

Trade, Value Added, and Productivity Linkages: A Quantitative Analysis

Online Appendix (OA)

For Online Publication Only

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This appendix includes additional exercises examining the empirical relationship between trade linkages and cross-country GDP correlations. It also provides detailed information on the quantitative model and conducts extensive robustness checks that compare this model with previous frameworks documented in the literature.

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OA1 Empirical Appendix

We present additional information about the datasets, along with a series of sensitivity analyses of the empirical Trade Comovement Slope. Although the core paper uses a balanced panel of 30 countries in both the empirical section and the many-country model to limit computational complexity and focus on countries for which most variables are available, some of our analyses below demonstrate that our empirical results are robust to a wider range of countries, time periods, and alternative datasets.

OA1.1 Data source

Trade Comovement Puzzle. In this appendix, unless explicitly stated, the empirical analysis is performed on 40 OECD countries and major emerging markets, which account for around 90% of world GDP.¹ Our bilateral trade flows data come from [Johnson and Noguera \(2017\)](#) who separated between final and intermediate goods for 42 countries between 1970 and 2009. According to their data appendix A.2, they construct their data as follows. For bilateral goods trade, they use the NBER-UN Database for 1970-2000 and the CEPII BACI Database for 1995-2009. This data is reported on a commodity-basis. They assign commodities to end uses and industries using existing correspondences from the World Bank. To assign commodities to end uses, they use correspondences between SITC (Revision 2) 4-digit or HS (1996 Revision) 6-digit commodities and the BEC end use classifications. To assign commodities to industries, they use

¹The list of countries is: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States and Vietnam.

correspondences between SITC and HS categories and ISIC (Revision 2) industries. GDP data comes from the 9th Penn World Tables. We use the output-side real GDP at chained PPPs (variable *rgdpo*), to compare relative productive capacity across countries and over time.² We also use OECD data and quarterly based price GDP data in some robustness checks. In total, we have 630 country-pairs appearing 4 times and 190 pairs appearing 2 times (both in the case of 10 years time windows), leading to a dataset with a total of 2900 observations.

OA1.2 Summary statistics

Table 1 shows the summary statistics of the data used throughout the empirical investigations of the core paper.

Table. 1. Summary statistics

Statistic	Min	Max	Mean	Pctl(25)	Median	Pctl(75)	N	St. Dev.
log_inx_tot_trade	-15.440	-2.412	-6.870	-7.685	-6.756	-5.878	2,380	1.653
log_inx_int_trade	-15.814	-2.851	-7.407	-8.229	-7.298	-6.415	2,380	1.682
log_inx_fin_trade	-16.647	-3.451	-7.875	-8.738	-7.716	-6.767	2,380	1.748
corr_GDP_HP	-0.933	0.964	0.265	-0.013	0.315	0.583	2,380	0.399
corr_GDP_BK	-0.928	0.961	0.263	-0.010	0.312	0.573	2,380	0.389
corr_GDP_FD	-0.976	0.978	0.245	-0.026	0.285	0.542	2,380	0.382
third_tot	0.123	0.942	0.538	0.411	0.544	0.674	2,380	0.172
sector_ex	0.023	0.722	0.304	0.208	0.291	0.386	2,380	0.134

OA1.3 Cross-sectional TC-slope by time-windows

As a background, Figure 1 provides scatterplots relating GDP co-movements to trade intensity for each individual time window. In each time window, there is a cross-sectional positive relationship between the two variables. Therefore, the main analysis consists of disentangling effects stemming from unobserved heterogeneity (such as common borders, shared language, etc.) and specific time effects (such as the global rise in GDP correlation over time).

OA1.4 Sensitivity Analysis of the Empirical Trade Comovement Slope

We now compile several sensitive analyses related to our main empirical findings, regarding the trade-comovement slope. Table 2 gathers the results. The first row simply restates our baseline results for reference. More details of those experiments are provided in the following.

²We drop Romania and South Africa from their sample because of lack of GDP series in the Penn World Tables. Moreover, in [Johnson and Noguera \(2017\)](#)'s data for Russia starts only in 1990 while data for Estonia, Slovak Republic, Slovenia and Czech Republic start only in 1993. All country-pairs involving one of those five countries appears only two times in the case of 10 years time-windows and cannot be used at all in the case of 20 years time-windows.

Figure 1. Bilateral trade intensity and GDP correlation (HP-filter) for the four time windows from 1970 to 2009. Blue: 1970-1979, Red: 1980-1989, Green: 1990-1999, Orange: 2000-2009.

