

$$(F.135) \overline{sd} \equiv \frac{\overline{i}^g - \overline{i}^b}{2}$$

Both of these are calibrated to long-run moments from the Moneyfacts data, as described in Appendix E.2.

From the attention first order condition (combining equations F.13 and F.14), we have

$$\bar{\lambda} = \frac{\mu}{\beta b \bar{u}_c}$$

Rearranging equation F.60 yields

(F.137)
$$\bar{p}^g = \frac{\exp\left(\frac{2\overline{sd}}{\lambda}\right)}{\exp\left(\frac{2\overline{sd}}{\lambda}\right) + 1}$$

Using $\bar{i}^e = \beta^{-1}$ and the definition of \overline{sd} , equation F.61 can be written as

$$(F.138) \bar{i}^b = \beta^{-1} - 2\bar{p}^g \overline{sd}$$

Substituting this into equation F.135 gives

(F.139)
$$\bar{i}^g = \beta^{-1} + 2(1 - \bar{p}^g)\overline{sd}$$

Having solved for each offered interest rate, we now use equation F.134 to back out the steady state policy rate:

$$(F.140) \bar{i}^{CB} = \overline{mn} + \frac{\bar{i}^g + \bar{i}^b}{2}$$

Finally, we use the bank first order conditions (equations F.62 and F.63) to back out the cost parameters χ^g and χ^b required to hit the calibration targets $\overline{mn}, \overline{sd}$ in steady state. Specifically, rearranging these first order conditions in steady state gives:

(F.141)
$$\bar{i}^g = \bar{i}^{CB} - \chi^g - \frac{\bar{\lambda}}{1 - \bar{p}^g}$$

(F.142)
$$\bar{i}^b = \bar{i}^{CB} - \chi^b - \frac{\bar{\lambda}}{\bar{p}^g}$$

Substituting these optimality conditions into the definitions of \overline{mn} and \overline{sd} and rearranging

we obtain two conditions pinning down χ^g, χ^b . The unique solution to these conditions is

$$\chi^g = \overline{mn} - \overline{sd} - \frac{\bar{\lambda}}{1 - \bar{p}^g}$$

$$\chi^b = \overline{mn} + \overline{sd} - \frac{\overline{\lambda}}{\overline{p}^g}$$

Exports From equation F.69 we have

$$\bar{y}^v = \kappa^{hv} \bar{y}^h + \kappa^{xv} \bar{y}^x$$

Substituting out for \bar{y}^h using equation F.67, using that $\bar{y}^v = 1$, and rearranging gives

(F.146)
$$\bar{y}^x = \frac{1 - \kappa^{hv}(\bar{c}^h + \overline{inv} + \bar{g})}{\kappa^{xv}}$$

Note that from equation F.68, $\bar{x} = \bar{y}^x$. Equation F.38 then implies

$$\overline{mi}^x = (1 - \kappa^{xv})\overline{y}^x$$

Government I set steady state government spending \bar{g} to match the share of government spending in GDP from UK national accounts. Steady state lump sum taxes are then pinned down by equation F.65, which in real terms in steady state is

$$\bar{\tau} = \bar{g} + \bar{i}^{CB}b$$

Comparison to Harrison and Oomen (2010)

Aside from the introduction of inattention to savings and a banking sector, I only make minimal changes to the model in Harrison and Oomen (2010). In the shocks, I use a risk premium shock rather than a discount factor shock, and I assume that the labor disutility shock is i.i.d. (Harrison and Oomen estimate its persistence at 0.001). In price setting, I model labor unions and foreign exporters as facing quadratic adjustment costs of price changes, rather than Calvo-style staggered contracts. This makes no difference to the log-linearized equations, but makes the exposition simpler and brings them into line with the price-setting problem of domestic intermediate goods producers. Finally, Harrison and Oomen allow households to invest in foreign bonds, subject to a quadratic cost of holding a portfolio that deviates from steady state net foreign assets. In contrast, I do not allow households to access these bonds, and instead impose UIP and the balance of payments separately. The reason for this is that, if I followed the Harrison and Oomen

approach, UIP would depend on i_t^e rather than the policy rate i_t^{CB} . It is not plausible that arbitrageurs in foreign exchange markets are subject to the same information frictions as households, and so I impose UIP separately. The log-linearized versions of (F.78) and (F.79) therefore correspond exactly to those in Harrison and Oomen, but they are not derived from the household problem.

