Spatial Externalities, Inefficiency, and Sufficient Statistics

Gabriel Kreindler and Kartik Patekar

AEA P&P Developments in Infrastructure and Transportation Session

Spatial Externalities in Complex Models

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  - Positive: firm agglomeration, positive amenities
  - Negative: EV charging queues, traffic congestion delays, local air pollution

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  - Are results driven by functional form or empirical moments?
  - What can we learn with objects estimated around an unpriced equilibrium?

This paper

- Sufficient statistics in model with spatial externalities
  - Show large externalities, large own- elasticity, and **small** deadweight loss can coexist
  - Derive appropriate version of “Harberger triangle”
  - Highlight flexible substitution patterns essential for modeling and empirical work

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Related Literature:

- Builds on classic Harberger (1964) *The Measurement of Waste*
- "Sufficient statistics" approach in public finance (Chetty, 2009; Kleven, 2021)
- Non-parametric impact of shocks in macro, trade, spatial (Baqaee & Farhi, 2020; Donald et al., 2024; Hulten, 1978)

Equilibrium Model of Location Choice with Spatial Externalities

- Unit mass of agents. Agent ω picks location $k \in \{1, \ldots, M\}$ with indirect utility

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- Assumptions: F continuously differentiable and $\sum_k \pi_k = 1$
- Allow for flexible substitution patterns across locations (not only logit)

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- Deterministic utility u_k sum of three components

$$u_k = u_k^0 + e_k(\pi) + c_k$$

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3. **transfers:** c_k, location-specific, redistributed lump sum

Equilibrium and Welfare

- Equilibrium (u, π) given transfers c and amenities u^0 (vector notation, e.g. $u = (u_k)_k$)

$$u = u^0 + e(\pi) + c$$
$$\pi = F(u)$$

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- Welfare: utilitarian with transfers redistributed lump-sum

$$W(c) = \mathbb{E} \max_k [u_k + \epsilon_k(\omega)] - c'\pi$$

Model Applications and Limitations

- Applications: settings with *multi-dimensional* and *spatial* externalities:
    - road traffic congestion, peak-hour traffic congestion (departure time)
    - queuing (local externality): EV charging stations, port choice.
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    - local prices change economics of lump-sum redistribution (Donald et al., 2024; Fajgelbaum & Gaubert, 2020)
  - no supply side market prices for locations/ goods (such as housing prices)

Two Key Matrices Help Describe Local Changes Around Eqm

1. Slutsky demand substitution matrix

$$D \equiv \frac{d\pi}{du} = \left(\frac{d\pi_i}{du_j}\right)_{ij}$$

   - How much does location i share change from *increase* in transfer at j?
   - D positive semi-definite, positive diagonal, non-positive off-diagonal, zero rows sum

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2. Marginal externalities

$$E' \equiv \frac{de(\pi)}{d\pi} = \left(\frac{de_i(\pi)}{d\pi_j}\right)_{ij}$$

   - E_{ji} = marginal externality of an agent at *j* on any agent at *i*
   - no restrictions on sign of E_{ji}

Equilibrium Impact of Transfers

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- **Assumption 1.** All eigenvalues of $E'D$ are less than 1 in absolute value.
  - *Note: can be verified empirically*

Intuition

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  with *equilibrium elasticity* matrix:

$$K \equiv D(I - E'D)^{-1}$$

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  The change in welfare is:

$$dW = \underbrace{(E\pi - c)'}_{\text{net externality}} d\pi$$

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Key implications:

- For empirics: estimate matrices E and D (or K) with variation around current equilibrium

⇒ *Optimal Small Charges*

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Key implications:

- For empirics: estimate matrices E and D (or K) with variation around current equilibrium
- For modeling: allow flexible D, E matrices (logit: $D = \mathrm{diag}(\pi) - \pi\pi'$)

Optimal Small Charges

Approximating Deadweight Loss

- Move further away from current eqm with 2nd order Taylor approx at $c = 0$

⇛ *More general statement*

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and deadweight loss is approximately

$$DWL \equiv W(c^{so}) - W(0) \approx -\frac{1}{2}(c^{so})' \underbrace{(EK - I)'K}_{R} c^{so}$$

⊜ *More general statement*

Relation to Harberger (1964)

- Harberger (1964) studies DWL in an efficient economy with taxes and finds

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- We offer two intuitive ways to interpret our result (and Harberger's)

Intuition for the Magnitude of Deadweight Loss 1/2

- R is negative-semidefinite with row sum 0. We can rewrite

$$DWL \approx -\frac{1}{2}\tau' R\tau = \frac{1}{4}\sum_{i,j}(c_i^{so} - c_j^{so})^2 R_{ij}$$

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    - Just one condition not enough!
- Still "$DWL \approx \frac{1}{2}$elasticity $\times$ externality" but with appropriate definition of these terms

Intuition for the Magnitude of Deadweight Loss 2/2

- With $\Delta\pi = \pi^{so} - \pi^0$

$$DWL \approx \frac{1}{2}(c^{so})'\Delta\pi - \frac{1}{2}(\Delta\pi)'E\Delta\pi$$

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    - In general, correction term can be positive/negative

Example 1: Local Congestion and Capacity Expansions

- **Local congestion:** $e_j(\pi) = e(\pi_j/\kappa_j)$ where κ_j is **capacity** at site j, $e' < 0$
- Examples: queues to access port services, EV chargers

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where $E = \text{diag}((e'(\pi_j/\kappa_j)/\kappa_j)_j)$.