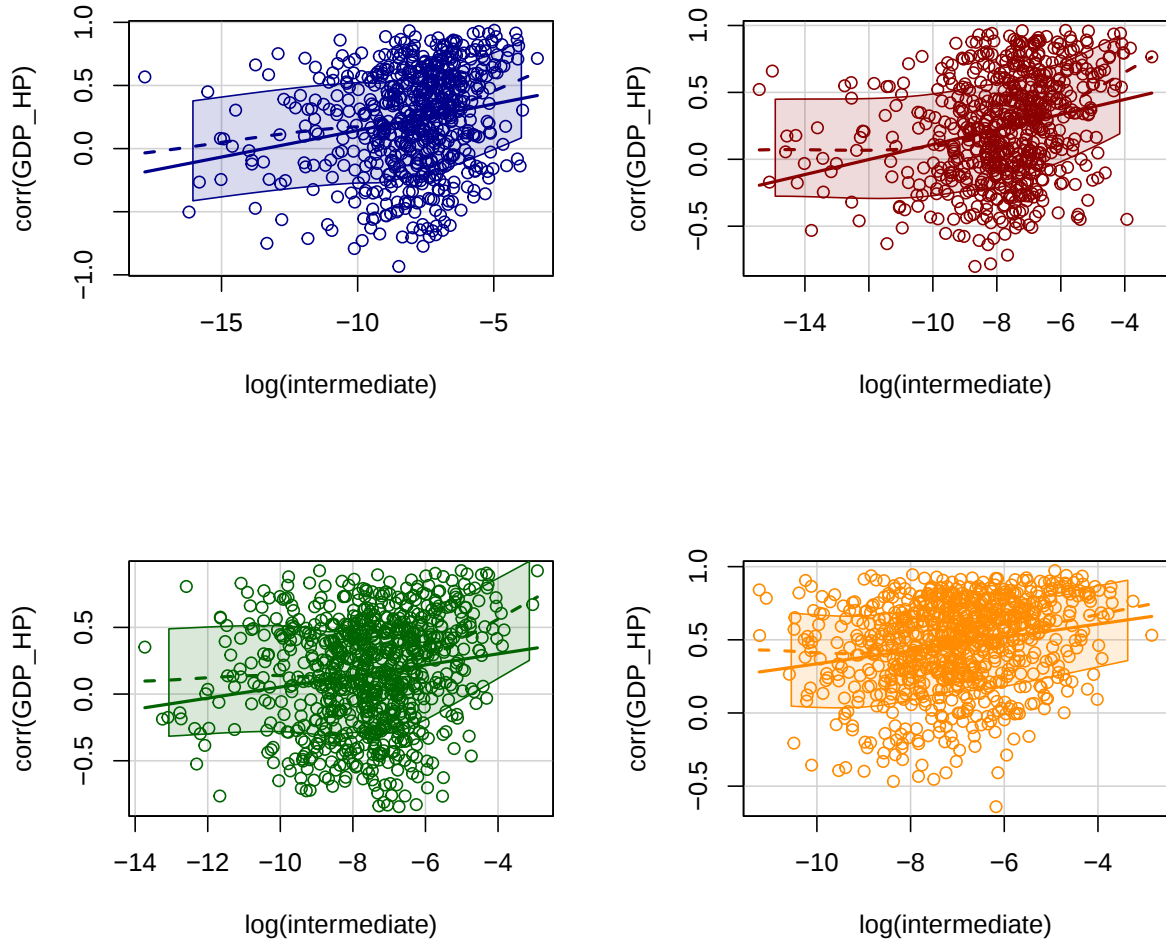


Table. 2. Robustness exercises on estimated Trade Comovement-slope in the data

	Coeff. trade inputs	Coeff. in trade in fi- nal goods	GDP Filter	Countries Obs.	Period	TW	CP
<i>Sample selection</i>							
1. Whole Sample	0.061**	-0.037	HP	40 2,900	1970-2009	Yes	Yes
1bis. 30 countries (cf. paper)	0.066**	-0.014	HP	30 1,740	1970-2009	Yes	Yes
2. 20 years TW	0.084**	-0.061*	HP	40 1,450	1970-2009	Yes	Yes
3. Excluding EU CP	0.070**	-0.046*	HP	40 2,280	1970-2009	Yes	Yes
4. Excluding USSR	0.072***	-0.031	HP	34 2,244	1970-2009	Yes	Yes
5. Alternative period	0.084***	-0.045	HP	40 2,080	1970-1999	Yes	Yes
<i>Alternative controls for sectoral composition</i>							
6. 4Digits SITC	0.062**	-0.040	HP	36 2,520	1970-2009	Yes	Yes
7. ISIC classification	0.061**	-0.039	HP	36 2,520	1970-2009	Yes	Yes
8. 1Digit Agg. sectors	0.083	-0.018	HP	38 1,291	1970-2009	Yes	Yes
<i>Alternative indexes</i>							
9. $level(trade)^a$	34.95***	-31.24*	HP	40 2,900	1970-2009	Yes	Yes
10. $\log(mean(trade))$	0.056**	-0.034	HP	40 2,900	1970-2009	Yes	Yes
11. $\max\left(\frac{T_{i \leftrightarrow j}}{GDP_i}, \frac{T_{i \leftrightarrow j}}{GDP_j}\right)$	0.062**	-0.036	HP	40 2,900	1970-2009	Yes	Yes
12. OEC + WID data	0.060***	-0.039**	HP	33 2,130	1970-2009	Yes	Yes
<i>Other robustnesses</i>							
13. Weighted by trade linkages	0.078*	-0.060	HP	40 2,900	1970-2009	Yes	Yes
14. Weighted by trade linkages	0.139***	-0.125**	FD	40 2,900	1970-2009	Yes	Yes
15. Whole sample	0.049**	-0.039	BK	36 2,520	1970-2009	Yes	Yes
16. SR -slope	0.051**	-0.024	BK	40 2,380	1970-2009	Yes	Yes
17. With SR -corr. as control	0.023	-0.014	HP	40 2,380	1970-2009	Yes	Yes
18. BIS data, without BIS control	0.196***	-0.013	HP	39 1,030	1980-2009	Yes	Yes
19. With BIS control	0.200***	-0.040	HP	39 1,030	1980-2009	Yes	Yes
20. FDI data, without FDI control	0.373***	-0.301***	HP	38 728	1980-2009	Yes	Yes
21. With FDI control	0.373***	-0.286***	HP	38 728	1980-2009	Yes	Yes
22. alternative SR measure 1	0.052**	-0.029	HP	34 2,380	1970-2009	Yes	Yes
23. alternative SR measure 2	0.044*	-0.020	HP	34 2,380	1970-2009	Yes	Yes
24. alternative SR measure 3	0.050*	0.008	HP	30 1,860	1970-2009	Yes	Yes

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. TW stands for Time Windows Fixed Effects while CP stands Country-Pairs Fixed Effects, which are included in all our analysis.

OA1.4.1 Sample selection

In the first series of robustness analysis, we ask how our results change when we restrict our sample in different ways. For all these experiments, the results persist when using first differences instead of the HP filter.

Alternative time-windows. In the second row, we use 20 years time windows when computing GDP correlation. In this case, our main results of a strong TC-slope for intermediate inputs is unchanged.

