

SUPPLEMENTAL APPENDIX

Did the U.S. Really Grow Out of Its World War II Debt?

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ONLINE APPENDIX

This Appendix describes various details of our procedures and our data sources.

A1. Exact Equations for the Evolution of Debt

AGGREGATE DEBT DYNAMICS. The analysis in the text assumes that equation (1), which relates the evolution of the debt to interest rates and the primary balance, holds exactly. In fact, there is a residual in this relationship:

$$(A1) \quad D_t = (1 + i_t)D_{t-1} - P_t + \epsilon_t.$$

The residual ϵ_t arises from a number of factors that add to or subtract from the debt besides interest on the debt and the primary balance, including changes in the level of operating cash held by the Treasury and interest paid to the government on assets such as student loans. We measure ϵ_t using (A1) and our series for D_t , i_t , and P_t . For years before 1962, the residual includes modest errors in our measures of interest rates arising from the need to approximate interest net of intragovernmental payments. In our counterfactuals, when we adjust i_t and P_t , we hold the path of ϵ_t constant. Part 6 of this Appendix reports the series for ϵ_t and analyzes its role in debt dynamics.

ACCOUNTING FOR TREASURY BILLS AND TIPS. For most debt issued after the Fed-Treasury Accord, our counterfactuals adjust the interest rate in year t on debt issued at τ by the inflation surprise $\pi_t - \mathbb{E}_\tau[\pi_t]$. However, as discussed in the main text, we assume that inflation surprises do not affect the real returns on Treasury bills or TIPS (inflation-indexed debt). That means we must modify the interest-rate adjustments in equation (6) so that adjustments for surprise inflation apply only to the fraction of the debt that is not T-bills or TIPS. The equation becomes:

$$(A2) \quad x_t^\tau = \begin{cases} 0 & \text{for } \tau \leq 1942 \\ r_t^{\star\tau} - (i_t^\tau - \pi_t) & \text{for } 1943 \leq \tau \leq 1950 \\ r_t^{\star\tau} - (i_t^\tau - \pi_t) & \text{for } \tau = 1951 \text{ and } t > \tau + 1 \\ (1 - s_{t-1}) (r_t^{\star\tau} - (\tilde{i}_t^\tau - \pi_t)) & \text{for } \tau = 1951 \text{ and } t = \tau + 1 \\ \pi_t - \mathbb{E}_\tau[\pi_t] & \text{for } 1952 \leq \tau \leq 1996 \text{ and } t > \tau + 1 \\ (1 - s_{t-1}) (\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } 1952 \leq \tau \leq 1996 \text{ and } t = \tau + 1 \\ (1 - z_{t-1}^\tau) (\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } \tau \geq 1997 \text{ and } t > \tau + 1 \\ (1 - s_{t-1} - z_{t-1}^\tau) (\pi_t - \mathbb{E}_\tau[\pi_t]) & \text{for } \tau \geq 1997 \text{ and } t = \tau + 1 \end{cases}$$

where s_{t-1} is the fraction of debt outstanding at the end of year $t - 1$ and issued during $t - 1$ that is T-bills, z_{t-1}^τ is the fraction of the debt outstanding at the end of $t - 1$ and issued during τ that is TIPS, and $\tilde{i}_t^\tau$ is the average interest rate paid at t on securities other than T-bills that are outstanding at the end of $t - 1$ and issued during τ .

In this equation, for $\tau \geq 1952$ we reduce each interest-rate adjustment by the fraction of debt that is T-bills or TIPS. The adjustment for T-bills is only relevant for $\tau = t - 1$, because all T-bills outstanding at the end of $t - 1$ were issued during $t - 1$, and the adjustment for TIPS is only relevant starting in 1997, when TIPS

were introduced. For most of the peg period, we do not make any adjustment for the fraction of debt that is T-bills, because T-bills were rolled over at the pegged interest rate. An exception is the last year of the peg, fiscal year 1951, because T-bills outstanding at the end of 1951 were rolled over in 1952, after the peg ended.

THE TRANSITIONAL QUARTER. In the main text, we assume that a period is a fiscal year. A nuisance feature of the data is the Transitional Quarter (TQ), the third quarter of calendar year 1976. This quarter is special because the government changed the start of its fiscal year from July to October for fiscal year 1977. In our simulations, we treat the TQ as a period between fiscal years 1976 and 1977. The debt dynamics equation (A1) holds for all periods including the TQ with i_t and P_t in the TQ measured as the non-annualized interest rate and primary balance. The adjustments x_t^τ in equation (A2) also hold with π_t in the TQ measured as the non-annualized inflation rate. The existence of the TQ complicates the measurement of inflation expectations in some periods, as described below.

A2. Measurement of Fiscal Variables

DEBT AND PRIMARY BALANCE. Our series for debt held by the public at the end of each fiscal year comes from the Office of Management and Budget (OMB) database. We compute debt/GDP ratios using the OMB's series for nominal GDP by fiscal year.

We measure debt at its par value, whereas some analyses of debt measure it at its market value (e.g., work on the fiscal theory of the price level by Cochrane 2021). However, this distinction is not important for our purposes. Unexpected changes in interest rates cause market and par values to diverge in the short run, but the two measures of debt move together closely at the horizons relevant for our analysis; see Figure A1. The closeness of the two debt paths reflects the fact

that a revaluation of a security is offset over time by the corresponding change in its market yield.

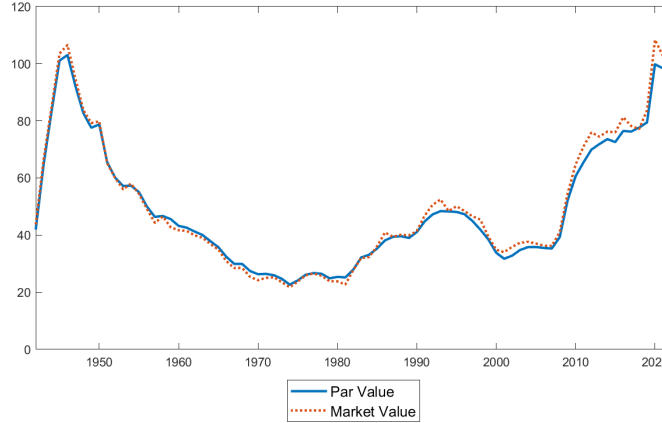


Figure A1. Par Value and Market Value of Federal Debt Held by the Public as a Percent of GDP

Note: The lines represent the ratio of the federal debt held by the public to GDP with debt measured at its par value and at its market value.

Source: Hall, Authors' calculations.

We also use the OMB data to compute the primary balance P_t as the sum of the total fiscal surplus (which is usually negative) and total interest payments, with total interest calculated as described below.

AGGREGATE INTEREST RATES. The aggregate interest rate i_t is defined as total interest payments during period t divided by the stock of debt at the end of $t - 1$. The debt is debt held by the public, and interest payments are the payments on that debt: they exclude intragovernmental payments on debt held by entities such as the Social Security Trust Fund.

For $t \geq 1962$, we compute the appropriate series for interest payments as gross interest on the debt minus intragovernmental interest payments, using the OMB historical database.

For $t < 1962$, OMB does not report intragovernmental interest payments. It reports gross interest and “net interest,” but the latter understates the interest paid on debt held by the public because interest received by the government, as well as intragovernmental payments, are subtracted from gross interest (a problem noted by Hall and Sargent (2011)). We can gauge the extent of this understatement by comparing net interest to the correct series for interest on debt held by the public for $t \geq 1962$, when both series exist. Figure A2 shows the interest rates computed by dividing each of these series by debt held by the public at the end of $t - 1$.