The attention and bank problems introduce 5 new variables not in the Harrison and Oomen model: i_t^e , λ_t , p_t^g , i_t^g , i_t^g . The new equations are the first order condition on attention (F.14), the choice probability rule (F.60), the definition of i_t^e (F.61), and the two bank first order conditions (F.62 and F.63). There are three new shocks, to attention (ζ_t^{μ}) , the level of bank interest rates (ζ_t^{χ}) and their dispersion $(\zeta_t^{\chi b})$.

Two-agent model extension

In Appendix E.3, I introduce an extension to the quantitative model to include borrowers. This section sets out this extended model.

Households

A fraction $1 - q_d$ of households are savers. They face exactly the same utility function (equation F.1) as in the representative-agent model. Their budget constraint is also unchanged, except for a lump sum tax which will be transferred to debtor households. The saver budget constraint is therefore:

(F.149)
$$PC_{t}c_{t} + PI_{t}inv_{t} + B_{t} - (1 + i_{t-1}^{e})B_{t-1} = W_{t}h_{t} + R_{t}k_{t}^{s} + \Pi_{t}^{v} + \Pi_{t}^{b} - PC_{t}\tau_{t} - PC_{t}\frac{q_{d}}{1 - q_{d}}\tau_{0}$$

where the final term is the new lump-sum tax, set to ensure the real transfer to each debtor household is equal to τ_0 . Including this tax does not affect the first order conditions (equations F.13-F.18). Note that bank profit Π_t^b now also includes any profit made by banks engaged in lending, though again since this is lump sum it does not affect the first order conditions.

The remaining q_d households are debtors. Their instantaneous utility function is identical to that of savers, but they have a lower discount factor $\beta^d < \beta$. This means they will borrow in equilibrium, and will hold no capital or shares in firms. Their problem

is given by

(F.150)

$$\max_{c_t^d, D_t, i_t^e} E_t \sum_{s=0}^{\infty} (\beta^d)^s \left[\frac{1}{1 - \frac{1}{\sigma^c}} \left(\frac{c_t^d}{\tilde{c}_{t-1}^{\psi^{hab}}} \right)^{1 - \frac{1}{\sigma^c}} - (\kappa^h)^{-\frac{1}{\sigma^h}} e^{\zeta_t^{\kappa h}} \frac{1}{1 + \frac{1}{\sigma^h}} (h_t)^{1 + \frac{1}{\sigma^h}} - \mu e^{\zeta_t^{\mu}} \mathcal{I}^d(i_t^{ed}) \right]$$

subject to

(F.151)
$$PC_t c_t^d - D_t + (1 + i_{t-1}^{ed}) D_{t-1} = W_t h_t - PC_t \tau_t + PC_t \tau_0$$

$$(F.152) D_t \le PC_t d$$

(F.153)
$$\mathcal{I}^{d\prime}(i_t^{ed}) < 0, \quad \mathcal{I}^{d\prime\prime}(i_t^{ed}) > 0$$

where c_t^d is debtor consumption, D_t is nominal debt, i_t^{ed} is the effective interest rate on debt, and $\mathcal{I}^d(i_t^{ed})$ is the information processing required to achieve that effective interest rate. Equation F.152 is the borrowing constraint: real debt cannot exceed the exogenous limit d. Note that in the budget constraint debtors have the same labor income $w_t h_t$ as savers, which is explained in the labor union section below. Habits for both savers and debtors depend on \tilde{c}_{t-1} , which is aggregate consumption across both household types, defined as:

(F.154)
$$\tilde{c}_t = (1 - q_d)c_t + q_d c_t^d$$

In real terms, the budget constraint of debtors is:

(F.155)
$$c_t^d - d_t + \frac{(1 + i_{t-1}^{ed})}{\pi_t} d_{t-1} = w_t h_t - \tau_t + \tau_0$$

where $d_t \equiv D_t/PC_t$ is real debt.