Excluding EU and USSR country-pairs. In the third and fourth row, we reduce our sample by excluding country pairs in the European Union and the USSR respectively. This is motivated by the fact that the European Union (or trade unions) have made correlated policies to improve trade each other, which may influence the correlation between trade intensity and GDP correlation. Perhaps surprisingly, dropping country-pairs in the European Union from the sample increases the correlation between trade in intermediate goods and GDP comovement. We find a similar conclusion when excluding USSR countries.

Alternative time periods. In the fifth row, we restrict our sample to the first three time windows (from 1970 to 1999) so that our time coverage is in line with original [Kose and Yi \(2006\)](#) analysis. In that case, we still find a significant trade co-movement slope, with a larger effect of trade intensity on GDP comovement.

OA1.4.2 Alternative controls for sectoral composition

A second series of analysis relates to the inclusion of alternative controls for sectoral decomposition. Recall that our regressions include controls for similarity in specialization patterns which could impact GDP synchronization over and beyond any trade effect. In rows six, seven and eight, we use different sector categorization and dis-aggregation to compute our sectoral proximity measure.

Using ISIC and SITC 4-digits. The results of an analysis where the index of similarity in sectoral composition is constructed using first the ISIC classification and then using the SITC 4-digits classification do not alter the main estimates.

Using WDI data. We then use the data from the World Development Indicators (WDI) of the World Bank to construct our index of proximity in sectoral composition. We use the share in value added of main sectors: service and agricultural sectors and we decompose manufacturing sectors into 7 main sub-sectors.³ We then compute the following index:

$$index_sector_{ij} = 1 - \frac{1}{2} \sum_k \left| share_GDP_i^k - share_GDP_j^k \right| \quad (1)$$

where k refers to a particular sector. Pairs of country with very similar sectoral composition have an index close to 1, while countries that completely specialize in different sectors would have an index of 0. Results show a positive relationship between trade intensity in inputs

³This includes textile, industry, machinery, chemical, high-tech, food and tobacco, other. Data are available here: <https://databank.worldbank.org/data/source/>.

and GDP comovement, with a similar magnitude when controlling for third index and sectoral composition. Note that the sample size is drastically reduced when using this data. Results are not significant with this sub-sample using HP-filtered data.

OA1.4.3 Alternative indexes

A third series of robustness exercises relate to the definition of our trade proximity indices. In our main results, we construct, for each country-pair and time window, the average over all years in the time window of the log of bilateral trade divided by the sum of GDP.

Using trade level. In row nine, we do not take the log of the trade over (bilateral) GDP ratios and instead use simply the levels. We find consistent results.

Using log(mean) regressions. In row ten, we calculate the logarithm of the average ratio rather than constructing the average of the log ratio. This adjustment leads to slightly smaller magnitudes in the results, but they remain fairly consistent.

Using $\max\left(\frac{T_{i \leftrightarrow j}}{GDP_i}, \frac{T_{i \leftrightarrow j}}{GDP_j}\right)$. Row eleven presents results where instead of using the ratio of bilateral trade over sum of GDP, we use the maximum of bilateral trade divided by each country in the pair ($\max\left(\frac{T_{i \leftrightarrow j}}{GDP_i}, \frac{T_{i \leftrightarrow j}}{GDP_j}\right)$). This alternative measure is commonly used in the Trade Comovement Puzzle literature and serves as a natural alternative. We find nearly identical results.

Using alternative datasets: OEC trade flows and WID GDP data. Finally, row twelve presents results where we change our database and use the Observatory of Economic Complexity (MIT) database to compute trade flows. Annual GDP data come from the World Development Indicators (WDI). These estimates are taken from [de Soyres and Gaillard \(2022\)](#). Again, our main message is not altered by these variations, and the point estimates remain close to our benchmark analysis.

OA1.4.4 Other robustness

The final set of sensitivity investigations pertains to alternative definitions, weighting, and other robustness exercises.

Weighting by the importance in trade linkages. In rows thirteen and fourteen, we conduct a weighted regression, where each observation is weighted by the sum of trade linkages between both countries in the pair. Similar to previous analyses, our results are not significantly affected by these changes, but they appear to be more robust.

BK filter. In row fifteen and sixteen, we employ the Baxter and King (BK) filter for GDP and Solow Residual (SR) and find results consistent with our main analysis using the HP filter.

The absorption of the TC-slope when controlling for measured productivity. In row seventeen, we investigate whether including bilateral Solow Residual correlation in the main specification of the paper significantly affects the relationship between real GDP co-movement and intermediate inputs trade. This result is particularly interesting as the main theory predicts that the Solow Residual should capture most of the correlation through profit and extensive margins. Upon controlling for the cross-country correlation in measured productivity (SR), the coefficient associated with trade in intermediate inputs becomes statistically insignificant, though still positive. Section [OA3.2](#) of the Online Appendix discusses at length how mismeasurement in labor might explain why adding the correlation of the Solow Residual might not entirely absorb the correlation of RGDP.

Trade comovement slope with financial controls. Previous studies found that financial interconnection is significantly (and negatively) associated with GDP comovements. [Kalemli-Ozcan et al. \(2013\)](#) identifies a strong negative effect of banking integration on output synchronization, conditional on global shocks and country-pair heterogeneity. We test the validity of our results controlling for such linkages.

In row 18 to 21, we provide additional robustness of the trade comovement slope using financial controls. We construct two additional variables capturing the financial interconnection between every country-pairs. First, we construct an index of financial integration (FI) using Foreign Direct Investment (FDI) data, as follows: $FI_{ijt} = \frac{FDI_{i \rightarrow j,t} + FDI_{j \rightarrow i,t}}{GDP_{it} + GDP_{jt}}$. Second, we use the total bilateral cross-border claims (including bank and non-bank sectors for all maturities) from the consolidated banking statistics from the Bank for International Settlement to construct an index of financial proximity (FP) between a country i and j : $FP_{ijt} = \frac{C_{i \rightarrow j,t} + C_{j \rightarrow i,t}}{GDP_{it} + GDP_{jt}}$, where here $C_{i \rightarrow j,t}$ refers to total cross-border claims from country i to country j .