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- Equilibrium utility changes smaller than partial-eqm (in L2 norm)
- Highlights importance of demand substitution matrix D.

Example 2: Peak-Hour Road Traffic Congestion

- Kreindler (2024) studies peak-hour road traffic congestion
- Locations $k \Leftrightarrow$ **departure time** h

$$\text{Departure rates } \pi(h) \rightarrow \text{travel times } T(h) \rightarrow \text{utilities } u(h) \rightarrow \pi$$

⇒ *Utility function*

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  - Externality $e(\pi, \zeta^A)$. Longer travel time effect depends on already arriving early vs late
- Equilibrium conditions depend on ideal arrival time ζ^A

$$u(\zeta) = u^0(\zeta) + e(\pi, \zeta) + c, \forall \zeta$$

$$\pi = \int F(u(\zeta), \zeta) dG(\zeta)$$

⇨ *Utility function*

Example 2: Peak-Hour Road Traffic Congestion

- Similar method to approximate deadweight loss: integrate over ζ^A

$$DWL \approx (c^{so})'(I - M_{ED} - M_{EDE'}K)'Kc^{so}$$

where

$$\begin{aligned}
c^{so} &= (I - M_{ED} - M_{EDE'}K)^{-1} M_{E\pi^0} \\
K &= (I - M'_{ED})^{-1} M_D \\
M_D &= \int D(\zeta^A) dG(\zeta^A) \\
M_{E\pi^0} &= \int E(\zeta^A)\pi^0(\zeta^A) dG(\zeta^A) \\
M_{ED} &= \int E(\zeta^A) D(\zeta^A) dG(\zeta^A) \\
M_{EDE'} &= \int E(\zeta^A) D(\zeta^A) E(\zeta^A)' dG(\zeta^A)
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- Full model estimated using demand side experiment and externality calibration

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- Small DWL with both approaches

	(1) Welfare (INR)	(2) DWL (INR)	(3) DWL (%)
Nash	-405		
Social Optimum (Full Model)	-399	5.8	1.4%
Social Optimum (2nd Order)	-412	7.7	1.9%

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- Small DWL with both approaches
- We slightly overestimate DWL

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Discussion

- What can we learn about externalities with **variation around unpriced eqm**?
- **Sufficient statistics** for small charges, optimum, and deadweight loss
- Expressions highlight importance of allowing for **flexible substitution patterns**
- Future work: use these expressions to evaluate robustness of model and variation

Impact of Local Charges — Intuition

Back

- What is the local effect of imposing charges?
- Imagine that agents observe current charges and externalities at discrete intervals, and modify preferences
  - at time t = 0, locations get additional charges $c = (c_k)_k$
  - In period 1, agents move across locations by $\Delta\pi^1 = Dc$
  - In period 2, they observe additional externalities $E'\Delta\pi^1$ and further move by by $\Delta\pi^2 = DE'\Delta\pi^1 = DE'Dc$
  - So on... In period n, agents move by $\Delta\pi^n = D(E'D)^{n-1}c$
  - Overall change in preferences is $\Delta\pi = \Delta\pi^1 + \Delta\pi^2 + \Delta\pi^3 + \ldots$
- For convergence, we require $\Delta\pi^n \to 0$ or equivalently $(E'D)^n \to 0$

Small Optimal Charges

Back

- Can use Prop. 1 to identify optimal "small" charges
- Under restriction that the induced "migration" $d\pi = Kdc$ is small ($||d\pi||_2 \leq \eta$), optimal charges that maximally increase welfare satisfy $d\pi = Kdc \propto E\pi - c$
- Can also solve for bounded $||c||_2$

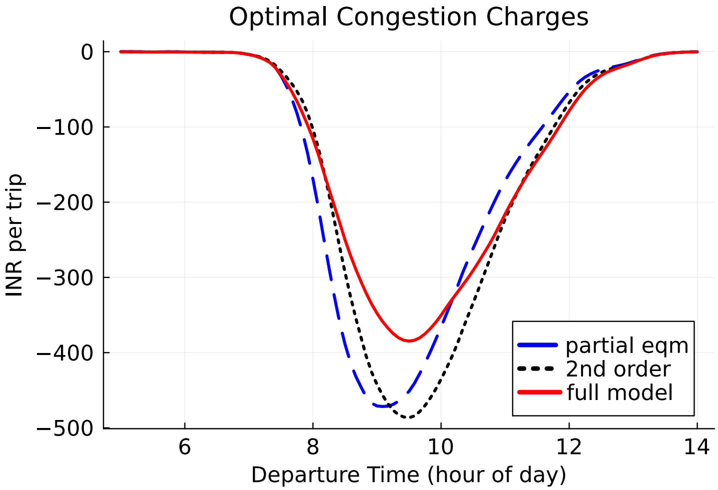
Kreindler (2024) Utility Function ⇔ *Back*

