Appendix for: Inflation Targeting under Fiscal Fragility

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B Supplemental Appendix: Productivity Cost of Inflation

In this section we present an alternative model in which inflation, instead of affecting utility directly, reduces the endowment of agents as inflation becomes higher. The rest of the model is identical, and results are nearly identical.

The policymaker maximizes an intertemporal utility with the same assumptions as the main model:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{1}$$

but is now subject to the following budget constraint:

$$g_t + (1 + r_t) D_t \leq D_{t+1} + \alpha_t \tau e \ , \tag{2}$$

The fixed endowment is subject to a penalty α_t that depends on the policymaker's choice of inflation. The penalty function α is divided into two components, α^p and α^c. The first component, α^p, depends on the inflation level and reflects the productivity cost of the inflation level on output. We assume a productivity cost of inflation function of the form of

$$\alpha^p(\pi) = (1 - \sigma) + \sigma e^{-\lambda \pi^2} \ ,$$

where $1 - \sigma$ is the lower limit on the inflation cost and λ is a fixed parameter. In this setup, $\alpha(0) = 1$, so the optimal inflation level considering only the productivity cost of inflation is zero. The second component is analogous to the violation cost in the main model:

$$\alpha_t^c = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \alpha_{t-1}^c = 0 \\ -\epsilon & \text{if } \pi_t \neq \pi^a, \alpha_{t-1}^c = 0 \\ \alpha_{t-1} & \text{otherwise} \end{cases}$$

We define $\alpha^a = \alpha^p(\pi^a)$, the productivity cost of committing to the inflation target. The optimal discretionary inflation problem can be rewritten as

$$\max_{\pi, D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g)$$

subject to

$$\begin{aligned} g_T &= \alpha(\pi)\tau e - (1 + r_T^D) D_T + D \\ g &= \alpha(\pi)\tau e - D \left(\frac{1}{\beta} - 1 \right) \\ c_T &= \alpha(\pi) e - g_T \\ c &= \alpha(\pi) e - g \\ 1 + r_T^D &= \frac{1}{\beta} \frac{1}{q_T^e} \frac{1}{1+\pi}. \end{aligned} \tag{3}$$

The solution for the optimal deviation D is identical to the one given by equation (8), but the first-order condition for the discretionary inflation choice changes to

$$(\rho - (1-\rho) v'(g)) \frac{\partial (1 + r_T^D)}{\partial \pi} D_T + (\rho(1-\tau) + (1-\rho) v'(g)\tau)\, \alpha' e = 0 \tag{4}$$

which has the same intuition as before, but now raising the inflation level affects utility by reducing the available consumption in the steady-state, both for the private and the government consumption goods.

B.1 Calibration and Results

As before, we consider that $v(g) = log(g)$ and calibrate the parameters of the model following the same reasoning of the main model, but we now have an additional parameter σ to calibrate to match the 70% debt-to-GDP crisis zone observed in the Brazilian 2002 inflation crisis:

Parameter	Value	Meaning	Calibration
β	.914	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
σ	20%	Limit to TFP cost	Brazilian 2002 crisis
ϵ	.002	Fixed cost	Brazilian 2002 crisis
λ	1.77	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table I: Parameters of the Baseline Model

The main results are presented below. The policy function is nearly unchanged, as the calibration ensures that the crisis zone starts at around 70% of GDP:

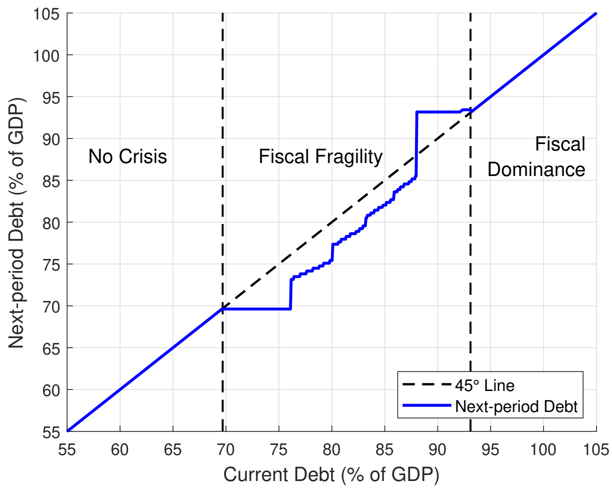


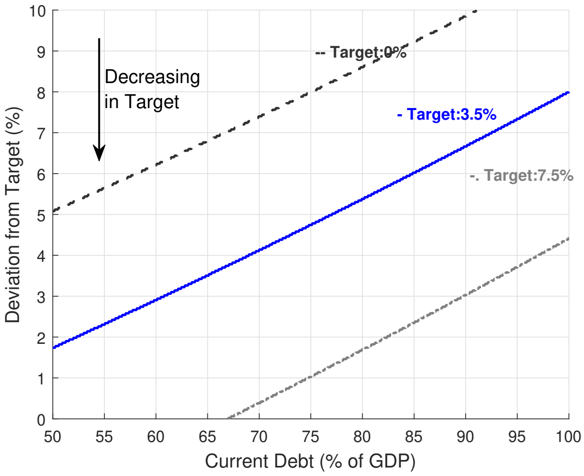
Figure I: Debt Policy Function

Expected inflation is lower for the model with productivity penalty, which imply a lower benefit of reducing debt rapidly and as a result the policymaker takes longer to exit the FFZ for moderate debt levels.

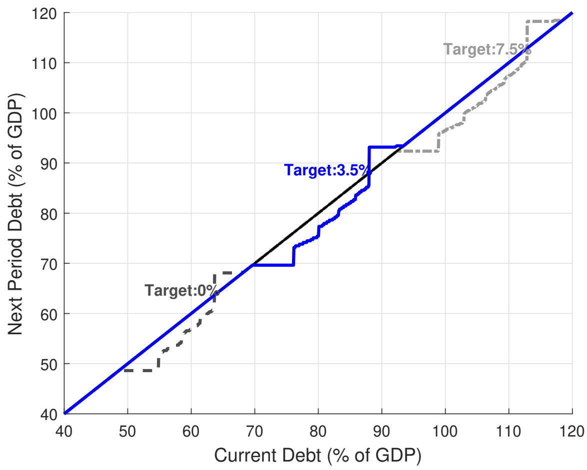
Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.5%	1 year
85%	Fiscal Fragility	4.7%	4 years
100%	Fiscal Dominance	11.5%	-

Table II: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

As before, the deviation from target is decreasing on the target level, and the inflation target serves the same coordinating role of expectations as the model with inflation disutility, raising the FFZ floor as the target is increased. Given the lower discretionary inflation, the optimal inflation target is slightly lower than the previous model.



(a) Sensitivity: Deviations to Inflation Target



(b) Sensitivity: Inflation Target and the Fiscal Fragility Zone

Figure II: Sensitivity Analysis for Inflation Targets

Initial Debt Level	Optimal Inflation Target
60%	2%
70%	4%
80%	5.5%

Table III: Optimal Inflation Target for Each Debt Level in the Calibrated 2002 Brazilian Case

C Supplemental Appendix: Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability of deviating from the target would increase with debt and decrease with higher target levels. The present section investigates whether there is empirical evidence for the predictions based on our model. We construct a dataset that includes 20 countries with at least 15 years of inflation targeting[1] covering the period from 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regard to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table IV. The variables present both inter- and intracountry variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle-income countries.[2]

	Debt/GDP	Revenue/GDP	CPI	CPI Target
Average	43.1	32.9	4.2	3.3
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table IV: Data Description

Real effective exchange rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[3] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalents are seasonally adjusted with the Arima X-11 procedure.[4] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

[1]The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, the Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom. All macroeconomic data used in this section are sourced from the IMF (International Monetary Fund, 2025), World Bank (World Bank, 2025), and BIS (Bank for International Settlements, 2025), as documented in the replication package.