The results are robust to the inclusion of financial controls. However, it's worth noting that the same specification, which includes the correlation of first difference GDP along with the financial proximity index, does not yield significant results. [de Soyres and Gaillard \(2022\)](#) demonstrate consistent findings using a larger sample that includes both high and low-income countries.

Alternative measure of SR correlation. We finally investigate the results if we use alternative measures of the Solow Residual. In the core paper, the Solow Residual is constructed using

the variables of real GDP ($rgdpna$), real capital service ($rkna$), and total employment (emp), such that: $SR_{it} = \log(rgdpna_{it}) - \alpha \log(rkna_{it}) - (1 - \alpha) \log(emp_{it})$, with $\alpha = 1/3$.

Row twenty-two utilizes the Solow Residual constructed using the real capital stock ($rkna$), such that: $SR_{it} = \log(rgdpna_{it}) - \alpha \log(rkna_{it}) - (1 - \alpha) \log(emp_{it})$, with $\alpha = 1/3$.

Row twenty-three utilizes the Solow Residual constructed using the time-varying labor share of the PWT9.1, such that $SR_{it} = \log(rgdpna_{it}) - \alpha_{it} \log(rkna_{it}) - (1 - \alpha_{it}) \log(emp_{it})$.

Row twenty-four uses the Solow Residual constructed using, in addition to employment, the number of hours worked (avh), such that: $SR_{it} = \log(rgdpna_{it}) - \alpha \log(rkna_{it}) - (1 - \alpha) \log(emp_{it} * avh_{it})$.

In these robustness checks, the TC-slope is positive and significant when using trade in intermediate inputs, while it is either not significant or even negative when using trade in final goods.

OA1.5 Trade and GDP components (investment and labor)

To better understand the source of the positive association between GDP comovement and trade, we now propose a simple refinement to the empirical analysis performed so far.

We decompose GDP fluctuations into changes in factor supply (labor and capital) and variations of the Solow Residual (SR). A natural question to ask is: if trade is associated with higher GDP correlation, is it also associated with higher synchronization of labor and/or capital movements? This is an important question because its answer should guide our theoretical construction. For example, [Johnson \(2014\)](#) notes that, in a perfectly competitive framework with constant returns to scale, “*real value added depends on productivity and factor inputs alone*”. In his framework, when the correlation of technology shocks across countries is independent of trade, then the trade-GDP comovement slope can only be generated by an increased correlation of factor supply.

However, in a framework where the Solow Residual does not only fluctuate due to changes in technology, one should not only focus on the comovement in factor supply. Indeed, as discussed in the core paper, a key element in solving the Trade Comovement Puzzle lies in our recognition that real GDP fluctuations are not restricted to movements in technology, labor and capital.

We examine this issue by investigating the relationship between trade and factor supply synchronization. Denoting $\text{Corr } L_{ijt}$ and $\text{Corr } I_{ijt}$ the correlation of labor and investment between

countries i and j at time t , we estimate:

$$\text{Corr } L_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (2)$$

$$\text{Corr } I_{ijt} = \beta_1 \ln(\text{Trade}_{ijt}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijt}^{\text{final}}) + \text{controls}_{ijt} + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (3)$$

Investment and labor are measured using the variable total investment and employment in the PWT 9.1. We acknowledge that using total employment as a proxy for labor supply is subject to a number of limitations, with issues regarding differences in skills and un-observable hours worked for example.

Results in Table 3 reveal two key insights into the relationship between trade and real GDP synchronization. First, we note that higher trade integration in intermediate inputs is not statistically significantly associated with an increase in labor comovement, although the estimates are positive. Regarding investment correlation, the result is positive and significant using the HP filter. This weak increased correlation in factor supply for country-pairs increasing their trade links suggests that models where GDP synchronization is achieved by inducing a strong factor supply reaction to a foreign shock are likely to be at odds with the data. Comparatively, models in which an increase in trade linkages is associated with higher co-movement of the Solow Residual are in line with the data.

Once again, it is important to remember that our panel specifications are not intended to use information about the *level* of GDP comovement across countries, but rather to account for the *change* in GDP comovement when countries are more integrated through trade. Hence, although the *level* of factor supply synchronization is high in the data, this synchronization does not seem to systematically increase with trade proximity. As a result, we argue that an important part of the high value of the observed trade-comovement slope comes from an increase in the synchronization of the Solow Residual, as shown in the core paper, which can arise from a synchronization of aggregate profits.

OA1.6 The Role of the Extensive Margin

We now conduct two robustness checks regarding the role of the extensive margin. We first focus on firm-level data, and then use the HS6 classification of exported goods as a variety.

Table. 3. TC-slope with respect to GDP components (investment and labor)

	Coeff. trade in inputs	Coeff. trade in fi- nal goods	Filter	Countries Obs.	Period	TW	CP
Investment correlation	0.045*	−0.102***	HP	40 2,380	1970-2009	Yes	Yes
Investment correlation	0.013	−0.062**	FD	40 2,380	1970-2009	Yes	Yes
Labor correlation	0.020	−0.045*	HP	40 2,380	1970-2009	Yes	Yes
Labor correlation	0.007	−0.017	FD	40 2,380	1970-2009	Yes	Yes

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. TW stands for Time Windows Fixed Effects while CP stands Country-Pairs Fixed Effects, which are included in all our analysis.

