

Supplemental Appendix of Cursed Sequential Equilibrium

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1 Pooling Equilibria in Signaling Games

Proof of Proposition 6

Here we provide a characterization of χ -CSE of Game BH 3 and Game BH 4. In the following, we denote $\mu_I \equiv \mu_2^\chi(\theta_1|m = I)$ and $\mu_S \equiv \mu_2^\chi(\theta_1|m = S)$.

Part 1 (Analysis of BH 3): At information set S , given μ_S , the expected payoffs of C , D , E are $90\mu_S$, $30 - 15\mu_S$ and 15 , respectively. Therefore, for any μ_S , E is never a best response. Moreover, C is the best response if and only if $90\mu_S \geq 30 - 15\mu_S$ or $\mu_S \geq \frac{2}{7}$. Similarly, at information set I , given μ_I , the expected payoffs of C , D , E are 30 , $45 - 45\mu_I$ and 15 , respectively. Therefore, E is strictly dominated, and C is the best response if and only if $30 \geq 45 - 45\mu_I$ or $\mu_I \geq \frac{1}{3}$. Now we consider four cases.

Case 1 [$m(\theta_1) = I, m(\theta_2) = S$]: By Lemma 1, $\mu_I = 1 - \frac{\chi}{2}$ and $\mu_S = \frac{\chi}{2}$. Moreover, since $\mu_I = 1 - \frac{\chi}{2} \geq \frac{1}{2}$ for any χ , player 2 will choose C at information set I . To support this equilibrium, player 2 has to choose C at information set S . In other words, $[(I, S); (C, C)]$ is separating χ -CSE if and only if $\mu_S \geq \frac{2}{7}$ or $\chi \geq \frac{4}{7}$.

Case 2 [$m(\theta_1) = S, m(\theta_2) = I$]: By Lemma 1, $\mu_I = \frac{\chi}{2}$ and $\mu_S = 1 - \frac{\chi}{2}$. Because $\mu_S \geq 1 - \frac{\chi}{2} \geq \frac{1}{2}$, it is optimal for player 2 to choose C at information set S . To support this as an

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equilibrium, player 2 has to choose D at information set I . Yet in this case, type θ_2 player 1 will deviate to S . Therefore, this profile cannot be supported as an equilibrium.

Case 3 $[m(\theta_1) = I, m(\theta_2) = I]$: Since player 1 follows a pooling strategy, player 2 will not update his belief at information set I , i.e., $\mu_I = \frac{1}{2}$. Moreover, χ -dampened updating property implies $\frac{\chi}{2} \leq \mu_S \leq 1 - \frac{\chi}{2}$. Since $\mu_I > \frac{1}{3}$, player 2 will choose C at information set I . To support this profile to be an equilibrium, player 2 has to choose D at information set S , and hence, it must be the case that $\mu_S \leq \frac{2}{7}$. Coupled with the requirement from χ -dampened updating, the off-path belief has to satisfy $\frac{\chi}{2} \leq \mu_S \leq \frac{2}{7}$. That is, $[(I, I); (C, D)]$ is pooling χ -CSE if and only if $\frac{\chi}{2} \leq \frac{2}{7}$ or $\chi \leq \frac{4}{7}$.

Case 4 $[m(\theta_1) = S, m(\theta_2) = S]$: Similar to the previous case, since player 1 follows a pooling strategy, player 2 will not update his belief at information set S , i.e., $\mu_S = \frac{1}{2}$. Also, the χ -dampened updating property suggests $\frac{\chi}{2} \leq \mu_I \leq 1 - \frac{\chi}{2}$. Because $\mu_S > \frac{2}{7}$, it is optimal for player 2 to choose C at information set S . To support this as an equilibrium, player 2 has to choose D at information set I . Therefore, it must be that $\mu_I \leq \frac{1}{3}$. Combined with the requirement of χ -dampened updating, the off-path belief has to satisfy $\frac{\chi}{2} \leq \mu_I \leq \frac{1}{3}$. As a result, $[(S, S); (D, C)]$ is a pooling χ -CSE if and only if $\chi \leq \frac{2}{3}$.

Part 2 (Analysis of BH 4): At information set I , given μ_I , the expected payoffs of C , D , E are 30, $45 - 45\mu_I$ and $35\mu_I$. Hence, D is the best response if and only if $\mu_I \leq \frac{1}{3}$ while E is the best response if $\mu_I \geq \frac{6}{7}$. For $\frac{1}{3} \leq \mu_I \leq \frac{6}{7}$, C is the best response. On the other hand, since player 2's payoffs at information set S are the same as in BH 3, player 2 will adopt the same decision rule—player 2 will choose C if and only if $\mu_S \geq \frac{2}{7}$, and choose D if and only if $\mu_S \leq \frac{2}{7}$. Now, we consider the following four cases.

Case 1 $[m(\theta_1) = I, m(\theta_2) = S]$: In this case, by Lemma 1, $\mu_I = 1 - \frac{\chi}{2}$ and $\mu_S = \frac{\chi}{2}$. To support this profile to be an equilibrium, player 2 has to choose E and C at information set I and S , respectively. To make it profitable for player 2 to choose E at information set I , it must be that: $\mu_I = 1 - \frac{\chi}{2} \geq \frac{6}{7} \iff \chi \leq \frac{2}{7}$. On the other hand, player 2 will choose C at information set S if and only if $\frac{\chi}{2} \geq \frac{2}{7}$ or $\chi \geq \frac{4}{7}$, which is not compatible with the previous inequality. Therefore, this profile cannot be supported as an equilibrium.

Case 2 $[m(\theta_1) = S, m(\theta_2) = I]$: In this case, by Lemma 1, $\mu_I = \frac{\chi}{2}$ and $\mu_S = 1 - \frac{\chi}{2}$. To support this as an equilibrium, player 2 has to choose D at both information sets. Yet, $\mu_S = 1 - \frac{\chi}{2} > \frac{2}{7}$, implying that it is not a best reply for player 2 to choose D at information set S . Hence this profile also cannot be supported as an equilibrium.

Case 3 [$m(\theta_1) = I, m(\theta_2) = I$]: Since player 1 follows a pooling strategy, player 2 will not update his belief at information set I , i.e., $\mu_I = \frac{1}{2}$. The χ -dampened updating property implies $\frac{\chi}{2} \leq \mu_S \leq 1 - \frac{\chi}{2}$. Because $\frac{1}{3} < \mu_I = \frac{1}{2} < \frac{6}{7}$, player 2 will choose C at information set I . To support this profile as an equilibrium, player 2 has to choose D at information set S , and hence, it must be the case that $\mu_S \leq \frac{2}{7}$. Coupled with the requirement of χ -dampened updating, the off-path belief has to satisfy $\frac{\chi}{2} \leq \mu_S \leq \frac{2}{7}$. That is, $[(I, I); (C, D)]$ is pooling χ -CSE if and only if $\frac{\chi}{2} \leq \frac{2}{7}$ or $\chi \leq \frac{4}{7}$.

Case 4 [$m(\theta_1) = S, m(\theta_2) = S$]: Similar to the previous case, since player 1 follows a pooling strategy, player 2 will not update his belief at information set S , i.e., $\mu_S = \frac{1}{2}$. Also, the χ -dampened updating property implies $\frac{\chi}{2} \leq \mu_I \leq 1 - \frac{\chi}{2}$. Because $\mu_S > \frac{2}{7}$, it is optimal for player 2 to choose C at information set S . To support this as an equilibrium, player 2 can choose either C or D at information set I .

Case 4.1: To make it a best reply for player 2 to choose D at information set I , it must be that $\mu_I \leq \frac{1}{3}$. Combined with the requirement from χ -dampened updating, the off-path belief has to satisfy $\frac{\chi}{2} \leq \mu_I \leq \frac{1}{3}$. As a result, $[(S, S); (D, C)]$ is a pooling χ -CSE if and only if $\chi \leq \frac{2}{3}$.

