Accounting for Wealth Concentration in the United States

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Online Appendix

A. ENTREPRENEURSHIP AND THE DISTRIBUTION OF LABOR AND CAPITAL INCOME

Consider the following portfolio allocation problem for an entrepreneur endowed with a units of assets and a diminishing-return-to-scale business income production function $y^b = \theta k^\alpha$, where θ represents the productivity of the entrepreneur. We implicitly assume that a unit of entrepreneurial labor is supplied inelastically as long as the business is in operation. The entrepreneur's problem is to

$$\max_k y = \theta k^\alpha + r(a-k),$$

where the first term is business income and the second term is market income on excess assets (or debt service if $a < k$ in equilibrium). The optimal business investment k^* solves $\theta\alpha k^{\alpha-1} = r$. Substituting the optimality condition back into the objective function gives:

$$y^* = ra + \underbrace{(1-\alpha)\theta_i^{\frac{1}{1-\alpha}}(r/\alpha)^{\frac{\alpha}{\alpha-1}}}_{non-capital\ income}$$

Note that $ra = \alpha y^b + r(a-k)$, the sum of capital's share of business income and capital income on excess assets.

This setting is observationally equivalent to a version of our model with a common return on assets and labor income heterogeneity, which here is driven by differences in entrepreneurial ability, θ. In this setting, our calibration procedure correctly interprets differences in business income as reflecting labor income heterogeneity.

Next, consider the case where entrepreneurs are constrained by their assets when investing in their business: $k \leq a$. For entrepreneurs with sufficient assets, given their productivity θ, this constraint does not bind, and the argument above applies all the same. If an entrepreneur is constrained, then the optimal investment is $k^* = a$. Let $r_i = \theta\alpha a^{\alpha-1} > r$ denote the marginal return on business capital of a constrained en-

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trepreneur. Then total income of an entrepreneur can be written as:

$$y^* = r_i a + \underbrace{(1-\alpha)\theta_i^{\frac{1}{1-\alpha}}(r_i/\alpha)^{\frac{\alpha}{\alpha-1}}}_{non-capital\ income}$$

Our accounting approach then attributes variation in the first term across households to differences in wealth and in the return on assets, and variation in the second term to differences in labor productivity. The relative shares of labor and capital income are correctly identified.

Finally note that for constrained entrepreneurs, heterogeneity in the rate of return affects not only the capital income component, but also the labor income component of income. In particular, constrained entrepreneurs have lower earnings conditional on productivity, since they cannot scale up their ideas to full capacity. Therefore, eliminating differences in asset returns also raises labor income dispersion. As a consequence, eliminating rate of return differences while keeping earnings heterogeneity unchanged, as we do in our analysis, may overstate the importance of rate of return differences. Similarly, eliminating differences in calibrated productivity levels reduces dispersion in rates of return across households, given the definition of r_i. This implies that eliminating earnings differences while keeping rate of return differences unchanged, as we do in our analysis, may understate the importance of productivity differences.

B. Details of Model Calibration

B1. Demographics

Following Halliday et al. (2019), we model survival probability as a logistic function of age: $s(j) = [1 + \exp(\omega_0 + \omega_1 j + \omega_2 j^2)]^{-1}$ and use their recommended parameter values. Halliday et al. (2019) calibrate to three moment conditions: the dependency ratio (population aged 65 and over divided by population aged 20-64), which is 39.7% in the data; the age-weighted death rate for 20 to 100 year olds of 8.24%, and the ratio of the change in the survival probability between ages 65-69 and 75-79 to the change in survival probability between ages 55-59 and 65-69, which is 2.27 in the data.

B2. US Personal Income Tax System

Taxable personal income is given by:

$$y_f = (zw\varepsilon_j h)\,\mathbb{I}_{j<J_r} + b(z)\,\mathbb{I}_{j\geq J_r} + \min\{r\kappa k, d_c\},$$

where $\mathbb{I}_{j\geq J_r} \in \{0,1\}$ indicates retirement status. Disposable income is obtained by deducting corporate and personal income taxes and adding transfers:

$$y^d = \lambda \min\{y_b, y_f\}^{1-\tau} + (1-\tau_{max})\max\{0, y_f - y_b\} + (1-\tau_c)\max(r\kappa k - d_c, 0) + Tr$$

The first two terms above represent our formulation of the current US income tax system, which can be approximated by a log-linear form for income levels outside the top of the income distribution (Bénabou, 2002), augmented by a flat rate for the top income tax bracket. The power parameter $\tau \in [0, 1]$ controls the degree of progressivity of the tax system, while λ adjusts to meet the government's budget requirement.