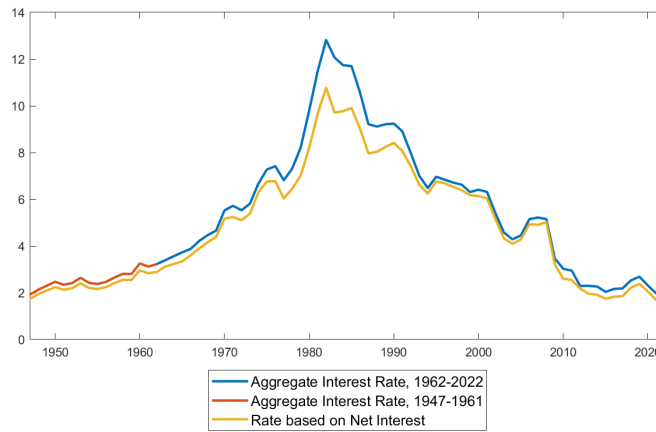


Figure A2. Alternative Measures of the Aggregate Interest Rate

Note: The aggregate interest rate is our measure of the interest rate on the public debt. We compare it to an alternative measure computed from net interest payments as reported by OMB. For 1962-2022, our aggregate interest rate is computed from gross interest payments minus intra-governmental payments. For 1947-1961, our interest rate is computed as 1.1 times the rate based on net interest.
Source: OMB historical database, authors' calculations.

For the period from 1962 to 2022, we find that the ratio of our correct measure of interest payments to the net interest reported by OMB is equal on average to 1.1, and is fairly stable over that period. Therefore, we estimate the average interest rate before 1962 by multiplying net interest by 1.1 and dividing by the stock of debt at the end of $t - 1$.

Our interest rate series for $t < 1962$ is an approximation based on incomplete data, but our results are not very sensitive to the exact approximation. If for $t < 1962$ we measured interest payments with net interest from OMB, then the debt/GDP ratio in our combined counterfactual would be 73% in 1974 (compared to 74% in our baseline case) and 83% in 2022 (compared to 84%).

THE REVERSE MATURITY STRUCTURE. We construct the reverse maturity structure of the debt from the Hall, Payne and Sargent (2018) database for the period from 1942 through 1960, and from the CRSP Monthly U.S. Treasury Database for 1961 through 2022. For every month, these databases provide an accounting of individual Treasury securities outstanding, including issue dates and quantities. We use the data for the final month of each fiscal year to construct D_t^τ , the amount of debt outstanding at the end of year t that was issued in year τ .

The Hall, Payne and Sargent (2018) data set includes almost every outstanding security. Therefore, for $t - 1 \leq 1960$, the weights w_{t-1}^τ that define the reverse maturity structure can be computed simply as:

$$(A3) \quad w_{t-1}^\tau = D_{t-1}^\tau / D_{t-1},$$

where here D_{t-1} is the total stock of debt reported in the Hall, Payne and Sargent (2018) data set (which is extremely close to the stock of debt reported by OMB that we use elsewhere). Our measurement of the weights implies that they sum exactly to one.

The CRSP data set that we use for 1961-2022 has two limitations: it excludes non-marketable debt and it excludes Treasury bills. We proceed as follows. First, we divide the total debt D_t into Treasury bills, marketable debt excluding Treasury bills, and non-marketable debt. We use data from the Monthly Statement of the Public Debt (MSPD) database for Treasury bills and Hall and Sargent (2022) for aggregate marketable debt and non-marketable debt.

We derive a reverse maturity structure for all marketable debt using the quantity of T-bills and the data on other marketable debt from CRSP. The weights $w_{t-1}^{\tau,m}$ for $t-1 > 1960$ are defined by:

$$(A4) \quad w_{t-1}^{\tau,m} = D_{t-1}^{\tau,m} / D_{t-1}^m,$$

where $D_{t-1}^{\tau,m}$ is the stock of marketable debt at the end of $t-1$ that was issued at τ and D_{t-1}^m is total marketable debt at the end of $t-1$ (the sum of $D_{t-1}^{\tau,m}$ for all τ). For $t-1 > \tau$, $D_{t-1}^{\tau,m}$ is the sum of all securities in the CRSP data set that were issued at τ and are outstanding at the end of $t-1$. For $t-1 = \tau$, $D_{t-1}^{\tau,m}$ is the sum of two components: the CRSP securities that were issued during $t-1$ and are outstanding at the end of $t-1$, and the stock of Treasury bills outstanding at the end of $t-1$, which we assume were also issued during $t-1$. We checked that the sum D_{t-1}^m is extremely close to the stock of marketable debt that we compute from the MSPD database and Hall's website.

Based on the Hall, Payne and Sargent (2018) data set, which includes both marketable and non-marketable debt before 1961, we know that non-marketable securities tend to have longer maturities. (See Figures A3 and A4 for the reverse maturity structure of marketable and non-marketable debt.) Lacking granular data on non-marketable debt, we simply assume that the reverse maturity structure of that part of the debt is the same in all years after 1960 as it is in 1960:

$$(A5) \quad w_{t-1}^{\tau,nm} = w_{1960}^{\tau,nm} \quad \forall t > 1961 \text{ and } \forall \tau \leq t-1,$$

where $w_{1960}^{\tau,nm}$ for all τ 's is the reverse maturity structure for non-marketable debt in 1960, which we obtain from the Hall, Payne and Sargent (2018) database. Assuming that the reverse maturity structure of non-marketable debt does not change after 1960 introduces some error in our calculations, but we believe the impact is modest because after 1960 non-marketable debt was a fairly small part

of total debt: as shown in Figure A5, it declined from 23% of total debt in 1960 to 3% percent in 2022.

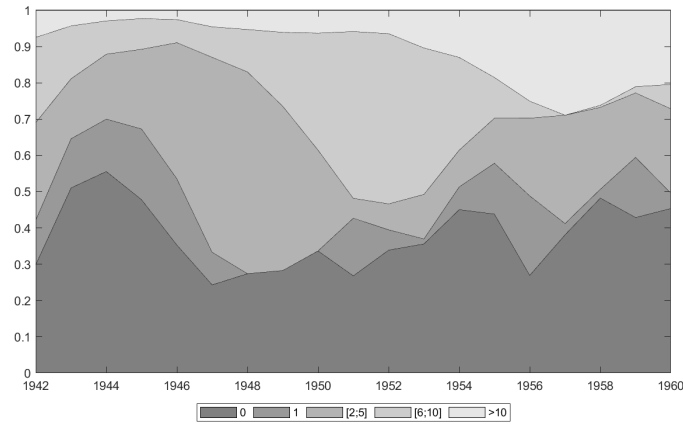


Figure A3. Reverse Maturity Structure of Marketable Public Debt, 1942-1960

Note: This chart shows the reverse maturity structure of marketable debt held by the public between 1942 and 1960. The different shades represent the share of the debt at the end of the fiscal year which was issued in the same year, the previous year, 2 to 5 years earlier, 6 to 10 years earlier, and more than 10 years earlier. Lighter shades indicate longer reverse maturities.

Source: Authors' calculations.