OA1.6.1 EM-IM decomposition using firm-level data

We now use the Exporter Dynamics Database (EDD) from the World Bank and test whether a change in the number of exporters (EM) and a change in the average value added per exporter (IM) are correlated with changes in real GDP comovement. This database provides measures of micro-characteristics of the export sector; number of exporters (their size and growth), their dynamics in terms of entry, exit and survival, and the average unit prices of the products they trade, across 70 countries from 1997 to 2014. Over this time period, we average real GDP (transformed with log and HP-filter or first difference) correlations between country-pairs at quarterly frequency over 3 time-windows of 5 years, starting in 1997-Q1.⁴ Due to the lack of coverage of the EDD, we use the only reported information of a reference country within a country-pair as direct measure for the EM and the IM.⁵ Unfortunately, it is not possible to distinguish trade in intermediate goods and trade in final goods.

We measure the EM using the number of new exporters net of exiting firms between country i and country j , normalized by the total number of exporters. For the IM, we use the natural logarithm of the average value added per exporter. We test:

$$\text{Corr GDP}_{ijt} = \beta_1 \left[\frac{\text{Entry} - \text{Exit}}{\text{Nb Exp}} \right]_{ijt} + \beta_2 \ln \left(\left[\frac{\text{value}}{\text{exporter}} \right]_{ijt} \right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (4)$$

Table 4, column (Avg.), summarizes the results. Point estimates imply that an increase of 1% of the number of new net exporters is associated with an increase in real GDP correlation of about 3.5%. On the contrary, we find that the relationship between the IM and GDP correlation is not statistically significant. We then investigate in column (Std.) whether more variability along

⁴Unfortunately, OECD real GDP at quarterly frequency is not available for all the countries. We therefore reduce the sample.

⁵For instance, the database contains information about exports from Belgium to many destinations, but there is no information about Belgium's imports. It is therefore not possible to compute symmetric measures.

the extensive and intensive margins are associated with more real GDP correlation within the considered time-windows. We test:

$$\text{Corr GDP}_{ijt} = \beta_1 \ln(\text{std nb exp}_{ijt}) + \beta_2 \ln \left(\left[\text{std} \frac{\text{value}}{\text{exporter}} \right]_{ijt} \right) + \text{CP}_{ij} + \text{TW}_t + \epsilon_{ijt} \quad (5)$$

Results feature a positive relationship between variations in the number of exporters and GDP correlation, while variations along the intensive margin is negatively correlated with GDP co-movement. The results are not significant at the 10% level in this case.

Table. 4. Cross-country GDP correlations and the margins of trade.

	Corr GDP ^{HP filter}		Corr Δ GDP	
	(Avg.) specification	(Std.) specification	(Avg.) specification	(Std.) specification
EM measure	3.558*** (1.223)	0.091 (0.057)	2.586*** (1.289)	0.050 (0.054)
IM measure	0.013 (0.166)	-0.078** (0.038)	-0.159 (0.195)	-0.062 (0.050)
CP + TW FE	Yes	Yes	Yes	Yes
N	120	120	120	120
R^2	0.205	0.093	0.090	0.032

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. We use EDD data from 1997 to 2014.

Avg. refers to specifications where we assess the link between GDP comovement and the average of each margin. Std. refers to specifications where we assess the link between GDP comovement and the volatility (standard deviation) of each margin in each configuration.

OA1.6.2 Robustness: HK decomposition using HS6 classification

In table 5, we provide the results of the estimation performed in section 6 of the paper using the [Hummels and Klenow \(2005\)](#) (HK) decomposition with the HS6 classification. It turns out that our results are consistent with this alternative specification (i.e. the slope is positive with respect to EM and negative with respect to IM). Only results with volatility of the EM measures is statistically significant. This could be due to the fact that we are not able to identify enough variation in the data with only two time windows, since our sample with the HS6 classification started from 1995 to 2006.

OA2 Theory: An Economy Without Imported Inputs

In order to clarify the importance of international input-output linkages in generating a link between foreign shocks and domestic productivity, we review the intuitions presented in the

Table. 5. Extensive and Intensive margins and GDP correlation with 5 years time windows using HS6

	Corr GDP ^{HP filter}		Corr Δ GDP	
	(1)	(2)	(3)	(4)
ln(EM)	0.057 (0.087)		0.059 (0.098)	
ln(IM)	-0.103** (0.051)		-0.204*** (0.059)	
ln(std(EM))		0.050** (0.024)		0.031 (0.027)
ln(std(IM))		-0.034* (0.017)		-0.123*** (0.019)
Country-Pair FE	Yes	Yes	Yes	Yes
Time Window FE	No	Yes	No	Yes
N	1,122	1,122	1,122	1,122
R^2	0.238	0.239	0.209	0.066

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation.

paper in a situation with no imported input. In such a case, we show that markups (μ_{nt}) impact the measurement of GDP fluctuations only insofar that they vary over time. In other words, constant markups do not introduce any link between foreign shocks and domestic productivity.

Basic Setup. In absence of imported inputs, the production function in country n at time t is directly expressed in value added terms as:

$$Y_{nt} = Z_{nt} K_{nt}^{\alpha} L_{nt}^{1-\alpha}$$

where Y_{nt} is the quantity of goods produced, K_{nt} and L_{nt} are the capital and labor inputs and Z_t is a measure of a country's efficiency at transforming inputs into output. Real GDP change between $t - 1$ and t is constructed using previous period prices as base period prices, such that:

$$\widehat{GDP}_{nt} = \frac{P_{nt-1} \Delta Y_{nt}}{P_{nt-1} Y_{nt-1}} = \hat{Y}_{nt} \approx \hat{Z}_{nt} + \alpha \hat{K}_{nt} + (1 - \alpha) \hat{L}_{nt} \quad (6)$$

Equation (6) illustrates that without international input-output linkages, the presence of markups in the base period prices used in RGDP construction does not introduce a term that creates a link between foreign goods and real GDP fluctuation. Keeping the TFP as an exogenous variable, this means that any endogenous change in real GDP in response to a foreign shocks comes from a change in factor supply ($\hat{K}_{nt}$ or $\hat{L}_{nt}$). If foreign goods and domestic inputs are complement, domestic real GDP can increase as a response to a positive foreign shock. However, this response

is disciplined by the elasticity of factor supply.