[2]We used the World Bank classification.

[3]BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[4]This was the case for Peru and Turkey.

Deviations from the Target

The FOC of the discretionary inflation problem from 7 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^a_{i,t}$ to observable and latent variables for each country i. We estimate the following model,

$$\pi_{i,t} - \pi^a_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^a_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{5}$$

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time t for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map the model variables D, τe, and π^a to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservable variables e, f, ϵ and κ are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator because it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0\ \forall t$ as required for a random effect estimator.[5]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in (5), one would have to examine the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative because the policymaker could help coordinate private agents' expectations by adopting a more believable (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means that the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation, leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table V is the basic model from (5). The remaining estimations, II-V, are robustness checks.

In estimation I, deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation, the coefficient should not be significantly

[5] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

	I	II	III	IV	V
Revenue	0.171^{**}	0.098	0.051	0.125^{*}	0.076
	(0.076)	(0.076)	(0.079)	(0.070)	(0.073)
Debt	0.069^{*}	0.074^{**}	0.074^{**}	0.062^{**}	0.060^{*}
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt × Revenue/100	-0.194^{*}	-0.168^{*}	-0.149	-0.163^{*}	-0.136
	(0.099)	(0.096)	(0.097)	(0.088)	(0.090)
Target	-0.403^{***}	-0.458^{***}	-0.441^{***}	-0.360^{***}	-0.343^{***}
	(0.063)	(0.062)	(0.062)	(0.059)	(0.059)
GDP Gap			0.389^{***}		0.367^{***}
			(0.104)		(0.096)
Reer YoY				-13.956^{***}	-13.583^{***}
				(1.648)	(1.666)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: *p<0.1; **p<0.05; ***p<0.01

Table V: Deviations from the Inflation Target

different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP, the net impact of debt is positive. This result applies to the middle-income countries in our sample. The result goes in the direction of what the theoretical model predicted, as both the probability of deviating and deviations from the target are positively related to debt levels. On average, countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of 88%, above the maximum in our sample. Hence, the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated with preferences for public spending that in turn could lead to inflationary pressure.

The results remain after accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect to account for global shocks such as commodity prices. In our sample, 2008 stands out, as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global comovements in inflation into account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Deviating from the Target

The policymaker deviates from the inflation target when end-of-year inflation exceeds the upper bound of the target.[6] In the theoretical model, the policymaker had more incentive to deviate when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to (5) but with regard to the probability of deviating:

$$I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \quad (6)$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ exceeds the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$. The probability of deviating will then be a logistic function:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \quad (7)$$

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of deviating. Each year in the sample, at least two countries deviate from their respective inflation targets. The years 2007 and 2008 stand out, as over half of the countries deviated. A time dummy is likely to capture this effect. Additionally, virtually all countries except two deviated from their targets at least once, with some countries such as Turkey close to being serial deviators. Overall, middle-income countries exceed the target more often than high-income countries. Nevertheless, high-income countries exceeded the target 39 times.

The first column of Table VI is the baseline model, while the remaining columns represent robustness checks similar in spirit to the previous section. When considering the net impact of debt on the probability of deviating from the target, the coefficients have similar signs to the previous estimates with regard to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP, the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

[6] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case for the UK and Norway. In such cases, we used the average upper tolerance limit from the rest of the sample (1.2%).

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Inflation Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	-178.526	-151.367	-149.281	-139.619	-137.954

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table VI: Results – Probability of Overshooting the Target

The net impact of revenue remains positive for debt levels in the sample, not in the same direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, which decreases the incentive to use discretionary inflation to transfer resources away from private agents' debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, which would increase incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of deviating is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, making it more likely to exceed the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to deviate. The output gap is not significant.

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