Figure A4. Reverse Maturity Structure of Non-Marketable Public Debt, 1942-1960

Note: This chart shows the reverse maturity structure of non-marketable debt held by the public between 1942 and 1960. The different shades represent the share of the debt at the end of the fiscal year which was issued in the same year, the previous year, 2 to 5 years earlier, 6 to 10 years earlier, and more than 10 years earlier. Lighter shades indicate longer reverse maturities.

Source: Authors' calculations.

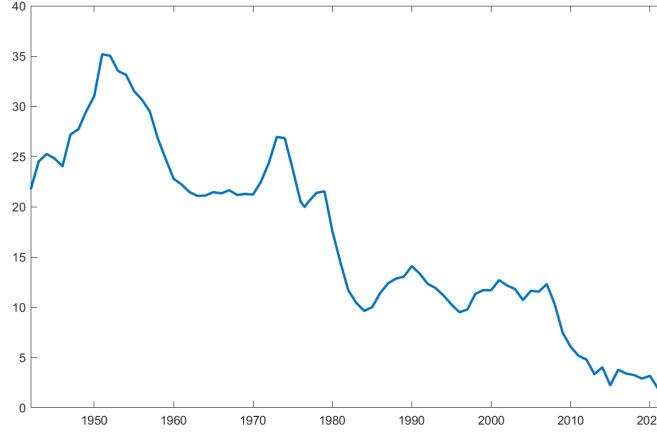


Figure A5. Non-Marketable Debt as a Share of Total Debt (%)

Note: The line represents the ratio of the par value of non-marketable Treasury securities held by the public to total debt held by the public.

Source: MSPD, Hall.

Given the reverse maturity structures of marketable and non-marketable debt after 1960, we construct the w_{t-1}^τ 's defining the reverse maturity structure of total debt as the average of $w_{t-1}^{\tau,m}$ and $w_{t-1}^{\tau,nm}$ weighted by the shares of the two types of debt:

$$(A6) \quad w_{t-1}^\tau = w_{t-1}^{\tau,m} m_{t-1} + w_{t-1}^{\tau,nm} (1 - m_{t-1})$$

where m_{t-1} is the share of marketable debt in total debt outstanding at the end of $t - 1$. We compute the weights m_{t-1} from Hall's data on aggregate outstanding marketable and non-marketable debt held by the public. We checked that outstanding marketable debt is extremely close to the sum of outstanding debt reported by CRSP and outstanding Treasury bills reported in the Treasury Bulletins.

TIPS AND TREASURY BILLS. We compute s_{t-1} , the share of Treasury bills in outstanding debt that was issued in year $t - 1$, as:

$$(A7) \quad s_{t-1} = \frac{D_{t-1}^{bills}}{D_{t-1}^{t-1}}$$

where D_{t-1}^{bills} is the stock of Treasury bills outstanding at the end of $t - 1$ from the MSPD. We compute z_{t-1}^τ , the share of TIPS securities in outstanding debt that was issued in year τ , as:

$$(A8) \quad z_{t-1}^\tau = \frac{D_{t-1}^{\tau, tips}}{D_{t-1}^\tau}$$

where $D_{t-1}^{\tau, tips}$ is the stock of TIPS outstanding at the end of $t - 1$ that were issued in τ . In calculating z_{t-1}^τ , we use the securities included in CRSP (and T-bills for $t - 1 = \tau$) in both the numerator and denominator. The denominator also includes non-marketable debt.

A3. *Measuring Inflation and Inflation Expectations*

ACTUAL INFLATION. We measure the inflation rate in fiscal year t as the growth rate in the GDP deflator from the last quarter of year $t - 1$ to the last quarter of year t , from the National Income and Product Accounts.

As described below, we also use data on the CPI inflation rate before 1970. We measure CPI inflation in fiscal year t with the inflation rate from the last month of $t - 1$ to the last month of t (from June to June given the dating of fiscal years before 1970). We use CPI data from the BLS.

ONE-YEAR EXPECTATIONS. For $t \geq 1970$, we measure one-year expected inflation $\mathbb{E}_t[\pi_{t+1}]$ with the median forecast of inflation over the next four quarters reported in the Survey of Professional Forecasters for the last quarter of fiscal year t .

For $t < 1970$, we create a proxy for expected GDP deflator inflation from forecasts of CPI inflation in the Livingston survey of business economists. We use forecasts from the first of each calendar year's semi-annual surveys, which are published in June. Before 1970, June is the last month of the fiscal year. The raw data are forecasts of the CPI level in the following June. The FRB of Philadelphia, which maintains the SPF, computes an inflation rate forecast following the methodology of Carlson (1977). The method assumes that forecasters have observed the actual CPI for April, and therefore uses the CPI for April of year t and the forecast for June of $t + 1$ to compute a forecast for annualized inflation over 14 months. We use this as a proxy for expected CPI inflation over fiscal year $t + 1$, from June of t to June of $t + 1$.

As discussed in the text, we compute a forecast of GDP deflator inflation for $t < 1970$ by assuming that the expectation error $\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]$ is the same for the GDP deflator as for the CPI (even though the level of CPI inflation is on average several tenths of a percentage point higher than that of GDP deflator inflation). We can see that this approximation is reasonable from Figure A6, which plots the expectation errors for the two inflation rates from 1970 to 1998, when we can compute both using the SPF and Livingston data. The two series are usually close.

TEN-YEAR EXPECTATIONS. As discussed in the text, for $t \geq 1968$, we measure ten-year expected inflation $\mathbb{E}_t[\pi^{10}]$ with long-term expected inflation from the Fed's database for its FRB/US Model¹. These expectations are forecasts of inflation in the PCE deflator, but we use them as expectations of GDP deflator inflation because the actual inflation rates for these two deflators move closely together;

¹More specifically, we use the historical values of the PTR variable, which come from several sources. Since 1991Q4, the source is the Survey of Professional Forecasters (SPF), first for expected CPI inflation and then, when it becomes available in 2007, for expected PCE deflator inflation. PTR data from 1981Q1 to 1991Q3 is primarily from a survey conducted by Richard Hoey. The Hoey and SPF CPI observations are reduced by 40 basis points to account for the average difference between CPI and PCE inflation. Values of PTR before 1981 are constructed in a manner similar to the one described in Kozicki and Tinsley (2001).

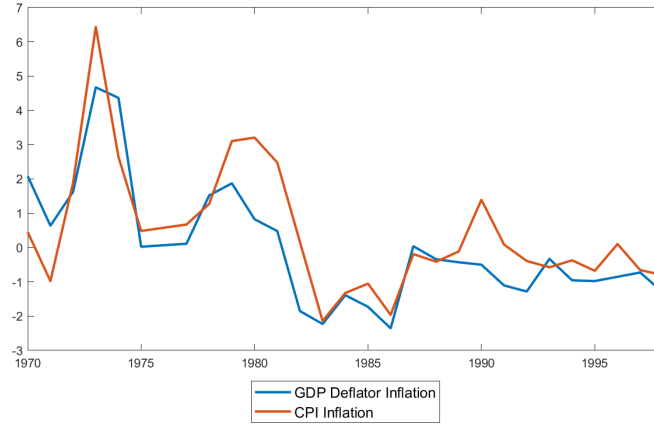


Figure A6. One-year Inflation Expectation Errors $\mathbb{E}_{t-1}[\pi_t] - \pi_t$, 1970 - 1998

Note: The line for CPI inflation expectation errors is computed as the actual CPI inflation rate minus the expected CPI inflation rate (Livingston Survey), from FY 1970 to FY 1998. The line for GDP inflation expectation errors is computed as the GDP deflator inflation rate minus the expected GDP deflator inflation rate (Survey of Professional Forecasters).