$$u(h, T, \zeta^A) = -\alpha T - \beta_E |h + T - \zeta^A|_- - \beta_L |h + T - \zeta^A|_+$$
$$u(h, \zeta^A) = \mathbb{E}_{T \sim \mathcal{T}(h)} u(h, T, \zeta^A) - p(h) + \varepsilon(h)$$

- departure time h, ideal arrival time ζ^A
- travel time T drawn from distribution $\mathcal{T}(h)$
    - $\mathcal{T}$ lognormal, $\mathbb{E}_{T \sim \mathcal{T}(h)} T$ depends on full traffic profile π, variance depends on mean.
- $p(h)$ are departure-time charges
- $\varepsilon(h)$ are Gumbel(σ) iid shocks

Optimal Congestion Charges in Full vs Approximation Models

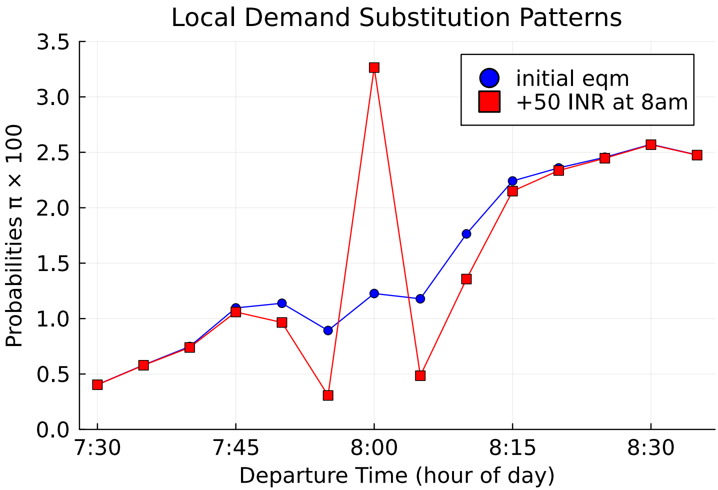
Back



Example of Local Demand Substitution

Back

"Experiment:" pay bonus of 50 INR ($\approx$ 0.75 USD) for leaving at 8am



Deadweight loss - full expression

Back

Proposition: Adding charges c to an unpriced initial equilibrium modifies welfare according to (up to second order in charges)[1]

$$W(c) - W(0) = (E^0\pi^0)'K^0 c + \frac{1}{2}c'Ac + o(c^3)$$

$$A = (E^0\pi^0)'\frac{dK^0}{dc} + (\frac{dE^0}{d\pi}\pi^0 K^0 + E^0K^0 - 1)'K^0$$

All terms in A are evaluated at the unpriced equilibrium. When A is positive semi-definite, welfare is maximized by socially optimal charges c^{so} satisfying $(K^0)'E^0\pi^0 + Ac^{so} \approx 0$. The deadweight loss up to the second order in c^{so} is

$$DWL \equiv W(c^{so}) - W(0) \approx -\frac{1}{2}(c^{so})'\left[(E^0\pi^0)'\underbrace{\frac{dK^0}{dc}}_{\text{tensor}} + [\underbrace{\frac{dE^0}{d\pi}}_{\text{tensor}}\pi^0K^0 + (E^0K^0 - I)']K^0\right]c^{so}$$

1 $\frac{dK^0}{dc}$ and $\frac{dE^0}{d\pi}$ are tensors such that $[\frac{dK^0}{dc}e]_{ij} = \sum_k \frac{dK^0_{ik}}{dc_j}e_k$

Deadweight loss - approximation ⊜ *Back*

Assumption 2: All eigenvalues of ED + E D less than 1 in absolute value.
Assumption 3: There exists a unique equilibrium for any charges c.
Assumption 4: E and D are approximately constant around $c = 0$. Precisely, there exists a ball B_r of radius r such that for any $c_1, c_2 \in B_r$, $\frac{dE^{c_1}}{d\pi}\pi^{c_1} << E^0$ and $\frac{dK^{c_1}}{dc}c_2 << K^0$
Proposition: Let assumptions 1,2,3,4 hold for E, D evaluated at unpriced equilibrium. Additionally, assume that $E\pi^0, (1 - EK)^{-1}E\pi^0 \in B_r$. Then, the socially optimal charges that maximize welfare are given by

$$c^{so} \approx (1 - EK)^{-1}E\pi^0$$

and the deadweight loss is

$$DWL \approx -\frac{1}{2}(c^{so})'(EK^0 - 1)'K^0 c^{so}$$