Supplemental Appendix
“Nature Loss and Climate Change: The Twin-Crises Multiplier”

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The planner chooses physical capital K_1 and land conservation $c = 1 - u$ to maximize

$$W = U\left(F(K_0, (1-c)L, E_0) - K_1\right) + \beta U\left(Y_1(K_1, \bar{u}L, E_1, Z_1)\right) \tag{1}$$

where $Y_1 = (1 - D(Z_1, E_1))F(K_1, \bar{u}L, E_1)$ is output net of climate damages, subject to the two constraints

$$E_1 = G(c, Z_1)$$
$$Z_1 = H(K_1, E_1)$$

which capture how ecosystem services E_1 are affected by conservation and climate change, and climate change is affected by production, respectively. Define

$$\theta^K = \frac{\partial \log H}{\partial \log K_1}$$
$$\theta^E = \frac{\partial \log H}{\partial \log E_1}$$
$$\delta = \frac{\partial \log G}{\partial \log c}$$
$$\gamma = -\frac{\partial \log G}{\partial \log Z_1}$$

The Lagrangian is

$$W + \lambda\left\{G(c, Z_1) - E_1\right\} + \mu\left\{Z_1 - H(K_1, E_1)\right\}$$

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and the first-order conditions with respect to c, K_1, E_1, Z_1 are:

$$
\begin{aligned}
-U_0' F_{L,0} + \lambda G_c &= 0 \\
-U_0' + \beta U_1' F_{K,1} - \mu H_K &= 0 \\
\beta U_1' F_{E,1} - \lambda - \mu H_E &= 0 \\
\beta U_1' F_{Z,1} + \lambda G_Z + \mu &= 0
\end{aligned}
$$

Without climate change, we can ignore the last equation and replace $\lambda = \beta U_1' F_{E,1}$ to obtain two equations

$$
\begin{aligned}
U_0' &= \beta U_1' F_{K,1} \\
U_0' &= \beta U_1' \frac{F_{E,1} G_c}{F_{L,0}}
\end{aligned}
$$

that characterize the standard optimal consumption-saving decision and equalize the marginal returns on physical capital and natural capital.

With climate change, the solution becomes

$$
\begin{aligned}
U_0' &= \beta U_1' \left\{ F_{K,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_K \right\} \\
U_0' &= \beta U_1' \left\{ F_{E,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_E \right\} \frac{G_c}{F_{L,0}}
\end{aligned}
$$

Instead of the simple marginal returns on physical capital $F_{K,1}$ and on natural capital $\frac{F_{E,1} G_c}{F_{L,0}}$ we have modified marginal returns taking into account the feedback loop between nature, economic activity, and climate change. These can be further simplified as

$$
\begin{aligned}
F_{K,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_K &= \frac{Y_1}{K_1} \tilde{\eta}_{K,1} \\
F_{E,1} + \frac{F_{Z,1} + F_{E,1} G_Z}{1 - H_E G_Z} H_E &= \frac{Y_1}{E_1} \tilde{\eta}_{E,1}
\end{aligned}
$$

where we define the modified capital and ecosystem elasticities as

$$
\begin{aligned}
\tilde{\eta}_{K,1} &= \eta_{K,1} + \left(\eta_{Z,1} - \eta_{E,1} \gamma \right) \Phi \theta^K \\
\tilde{\eta}_{E,1} &= \Phi \left[\eta_{E,1} + \eta_{Z,1} \theta^E \right]
\end{aligned}
$$

respectively. Therefore, in the case of log-utility $U = \log$, the optimality conditions become

$$\frac{s^*}{1 - s^*} = \beta \tilde{\eta}_{K,1}$$
$$\frac{c^*}{1 - s^*} = \beta \tilde{\eta}_{E,1} \frac{\delta}{\eta_{L,0}}$$

where we denote $s^* = K_1/Y_0$ the optimal savings rate in physical capital, which yields

$$s^* = \frac{\beta \tilde{\eta}_{K,1}}{1 + \beta \tilde{\eta}_{K,1}}$$
$$c^* = 1 - u^* = \frac{\delta}{\eta_{L,0}} \frac{\beta \tilde{\eta}_{E,1}}{1 + \beta \tilde{\eta}_{K,1}}$$

Therefore the ratio of optimal savings rates is

$$\frac{1 - u^*}{s^*} = \frac{\delta}{\eta_{L,0}} \frac{\tilde{\eta}_{E,1}}{\tilde{\eta}_{K,1}}.$$