Source: FRED, Livingston Survey, Survey of Professional Forecasters.

see Figure A7.

We do not have data on long-term expectations before 1968, so we construct a proxy using the series on one-year expectations and the relation between one-year and ten-year expectations. In developing this procedure, we first smooth the series for one-year expectations using the HP filter with $\lambda = 100$. Figure A8 shows the smoothed series along with the actual series for one-year expectations and for ten-year expectations after 1968. We see that ten-year expectations generally follow the trend in one-year expectations, but lag behind somewhat as one-year expectations rise from 1968 to the early 1980s and then as they fall to the late 1990s. To capture this relationship, we regress the difference between ten-year and smoothed one-year expectations on the change in smoothed one-year expectations for the period 1968-1997, which yields the results shown in Table A1. Notice there is a negative coefficient on the change in one-year expectations, capturing the tendency of long-term expectations to lag behind short-term expectations when

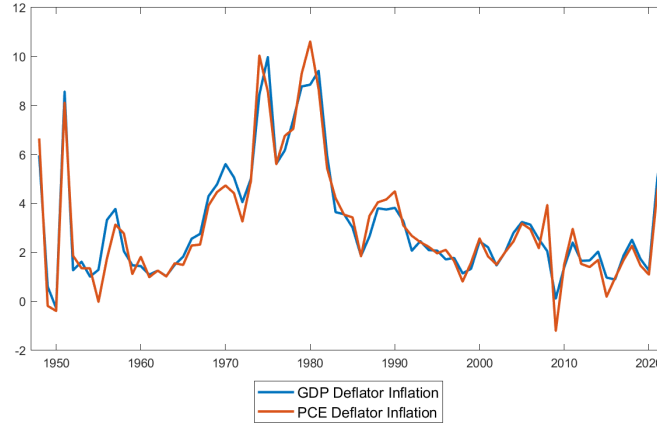


Figure A7. GDP Deflator and PCE Deflator Inflation Rates

Note: The blue and orange lines represent, respectively, the GDP deflator inflation rate and the PCE deflator inflation rate.

Source: FRED, Bureau of Economic Analysis.

the latter are trending up or down. Figure A9 shows the fitted values of long-term expected inflation based on the equation in Table A1 along with actual long-term expected inflation. We can see that the fitted values are close to actual long-term expectations over the estimation period. The Figure extends the fitted values back to 1952 and we use this fitted path as our proxy for ten-year expectations before 1968.

THE TERM STRUCTURE OF INFLATION EXPECTATIONS. Given our series on one-year and ten-year inflation expectations, we make assumptions about the shape of the term structure of expectations that allow us to estimate the entire term structure $\mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[\pi_{t+2}], \dots$. Here, we describe our approach for all fiscal years except those from 1972 through 1976. The proximity of those years to the Transitional Quarter produces a complication discussed below.

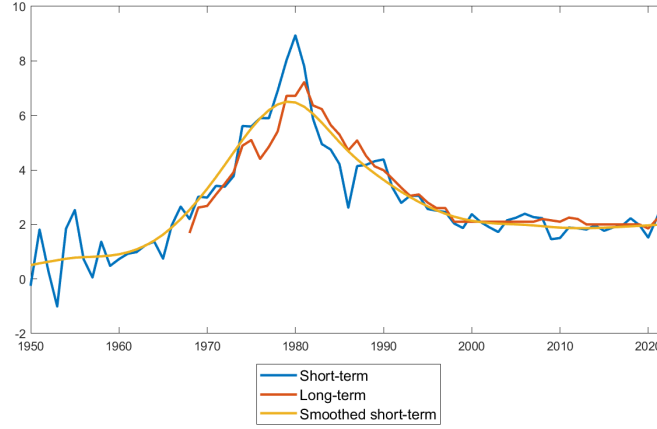


Figure A8. Short-term, Smoothed Short-term, and Long-term Inflation Expectations

Note: The lines represent the short-term, smoothed short-term, and long-term inflation expectations.

Source: Livingston Survey, Survey of Professional Forecasters, Federal Reserve FRB/US Model, authors' calculations.

For the years we consider here, we assume:

$$(A9) \quad \mathbb{E}_t[\pi_{t+j}] = \mathbb{E}_t[\pi_{t+1}] + (j-1)k_t \text{ for } 2 \leq j \leq 5$$

$$(A10) \quad \mathbb{E}_t[\pi_{t+j}] = \mathbb{E}_t[\pi_{t+5}] \text{ for } j > 5$$

The first equation says that inflation is expected to follow a linear path over the next five years, and the second says that inflation is then expected to remain constant. We view these assumptions as roughly consistent with term structures of expectations estimated by the Federal Reserve Banks of Philadelphia and Cleveland, which typically show that inflation is expected to change monotonically for roughly five years and then flatten out².

Given these assumptions, the term structure is determined by k_t , the rate at which inflation is expected to rise from $t+1$ to $t+5$. To determine k_t , we use the

²See the historical data on the term structure of expectations from the Federal Reserve Bank of Philadelphia, which uses the methodology developed in Aruoba (2020), and from the Federal Reserve Bank of Cleveland series based on Haubrich, Pennacchi and Ritchken (2012).

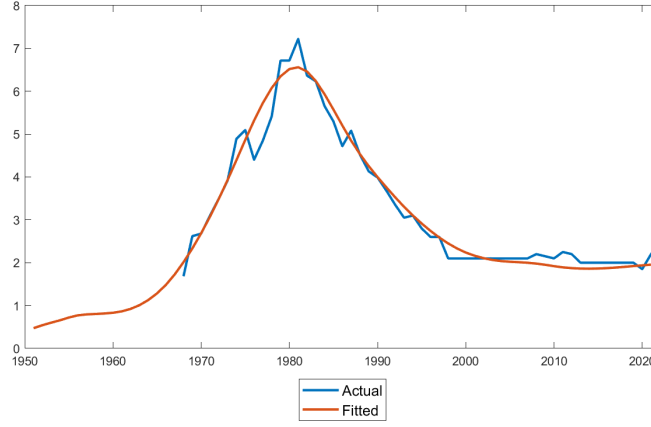


Figure A9. Actual and Fitted Long-term Inflation Expectations

Note: The lines represent the actual and fitted long-term inflation expectations.

Source: Livingston Survey, Survey of Professional Forecasters, Federal Reserve FRB/US Model, authors' calculations.

fact that long-term expected inflation $\mathbb{E}_t[\pi^{10}]$ is the average of one-year inflation rates expected over the next ten years:

$$(A11) \quad \mathbb{E}_t[\pi^{10}] = \frac{1}{10} \sum_{j=1}^{10} \mathbb{E}_t[\pi_{t+j}]$$

Substituting equations (A9) and (A10) for $j = 2, 3, \dots$ into the last equation yields an equation defining k_t in terms of $\mathbb{E}_t[\pi_{t+1}]$ and $\mathbb{E}_t[\pi^{10}]$, for which we have data. The solution is:

$$(A12) \quad k_t = \frac{\mathbb{E}_t[\pi^{10}] - \mathbb{E}_t[\pi_{t+1}]}{3}$$

This solution and equations (A9) and (A10) define the term structure of expectations for $t < 1972$, $t = TQ$, and $t \geq 1977$. For $t < 1972$, expected inflation in

Table A1— Long-term and Smoothed Short-term Expectations

VARIABLES	$\mathbb{E}_t[\pi^{10}] - \tilde{\mathbb{E}}_t[\pi_{t+1}]$
$\Delta\tilde{\mathbb{E}}_t[\pi_{t+1}]$	-1.549 [0.217]
Observations	30
R-squared	0.637

Standard errors in brackets

Note: $\tilde{\mathbb{E}}_t[\pi_{t+1}]$ indicates smoothed one-year expected inflation. The table shows the results of a regression of the difference between ten-year and smoothed one-year inflation expectations on the change in smoothed one-year inflation expectations for the period 1968-1997.

the TQ is the non-annualized rate in that quarter implied by the constant annual rate expected for $t + 5$ and later.