Case 4.2: To make it a best reply for player 2 to choose C at information set I , it must be that $\frac{1}{3} \leq \mu_I \leq \frac{6}{7}$. Combined with the requirement from χ -dampened updating, the off-path belief has to satisfy

$$\max \left\{ \frac{1}{2}\chi, \frac{1}{3} \right\} \leq \mu_I \leq \min \left\{ \frac{6}{7}, 1 - \frac{1}{2}\chi \right\}.$$

For any $\chi \in [0, 1]$, one can find μ_I that satisfies both inequalities. Hence $[(S, S); (C, C)]$ is a pooling χ -CSE for any χ . ■

2 Reputation Building, Entry Deterrence, and Predatory Pricing

Proof of Proposition 7

In this proof, we characterize the strategies of entrants and the weak monopolist in the χ -CSE. We start with the analysis of period 2, developing the backward induction argument underlying the χ -CSE.

Step 1: In period 2, the strong monopolist will fight and the weak monopolist will play soft. Let μ^x be the entrant's posterior belief about the monopolist being a strong type in period 2, and it is profitable for an entrant to enter if and only if $-\mu^x + (1 - \mu^x) \geq 0 \iff \mu^x \leq \frac{1}{2}$.

Therefore, if an entrant doesn't enter in period 1, $\mu^x = q \leq \frac{1}{4}$ and an entrant will enter in period 2.

Step 2: Let σ^x be the probability of the weak monopolist to fight in period 1 if there is entry. Given the weak monopolist adopting σ^x , if an entrant enters in period 1 and the monopolist fights, then by Lemma 1, an entrant's posterior belief in period 2 is

$$\mu^x = \chi q + (1 - \chi) \left(\frac{q}{q + (1 - q)\sigma^x} \right).$$

Yet if the monopolist plays soft in period 1, an entrant's posterior belief in period 2 becomes $\mu^x = \chi q < \frac{1}{2}$, and an entrant will enter the market. Moreover, if the weak monopolist thinks σ^x can deter an entrant from entering in period 2, the weak monopolist's expected payoff in period 1 is σ^x , indicating that the weak monopolist will choose σ^x such that $\mu^x = \frac{1}{2}$. Hence,

$$\chi q + (1 - \chi) \left(\frac{q}{q + (1 - q)\sigma^x} \right) = \frac{1}{2} \iff \sigma^x = \left[\frac{1 - 2(1 - q)\chi}{1 - 2q\chi} \right] \left(\frac{q}{1 - q} \right).$$

When $\chi > \frac{1}{2(1-q)}$, the weak monopolist believes that an entrant will enter in period 2 for all $\sigma^x \in [0, 1]$, and hence the weak monopolist will adopt $\sigma^x = 0$. In contrast, when $\chi < \frac{1}{2(1-q)}$, the weak monopolist will play a mixed strategy in period 1.

Step 3: To support the mixed strategy equilibrium, the weak monopolist has to be indifferent between fighting and playing soft if there is entry in period 1. Let τ^x be the probability of an entrant entering the market in period 2 upon seeing the monopolist fight in period 1. Therefore, in equilibrium, $-1 + 2(1 - \tau^x) = 0 \iff \tau^x = \frac{1}{2}$. Note that when $\chi > \frac{1}{2(1-q)}$, the weak monopolist will never fight, and as seeing the monopolist fight in period 1, an entrant will still enter in period 2 because $\mu^x = \chi q + (1 - \chi) < \frac{1}{2} \iff \chi > \frac{1}{2(1-q)}$.

Step 4: Lastly, for $\chi > \frac{1}{2(1-q)}$, the weak monopolist will play soft if there is entry in period 1 and an entrant will enter in period 1 as $q \leq \frac{1}{4}$. Additionally, for $\chi < \frac{1}{2(1-q)}$, given that the weak monopolist will fight in period 1 with probability σ^x , an entrant will enter in period 1 if and only if

$$-[q + (1 - q)\sigma^x] + (1 - q)(1 - \sigma^x) = 1 - \frac{4q - 4q\chi}{1 - 2q\chi} \geq 0 \iff q \leq \frac{1}{2(2 - \chi)}$$

which holds for any $\chi \in [0, 1]$ because $q \leq \frac{1}{4}$. We can hence conclude that an entrant will enter in period 1 for any $\chi \in [0, 1]$. ■

Reputation Building Game Experiment by Jung et al. (1994)

Table OA.1: Stage Game Payoffs in the Reputation Building Game (Table 1, Jung et al. 1994)

E 's strategy	M 's strategy	Payoffs		
		E	Weak M	Strong M
In	Fight	80	70	160
	Play Soft	150	160	70
Out	—	95	300	300

The reputation building game in Jung et al. (1994) consists of one monopolist (M) and eight entrants (E) over eight consecutive periods, all with the same stage game payoffs. At the beginning, the monopolist is randomly assigned a private type, either “strong” or “weak,” with the prior probability of being a strong type set at $\frac{1}{3}$. In each period, one of the eight E 's first decides whether to enter the market or not. If E does not enter the market, the period ends. Otherwise, it's M 's turn to decide whether to *fight* or *play soft*. The payoffs are summarized in Table OA.1 above.

Proposition 11 in the main text summarizes the properties of the χ -CSE of this game. The formal characterization is provided in Proposition OA.1 below.

Proposition OA.1. *The unique χ -CSE of the reputation building game in Jung et al. (1994) has the following properties:*

- let μ_t^χ be E 's posterior belief about M being a strong type in period t , and E will enter the market if and only if

$$\mu_t^\chi \leq \frac{11}{14} \left(\frac{11}{14 - 3\chi} \right)^{8-t} \equiv r_t^\chi;$$

- a strong M will fight with probability 1 whenever there is entry;
- a weak M will fight with probability 0 in period 8 if there is entry, and will fight with

probability σ_t^χ in period $t \leq 7$ if there is entry where

$$\sigma_t^\chi = \begin{cases} 1 & \text{if } \mu_t^\chi \geq r_{t+1}^\chi \\ \left[\frac{(1-r_{t+1}^\chi) - \chi(1-\mu_t^\chi)}{r_{t+1}^\chi - \chi\mu_t^\chi} \right] \left(\frac{\mu_t^\chi}{1-\mu_t^\chi} \right) & \text{if } \mu_t^\chi \in \left[1 - \left(\frac{1-r_{t+1}^\chi}{\chi} \right), r_{t+1}^\chi \right]; \\ 0 & \text{if } \mu_t^\chi \leq 1 - \left(\frac{1-r_{t+1}^\chi}{\chi} \right) \end{cases}$$

- once a weak M starts playing a mixed strategy, in periods after M fights, E will enter the market with probability $\frac{5}{14}$.

Proof: In the reputation building game in [Jung et al. \(1994\)](#), a strong M does not have any reputation concern and will always fight whenever there is entry since this is a strong M 's dominant strategy. In the following, we characterize the strategies of E 's and a weak M in χ -CSE. We start with the analysis of the last period of play first, developing the backward induction argument underlying the χ -CSE.

Step 1: In period 8, a strong M will fight and a weak M will play soft. Let μ_8^χ be E 's posterior belief about M being a strong type in period 8, and it is profitable for E to enter the market if and only if $80\mu_8^\chi + 150(1 - \mu_8^\chi) \geq 95 \iff \mu_8^\chi \leq \frac{11}{14}$.

Step 2: Let μ_t^χ be E 's posterior belief about M being a strong type in period t . In this step, we derive the upper bound of μ_t^χ for E to be willing to enter the market in period t . First, in period 7, let σ_7^χ be the probability of a weak M to fight, and E will enter if and only if

$$80[\mu_7^\chi + \sigma_7^\chi(1 - \mu_7^\chi)] + 150(1 - \sigma_7^\chi)(1 - \mu_7^\chi) \geq 95 \iff \mu_7^\chi + \sigma_7^\chi(1 - \mu_7^\chi) \leq \frac{11}{14}.$$

If E enters, a weak M believes that using σ_7^χ would induce μ_8^χ to be

$$\mu_8^\chi = \chi\mu_7^\chi + (1 - \chi) \left[\frac{\mu_7^\chi}{\mu_7^\chi + \sigma_7^\chi(1 - \mu_7^\chi)} \right] \quad (1)$$

by Lemma 1. Therefore, if a weak M thinks σ_7^χ can deter E from entering in the next period, a weak M 's expected payoff in period 7 is $\sigma_7^\chi(70 + 300) + (1 - \sigma_7^\chi)(160 + 160) = 320 + 50\sigma_7^\chi$ which increases with σ_7^χ , indicating that in the entry phase of χ -CSE, a weak M will choose σ_7^χ such that $\mu_8^\chi = \frac{11}{14}$. Combined with (1), we can obtain that

$$\chi\mu_7^\chi + (1 - \chi) \left[\frac{\mu_7^\chi}{\mu_7^\chi + \sigma_7^\chi(1 - \mu_7^\chi)} \right] = \frac{11}{14} \iff \sigma_7^\chi = \left[\frac{\frac{3}{14} - \chi(1 - \mu_7^\chi)}{\frac{11}{14} - \chi\mu_7^\chi} \right] \left(\frac{\mu_7^\chi}{1 - \mu_7^\chi} \right). \quad (2)$$