Productivity. Based on equation (6), we can construct proportional changes in the Solow Residual $\widehat{SR}_{nt}$ as:

$$\widehat{SR}_{nt} = \widehat{GDP}_{nt} - \alpha \widehat{K}_{nt} - (1 - \alpha) \widehat{L}_{nt} = \widehat{Z}_{nt}$$

Without international input-output linkages, the negative result from [Kehoe and Ruhl \(2008\)](#) holds both with and without markups: domestic productivity is only driven by domestic technology and does not react to foreign shocks. Note however that despite the fact that μ_{nt} does not appear directly in equation (6), the presence of *variable* markups would have an impact on real GDP through its impact on labor and capital supply.

OA3 Quantitative Appendix

OA3.1 Detailed parameter values

Table 6 reports the parameter values used throughout the quantitative analysis.

OA3.2 SR measurement and attenuation bias

A possible concern when evaluating the trade-productivity slope is whether the Solow Residual (SR) is properly measured. As is standard, we defined SR such that it captures the change in real GDP ($RGDP$) that are not explained by movements of Labor and Capital. However, [Huo et al. \(2020\)](#) highlighted that such an approach is not perfect, as standard measures of factor inputs might be biased due to the presence of unobserved elements. As discussed in their paper, effective labor used in production might include a level of “unobserved effort” which implies that measures such as hours worked, which is available in standard database, do not accurately describe the effective level of labor input.

To better understand how measurement error can affect our results, we use our model as a data generating process. In particular, we assume that labor input effectively used in production is the combination of an observed factor $\tilde{L}_{it}$ (for instance, total employment), and an unobserved factor u_{it} (for instance, labor utilization), so that $L_{it} = \tilde{L}_{it} * u_{it}$. To investigate how the presence of unobserved factors impact our results, we define the "true" and the "mis-measured" Solow

Table. 6. Parameter values

ISO	γ_i	f_i^E / f_{US}^E	$\sigma_i / (\sigma_i - 1)$	L_i^{ss}
ARG	0.29	4.38	1.56	13.60
AUS	0.31	0.45	1.19	8.90
AUT	0.30	3.93	1.16	3.76
BEL	0.24	0.71	1.23	4.12
BRA	0.33	14.93	1.59	68.13
CAN	0.35	0.98	1.28	14.95
CHE	0.31	1.79	1.72	3.93
CHL	0.29	1.34	1.58	5.54
CHN	0.10	5.25	1.31	735.72
DEU	0.32	2.59	1.20	39.60
DNK	0.31	1.07	1.47	2.76
ESP	0.30	2.50	1.23	16.61
FRA	0.33	0.80	1.21	25.63
GBR	0.33	1.07	1.33	27.38
GRC	0.44	2.32	1.20	4.45
IND	0.31	6.25	1.12	409.25
IRL	0.22	1.07	1.36	1.69
ITA	0.28	1.16	1.77	22.92
JPN	0.36	2.00	1.18	65.92
KOR	0.15	0.71	1.11	21.44
MEX	0.42	1.50	1.57	37.88
NLD	0.30	0.62	1.23	8.20
NOR	0.38	0.89	1.35	2.32
NZL	0.29	0.09	1.23	1.82
POL	0.26	6.61	1.33	14.48
PRT	0.30	1.07	1.19	5.08
SWE	0.29	2.86	1.16	842.13
THA	0.29	5.27	1.38	4.31
TUR	0.30	1.96	1.20	31.47
USA	0.43	1.00	1.47	20.09
RoW	0.29	4.03	1.36	138.64
Mean	0.30	2.62	1.33	83.96

Residuals as follows:

$$\text{"True" SR : } SR_{it} = \log(RGDP_{it}) - \alpha \log(L_{it}) - (1 - \alpha) * \log(K_{it}) \quad (7)$$

$$\text{"Mis-measured" SR : } SR_{it}^{measured} = \log(RGDP_{it}) - \alpha \log(\tilde{L}_{it}) - (1 - \alpha) * \log(K_{it}) \quad (8)$$

We model the unobserved part of labor input u_{it} as a stochastic variable and assume that it is positively synchronized with technology $Z_{i,t}$, i.e. $cov(Z_{i,t}, u_{it}) > 0$. This assumption captures the intuition that unobserved labor utilization (or effort level) is likely to co-move with the state of the aggregate economy. An econometrician only observes $\tilde{L}_{it}$ and hence takes $SR_{it}^{measured}$ as a definition of the Solow Residual, even though it is not an exact measure of SR_{it} . The higher the variance of the unobserved factor u_{it} , the higher the generated measurement error in the

evaluation of SR_{it} .

We investigate the consequences of measurement errors due to the presence of an unobserved factor on our results by setting $u_{it} = v_{it} + \mu\epsilon_{it}$, with ϵ_{it} the innovation used in the AR(1) generation of Z_{it} and $v_{it} \sim \mathcal{N}(\bar{v}, \sigma_v^2)$. The parameter μ captures the correlation between the unobserved factor with the aggregate TFP shock and v_{it} introduces additional movements in the unobserved factor that are not linked to the TFP shock. We set $\sigma_v = 0.032$, $\mu = 0.833$ and $\bar{v} = 0.17$. Table 7 provides the results and compares the result of using "true" and "mis-measured" SR in our regressions.