THE TERM STRUCTURE NEAR THE TRANSITIONAL QUARTER. We have a special procedure for determining expectations set in fiscal years from 1972 through 1976. In those years, the periods $t + 1, \dots, t + 5$ include the Transitional Quarter, and since that period is shorter than the others, it is no longer natural to assume that expected inflation changes linearly with the horizon measured in periods. Therefore, for 1972 through 1976, we use the quarterly data on expectations from the SPF to compute a term structure at the quarterly frequency. (This is not possible for our entire sample, because we have only the semi-annual Livingston survey of expectations before 1968.)

We index quarters by s . $\mathbb{E}_s[\pi_{s+j}]$ is the expectation in quarter s of annualized inflation in quarter $s + j$.

For each quarter, the SPF gives forecasts of the inflation rates in the next four quarters, $s + 1, \dots, s + 4$. To construct a term structure for later quarters, we assume that inflation is expected to change linearly from $s + 4$ through $s + 20$ and

then remain constant:

$$(A13) \quad \mathbb{E}_s[\pi_{s+j}] = \mathbb{E}_s[\pi_{s+4}] + (j-4)k_s \text{ for } 5 \leq j \leq 20$$

$$(A14) \quad \mathbb{E}_s[\pi_{s+j}] = \mathbb{E}_s[\pi_{s+20}] \text{ for } j > 20$$

We assume that long-term expected inflation equals the average of inflation expected over the next forty quarters:

$$(A15) \quad \mathbb{E}_s[\pi^{40}] = \frac{1}{40} \sum_{j=1}^{40} \mathbb{E}_s[\pi_{s+j}]$$

where $\mathbb{E}_s[\pi^{40}]$ is the expectation at s of inflation over the next forty quarters (ten years), which we measure with the FRB/US quarterly series for long-term expectations. These equations lead to:

$$(A16) \quad k_s = \frac{1}{456} \left[40 \mathbb{E}_s[\pi^{40}] - \sum_{j=1}^3 \mathbb{E}_s[\pi_{s+j}] - 37 \mathbb{E}_s[\pi_{s+4}] \right]$$

which defines the quarterly term structure of expectations.

For fiscal years from $t = 1972$ through $t = 1976$, $\mathbb{E}_t[\pi_{t+j}]$ is the expectation in the last quarter of t of cumulated inflation over the four quarters of fiscal year $t + j$. To write this expectation in terms of our quarterly series for expectations, let $s = (t, q)$ denote quarter q of fiscal year t . With this notation,

$$(A17) \quad \mathbb{E}_t[\pi_{t+j}] = \left[\prod_{q=1}^4 (1 + \mathbb{E}_{(t,4)}[\pi_{(t+j,q)}]) \right]^{1/4} - 1$$

We compute this expectation for $1972 \leq t \leq 1976$ and $j > 0$, accounting for which quarters belong to each fiscal year given the switch in timing in 1977.

Finally, we need to measure non-annualized expected inflation in the TQ for $1972 \leq t \leq 1976$ to compute the inflation surprises $\pi_{TQ} - \mathbb{E}_t[\pi_{TQ}]$ that determine the interest rate adjustments for TQ in our counterfactuals. We do so by converting the expected annualized inflation rate from the quarterly term structure into a non-annualized rate:

$$(A18) \quad \mathbb{E}_t[\pi_{TQ}] = [1 + \mathbb{E}_{(t,4)}[\pi_{TQ}]]^{1/4} - 1$$

A4. An Endogenous Reverse Maturity Structure

In our main analysis, we measure the reverse maturity structure of the debt, the weights w_t^r , with data on actual debt and hold the weights fixed in all counterfactuals. Here we consider a variation in which we account for the fact that a change in the path of aggregate debt affects the amount of new debt issued each period and therefore influences the reverse maturity structure. We capture this effect under the assumption that the forward maturity structure of new debt in each period is the same in a counterfactual as in reality. For example, if 20 percent more debt is issued in year τ in a counterfactual than in history, we assume that the amount of debt issued in τ and maturing at $\tau + j$ is 20 percent higher for all j .

DEBT DYNAMICS. Letting $\hat{D}_t$ equal the level of debt at time t in a counterfactual, the equation for debt dynamics in the counterfactual is:

$$(A19) \quad \hat{D}_t = (1 + \hat{i}_t)\hat{D}_{t-1} - \hat{P}_t + \epsilon_t$$

where $\hat{i}_t$ and $\hat{P}_t$ are the counterfactual interest rates and primary surpluses and we again hold the residual ϵ_t constant at its historical levels. The interest rate is a weighted average of rates on debt issued in different periods $\tau < t$, with the

weights given by the counterfactual reverse maturity structure $\hat{w}_{t-1}^\tau$:

$$(A20) \quad \hat{i}_t = \sum_{\tau=t-M}^{t-1} \hat{w}_{t-1}^\tau \hat{i}_t^\tau = \sum_{\tau=t-M}^{t-1} \hat{w}_{t-1}^\tau (i_t^\tau + x_t^\tau)$$

where the second equality uses our decomposition of the counterfactual $\hat{i}_t^\tau$ into the actual rate i_t^τ and the adjustment x_t^τ . We continue to measure x_t^τ with equation (A.2).

MEASURING i_t^τ . In our counterfactual analysis with a fixed maturity structure, we eliminate i_t^τ , the interest rates on debt with different reverse maturities, from our expression for the aggregate interest rate (see equation (5)). This simplification is not possible with an endogenous maturity structure, so we must measure the i_t^τ 's. We do so with our data on interest rates on individual securities. For fiscal years up to 1960, we use the coupon rates on securities in the Hall et al (2018) database (or, when the coupon rate is missing, the interest rates by maturity from Friedman and Schwartz (1963)), and for years after 1960 we use the interest rates on securities in the CRSP database. As an initial measure of the interest rate i_t^τ , we use the average of the rates on all securities issued at τ and outstanding at t .

These measures are imperfect because the Hall et al. and CRSP data sets do not include every government security; the most important omission is that CRSP does not include non-marketable debt. As a result, with i_t^τ measured with these data, equation (4) relating the aggregate interest rate i_t to the i_t^τ does not hold exactly. To ensure consistency of our interest rate measures, we multiply our initial rates for a given year t by a factor ξ_t such that equation (4) holds. That is, if $\tilde{i}_t^\tau$ is the interest rate calculated from Hall et al. or CRSP, our final measure of i_t^τ is $\xi_t \tilde{i}_t^\tau$ with ξ_t defined by the condition $i_t = \xi_t \sum_{\tau=t-M}^{t-1} w_{t-1}^\tau \tilde{i}_t^\tau$.