Note that if $\mu_7^\chi \leq 1 - \frac{3}{14\chi}$, then a weak M believes that E will enter in period 8 no matter how small σ_7^χ is. Hence, a weak M will choose $\sigma_7^\chi = 0$. On the other hand, if $\mu_7^\chi \geq \frac{11}{14}$, then E will not enter in period 8 unless M plays soft in period 7. Therefore, if E enters in period 7, a weak M will fight with probability 1. Lastly, for $\mu_7^\chi \in \left[1 - \frac{3}{14\chi}, \frac{11}{14}\right]$, combined with (2), we can obtain that E will enter in period 7 if and only if

$$\mu_7^\chi + \sigma_7^\chi (1 - \mu_7^\chi) \leq \frac{11}{14} \iff \mu_7^\chi \leq \frac{11}{14} \left(\frac{11}{14 - 3\chi} \right), \quad (3)$$

which is the upper bound of μ_7^χ for E to be willing to enter the market in period 7.

In the following, we prove by induction that in any period $8 - t$, E is willing to enter the market if and only if

$$\mu_{8-t}^\chi \leq \frac{11}{14} \left(\frac{11}{14 - 3\chi} \right)^t \equiv r_{8-t}^\chi.$$

Suppose there is $t' \leq 7$ such that the statement is true for any $1 \leq t \leq t' - 1$. In period $8 - t'$, E will enter if and only if

$$\begin{aligned} 80 [\mu_{8-t'}^\chi + \sigma_{8-t'}^\chi (1 - \mu_{8-t'}^\chi)] + 150 (1 - \sigma_{8-t'}^\chi) (1 - \mu_{8-t'}^\chi) &\geq 95 \\ \iff \mu_{8-t'}^\chi + \sigma_{8-t'}^\chi (1 - \mu_{8-t'}^\chi) &\leq \frac{11}{14}. \end{aligned}$$

where $\sigma_{8-t'}^\chi$ is the probability of a weak M to fight in period $8 - t'$. By a similar argument as before, in the entry phase of the χ -CSE, a weak M will choose $\sigma_{8-t'}^\chi$ such that $\mu_{9-t'}^\chi = r_{9-t'}^\chi$ if E enters. By Lemma 1, we can obtain that in the entry phase,

$$\begin{aligned} \chi \mu_{8-t'}^\chi + (1 - \chi) \left[\frac{\mu_{8-t'}^\chi}{\mu_{8-t'}^\chi + \sigma_{8-t'}^\chi (1 - \mu_{8-t'}^\chi)} \right] &= r_{9-t'}^\chi \\ \iff \sigma_{8-t'}^\chi &= \left[\frac{(1 - r_{9-t'}^\chi) - \chi(1 - \mu_{8-t'}^\chi)}{r_{9-t'}^\chi - \chi \mu_{8-t'}^\chi} \right] \left(\frac{\mu_{8-t'}^\chi}{1 - \mu_{8-t'}^\chi} \right). \end{aligned} \quad (4)$$

If $\mu_{8-t'}^\chi \leq 1 - \left(\frac{1 - r_{9-t'}^\chi}{\chi} \right)$, then a weak M believes that E will enter in the next period no matter how small $\sigma_{8-t'}^\chi$ is, so a weak M will choose $\sigma_{8-t'}^\chi = 0$. On the other hand, if $\mu_{8-t'}^\chi \geq r_{9-t'}^\chi$, then E will not enter in the next period unless M plays soft in period $8 - t'$. Thus, if E enters in period $8 - t'$, a weak M will fight with probability 1. Lastly, for $\mu_{8-t'}^\chi \in \left[1 - \left(\frac{1 - r_{9-t'}^\chi}{\chi} \right), r_{9-t'}^\chi\right]$, combined with (4), we can obtain that E will enter in period $8 - t'$ if and only if

$$\mu_{8-t'}^\chi + \sigma_{8-t'}^\chi (1 - \mu_{8-t'}^\chi) \leq \frac{11}{14} \iff \mu_{8-t'}^\chi \leq \left(\frac{11}{14} \right) \left(\frac{11}{14 - 3\chi} \right)^{t'} = r_{8-t'}^\chi. \quad (5)$$

In summary, we have shown in this step that E will enter the market if and only if $\mu_t^\chi \leq r_t^\chi$, and in any period $t \leq 7$, if there is entry, a weak M will fight with probability

$$\sigma_t^\chi = \begin{cases} 1 & \text{if } \mu_t^\chi \geq r_{t+1}^\chi \\ \left[\frac{(1-r_{t+1}^\chi) - \chi(1-\mu_t^\chi)}{r_{t+1}^\chi - \chi\mu_t^\chi} \right] \left(\frac{\mu_t^\chi}{1-\mu_t^\chi} \right) & \text{if } \mu_t^\chi \in \left[1 - \left(\frac{1-r_{t+1}^\chi}{\chi} \right), r_{t+1}^\chi \right] \\ 0 & \text{if } \mu_t^\chi \leq 1 - \left(\frac{1-r_{t+1}^\chi}{\chi} \right) \end{cases}$$

An immediate implication of this step is that there is a cutoff period

$$t_\chi^* = \min \left\{ t \mid \frac{11}{14} \left(\frac{11}{14-3\chi} \right)^{8-t} \geq \frac{1}{3} \right\}$$

which is the earliest period that E will enter the market on the equilibrium path. In periods 1 to $t_\chi^* - 1$, E will not enter the market on the equilibrium path. In any period $t \geq t_\chi^*$, if there is entry and M fights in period $t-1$, $\mu_t^\chi = r_t^\chi$ and a weak M will fight with probability

$$\sigma_t^\chi = \left[\frac{(1-r_{t+1}^\chi) - \chi(1-r_t^\chi)}{r_{t+1}^\chi - \chi r_t^\chi} \right] \left(\frac{r_t^\chi}{1-r_t^\chi} \right) = \frac{11(14-3\chi)^{8-t} - 11^{9-t}}{14(14-3\chi)^{8-t} - 11^{9-t}}$$

if there is entry in period t .

Step 3: In this step, we show that E will enter the market with probability $\tau^\chi = \frac{5}{14}$ in each period after a weak M starts playing a mixed strategy and M fights. First, suppose a weak M uses a mixed strategy in period 7. In this case, E has to enter the market in period 8 with probability τ^χ such that a weak M is indifferent between fighting and playing soft in period 7 if E enters. That is,

$$70 + [160\tau^\chi + 300(1-\tau^\chi)] = 160 + 160 \iff \tau^\chi = \frac{5}{14}.$$

More generally, suppose a weak M adopts a mixed strategy in period $8-t$. In period $9-t$, E has to enter with probability τ^χ such that a weak M is indifferent between fighting and playing soft in period $8-t$ if E enters. That is,

$$70 + [160t\tau^\chi + (300 + 160(t-1))(1-\tau^\chi)] = 160(t+1) \iff \tau^\chi = \frac{5}{14}.$$

Step 4: To complete the proof, this step characterizes the equilibrium strategies in off-path events. By Step 2, we know that E will not enter in period 1 to t_χ^* . If E enters in period $t_\chi^* - 1$, a weak monopolist will mix so that E is indifferent between entering or not in period

t_χ^* . Yet in periods $t < t_\chi^* - 1$, if E goes off-path and enters the market, a weak M will fight with probability 1. It is profitable for a weak M to fight in these events because if a weak M deviates to play soft, E 's belief about M being a strong type becomes $\frac{1}{3}\chi$ due to dampened updating and $\frac{1}{3}\chi \leq r_t^\chi$ for all $t \leq 8$. In other words, whenever E sees M plays soft, E will enter the market, making this deviation unprofitable for a weak M . More generally, if M plays soft in any period t , E 's belief about M being a strong type becomes $\chi\mu_t^\chi$ and E will enter in all later periods. ■

Notice that for any $t \leq 7$, the reputation cutoff r_t^χ increases with χ , suggesting that the entry phase starts earlier for higher χ . Additionally, to see how cursedness affects the behavior of a weak M , we characterize the probability of fighting for a weak M in the entry phase (conditional on entry in the previous period) in Corollary OA.1.