The second term caps the marginal tax rate at $\tau_{max} = 39.6\%$, as reported by the IRS. y_b denotes the critical level of taxable income at which the top marginal tax rate is reached: $\lambda(1-\tau)y_b^{-\tau} = 1 - \tau_{max}$. We calibrate the progressivity of the income tax system, τ, to the difference between the average income tax rate paid by the top 1% and the bottom 99% of the income distribution. Piketty and Saez (2007) report this value to be 12.4%.

B3. *Pension Benefits*

The formula that determines Social Security benefits features two bend points (bp_1 and bp_2 expressed as multiples of average earnings), three replacement rate brackets (0.90, 0.32, and 0.15), and a maximum benefit, b^{cap}. The benefit for an individual retiring with productivity z is

$$\begin{aligned} b(z) \quad = \quad & \xi \min\{b^{cap}, 0.9 \min(\tilde{e}(z), bp_1) + 0.32 \max[\min(\tilde{e}(z), bp_2) - bp_1, 0] \\ & +0.15 \max(\tilde{e}(z) - bp_2, 0)\}, \end{aligned}$$

where $\tilde{e}(z)$ are average earnings of working age agents of productivity z in the model's stationary equilibrium. The formula reported by the SSA is for an individual, whereas the model is based on households, which may contain non-working spouses or survivors. Therefore, we adjust benefits by a factor, ξ , and calibrate it to match the average ratio of Social Security expenditure to GDP in the data.

B4. *Labor Productivity Process*

Table B.1 summarizes the transition probabilities and the corresponding productivity levels for the stochastic process. The initial distribution represents the share of workers in each productivity state at labor market entry. Since the initial distribution of young workers is different from the invariant distribution, and because agents have finite lives, the population shares of workers across productivity states are different from the invariant distribution. The population shares for the working age population are reported in the last row of the table. Retired agents have zero labor productivity.

Note that our formulation implies that if a household loses its extraordinary status, then it is equally likely to transition to any one of the ordinary states. The effect of this assumption on our quantitative analysis is negligible. In particular, assuming instead that all those leaving z_7 downwards enter z_6 hardly affects our quantitative findings.

Table B.1—: Calibrated Productivity Process in the Benchmark Economy

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
$z_1 = 1.00$	0.874	0.119	0.004	0	0	0	0.002	0
$z_2 = 2.05$	0.060	0.878	0.060	0	0	0	0.002	0
$z_3 = 4.21$	0.004	0.119	0.874	0	0	0	0.002	0
$z_4 = 3.47$	0	0	0	0.874	0.119	0.004	0.002	0
$z_5 = 7.12$	0	0	0	0.060	0.878	0.060	0.002	0
$z_6 = 14.59$	0	0	0	0.004	0.119	0.874	0.002	0
$z_7 = 124.83$	0.021	0.021	0.021	0.021	0.021	0.021	0.850	0.021
$z_8 = 1034.51$	0	0	0	0	0	0	0.242	0.758
initial distribution	0.044	0.412	0.044	0.044	0.412	0.044	0	0
population share	0.097	0.303	0.097	0.097	0.303	0.097	0.0076	0.0003

Note: Table shows the calibrated productivity levels and the corresponding transition probabilities. The last row shows the fraction of working age population in each productivity state.

B5. Stochastic Process for Rates of Return

Table B.2 summarizes the transition probabilities and the corresponding annualized rates of return for the stochastic return process.

Table B.2—: The Transition Matrix for Rates of Return on Capital

from / to	κ_L	κ_H	κ_{top}
κ_L	0.96	$0.04 - \pi_{in}(z)$	π_{in}
κ_H	$0.06 - \pi_{in}(z)$	0.94	$\pi_{in}(z)$
κ_{top}	0.0	0.10	0.90
population share (%)	59.9	40.0	0.1
annual rate of return (%)	0.0	6.7	22.7

Probability of entering the top return state by z state of origin:

	z_1 to z_6	z_7	z_8
$\pi_{in}(z)$	2.4×10^{-4}	1.2×10^{-3}	3.6×10^{-3}

Note: The top panel shows the transition probabilities in the benchmark economy from the rate of return in Column 1 to rates of return in Columns 2 to 4. The annual rates of return associated with each state and the share of the population in each state are reported in the last two rows. Transition probabilities vary with the state z, as shown in the lower panel.