In all counterfactuals, we keep s_{t-1} , the shares of T-bills in debt issued during $t-1$, and z_{t-1}^τ , the shares of TIPS in debt issued at τ , fixed at their levels in actual history.

ITERATIVE CONSTRUCTION OF THE $\hat{w}_t^\tau$ 'S. The counterfactual weights $\hat{w}_{t-1}^\tau$ are determined by the interplay of the reverse maturity structure captured by the weights and the forward maturity structure of debt issued in each year. This forward maturity structure is defined by D_{t+j}^t , the amount of debt issued in year t that will still be outstanding at $t+j$, for $j \geq 0$. To derive the forward and reverse maturity structures in a counterfactual, we start with these maturity structures in 1946 and earlier, which we take from the data, and then perform an iterative procedure: Given the two maturity structures in years $t-1$ and earlier, we derive them for year t . Each iteration has the following steps:

- 1) Compute $\hat{D}_t$, the amount of aggregate debt in period t , from equations (A.19) and (A.20). (Note that (A.20) includes the weights $\hat{w}_{t-1}^\tau$ for $\tau \leq t-1$, which come from the previous iteration).
- 2) Compute $\hat{D}_t^t$, which is the amount of debt outstanding at t and issued during t , that is, the new debt at t . This quantity is given by $\hat{D}_t^t = (\hat{D}_t - \hat{D}_{t-1}) + \sum_{\tau=t-M}^{t-1} (\hat{D}_{t-1}^\tau - \hat{D}_t^\tau)$. In this expression, the first term is the amount of new debt that must be issued to accommodate the increase in total debt at t . The second term is the amount of new debt that must be issued to roll over debt that matures at t . It is the sum over issue dates τ of debt that was outstanding at $t-1$ but is no longer outstanding at t .
- 3) Compute $\hat{D}_{t+j}^t$ for all $j > 0$, which define the forward maturity structure of the debt issued at t . To perform this step, we assume that $\hat{D}_{t+j}^t = D_{t+j}^t \frac{\hat{D}_t^t}{D_t^t}$. That is, the amount of debt issued at t that is outstanding at $t+j$ is scaled up relative to actual history by the ratio of new debt at t in the counterfactual and in actual history.
- 4) The reverse maturity structure at t is defined by $\hat{D}_t$ and $\hat{D}_t^\tau$ for $\tau \leq t$. $\hat{D}_t$ and $\hat{D}_t^t$ are derived in steps 1 and 2. $\hat{D}_t^\tau$ for $\tau < t$ is available from the forward maturity structure at τ , which was derived in a previous iteration.

RESULTS. The results of this exercise prove somewhat anticlimactic. While the endogenous reverse maturity structures differ from the fixed maturity structures in our main analysis, the differences are modest. Figure A10 illustrates this result by comparing the two reverse maturity structures in three typical years.

Because the endogenous and fixed reverse maturity structures are similar, it does not matter much which we use in our counterfactual analysis. For our combined counterfactual, Figure A11 shows that the paths of debt/GDP with the two reverse maturity structures are almost indistinguishable. With the fixed structure, debt/GDP is 73.8% in 1974 and 84.1% in 2022. With an endogenous structure, debt/GDP is 74.3% in 1974 and 87.4% in 2022.

A5. Robustness to Alternative Assumptions About Equilibrium Interest Rates

A central part of our analysis is estimating undistorted real interest rates for various periods. Here we consider the robustness of our results to varying some of the assumptions we make in that exercise. In particular, we consider alternative assumptions about inflation expectations before 1943, equilibrium real rates during the peg period of 1943-1951, and the effects of quantitative easing since 2009.

INFLATION EXPECTATIONS BEFORE 1943. In our main counterfactual simulations, we do not adjust the interest rates on bonds issued before the start of the peg. In principle, it would be more accurate to treat the pre-1943 period in the same way as the post-peg period and adjust rates based on inflation surprises relative to expectations when securities were issued. Our main approach is equivalent to assuming that expectations before 1943 of inflation after 1946 (the start of our simulations) equal actual inflation: there were no surprises relative to pre-1943 expectations. Here we examine the implications of more reasonable conjectures about these expectations (which were not measured directly in surveys). These exercises yield somewhat different results from our main case, but the differences

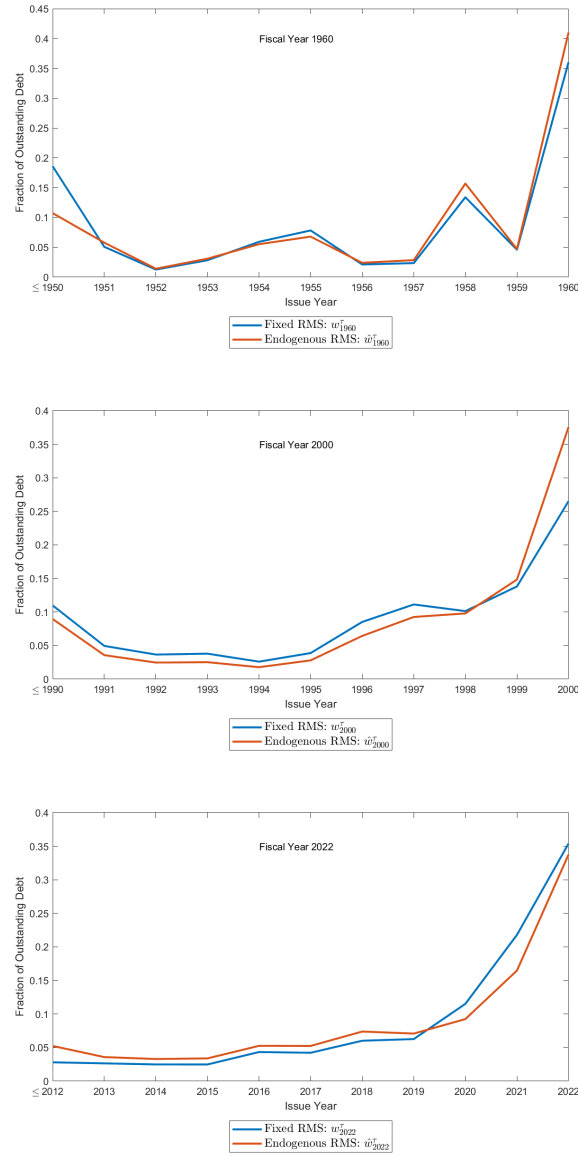


Figure A10. Fixed and Endogenous Reverse Maturity Structures (Selected Years)

Note: These graphs compare the fixed and endogenous reverse maturity structures for fiscal years 1960, 2000, and 2022. The graph for year t shows the fractions of outstanding debt issued at each $\tau \leq t$ in the two cases.
Source: Authors' calculations.

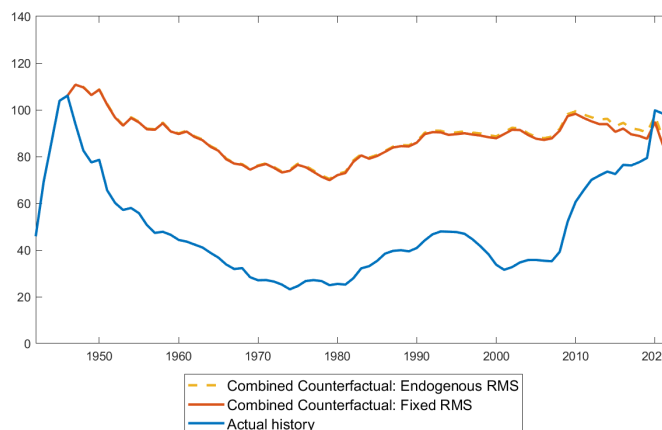


Figure A11. Debt/GDP Paths with Fixed and Endogenous Reverse Maturity Structures

Note: The lines represent the path of the debt/GDP ratio in actual history and in our combined counterfactual with fixed and endogenous reverse maturity structures.