The first order conditions of debtors are:

(F.156)
$$\frac{1}{\tilde{c}_{t-1}^{\psi^{hab}}} \left(\frac{c_t^d}{\tilde{c}_{t-1}^{\psi^{hab}}} \right)^{-\frac{1}{\sigma^c}} = PC_t \Lambda_t^d$$

(F.157)
$$\beta^d E_t \Lambda_{t+1}^d D_t = -\mu e^{\zeta_t^\mu} \mathcal{I}_t^{d\prime} (i_t^{ed})$$

$$\Lambda_t^d = \beta^d E_t (1 + i_t^{ed}) \Lambda_{t+1}^d + \Theta_t^d$$

where Λ^d_t and Θ^d_t are the Lagrange multipliers on the budget constraint and the borrowing

constraint respectively.

 β^d will be set sufficiently low that in the neighbourhood of steady state, $\beta^d(1+i_t^{ed}) < 1$. Equation F.158 implies that $\Theta^d_t > 0$, i.e. that the borrowing constraint binds. This means that $d_t = d$, and debtor consumption is determined by the budget constraint (equation F.155) alone. Debtors are therefore hand-to-mouth, with a marginal propensity to consume of 1.

For both types of household, the consumption index is defined using the same CES aggregator over home and imported goods (equation F.3), so the price index remains as in equation F.4, and the allocation of expenditure between home and imported goods for savers is as in equations F.6 and F.7. For debtors, the equivalent allocation equations are

(F.159)
$$c_t^{hd} = (1 - \psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} c_t^d$$

(F.160)
$$c_t^{md} = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} c_t^d$$

Since the coefficients are the same for both household types, we can write expressions for aggregate domestic and imported consumption as

(F.161)
$$\tilde{c}_t^h = (1 - \psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^h)^{-\sigma^m} \tilde{c}_t$$

(F.162)
$$\tilde{c}_t^m = (\psi^m)^{\sigma^m} (\kappa^c)^{\sigma^m - 1} (p_t^m)^{-\sigma^m} \tilde{c}_t$$

Individuals

Individuals within saver households are as in the representative-agent model. Individuals within the debtor households are as described in Appendix C.2.1. Specifically, since we will assume the number of lending banks is $N^d = 2$, we can denote the probability of choosing the lower interest rate lender as p_t^{gd} . Solving the individual's rational inattention problem yields:

$$(F.163) p_t^{gd} = \frac{\exp\left(-\frac{i_t^{gd}}{\lambda_t^d}\right)}{\exp\left(-\frac{i_t^{gd}}{\lambda_t^d}\right) + \exp\left(-\frac{i_t^{bd}}{\lambda_t^d}\right)}$$

where i_t^{gd} , i_t^{bd} are the good (low) and bad (high) interest rates on loans offered by the two banks, and λ_t^d is the shadow value of information about borrowing.

The effective interest rate experienced by borrowers is then

(F.164)
$$i_t^{ed} = p_t^{gd} i_t^{gd} + (1 - p_t^{gd}) i_t^{bd}$$

Using equation C.22, $\mathcal{I}^{d\prime}(i_t^{ed}) = -(\lambda_t^d)^{-1}$.

Banks

Deposit-taking banks are as in the representative-agent model. Lending banks are modeled as in Appendix C.2.1, with $N^d = 2$. As in that Appendix, we have that the first order conditions for profit maximization for good and bad banks respectively are

(F.165)
$$\frac{dp_t^{gd}}{di_t^{gd}} \cdot (i_t^{gd} - i_t^{CB} - \chi_t^{gd}) = -p_t^{gd}$$

(F.166)
$$-\frac{dp_t^{gd}}{di_t^{bd}} \cdot (i_t^{bd} - i_t^{CB} - \chi_t^{bd}) = -(1 - p_t^{gd})$$

Differentiating equation F.163 with respect to each interest rate and substituting into these first order conditions, they become

(F.167)
$$(1 - p_t^{gd}) \cdot (i_t^{gd} - i_t^{CB} - \chi_t^{gd}) = \lambda_t^d$$

(F.168)
$$p_t^{gd} \cdot (i_t^{bd} - i_t^{CB} - \chi_t^{bd}) = \lambda_t^d$$

The costs χ_t^{gd} and χ_t^{bd} are specified in Appendix E.3, and are reproduced here as equations F.169 and F.170

$$\chi_t^{gd} = \chi_0^{gd} + \zeta_t^{\chi}$$

(F.170)
$$\chi_t^{bd} = \chi_0^{bd} + \chi_1(i_t^{CB} - \bar{i}^{CB}) + \zeta_t^{\chi} + \zeta_t^{\chi b}$$

where $\chi_0^{gd}, \chi_0^{bd}, \chi_1$ are constants, and $\zeta_t^{\chi}, \zeta_t^{\chi b}$ are AR(1) exogenous shocks.