B6. Calibration Approach

Our calibration aims to minimize the equally weighted sum of squared deviations between model moments and data moments.

The age-wealth profile enters our calibration objective function as the mean squared distance between wealth by age group in the model and in data.

Note that in computing model measures of intergenerational wealth persistence, we exclude model parent-child pairs where either the child or the parent is in the top 1% of wealth. Results are similar when they are included.

B7. Summary Tables

Table B.3 shows the preset parameters. Table B.4 shows the values for internally (and jointly) calibrated parameters. Table B.5 shows a summary list of calibration targets along with their sources and the associated values obtained in the benchmark economy.

Table B.3—: Calibration of the Model: Preset Parameters

Parameter	Description	Value	Source
		Demographics	
J	Maximum life span	16	corresponds to age 100
J_R	Mandatory retirement age	10	corresponds to age 65
$\omega_0, \omega_1, \omega_2$	Survival probability by age	-5.49, 0.15, 0.016	Halliday et al. (2019)
		Preferences	
σ_c	Risk aversion	1.5	
σ_l	Inverse Frisch elasticity	1.22	Blundell, Pistaferri and Saporta-Eksten (2016)
		Technology	
δ	Depreciation (annual)	0.045	
		Labor productivity	
		See Sections IV.C and Online Appendix B.4	
		Taxes and transfers	
τ_c	Marginal corporate tax rate	0.236	Gravelle (2014)
τ_s	Consumption tax rate	0.05	Kindermann and Krueger (2022)
Tr	Government transfers/GDP	2.7%	NIPA Table 3.12

Table B.4—: Calibration of the Model: Jointly Calibrated Parameters

Parameter	Description	Value	Parameter	Description	Value
β	Annual Discount rate	0.974	θ	Labor disutility	6.0
α	Capital elasticity	0.27			
z_7, z_8	Top productivity states	Table B.1	$\lambda_{in}, \lambda_{ll}, \lambda_{lh}, \lambda_{hh}$	Transition rates	Table B.1
$\kappa_L, \kappa_H, \kappa_{top}$	Rates of return	Table B.2	$\pi_{ll}, \pi_{hh}, \pi_{in}, \pi_{top,top}$	Transition rates	Table B.2
τ_l	Tax progressivity	0.17	d_c	Corporate asset threshold	2.21
ξ	Pension / Earnings	0.62	G/Y	Expenditures / GDP	11.6%
ϕ_1, ϕ_2, σ_b	Bequest utility	-1.4, 0.1, 1.35	$\bar{\gamma}_z, \bar{\gamma}_\kappa$	Bequest correlations	0.65, 0.99

Table B.5—: Summary of Target Moments

Moment	Source	Data Value	Model Fit	Moment	Source	Data Value	Model Fit
Mean hours worked		0.35	0.34	Soc. Sec. Pay / GDP	NIPA	7.9%	8.1%
Top 0.1%,1% earning shares	SCF 2001–2019	Figure 3	Figure 3	Probability of staying in top 1% earners	Kopczuk, Saez and Song (2010)	0.62	0.63
Top 0.1%,1%,5%,10% wealth shares	SCF 2010–2019	Figure 3	Figure 3	Log wealth correlation between parents and kids	Charles and Hurst (2003)	0.37	0.39
Bequest/Wealth	Guvenen et al. (2023)	$\sim 1\%$	2%	Top 2% bequest dist.	Feiveson and Sabelhaus (2018)	40%	39%
Age-Wealth profile	SCF 2001–2019	Figure 5c	Figure 5c				
Difference between average income tax rate for top 1% and 99%	Piketty and Saez (2007)	12.4%	13.0%	Corporate income tax revenue/GDP	NIPA	2.5%	2.3%
Overall labor income share	SCF 2010–2019	0.82	0.79	Top 0.1% labor income share	SCF 2010–2019	0.49	0.48
Top 1% labor income share	SCF 2010–2019	0.59	0.59	Top 10% labor income share	SCF 2010–2019	0.72	0.71
Intergenerational wealth persistence at 4th quintile	Charles and Hurst (2003)	0.26	0.19	Intergenerational wealth persistence at 5th quintile	Charles and Hurst (2003)	0.36	0.32
Mean return top 0.1% income / mean	SCF 2001–2019	2.5	2.0	Mean return top 1% income / mean	SCF 2001–2019	1.8	1.6
Mean return top 0.1% wealth / mean	SCF 2001–2019	1.3	1.6	Mean return top 1% wealth / mean	SCF 2001–2019	1.2	1.2