Corollary OA.1. *In χ -CSE of the reputation building game in Jung et al. (1994),*

- *in any period $t \leq 7$, for $\chi \geq \bar{\chi}_t$ such that $\bar{\chi}_t$ solves $r_{t+1}^\chi = 1 - \frac{2}{3}\chi^t$, a weak M will fight with probability 0 if there is entry;*
- *in any period $t \geq t_\chi^*$, if there is entry and M fights in period $t - 1$, a weak M fights with probability*

$$\sigma_t^\chi = \frac{11(14 - 3\chi)^{8-t} - 11^{9-t}}{14(14 - 3\chi)^{8-t} - 11^{9-t}}$$

if there is entry in period t .

proof: We first prove by induction that in any period $t \leq 7$, a weak M will fight with probability 0 if $\chi \geq \bar{\chi}_t$ where $\bar{\chi}_t$ solves $r_{t+1}^\chi = 1 - \frac{2}{3}\chi^t$. If there is entry in period 1, a weak M will fight with probability

$$\sigma_1^\chi = \frac{1}{2} \left[\frac{(1 - r_2^\chi) - \frac{2}{3}\chi}{r_2^\chi - \frac{1}{3}\chi} \right]$$

and $\sigma_1^\chi \leq 0 \iff r_2^\chi \geq 1 - \frac{2}{3}\chi$. Because r_2^χ strictly increases with χ , we can obtain that $\sigma_1^\chi \leq 0 \iff \chi \leq \bar{\chi}_1$ where $\bar{\chi}_1$ solves $r_2^\chi = 1 - \frac{2}{3}\chi$. This proves the base case.

Suppose there is $t' \leq 7$ such that the statement is true for all period $t \leq t' - 1$. By induction hypothesis, if $\chi \geq \bar{\chi}_{t'} > \bar{\chi}_{t'-1}$, a weak M will fight with probability 0 if there is entry in period $t \leq t' - 1$. Therefore, by Lemma 1, $\mu_{t'}^\chi \leq 1 - \frac{2}{3}\chi^{t'-1}$ where the maximum is attained when there is entry in all previous periods and M always fights. By Proposition OA.1, if there is entry in period t' , a weak M will fight with probability 0 if

$$1 - \frac{2}{3}\chi^{t'-1} \leq 1 - \left(\frac{1 - r_{t'+1}^\chi}{\chi} \right) \iff r_{t'+1}^\chi \geq 1 - \frac{2}{3}\chi^{t'},$$

which is equivalent to that $\chi \geq \bar{\chi}_{t'}$ where $\bar{\chi}_{t'}$ solves $r_{t'+1}^\chi = 1 - \frac{2}{3}\chi^{t'}$.

Lastly, by Proposition OA.1, we know a weak M starts mixing in period $t_\chi^* - 1$. In any period $t \geq t_\chi^*$, if there is entry and M fights in the previous period, E 's posterior belief becomes $\mu_t^\chi = r_t^\chi$, and a weak M will fight with probability

$$\sigma_t^\chi = \left[\frac{(1 - r_{t+1}^\chi) - \chi(1 - r_t^\chi)}{r_{t+1}^\chi - \chi r_t^\chi} \right] \left(\frac{r_t^\chi}{1 - r_t^\chi} \right) = \frac{11(14 - 3\chi)^{8-t} - 11^{9-t}}{14(14 - 3\chi)^{8-t} - 11^{9-t}}$$

if there is entry in period t . ■

3 Durable Goods Monopoly and the Coase Conjecture

Proof of Proposition 9

Step 1 (Pricing Strategy in Period 2): Consider any cutoff value $\tilde{v}$. Let $\mu^\chi(v|\tilde{v}, 2)$ be the χ -cursed posterior belief density about the buyer's value given that the cutoff value is $\tilde{v}$ and the buyer does not buy in period 1. By Lemma 1, we can obtain that

$$\mu^\chi(v|\tilde{v}, 2) = \begin{cases} (1 - \chi) \left(\frac{1}{\tilde{v}} \right) + \chi & \text{if } v \leq \tilde{v} \\ \chi & \text{if } v > \tilde{v}. \end{cases}$$

In this case, the χ -cursed seller's expected profit of choosing p_2 , $\Pi_2^\chi(p_2)$, is simply

$$\Pi_2^\chi(p_2) = \begin{cases} p_2 - \chi p_2^2 - \left(\frac{1-\chi}{\tilde{v}} \right) p_2^2 & \text{if } p_2 \leq \tilde{v} \\ \chi p_2 - \chi p_2^2 & \text{if } p_2 > \tilde{v}. \end{cases}$$

If the seller chooses $p_2 \leq \tilde{v}$, then the optimal p_2^χ satisfies that

$$\frac{d}{dp_2} \Pi_2^\chi(p_2^\chi) = 1 - 2\chi p_2^\chi - 2 \left(\frac{1-\chi}{\tilde{v}} \right) p_2^\chi = 0 \iff p_2^\chi = \frac{\tilde{v}}{2(\chi\tilde{v} + (1-\chi))}, \quad (6)$$

which is an interior solution if and only if $\tilde{v} \geq 1 - \frac{1}{2\chi}$. In this case, the expected profit is

$$\frac{\tilde{v}}{2(\chi\tilde{v} + (1-\chi))} - \left(\chi + \frac{1-\chi}{\tilde{v}} \right) \left[\frac{\tilde{v}}{2(\chi\tilde{v} + (1-\chi))} \right]^2 = \frac{\tilde{v}}{4(\chi\tilde{v} + (1-\chi))}.$$

On the other hand, if the seller chooses $p_2 > \tilde{v}$, then the optimal p_2^χ satisfies that

$$\frac{d}{dp_2} \Pi_2^\chi(p_2^\chi) = \chi - 2\chi p_2^\chi = 0 \iff p_2^\chi = \frac{1}{2},$$

which is an interior solution if and only if $\tilde{v} \leq \frac{1}{2}$, yielding an expected profit of $\frac{\chi}{4}$. Note that

$$\frac{\tilde{v}}{4(\chi\tilde{v} + (1-\chi))} \geq \frac{\chi}{4} \iff \tilde{v} \geq \frac{\chi}{1+\chi} \geq 1 - \frac{1}{2\chi},$$

implying that if the equilibrium cutoff value $\bar{v}^\chi \geq \frac{\chi}{1+\chi}$, then we know $p_2^\chi = \frac{\bar{v}^\chi}{2(\chi\bar{v}^\chi + (1-\chi))}$.

Step 2 (Lower Bound of the Equilibrium Cutoff Value): In this step, we show that in χ -CSE, the cutoff value $\bar{v}^\chi \geq \frac{\chi}{1+\chi}$ by contradiction. If not, then $\bar{v}^\chi < \frac{\chi}{1+\chi} < \frac{1}{2}$ and $p_2^\chi = \frac{1}{2}$. For the buyer with value $\bar{v}^\chi$, he is indifferent between accepting p_1^χ and accepting $p_2^\chi = \frac{1}{2}$ in period 2. Therefore, in χ -CSE, $\bar{v}^\chi$ satisfies that

$$\bar{v}^\chi - p_1^\chi = \delta \left(\bar{v}^\chi - \frac{1}{2} \right) \iff \bar{v}^\chi = \frac{p_1^\chi - \frac{\delta}{2}}{1-\delta} < \frac{1}{2},$$

implying that $p_1^\chi < \frac{1}{2} = p_2^\chi$. This yields a contradiction because the buyer with value $\bar{v}^\chi$ strictly prefers to buy in period 1 rather than in period 2.