As expected, because L is not used directly in the definition of $RGDP$, the trade-comovement slope with respect to $RGDP$ is not affected by the presence of measurement error, as shown in the first row. More importantly, the presence of a measurement error in labor supply has consequences on both the SR -trade slope (second row) and the $RGDP$ -trade slope once controlling for $corr(SR)$ (third row). First, the SR -slope is positive and significant for trade intensity in intermediate inputs, but the point estimate associated with a "mis-measured" SR is lower than its counterpart using the "true" value for SR . This is intuitive: "mis-measured" SR is more noisy than its "true" counterpart, and the cross-country correlation of the SR is lower. As a result, the explanatory power associated with trade in intermediate inputs is reduced. Second, in the third row, we look at the inclusion of $corr(SR)$ as a control in the regression of $corr(RGDP)$ on trade intensity. When $corr(SR)$ is properly measured, its inclusion as a control brings the input-trade slope close to zero. However, when $corr(SR)$ is mis-measured, the input-trade slope is reduced but still remains at 0.01, ten times higher than the point estimate of 0.001 obtained when using "true" $corr(SR)$ as a control. The presence of measurement error generates an attenuation bias which reduces the estimated association between $corr(RGDP)$ and $corr(SR)$. This, in turn, increases the point estimate for input trade. Overall, this analysis highlights that the presence of measurement error can explain why, in our empirical investigation, the inclusion of $corr(SR)$ as a control does not bring the trade slope to zero.

Table. 7. SR-trade comovement with measurement error, using model simulations

	"True" value for SR		With measurement error	
	<i>Input</i>	<i>Final</i>	<i>Input</i>	<i>Final</i>
$corr(RGDP)$ - trade slope	0.056***	0.006***	0.056***	0.006***
$corr(SR)$ - trade slope	0.051***	0.003***	0.045***	0.005***
$corr(RGDP)$ - trade slope, controlling for $corr(SR)$	0.001***	0.003***	0.010***	0.001***

*p<0.1; **p<0.05; ***p<0.01. SE clustered by country-pairs. Note that the simulations used here are based on a calibration with 15 countries among the set of 35 countries in the baseline model of the core paper.

OA3.3 Comparison to [Liao and Santacreu \(2015\)](#)

In our paper, we delve deeper into the empirical relationship between trade linkages and cross-country GDP correlations, differentiating our approach from that of [Liao and Santacreu \(2015\)](#). While both studies explore the trade comovement slope (TC-slope), our findings diverge primarily due to differences in model specification and data treatment.

In Table 8, we conduct a comprehensive quantitative analysis to better understand these differences. These analyses are conducted, as in [Liao and Santacreu \(2015\)](#), using a two-country setup. To this end, we use the exact same model as the core paper, but calibrate the model for Europe and the United States. All other countries are bundled into the "Rest of the World". To be even more comparable to [Liao and Santacreu \(2015\)](#), we assume here that TFP shocks across countries are uncorrelated. The extended analysis and robustness checks provide a thorough validation of our findings, emphasizing the significance of modeling choices in determining the relationship between trade linkages and GDP correlations.

First, our model employs a nested CES production function, contrasting sharply with the CES production function utilized by [Liao and Santacreu \(2015\)](#). Our function aggregates intermediate inputs and final goods within a framework that ensures complementarity between domestic and foreign varieties, thanks to a Cobb-Douglas aggregation in the Armington elasticity setup. This specification allows us to calibrate our model with an elasticity of substitution (ρ_F and ρ_I) that supports stronger markup values while maintaining a positive trade-comovement slope, even when elasticity values increase—a scenario under which [Liao and Santacreu \(2015\)](#) observe a negative slope. As a first experiment, we show in Table 8 that higher values of the markups $\sigma/(\sigma - 1)$ tend to increase the TC-slope, while higher values of the final goods elasticity ρ_F or ρ_I tend to lower the TC-slope. When both ρ_F and σ increase in our two-period equivalent model, we find that the TC-slope increases relative to the benchmark, showing that the markup

effects seem to dominate the higher final goods elasticity of substitution. However, notice that the TC-slope regarding final goods is negative in that case, consistent with data.

Second, our model incorporates a complex network of global value chains (GVCs), which allows for intermediate goods produced in one country to be re-exported multiple times for production in other countries. This nuanced modeling of intermediate goods results in stronger trade linkages, contrasting with the assumptions found in [Liao and Santacreu \(2015\)](#), where intermediate goods are primarily considered final goods in their framework. In a model where there are no inputs traded (but solely trade in final goods), we find that the TC-slope is much weaker, with a slope of 0.013. The results even turn negative, as in [Liao and Santacreu \(2015\)](#), when the Armington elasticity is set to 2.8 or 3.1, consistent with their calibration.

Third, other assumptions such as bond market economy or Cobb-Douglas utility do not appear to be critical for the TC-slope. In both cases, we find that the TC-slope is stronger than that reported in [Liao and Santacreu \(2015\)](#).

To sum up, the crux of the difference comes from the specification of trade linkages and the CES structure of the economy.

Table. 8. Sensitivity Analysis using a two-country version.
Alternative specifications for comparison with [Liao and Santacreu \(2015\)](#)

Variation of the Model	ρ_I	ρ_F	σ	$\frac{\omega_i^I(j)}{\omega_{i,bench}^I(j)}$	TC_I^{slope}	TC_F^{slope}	TC_{tot}^{slope}
1. Benchmark	1.0	1.0	4.0	1.0	0.044***	0.013***	0.060***
2. Alt. σ elasticity	1.0	1.0	2.8	1.0	0.052***	0.011***	0.070***
3. Alt. σ elasticity	1.0	1.0	3.1	1.0	0.049***	0.012***	0.067***
4. Alt. ρ^F elasticity	1.0	2.8	4.0	1.0	0.046***	-0.021***	0.052***
5. Alt. ρ^F elasticity	1.0	3.1	4.0	1.0	0.045***	-0.023***	0.051***
6. Alt. ρ^I elasticity	1.5	1.0	4.0	1.0	0.030***	0.018***	0.044***
7. Alt. ρ^I elasticity	2.0	1.0	4.0	1.0	0.023***	0.017***	0.034***
8. Alt. ρ^F, σ elasticity	1.0	2.8	2.8	1.0	0.058***	-0.030***	0.064***
9. Alt. ρ^F, σ elasticity	1.0	3.1	3.1	1.0	0.053***	-0.028***	0.059***
10. Cobb-Douglas utility	1.0	1.0	4.0	1.0	0.042***	0.012***	0.058***
11. Complete Markets	1.0	1.0	4.0	1.0	0.032***	0.015***	0.047***
12. No inputs traded	1.0	1.0	4.0	0.0	—	0.013***	0.013***
13. No inputs traded – no EM	1.0	1.0	4.0	0.0	—	0.003***	0.003***
14. No inputs traded – no EM & $\sigma = \infty$	1.0	1.0	4.0	0.0	—	0.005***	0.005***
15. No inputs traded	1.0	2.8	2.8	0.0	—	-0.027***	-0.028***
16. No inputs traded	1.0	3.1	3.1	0.0	—	-0.026***	-0.026***