C. Supplementary Tables and Results

Table C.1 shows additional decomposition results for wealth concentration. The first row reports the benchmark results. Row (1a) removes non-homotheticity in altruism by setting $\phi_2 = 0$, which makes bequests proportional to wealth. In (1b), we remove the correlation of bequests with parental wealth by setting both $\bar{\gamma}_z$ and $\bar{\gamma}_\kappa$ to 0.5, so that all bequest recipients draw their bequest from the same distribution. While this eliminates the intergenerational correlation of wealth, its effect on top wealth concentration is modest, with top wealth shares decreasing by about 10 percent. In row (1c), we set σ_b equal to σ_c. Out of the three scenarios in this table, this one has the strongest effect on wealth inequality. Nevertheless, its effect (necessarily) is much weaker than that of fully equalizing bequests, as considered in the main text.

In (2), we set the value of the higher top productivity state, z_8, equal to the calibrated value for the lower top state, z_7. This lowers wealth concentration by about 10 percent for the top 1% and about a quarter for the top 0.1%.

In (3), we set the top excess return, κ_{top}, equal to the higher regular excess return, κ_H. This cuts the top 0.1% wealth share by about a half and the top 1% wealth share by about a fifth.

Table C.1—: Determinants of Wealth Concentration: Additional Decomposition Results

	wealth Gini	top wealth shares		top earnings shares	
		0.1%	1%	0.1%	1%
... (0) benchmark	0.80	0.15	0.35	0.050	0.16
counterfactual economies with . . .					
. . . (1a) homothetic bequests	0.78	0.15	0.34	0.050	0.16
. . . (1b) uncorrelated bequests	0.80	0.13	0.33	0.050	0.16
. . . (1c) $\sigma_b = \sigma_c$	0.77	0.11	0.30	0.049	0.16
. . . (2) $z_8 = z_7$	0.79	0.11	0.31	0.028	0.14
. . . (3) $\kappa_{\text{top}} = \kappa_H$	0.79	0.09	0.28	0.050	0.16

Note: Results from model simulations. In counterfactual economy (1a), we set ϕ_2 to zero, making the bequest motive homothetic. In (1b), we remove the correlation of bequests with parental wealth by setting both $\bar{\gamma}_z$ and $\bar{\gamma}_\kappa$ to 0.5, so that all bequest recipients draw their bequest from the same distribution. In (2), we set the value of the higher top productivity state, z_8, equal to the calibrated value for the lower top state, z_7. In (3), we set the top excess return, κ_{top}, equal to the higher regular excess return, κ_H. In (4), we do not allow top earners to transit into the top return state.

C1. Sensitivity to Labor Share Estimates

In Section 8.2, we calibrated the model to wage shares instead of labor income shares, that is, we allocated all business income to capital. The low LIS economy differs from the benchmark not only in lower top LIS (49% instead of 59% for the top 1% income group and 37% instead of 49% for the top 0.1% group), but also has lower earnings concentration and slightly lower relative rates of return at the top. To match these statistics, we recalibrate key parameters of the model. Because the aggregate *wage* income share is lower than the labor income share, we target the top LIS relative to the aggregate one.

Here, we report the main changes in parameters. The low-LIS economy has different top productivity parameters (z_7 is 24% lower than in the benchmark, z_8 is 9% higher), and substantially greater return dispersion. The low (medium) [high] return is 0.1% (7.7%) [24.4%]. In addition, the bequest parameter ϕ_1 is lower in absolute value, at -0.45.