Step 3 (Pricing Strategy in Period 1): From step 1 and 2, we can conclude that in χ -CSE, given p_1 , the cutoff value $\tilde{v}^\chi(p_1)$ is an implicit function defined by

$$\tilde{v}^\chi(p_1) - p_1 = \delta \left[\tilde{v}^\chi(p_1) - \frac{\tilde{v}^\chi(p_1)}{2(\chi\tilde{v}^\chi(p_1) + (1-\chi))} \right]. \quad (7)$$

Because the expected profit in period 2 is $\frac{\tilde{v}^\chi(p_1)}{4(\chi\tilde{v}^\chi(p_1) + (1-\chi))}$, the seller's optimization price in period 1 satisfies that

$$p_1^\chi = \operatorname{argmax}_{p_1} p_1 (1 - \tilde{v}^\chi(p_1)) + \rho \tilde{v}^\chi(p_1) \left[\frac{\tilde{v}^\chi(p_1)}{4(\chi\tilde{v}^\chi(p_1) + (1-\chi))} \right].$$

Once we pin down p_1^χ , we can plug it into (7) to obtain $\bar{v}^\chi$ and then plug $\bar{v}^\chi$ into (6) to obtain p_2^χ . ■

Proof of Corollary 3

By Proposition 9, we know in the fully cursed sequential equilibrium, $p_2^{\chi=1} = \frac{1}{2}$ and $p_1^{\chi=1} \geq \frac{1}{2}$. Therefore, given any p_1 , the cutoff value $\tilde{v}^{\chi=1}(p_1)$ satisfies that

$$\tilde{v}^{\chi=1}(p_1) - p_1 = \delta \left(\tilde{v}^{\chi=1}(p_1) - \frac{1}{2} \right) \iff \tilde{v}^{\chi=1}(p_1) = \frac{p_1 - \frac{\delta}{2}}{1 - \delta}. \quad (8)$$

This implies that when choosing p_1 , the seller's expected profit $\Pi^{\chi=1}(p_1)$ is

$$\Pi^{\chi=1}(p_1) = p_1 (1 - \tilde{v}^{\chi=1}(p_1)) + \frac{\rho}{4} \tilde{v}^{\chi=1}(p_1) = p_1 - \left(\frac{p_1 - \frac{\delta}{2}}{1 - \delta} \right) p_1 + \frac{\rho}{4} \left(\frac{p_1 - \frac{\delta}{2}}{1 - \delta} \right).$$

By taking the derivative, we can obtain that

$$\frac{d}{dp_1} \Pi^{\chi=1}(p_1) = 1 - \left(\frac{2}{1 - \delta} \right) p_1 + \frac{\delta}{2(1 - \delta)} + \frac{\rho}{4(1 - \delta)} \geq 0 \iff p_1 \leq \frac{4 - 2\delta + \rho}{8}.$$

Combined with the constraint that $p_1^{\chi=1} \geq \frac{1}{2}$, we can obtain that $p_1^{\chi=1} = \max \left\{ \frac{4 - 2\delta + \rho}{8}, \frac{1}{2} \right\}$, and $p_1^{\chi=1} = \frac{1}{2} \iff \delta \geq \frac{\rho}{2}$. Plugging $p_1^{\chi=1}$ into (8) gives us $\bar{v}^{\chi=1} = \max \left\{ \frac{4 - 6\delta + \rho}{8(1 - \delta)}, \frac{1}{2} \right\}$.

As derived by [Güth et al. \(1995\)](#), the sequential equilibrium price in period 1 is $p_1^* = \frac{(2 - \delta)^2}{2(4 - 2\delta - \rho)}$. We can first notice that $\frac{(2 - \delta)^2}{2(4 - 2\delta - \rho)} \geq \frac{4 - 2\delta + \rho}{8}$ for any δ and ρ , implying that $p_1^* \geq p_1^{\chi=1}$ for any $\delta < \frac{\rho}{2}$. On the other hand, if $\delta \geq \frac{\rho}{2}$, then $p_1^{\chi=1} = \frac{1}{2}$ and

$$\frac{(2 - \delta)^2}{2(4 - 2\delta - \rho)} \leq \frac{1}{2} \iff \delta^2 - 2\delta + \rho \leq 0 \iff \delta \geq 1 - \sqrt{1 - \rho} \geq \frac{\rho}{2}.$$

Therefore, we know $p_1^{\chi=1} \geq p_1^* \iff \delta \geq 1 - \sqrt{1 - \rho}$. Finally, because $\frac{2 - \delta}{2(4 - 2\delta - \rho)} \leq \frac{1}{2}$ for any δ and ρ , we can conclude that $p_2^{\chi=1} \geq p_2^*$ for any δ and ρ . ■

CSE Analysis for Three-Period Durable Goods Monopolistic Markets

Similar to the analysis of two-period games, χ -CSE of the three-period games are characterized by the sales prices in each period $\{p_t^{\chi}\}_{t=1,2,3}$ and the cutoff values $\{\bar{v}_t^{\chi}\}_{t=1,2}$. The buyer with value $v \in [\bar{v}_1^{\chi}, 1]$ purchases in period 1, the buyer with value $v \in [\bar{v}_2^{\chi}, \bar{v}_1^{\chi}]$ purchases in period 2, and the buyer with value $v \in [p_3^{\chi}, \bar{v}_2^{\chi}]$ purchases in period 3. Here we characterize the fully cursed sequential equilibrium ($\chi = 1$) in Proposition [OA.2](#).

Proposition OA.2. *In the three-period durable goods monopolistic market, in the fully*

cursed sequential equilibrium, the prices $\{p_t^{\chi=1}\}_{t=1,2,3}$ and the cutoff values $\{\bar{v}_t^{\chi=1}\}_{t=1,2}$ are

$$\begin{aligned} p_1^{\chi=1} &= \max \left\{ \frac{1}{2}, \frac{8 - 4\delta + 2\rho + \rho^2}{16}, \frac{8 - 4\delta - 2\delta^2 + \delta\rho + 8\rho\bar{\Pi}}{16} \right\}, \\ p_2^{\chi=1} &= \max \left\{ \frac{4 - 2\delta + \rho}{8}, \frac{1}{2} \right\}, \\ p_3^{\chi=1} &= \frac{1}{2} \\ \bar{v}_1^{\chi=1} &= \max \left\{ \frac{1}{2}, \frac{8 - 12\delta + 2\rho + \rho^2}{16(1 - \delta)}, \frac{8 - 12\delta + 2\delta^2 - \delta\rho + 8\rho\bar{\Pi}}{16(1 - \delta)} \right\} \\ \bar{v}_2^{\chi=1} &= \max \left\{ \frac{4 - 6\delta + \rho}{8(1 - \delta)}, \frac{1}{2} \right\} \quad \text{where} \quad \bar{\Pi} = \frac{4\delta^2 - 12\delta\rho - 16\delta + \rho^2 + 8\rho + 16}{64(1 - \delta)}. \end{aligned}$$

Proof: When $\chi = 1$, the seller never updates their belief about the buyer's value, and we can solve for $p_2^{\chi=1}$, $p_3^{\chi=1}$ and $\bar{v}_2^{\chi=1}$ using a similar argument as in Corollary 3. Therefore, if $\delta \leq \frac{\rho}{2}$, the fully cursed seller's expected profit at period 2 is

$$\begin{aligned} p_2^{\chi=1}(1 - \bar{v}_2^{\chi=1}) + \frac{\rho}{4}\bar{v}_2^{\chi=1} &= \left(\frac{4 - 2\delta + \rho}{8} \right) \left[1 - \frac{4 - 6\delta + \rho}{8(1 - \delta)} \right] + \frac{\rho}{4} \left[\frac{4 - 6\delta + \rho}{8(1 - \delta)} \right] \\ &= \frac{4\delta^2 - 12\delta\rho - 16\delta + \rho^2 + 8\rho + 16}{64(1 - \delta)} \equiv \bar{\Pi}. \end{aligned}$$

On the other hand, if $\delta > \frac{\rho}{2}$, $p_2^{\chi=1} = \frac{1}{2}$ and the expected profit at period 2 is simply $\frac{1}{4} + \frac{\rho}{8}$. In the following, we separate the discussion into two cases.