Labor unions

As in the representative agent model, households supply labor to unions, who set wages. Saver and debtor households are members of the same unions, and labor supply from debtors is a perfect substitute for labor supply from savers. This means that the wage and labor supply from each union is the same for both household types.

Unions set wages to maximize the average expected discounted utility of their members. The problem of labor unions is therefore:

$$\max_{W_{t}(i)} E_{t} \sum_{s=0}^{\infty} \left\{ \tilde{\Lambda}_{t+s} W_{t+s}(i) h_{t+s}(i) - \tilde{\beta}^{s} (\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t+s}^{\kappa h}} \frac{1}{1 + \frac{1}{\sigma^{h}}} (h_{t+s}(i))^{1 + \frac{1}{\sigma^{h}}} - \tilde{\Lambda}_{t+s} \frac{\chi^{w}}{2} \left(\frac{W_{t+s}(i)}{W_{t+s-1}(i) \Xi_{t+s}^{w}} - 1 \right)^{2} W_{t+s} \right\}$$
(F.171)

subject to labor demand (equation F.23) and the definition of wage indexation Ξ_t^w (equation F.26). This is exactly as in the representative-agent model, except that wages and wage adjustment costs are discounted using average preferences across both household types:

(F.172)
$$\tilde{\beta} \equiv (1 - q_d)\beta + q_d\beta^d$$

(F.173)
$$\tilde{\Lambda}_{t+s} \equiv (1 - q_d)\beta^s \Lambda_{t+s} + q_d(\beta^d)^s \Lambda_{t+s}^d$$

Following the steps as in the representative-agent model, union wage setting generates a wage Phillips curve given by:

$$(1 - \sigma^{w})h_{t} = -(\kappa^{h})^{-\frac{1}{\sigma^{h}}} e^{\zeta_{t}^{\kappa h}} \sigma^{w} \frac{h_{t}^{1 + \frac{1}{\sigma^{h}}}}{w_{t}\tilde{u}_{ct}} + \frac{\chi^{w}w_{t}\pi_{t}}{w_{t-1}\Xi_{t}^{w}} \left(\frac{w_{t}}{w_{t-1}\Xi_{t}^{w}}\pi_{t} - 1\right)$$

$$(F.174) \qquad -E_{t} \frac{(\beta(1 - q_{d})u_{ct+1} + \beta^{d}q_{d}u_{ct+1}^{d})\chi^{w}w_{t+1}^{2}\pi_{t+1}}{w_{t}^{2}\Xi_{t+1}^{w}\tilde{u}_{ct}} \left(\frac{w_{t+1}}{w_{t}\Xi_{t+1}^{w}}\pi_{t+1} - 1\right)$$

where $\tilde{u}_{ct} = \tilde{\Lambda}_t PC_t$ is the average marginal utility of consumption across all households.

Firms and price setting

Firms are unchanged from the representative-agent model. In particular, firms are owned by savers only, so continue to discount future profits based on saver preferences alone. The price-setting problem is therefore unchanged from the representative-agent model.

Monetary and fiscal policy

Monetary policy is as in the representative-agent model. The government budget constraint is however different in two respects. First, I change the supply of real bonds to $(1-q_d)b$, so that the equilibrium quantity of bonds held by savers remains at b as in the representative-agent model. Second, the government is the source of funds for the lending banks. This ensures that saving and borrowing are treated symmetrically from the bank side. This means that the government lends out $q_d d$ real bonds in period t, and is repaid $q_d d(1+i_t^{CB})$ in period t+1. The government budget constraint in real terms therefore becomes

(F.175)
$$\tau_t = p_t^h g_t + ((1 - q_d)b - q_d d)i_{t-1}^{CB}$$

Market clearing

Domestic goods market clearing is

$$(F.176) y_t^h = \tilde{c}_t^h + inv_t + g_t$$

Export goods market clearing is as in the representative-agent model (equation F.68). Total domestic output is as in equation F.69.