The results reported in this table are based on simulations of a two-country version of our model with no correlated shocks across countries. We take Europe and United States to calibrate the steady-state trade linkages. In order to be comparable to [Liao and Santacreu \(2015\)](#), price indices are not corrected for variety effect here.

OA3.4 How the Number of Countries in Models' Simulation Influences the TC slope?

Previous research typically employs a two- or three-country model to assess the TC-slope. Our findings, however, demonstrate that the scale of the economy significantly influences the TC-slope, which we illustrate using a partitioning method.

The partitioning method involves dividing the 30 countries from our main analysis into subgroups. We then run the models with two subgroups at a time, repeating the process until we have covered all the country pairs in our empirical analysis. Note that we maintain the same steady-state calibration as in the core paper but adjust the partitioning to include only a few countries at a time from the initial 30 countries. All remaining countries are grouped into the 'Rest of the World' (RoW) category.

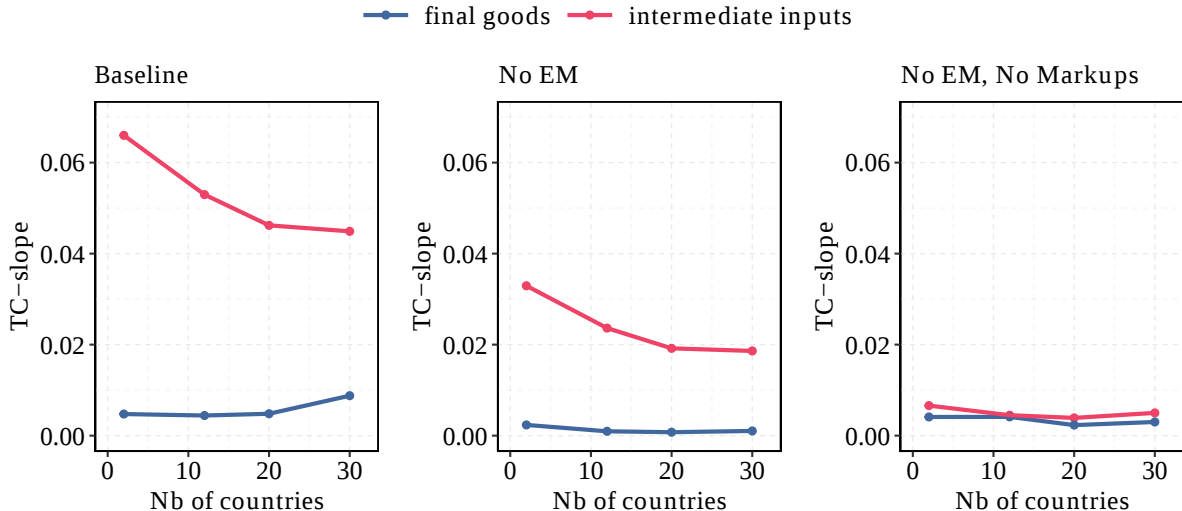
For instance, let us take the example of a 3 countries partitioning. If a first group of countries, such as Germany (GER), France (FRA), and Italy (ITA), is partitioned with a second group, namely the USA, Canada (CAN), and Mexico (MEX), then we use all the trade shares for those country-pairs in the calibration and bundle all remaining countries into a RoW category. Thus, in this example of partitioning, there are $3 + 3$ countries, plus the RoW (+1). Thus, the size of the RoW country depends on the size of the partitioning. However, note that we *do not include* country-pairs with RoW in the model-based estimation of the TC-slope. Because all countries fall in one of the group, we always recover all the country-pairs of our empirical sample. In this example, with 3 countries per group, there are a total of 10 groups.

To further explore the effect of model scale, we first employ our partitioning method to sequentially simulate the model with two countries at a time from our empirical sample (thus with 30 subgroups of one country), grouping the rest into a RoW category. This exercise is crucial as the two- to three-country model represents an important benchmark in this literature. Specifically, we start by simulating a two-country model version with, say, FRA + GER and the rest as RoW. We then save the resulting GDP correlation and trade linkages. We then repeat this two-country setup for every other possible pair of countries, aiming to produce the same statistics used in our empirical analysis. This repeated two-country simulation yields a TC-slope of 0.066, which is significantly higher than the 0.045 reported in the full-size model.

The impact of model scale for various partitioning is depicted in Figure 2, where we plot the TC-slope against the number of countries in each simulation. The 30-country scenario represents a non-partitioned model, whereas other data points reflect varying degrees of partitioning. For instance, a scenario with 20 countries involves partitioning into three groups of 10 countries

each. It becomes evident that the TC-slope decreases as the number of countries in the simulation increases, and the influence of scale diminishes as more country pairs are added to the simulation.

Figure 2. TC-slope as a function of the number of countries in the simulation



In conclusion, our analysis underscores the importance of considering the scale of the economy in quantitative models. We demonstrate that smaller-scale models might overestimate the TC-slope, particularly in models that incorporate profits and extensive margins like ours. This occurs because scaling up the model introduces a broader array of simultaneous shocks, which in turn reduces the GDP correlation within a given country pair, leading to a form of attenuation bias.

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