Case 1 ($\delta > \frac{\rho}{2}$): In this case, $p_2^{\chi=1} = p_3^{\chi=1} = \frac{1}{2}$, and when choosing p_1 , the cutoff value $\tilde{v}_1^{\chi=1}(p_1)$ satisfies that

$$\tilde{v}_1^{\chi=1}(p_1) - p_1 = \delta \left(\tilde{v}_1^{\chi=1}(p_1) - \frac{1}{2} \right) \iff \tilde{v}_1^{\chi=1}(p_1) = \frac{p_1 - \frac{\delta}{2}}{1 - \delta}. \quad (9)$$

Therefore, when choosing p_1 , the seller's expected profit $\Pi^{\chi=1}(p_1)$ is

$$\begin{aligned} \Pi^{\chi=1}(p_1) &= p_1 (1 - \tilde{v}_1^{\chi=1}(p_1)) + \rho \left(\frac{1}{4} + \frac{\rho}{8} \right) \tilde{v}_1^{\chi=1}(p_1) \\ &= p_1 - \left(\frac{p_1 - \frac{\delta}{2}}{1 - \delta} \right) p_1 + \rho \left(\frac{1}{4} + \frac{\rho}{8} \right) \left(\frac{p_1 - \frac{\delta}{2}}{1 - \delta} \right). \end{aligned}$$

As we take the derivative, we can obtain that

$$\begin{aligned} \frac{d}{dp_1} \Pi^{x=1}(p_1) &= 1 - \left(\frac{2}{1-\delta} \right) p_1 + \frac{\delta}{2(1-\delta)} + \left(\frac{\rho}{1-\delta} \right) \left(\frac{1}{4} + \frac{\rho}{8} \right) \geq 0 \\ &\iff p_1 \leq \frac{8 - 4\delta + 2\rho + \rho^2}{16}. \end{aligned}$$

Since in equilibrium, $p_1^{x=1} \geq p_2^{x=1} = \frac{1}{2}$, if $\delta > \frac{\delta}{2}$, $p_1^{x=1} = \max \left\{ \frac{1}{2}, \frac{8-4\delta+2\rho+\rho^2}{16} \right\}$.

Case 2 ($\delta \leq \frac{\rho}{2}$): In this case, $p_2^{x=1} = \frac{4-2\delta+\rho}{8}$, $p_3^{x=1} = \frac{1}{2}$, and when choosing p_1 , the cutoff value $\tilde{v}_1^{x=1}(p_1)$ satisfies that

$$\tilde{v}_1^{x=1}(p_1) - p_1 = \delta \left[\tilde{v}_1^{x=1}(p_1) - \left(\frac{4-2\delta+\rho}{8} \right) \right] \iff \tilde{v}_1^{x=1}(p_1) = \frac{p_1 - \delta \left(\frac{4-2\delta+\rho}{8} \right)}{1-\delta}, \quad (10)$$

and in equilibrium, $p_1^{x=1} \geq p_2^{x=1} = \frac{4-2\delta+\rho}{8}$. Thus, the seller's optimization problem becomes

$$\max_{p_1} p_1 \left[1 - \frac{p_1 - \delta \left(\frac{4-2\delta+\rho}{8} \right)}{1-\delta} \right] + \rho \bar{\Pi} \left[\frac{p_1 - \delta \left(\frac{4-2\delta+\rho}{8} \right)}{1-\delta} \right].$$

As we take the first order condition, we can obtain that $p_1^{x=1} = \frac{1}{2} - \frac{\delta}{4} - \frac{\delta^2}{8} + \frac{\delta\rho}{16} + \frac{\rho}{2}\bar{\Pi}$, and it is an interior solution because

$$\begin{aligned} \frac{1}{2} - \frac{\delta}{4} - \frac{\delta^2}{8} + \frac{\delta\rho}{16} + \frac{\rho}{2}\bar{\Pi} &\geq \frac{1}{2} - \frac{\delta}{4} - \frac{\delta^2}{8} + \frac{\delta\rho}{16} + \frac{\rho}{2} \left(\frac{1}{4} + \frac{\rho}{8} \right) \\ &= \frac{1}{2} - \frac{\delta}{4} + \frac{\rho}{8} + \underbrace{\left[-\frac{\delta^2}{8} + \frac{\delta\rho}{16} + \frac{\rho^2}{16} \right]}_{\geq \frac{\rho^2}{16} \text{ for } \delta \leq \frac{\rho}{2}} \geq \frac{1}{2} - \frac{\delta}{4} + \frac{\rho}{8} + \frac{\rho^2}{16}. \end{aligned}$$

Finally, as we plug p_1^x into $\tilde{v}^{x=1}(p_1)$, we can obtain the cutoff value

$$\bar{v}_1^{x=1} = \max \left\{ \frac{1}{2}, \frac{8 - 12\delta + 2\rho + \rho^2}{16(1-\delta)}, \frac{8 - 12\delta + 2\delta^2 - \delta\rho + 8\rho\bar{\Pi}}{16(1-\delta)} \right\}. \blacksquare$$

4 The Dirty Faces Game

Proof of Proposition 10

When observing a clean face, a player will know that he has a dirty face immediately. Therefore, choosing 1 (i.e., claiming in period 1) when observing a clean face is a strictly

dominant strategy. In other words, for any $\chi \in [0, 1]$, $\hat{\sigma}^\chi(O) = 1$. The analysis of the case where the player observes a dirty face is separated into two cases.

Case 1: $\chi > \bar{\alpha} = \frac{\alpha}{(1+\alpha)(1-p)}$. In this case, we show that $\hat{\sigma}^\chi(X) = T + 1$ is the only χ -CE. If not, suppose $\hat{\sigma}^\chi(X) = t$ where $t \leq T$ can be supported as a χ -CE. First, observe that $\hat{\sigma}^\chi(X) = 1$ cannot be a χ -CE because it is strictly dominated by $\hat{\sigma}^\chi(X) = T + 1$ when observing a dirty face. For $2 \leq t \leq T$, given the other player $-i$ plays the strategy $\hat{\sigma}_{-i}^\chi(X) = t$, we can find player $-i$'s *average strategy* is

$$\bar{\sigma}_{-i}(j) = \begin{cases} 1-p & \text{if } j = 1 \\ p & \text{if } j = t \\ 0 & \text{if } j \neq 1, t. \end{cases}$$

Therefore, the other player $-i$'s χ -cursed strategy is:

$$\begin{aligned} \sigma_{-i}^\chi(j|x_i = O) &= \begin{cases} \chi(1-p) + (1-\chi) & \text{if } j = 1 \\ \chi p & \text{if } j = t \\ 0 & \text{if } j \neq 1, t, \end{cases} \quad \text{and} \\ \sigma_{-i}^\chi(j|x_i = X) &= \begin{cases} \chi(1-p) & \text{if } j = 1 \\ \chi p + (1-\chi) & \text{if } j = t \\ 0 & \text{if } j \neq 1, t. \end{cases} \end{aligned}$$

In this case, given (player i perceives that) player $-i$ chooses the χ -cursed strategy, player i 's expected payoff from using any strategy $\hat{\sigma}^\chi(X) = j$ with $2 \leq j \leq t$ when observing a dirty face is:

$$(1-p) [-\delta^{j-1}\chi p] + p \{ \delta^{j-1}\alpha [\chi p + (1-\chi)] \} = p\delta^{j-1} \underbrace{[\alpha - \chi(1+\alpha)(1-p)]}_{<0 \iff \chi > \bar{\alpha}} < 0.$$

Hence, if the other player uses the strategy $\hat{\sigma}_{-i}^\chi(X) = t$ when observing a dirty face, $\hat{\sigma}_i^\chi(X) = j$ is strictly dominated for i by $\hat{\sigma}_i^\chi(X) = T + 1$ for all $j \leq t$. Therefore, the only χ -CE is $\hat{\sigma}^\chi(X) = T + 1$.

Case 2: $\chi < \bar{\alpha} = \frac{\alpha}{(1+\alpha)(1-p)}$. In this case, we show that $\hat{\sigma}^\chi(X) = 2$ is the only χ -CE. Again, first notice that $\hat{\sigma}^\chi(X) = 1$ is strictly dominated by $\hat{\sigma}^\chi(X) = T + 1$ when observing a dirty face. To show that $\hat{\sigma}^\chi(X) = 2$ is the only χ -CE, by way of contradiction suppose $\hat{\sigma}^\chi(X) = t$ for some $t \geq 3$ can be supported as a χ -CE. Moreover, given player $-i$ chooses $\hat{\sigma}^\chi(X) = t$,

the expected payoff to choose $2 \leq j \leq t$ is:

$$p\delta^{j-1} \underbrace{[\alpha - \chi(1 + \alpha)(1 - p)]}_{>0 \iff \chi < \bar{\alpha}} > 0,$$

which is decreasing in j . Therefore, the best response to $\hat{\sigma}^\chi(X) = t$ is to choose 2 when observing a dirty face. As a result, the only χ -CE in this case is $\hat{\sigma}^\chi(X) = 2$. ■

Proof of Proposition 11

When observing a clean face, a player will know that he has a dirty face immediately. Therefore, choosing C in period 1 when observing a clean face is a strictly dominant strategy, and thus $\tilde{\sigma}^\chi(O) = 1$ for all $\chi \in [0, 1]$. On the other hand, the analysis for the case where the player observes a dirty face consists of several steps.