Table C.2—: Determinants of Wealth Concentration: Excluding Imputed Wages for Entrepreneurs

	wealth Gini	top wealth shares		top earnings shares	
		0.1%	1%	0.1%	1%
benchmark	0.83	0.15	0.34	0.05	0.14
Counterfactual economies with …					
…(1) equal bequests	0.75	0.10	0.25	0.05	0.14
…(2) no top earners	0.78	0.08	0.21	0.004	0.04
…(3) common returns	0.75	0.09	0.26	0.05	0.14

Note: Results from model simulations, for a calibration targeting lower top labor income shares (see text). Economy (2) sets the extraordinary productivity levels to that of the highest "regular" earnings category ($z_8 = z_7 = z_6$). Economy (3) sets κ to its value-weighted average in the benchmark economy.

Table C.2 shows the main decomposition results for this economy. All results are summarized in Figure 8 in the main text.

C2. An Economy with Low Earnings Concentration

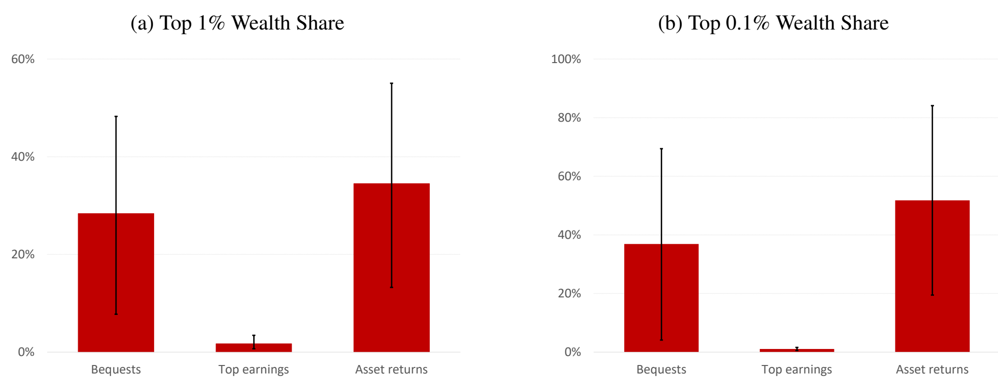
Figure C.1 and Table C.3 show decomposition results for an economy with a counterfactually low earnings inequality, as discussed in Section 9 in the main text. In particular, we recalibrate the earnings process in our model by setting the two top productivity states z_7 and z_8 to a common value that implies an earnings Gini of 0.41 as in De Nardi, Fella and Pardo (2016). This change also implies a reduction in the top 1% (0.1%) earnings share to 5% (1%), much below the data value of 17% (6%), which our benchmark model matches closely. The recalibrated model with low earnings inequality also has much lower wealth concentration, with a wealth Gini of 0.74 and a top 1% wealth share of 20%. This Gini coefficient is close to that found by Huggett (1996), and by De Nardi, Fella and Pardo (2016) when using an AR(1) process for earnings. In this economy, further reducing top earnings by setting z_7 and z_8 to equal the top ordinary productivity state, z_6, hardly reduces wealth inequality. This is not surprising, since the gap between top and ordinary productivity is small in this low-earnings-inequality economy. Instead, heterogeneous returns have a major effect on wealth concentration. Eliminating them reduces top wealth shares dramatically, and takes them to levels similar to those obtained in the models of Huggett (1996) and De Nardi, Fella and Pardo (2016; 2020), with a top 1% wealth share of about 10%.

Table C.3—: Determinants of Wealth Concentration – Alternative Economy with Low Earnings Inequality

	wealth Gini	top wealth shares		top earnings shares	
		0.1%	1%	0.1%	1%
benchmark	0.80	0.15	0.35	0.05	0.16
Alternative economy with earnings Gini = 0.41					
	0.74	0.09	0.20	0.01	0.05
Counterfactual economies with low earnigns inequality and ...					
... (1) equal bequests	0.67	0.03	0.10	0.01	0.05
... (2) no top earners	0.74	0.08	0.20	0.004	0.04
... (3) common returns	0.66	0.01	0.09	0.01	0.05

Note: The alternative economy has the same parameters as the benchmark, except that the top productivity levels z_7 and z_8 are equal and set to generate a Gini coefficient of earnings of 0.41. Counterfactual economies are generated as in Table 5 in the main text.

Figure C.1. : Factors of Wealth Concentration – Alternative Economy with Low Earnings Inequality



Note: Figure shows the marginal contribution of each factor to the concentration of net worth relative to the alternative economy with low earnings concentration. The whiskers show the range of marginal effects obtained by permuting the order in which factors are eliminated from the benchmark economy. The column height represents the average across permutations.

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