Foreign variables, exchange rates, and the balance of payments

These are all as in the representative-agent model.

Steady state

As in Galí et al. (2007), I set the steady-state inter-household transfer τ_0 such that steady state consumption is identical across saver and debtor households. As a result, the steady states of all variables that appear in the representative-agent model are unchanged by the introduction of debtors, with the exception of steady state taxes $\bar{\tau}$, which become

(F.177)
$$\bar{\tau} = \bar{g} + ((1 - q_d)b - q_d d)\bar{i}^{CB}$$

The steady states of the new variables c_t^d , \tilde{c}_t , i_t^{ed} , p_t^{gd} , λ_t^d , i_t^{gd} , i_t^{bd} are given by the following equations, in which \bar{x} refers to the steady state of the corresponding variable x_t .

$$(F.178) \bar{c}^d = \bar{\tilde{c}} = \bar{c}$$

where \bar{c} is unchanged from the representative-agent model.

As in the attention to saving block of the representative-agent model, I now define two new steady state objects, which are calibration targets. First, \overline{mn}^d is the spread between the policy rate and the effective interest rate on debt:

$$\overline{mn}^d \equiv \overline{i}^{ed} - \overline{i}^{CB}$$

Second, \overline{sd}^d as the standard deviation of available interest rates on debt:

(F.180)
$$\overline{sd}^d \equiv \frac{\overline{i}^{bd} - \overline{i}^{gd}}{2}$$

Both of these are calibrated to moments from the data, as described in Appendix E.3.

Since \bar{i}^{CB} is already pinned down by the deposit bank block (as in the representative-agent model), we can use equation F.179 to obtain $\bar{i}^{ed} = \bar{i}^{CB} + \overline{mn}^d$.

From the attention first order condition (combining equations F.156 and F.157), we have

$$\bar{\lambda}^d = \frac{\mu}{\beta^d d\bar{u}_c^d}$$

where the marginal utility in steady state is $\bar{u}_c^d = \bar{u}_c$, because consumption is the same in steady state across households.

Rearranging equation F.163 yields

(F.182)
$$\bar{p}^{gd} = \frac{\exp\left(\frac{2\overline{sd}^d}{\overline{\lambda}^d}\right)}{\exp\left(\frac{2\overline{sd}^d}{\overline{\lambda}^d}\right) + 1}$$

Using equation F.164 and F.180, we obtain expressions for each of the borrowing interest rates available in steady state

(F.183)
$$\bar{i}^{gd} = \bar{i}^{ed} - 2(1 - \bar{p}^{gd})\overline{sd}^d$$

$$\bar{i}^{bd} = \bar{i}^{ed} + 2\bar{p}^{gd}\bar{s}\bar{d}^d$$

Finally, the lending bank first order conditions (F.167 and F.168) imply

(F.185)
$$\bar{i}^{gd} = \bar{i}^{CB} + \chi_0^{gd} + \frac{\bar{\lambda}^d}{1 - \bar{p}^{gd}}$$

(F.186)
$$\bar{i}^{bd} = \bar{i}^{CB} + \chi_0^{bd} + \frac{\bar{\lambda}^d}{\bar{p}^{gd}}$$

Substituting these into the definitions of \overline{mn}^d and \overline{sd}^d (F.179 and F.180) gives two conditions pinning down the steady state bank costs required to meet those calibration targets. Solving those conditions yields

(F.187)
$$\chi_0^{gd} = \overline{m}\overline{n}^d - 2(1 - \overline{p}^{gd})\overline{s}\overline{d}^d - \frac{\overline{\lambda}^d}{1 - \overline{p}^{gd}}$$

(F.188)
$$\chi_0^{bd} = \overline{m}\overline{n}^d + 2\overline{p}^{gd}\overline{s}\overline{d}^d - \frac{\overline{\lambda}^d}{\overline{p}^{gd}}$$

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