Step 1: Suppose that both players choose C in some period $\bar{t}$ when seeing a dirty face. Let $\mu^\chi(X|t, X)$ be the χ -cursed belief about having a dirty face conditional on seeing a dirty face and the game reaching period t . In this step, we prove that in period $t \leq \bar{t}$, the cursed belief $\mu^\chi(X|t, X) = 1 - (1 - p)\chi^{t-1}$ by induction. In period $t = 1$, the belief about having a dirty face is simply the prior belief p . This establishes the base case. Now suppose the statement holds for any period $1 \leq t \leq t'$ (and $t' < \bar{t}$). In period $t' + 1$, by Lemma 1,

$$\begin{aligned} \mu^\chi(X|t' + 1, X) &= \chi\mu^\chi(X|t', X) + (1 - \chi) \\ &= \chi \left[1 - (1 - p)\chi^{t'-1} \right] + (1 - \chi) = 1 - (1 - p)\chi^{t'} \end{aligned}$$

where the second equality holds by the induction hypothesis.

Step 2: Given the cursed belief, the expected payoff to choose C in period t is:

$$\begin{aligned} \mu^\chi(X|t, X)\alpha - [1 - \mu^\chi(X|t, X)] &= [1 - (1 - p)\chi^{t-1}] \alpha - [(1 - p)\chi^{t-1}] \\ &= \alpha - (1 - p)(1 + \alpha)\chi^{t-1}, \end{aligned}$$

which increases with t . Note that in period 1, the expected payoff is $\alpha - (1 - p)(1 + \alpha) < 0$ by Assumption (1), so choosing C in period 1 is strictly dominated. Furthermore, suppose the opponent chooses to always wait when seeing a dirty face, the player would wait in every period when seeing a dirty face if and only if

$$\mu^\chi(X|T, X)\alpha - [1 - \mu^\chi(X|T, X)] \leq 0 \iff \alpha - (1 - p)(1 + \alpha)\chi^{T-1} \leq 0 \iff \chi \geq \bar{\alpha}^{\frac{1}{T+1}}.$$

As a result, both players choosing $\tilde{\sigma}^x(X) = T + 1$ is a χ -CSE if and only if $\chi \geq \bar{\alpha}^{\frac{1}{T-1}}$.

Step 3: In this step, we show that both players choosing $\tilde{\sigma}^x(X) = 2$ is a χ -CSE if and only if $\chi \leq \bar{\alpha}$. Suppose that the opponent will choose C in period 2 when seeing a dirty face. In this case, it is optimal to choose C in period 2 as long as the expected payoff of C in period 2 is positive because the game can never proceed beyond period 2. Consequently, both players choosing $\tilde{\sigma}^x(X) = 2$ is a χ -CSE if and only if

$$\mu^x(X|2, X)\alpha - [1 - \mu^x(X|2, X)] \geq 0 \iff \alpha - (1 - p)(1 + \alpha)\chi \geq 0 \iff \chi \leq \bar{\alpha}.$$

Step 4: Suppose the opponent chooses $\tilde{\sigma}^x(X) > t$. In this case, when the game reaches period t , the belief about the other player choosing W in period t is:

$$\underbrace{\mu^x(X|t, X)}_{\text{prob. of dirty}} [\chi\mu^x(X|t, X) + (1 - \chi)] + \underbrace{[1 - \mu^x(X|t, X)]}_{\text{prob. of clean}} [\chi\mu^x(X|t, X)] = \mu^x(X|t, X).$$

Furthermore, we denote the expected payoff of choosing C in period t as

$$\mathbb{E}[u^x(C|t, X)] \equiv \mu^x(X|t, X)\alpha - (1 - \mu^x(X|t, X)).$$

In the following, we claim that in any period $2 \leq t \leq T - 2$, given the opponent will claim in some period later than period $t + 2$ or never claim, if it is optimal for the player to choose W in period $t + 1$, then it is also optimal for the player to choose W in period t . That is,

$$\begin{aligned} \mathbb{E}[u^x(C|t + 1, X)] &< \delta\mu^x(X|t + 1, X)\mathbb{E}[u^x(C|t + 2, X)] \\ \implies \mathbb{E}[u^x(C|t, X)] &< \delta\mu^x(X|t, X)\mathbb{E}[u^x(C|t + 1, X)]. \end{aligned}$$

To prove this claim, we can first observe that

$$\begin{aligned} \mathbb{E}[u^x(C|t + 1, X)] &< \delta\mu^x(X|t + 1, X)\mathbb{E}[u^x(C|t + 2, X)] \\ \iff (1 + \alpha)\mu^x(X|t + 1, X) - 1 &< \delta\mu^x(X|t + 1, X)[(1 + \alpha)\mu^x(X|t + 2, X) - 1]. \end{aligned}$$

After rearrangement, the inequality is equivalent to

$$\delta\chi[\mu^x(X|t + 1, X)]^2 + \left[\delta(1 - \chi) - \frac{\delta}{1 + \alpha} - 1 \right] \mu^x(X|t + 1, X) + \frac{1}{1 + \alpha} > 0.$$

Consider a function $F : [0, 1] \rightarrow \mathbb{R}$ where

$$F(y) = \delta\chi y^2 + \left[\delta(1 - \chi) - \frac{\delta}{1 + \alpha} - 1 \right] y + \frac{1}{1 + \alpha}.$$

Since $\mu^\chi(X|j, X) = 1 - (1 - p)\chi^{j-1}$ increases with j , it suffices to complete the proof of the claim by showing there exists a unique $y^* \in (0, 1)$ such that F is single-crossing on $[0, 1]$ where $F(y^*) = 0$, $F(y) < 0$ for all $y > y^*$, and $F(y) > 0$ for all $y < y^*$. Because F is continuous, $\frac{1}{1+\alpha} > 0$ and $F(1) = -\frac{\alpha(1-\delta)}{1+\alpha} < 0$, by the intermediate value theorem, there exists a $y^* \in (0, 1)$ such that $F(y^*) = 0$. Moreover, y^* is the unique root of F on $[0, 1]$ because F is a strictly convex parabola and $F(1) < 0$. This establishes the claim.

Step 5: For any $3 \leq t \leq T$, in this step, we find the conditions to support both players choosing $\tilde{\sigma}^\chi(X) = t$ as a χ -CSE. We can first note that both players choosing $\tilde{\sigma}^\chi(X) = t$ is a χ -CSE if and only if

1. $\mathbb{E}[u^\chi(D|t, X)] \geq 0$, and
2. $\mathbb{E}[u^\chi(D|t-1, X)] \leq \delta\mu^\chi(X|t-1, X)\mathbb{E}[u^\chi(D|t, X)]$.

Condition 1 is necessary because if it fails, then it is better for the player to choose W in period t and get at least 0. Condition 2 is also necessary because if the condition doesn't hold, it would be profitable for the player to choose C before period t . Furthermore, these two conditions are jointly sufficient to support $\tilde{\sigma}^\chi(X) = t$ as a χ -CSE by the same argument as step 3. From condition 1, we can obtain that

$$\mathbb{E}[u^\chi(C|t, X)] \geq 0 \iff (1 + \alpha)\mu^\chi(X|t, X) - 1 \geq 0 \iff \chi \leq \bar{\alpha}^{\frac{1}{t-1}}.$$

In addition, by the calculation of step 4, we know

$$\mathbb{E}[u^\chi(C|t-1, X)] \leq \delta\mu^\chi(X|t-1, X)\mathbb{E}[u^\chi(C|t, X)] \iff F(\mu^\chi(X|t-1, X)) \geq 0,$$

which is equivalent to

$$\begin{aligned} \mu^\chi(X|t-1, X) &\leq \frac{\left[1 + \frac{\delta}{1+\alpha} - \delta(1 - \chi)\right] - \sqrt{\left[1 + \frac{\delta}{1+\alpha} - \delta(1 - \chi)\right]^2 - 4\delta\chi\left(\frac{1}{1+\alpha}\right)}}{2\delta\chi} \\ &= \frac{[(1 + \alpha)(1 + \delta\chi) - \alpha\delta] - \sqrt{[(1 + \alpha)(1 + \delta\chi) - \alpha\delta]^2 - 4\delta\chi(1 + \alpha)}}{2\delta\chi(1 + \alpha)} \equiv \kappa(\chi). \end{aligned}$$