Source: Authors' calculations.

are not large, reflecting the fact that only 20% of the debt outstanding in 1946 had been issued before 1943.

One reasonable conjecture is that, for $\tau < 1943$ and $t > 1946$, $\mathbb{E}_\tau[\pi_t] = 0$: no inflation was expected. This idea is suggested by the behavior of actual inflation in the decades before 1942. Barsky (1987) estimates the univariate process for inflation over the period 1860-1939 and finds it is close to a random walk without drift, implying zero expected inflation five or more years in the future. Before the peg there were episodes of inflation or deflation over several years, which produced substantial changes in the price level, but no period of persistently positive inflation.

We also consider an alternative assumption of $\mathbb{E}_\tau[\pi_t] = 3\%$ for $\tau < 1943$ and $t > 1946$. We view this as a generous upper bound on expected inflation: it means that markets expected a shift to a regime of persistent inflation that had not been seen in the past.

Table A2 shows the implications of these assumptions. For our combined

counterfactual with interest rate adjustments and primary balance, we compare debt/GDP in 1974 and 2022 in our main analysis to debt/GDP with the two alternative assumptions about pre-1943 expectations. The assumption of zero expected inflation raises D/Y in both 1974 and 2022 by about 2.5 percentage points, and 3% expected inflation raises D/Y by about 1 point. These results reflect the fact that the average inflation rate over 1947-1952, the period when ten-year bonds issued before 1943 were still outstanding, was about 4.5%. Any reasonable calibration of expected inflation before 1943 is below 4.5%, so adjusting for surprises relative to those expectations yields higher interest rates and debt levels.

Table A2— Robustness Check - Alternative Assumptions About Pre-1943 Inflation Expectations

		Debt/GDP (%)		
Year	Actual	Combined Counterfactual		
		Baseline	Expected Inflation	
			3%	0%
1974	23.2	73.9	74.8	76.5
2022	97.0	84.2	85.0	86.8

Note: This table examines the implications for our counterfactuals of different treatments of debt issued before 1942. In our baseline we do not adjust the interest rates on this debt. Alternatively, we adjust these rates to eliminate the effects of surprise inflation, for the cases where expectations before 1943 of inflation after 1946 are either 3% or 0%. For each of these cases, the table shows the levels of debt/GDP in 1974 and 2022 in the combined counterfactual.

EQUILIBRIUM INTEREST RATES IN THE PEG PERIOD. As discussed in the text, we do not have direct evidence on the real interest rates that would have prevailed on securities issued during the pre-Accord period if the Fed had not pegged rates. As a baseline measure, we assume that the rate for any security of a given maturity would have equaled the average of the ex-ante real rates on securities with that maturity issued over 1952-1961, the decade after the peg ended. This assumption

yields a term structure of undistorted real interest rates that ranges from 1.7% at a one-year horizon to 2.7% at thirty years.

As a robustness check, we calibrate r^* under the peg with average real rates over longer time periods: 1952-1980 and 1952-2022. We also consider the individual decades of the 1960s and 1970s. Our estimates of equilibrium real rates based on those periods are generally close to or a bit higher than those for 1952-1961: one-year rates range from 1.4 to 2.7 (for the 70s and 60s, respectively) and thirty-year rates range from 3.1 to 3.4 (for 1952-2022 and the 70s).

For each of these calibrations, Table A3 shows D/Y in 1974 and 2022 in our combined counterfactual. Because the assumed interest rates are mostly higher than in our main case, the levels of D/Y are somewhat higher. The calibrations based on 1952-1980 and 1952-2022 raise D/Y by 2 or 3 percentage points.

As another robustness check, we simply increase or decrease the real rates assumed in our main case by 1 or 2 percentage points at all maturities. Table A3 shows that these changes have significant effects on our results. Reducing the assumed rates by 2 points reduces the levels of D/Y in our combined counterfactual by more than 10 percentage points. However, the assumed real interest rates in that case are extremely low by historical standards: they range from -0.3% at one year to 0.7% at thirty years. There is no reason to think that r^* was unusually low during the peg period; if anything, we presume that wartime spending raised r^* above typical levels.

QUANTITATIVE EASING. Starting in 2009, the Fed addressed the constraints arising from the zero bound on the federal funds rate through quantitative easing, and there is a consensus that this policy reduced long-term interest rates somewhat. As discussed in footnote 6, it is questionable that this effect was a distortion of interest rates in the sense of our analysis, but here we suppose that it is and add estimates of the effects of QE to the interest-rate adjustments in our counterfactuals.

Table A3— Robustness Check - Alternative Assumptions About Undistorted Real Interest Rates Under the Peg

		Debt/GDP (%)								
Year	Actual	Combined Counterfactual								
		Baseline				Robustness				
			1952-1980	1952-2022	1960s	1970s	(-2%)	(-1%)	(+1%)	(+2%)
1974	23.2	73.9	76.9	76.0	79.3	77.4	61.6	67.5	80.9	88.5
2022	97.0	84.2	87.1	86.3	89.6	87.6	71.9	77.8	91.1	98.7

Note: This table examines the implications of assuming higher or lower levels of undistorted real interest rates on securities issued during the peg period. Specifically, we measure undistorted rates with the average values of ex-ante real rates during different historical periods (1952-1980, 1952-2022, 1960s, and 1970s). We also add or subtract 1% or 2% to the entire term structure of undistorted rates in our baseline case, which is based on ex-ante real rates for 1952-1961. For each of these adjustments, the table shows the levels of debt/GDP in 1974 and 2022 in the combined counterfactual.

We focus on the effects of QE over the period 2009-2015, when the federal funds rate was at its lower bound. Based on a survey of estimates, Ball et al. (2016) conclude that QE reduced ten-year interest rates by 50 basis points in calendar years 2009 and 2010, 75 points in 2011, 100 points in 2012, and 125 points from 2013 through 2015. We estimate the effect on the ten-year rate on bonds issued in fiscal year τ by $A^\tau = (3/4)A^{C,\tau} + (1/4)A^{C,\tau-1}$, where $A^{C,\tau}$ is the effect in calendar year τ . We assume these effects apply to all bonds with maturities of 10 years or more. For bonds with shorter maturities, we assume a linear effect on the term structure from the overnight rate to the ten-year rate. The effect for a bond issued at τ with maturity m is:

$$(A21) \quad A^{\tau,m} = \begin{cases} A^\tau & \text{for } m \geq 10 \\ A^\tau \times \frac{m}{10} & \text{for } 1 \leq m < 10 \end{cases}$$

In our counterfactuals, the adjustment x_t^τ applies to the average interest rate on all bonds issued at τ and outstanding at t . The addition to x_t^τ to account for

QE is a weighted average of the adjustments $A^{\tau,m}$:

$$(A22) \quad \bar{A}_t^\tau = \sum_{m=t-\tau}^M f_t^{\tau,m} \times A^{\tau,m}$$

where $f_t^{\tau,m}$ is the fraction of debt issued at τ and outstanding at t with maturity m .