Therefore, condition 2 holds if and only if

$$1 - (1 - p)\chi^{t-2} \leq \kappa(\chi) \iff \chi \geq \left(\frac{1 - \kappa(\chi)}{1 - p} \right)^{\frac{1}{t-2}}.$$

In summary, both players choosing $\tilde{\sigma}^x(X) = t$ is a χ -CSE if and only if

$$\left(\frac{1 - \kappa(\chi)}{1 - p} \right)^{\frac{1}{t-2}} \leq \chi \leq \bar{\alpha}^{\frac{1}{t-1}}. \blacksquare$$

Estimation of Quantal Cursed Sequential Equilibrium

This section presents the estimation results of the Quantal Cursed Sequential Equilibrium (QCSE) based on the pooled data from [Lin \(2023\)](#). Specifically, in QCSE, players are assumed to make logit quantal responses, with precision governed by the parameter $\lambda \in [0, \infty)$. When $\lambda = 0$, players become insensitive to the payoffs and choose randomly at every information set. As λ increases, players' behavior becomes more sensitive to the payoffs. In the limit as $\lambda \rightarrow \infty$, players become fully rational and make best responses, and QCSE reduces to χ -CSE. In addition, when $\chi = 0$, players make perfect Bayesian inferences and QCSE simplifies to the agent quantal response equilibrium (AQRE) proposed by [McKelvey and Palfrey \(1998\)](#).

Table [OA.2](#) presents the estimation results for QCSE and AQRE. From the table, we observe that the estimated value $\hat{\chi} = 1$, suggesting that players are fully cursed, and their deviation from the unique Nash equilibrium cannot be solely attributed to quantal responses. The estimated level of cursedness is statistically significant, as indicated by the likelihood ratio test ($\chi^2 = 79$, p-value < 0.001).

Table OA.2: Estimation Results for QCSE and AQRE ([Lin 2025](#))

		QCSE	AQRE
Parameters	χ	1.00	
	S.E.	(0.11)	
	λ	7.67	4.69
	S.E.	(0.56)	(0.32)
Fit	lnL	-1823	-1863
	AIC	3651	3728
	BIC	3662	3733

5 Information Transmission in Sender-Receiver Games

In this appendix, we characterize the equilibrium conditions for K -partition χ -CSE in Proposition OA.3 with a sketch of the proof. Since the formulation of χ -CSE coincides with formulation of prior-biased inferences proposed by Lee et al. (2023) in this environment, interested readers can refer to their appendix for a detailed proof.

Proposition OA.3. *A K -partition χ -CSE where $K \geq 2$ is characterized by $K + 1$ cutpoints $\{\theta_k^\chi\}_{k=0}^K$ such that $0 = \theta_0^\chi < \theta_1^\chi < \dots < \theta_K^\chi = 1$ and can be supported by:*

1. *For any $1 \leq k \leq K - 1$, the following condition holds:*

$$(1 - \chi)\theta_{k+1}^\chi = 2(1 + \chi)\theta_k^\chi - (1 - \chi)\theta_{k-1}^\chi - 2\chi + 4b. \quad (11)$$

2. *For any $1 \leq k \leq K$, if $\theta \in [\theta_{k-1}^\chi, \theta_k^\chi]$, the sender will send m_k drawn uniformly from the interval $[\theta_{k-1}^\chi, \theta_k^\chi]$, and for all $m_k \in [\theta_{k-1}^\chi, \theta_k^\chi]$, the receiver will choose*

$$a^\chi(m_k) = \chi \left(\frac{1}{2} \right) + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right). \quad (12)$$

Proof: Given that the sender will send message m_k uniformly on $[\theta_{k-1}^\chi, \theta_k^\chi]$ when $\theta \in [\theta_{k-1}^\chi, \theta_k^\chi]$, by Lemma 1, when the receiver receives m_k , his χ -cursed belief density becomes:

$$\mu^\chi(\theta|m_k) = \begin{cases} \chi & \text{if } \theta \notin [\theta_{k-1}^\chi, \theta_k^\chi] \\ \chi + (1 - \chi) \left(\frac{1}{\theta_k^\chi - \theta_{k-1}^\chi} \right) & \text{if } \theta \in [\theta_{k-1}^\chi, \theta_k^\chi]. \end{cases}$$

Therefore, the receiver's conditional expectation about the true state if $m_k \in [\theta_{k-1}, \theta_k]$ is

$$\begin{aligned} \mathbb{E}_{\mu^\chi}[\theta|m_k] &= \int_0^{\theta_{k-1}^\chi} \theta \chi d\theta + \int_{\theta_{k-1}^\chi}^{\theta_k^\chi} \theta \left[\chi + (1 - \chi) \left(\frac{1}{\theta_k^\chi - \theta_{k-1}^\chi} \right) \right] d\theta + \int_{\theta_k^\chi}^1 \theta \chi d\theta \\ &= \chi \left(\frac{1}{2} \right) + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right). \end{aligned}$$

In other words, if the sender sends $m_k \in [\theta_{k-1}^\chi, \theta_k^\chi]$, the receiver's induced expectation is

$$\mathbb{E}_{\mu^\chi}[\theta|m_k] = \chi \left(\frac{1}{2} \right) + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right),$$

which is the receiver's optimal action. Moreover, at the boundary between two intervals of the partition, the sender must be indifferent between the action induced at the lower interval

and the action induced at the higher interval. That is, for any $1 \leq k \leq K - 1$,

$$\left[\frac{\chi}{2} + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right) - (\theta_k^\chi + b) \right]^2 = \left[\frac{\chi}{2} + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k+1}^\chi}{2} \right) - (\theta_k^\chi + b) \right]^2. \quad (*)$$

Similar to the proof in [Crawford and Sobel \(1982\)](#) and [Lee et al. \(2023\)](#), we can find that for any $1 \leq k \leq K - 1$,

$$\chi \left(\frac{1}{2} \right) + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right) \leq \theta_k^\chi + b \leq \chi \left(\frac{1}{2} \right) + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k+1}^\chi}{2} \right).$$

Consequently, we can simplify Equation [\(*\)](#) as:

$$\begin{aligned} (\theta_k^\chi + b) - \frac{\chi}{2} - (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k-1}^\chi}{2} \right) &= \frac{\chi}{2} + (1 - \chi) \left(\frac{\theta_k^\chi + \theta_{k+1}^\chi}{2} \right) - (\theta_k^\chi + b) \\ \iff (1 - \chi)\theta_{k+1} &= 2(1 + \chi)\theta_k - (1 - \chi)\theta_{k-1} - 2\chi + 4b. \end{aligned}$$

This completes the proof. ■

Illustrative Example: 2-Partition χ -CSE

To illustrate the effect of cursedness, we characterize the 2-partition χ -CSE in this example, which consists of three cutpoints: $\{0, \theta_1^\chi, 1\}$. By Proposition [OA.3](#), we know that the cutpoint θ_1^χ satisfies:

$$1 - \chi = 2(1 + \chi)\theta_1^\chi - 2\chi + 4b \iff \theta_1^\chi = \frac{1}{2} - \frac{2}{1 + \chi}b.$$

Therefore, this 2-partition χ -CSE exists if and only if $\theta_1^\chi \in (0, 1)$, which is equivalent to

$$\frac{1}{2} - \frac{2}{1 + \chi}b > 0 \iff b < \frac{1 + \chi}{4}.$$

When $b = 0.25$, sequential equilibrium predicts that the only equilibrium is the 1-partition babbling equilibrium. However, for any $\chi > 0$, there exists a 2-partition χ -CSE, illustrating how cursedness can make the sender send more informative messages. Moreover, if $\chi = 0.2$, $\theta_1^\chi = \frac{1}{12}$, $a^\chi(m_1) = \frac{2}{15}$ and $a^\chi(m_2) = \frac{8}{15}$. It can be verified that both the sender and the receiver are better off in this equilibrium than the babbling equilibrium. This example shows that cursedness could also potentially improve welfare. □

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