Adding $\bar{A}_t^\tau$ to our interest rate adjustment has no effect on our counterfactuals up to 2009, but it raises the debt/GDP ratio after that. In our combined counterfactual, debt/GDP in 2022 is 84.2 percent in our baseline case and 87.5 percent with the adjustment for QE.

CONCLUSION. All in all, we find that reasonable variations on our interest rate adjustments either have little effect on our results or modestly raise the levels of debt/GDP in our counterfactuals.

A6. *The Role of the Debt-Dynamics Residual*

The exact equation for the evolution of the debt, (A1), includes a residual ϵ that captures factors other than interest rates and primary surpluses. Figure A12 shows the series for ϵ as a share of GDP. This residual is small in most years, but it is sizable in 1947 and in some years since 2008. The large residuals are explained by changes in the operating cash held by the Treasury: an increase in cash holdings requires an increase in debt, and running down cash reduces debt. The 1947 residual is negative because cash holdings fell as military operations were wound down. The residuals since 2008 reflect two factors: changes in cash as the Treasury coped with debt ceiling crises, and the flow of stimulus payments during the 2008-2009 recession and the COVID pandemic.

The residual averaged -0.19 percent of GDP over 1947-1974 and 0.25 percent over 1975-2022. Therefore, the residual contributed somewhat to both the decline in debt/GDP in the first period and the rise in the second. To assess the

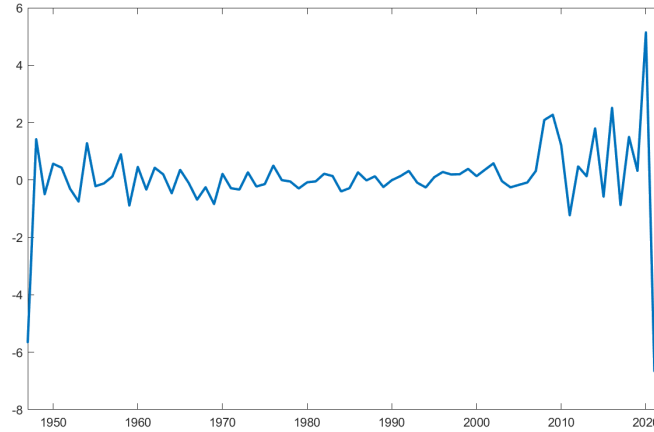


Figure A12. Residual in the Debt Dynamics Equation (% GDP)

Note: The line represents the residual ϵ_t such that equation (A1) holds exactly.

Source: Authors' calculations.

importance of this factor, Figure A13 compares the actual debt/GDP path to a counterfactual in which ϵ is set to zero in all years, but interest rates and primary surpluses are kept at their historical levels. The Figure also compares the combined counterfactual in our main analysis—a case with primary balance, no interest-rate distortions, and the historical values of ϵ —to a variation on that case with ϵ set to zero. This last counterfactual reveals the exact path that debt/GDP would have followed if the only factor driving it were $r^* - g$, the difference between the undistorted real interest rate and the growth rate.

In the combined counterfactual with $\epsilon = 0$, debt/GDP falls only to 78 percent in 1974. Thus the debt reduction from $r^* < g$ is even smaller than the debt reduction in the combined counterfactual with historical ϵ 's, in which debt/GDP is 74 percent in 1974. On the other hand, in the combined counterfactual with $\epsilon = 0$, debt/GDP reaches only 77 percent in 2022, somewhat lower than the 84 percent with historical ϵ 's.

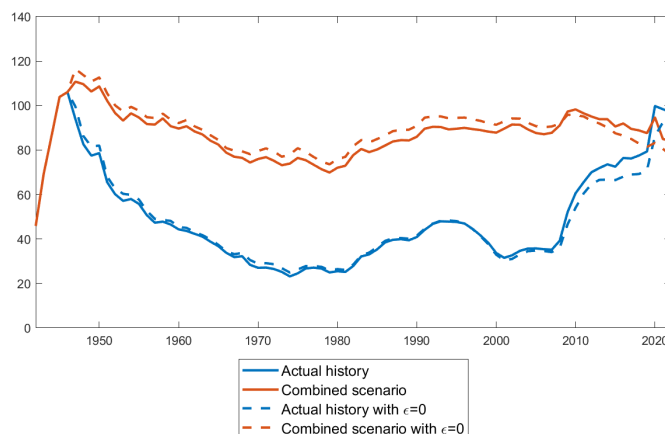


Figure A13. Debt/GDP Paths - Actual and Combined Counterfactual Scenario

Note: The lines represent the path of the debt-GDP ratio in actual history and our combined counterfactual scenario with ϵ equal to either its actual value or zero.

Source: Authors' calculations.

A7. Comparison to Blanchard (2019)

As discussed in the text, we find that the real interest rate has exceeded the growth rate on average since 1979, either with or without adjustments to the real rate for surprise inflation. This result appears to differ from Blanchard (2019), who reports that real rates have consistently been lower than growth rates. As discussed in the text, the different results are explained by two differences in how interest rates are measured:

- We measure the interest rate as the government's interest payments divided by outstanding debt, which yields the interest rates set when the debt was issued. Blanchard uses current market yields on debt, specifically a weighted average of the one-year and ten-year Treasury rates. Since 1979, these yields have usually been lower than the interest rates paid by the government because interest rates have trended downward.
- We ignore the taxation of interest income. In some of his analysis, Blanchard

examines after-tax interest rates that he calculates from estimates of the relevant tax rates.

Figure A14 shows how these differences matter. The Figure presents scenarios for the evolution of the debt/GDP ratio since 1979, with the initial level normalized to 100 as in Blanchard's Figures 5-6. In all cases, we assume a zero primary surplus and use actual interest rates without any adjustment for surprise inflation—our “primary balance” scenario—for comparability with Blanchard. We also set the residual ϵ to zero. With these assumptions, the path of debt/GDP is driven by $r - g$, the difference between the actual interest rate and the growth rate. We show the path of debt/GDP with our measure of interest rates and with Blanchard's market-yield measure with and without his tax adjustment (taken from the replication package available here).

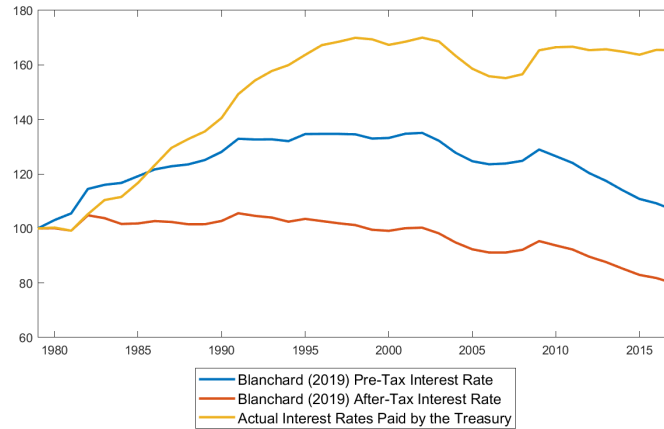


Figure A14. Debt/GDP with Zero Primary Balance and Alternative Measures of Interest Rates, 1979 - 2017

Note: The lines represent paths of debt/GDP with primary balance and the residual ϵ_t set to zero. Each line shows the path for a different measure of the interest rate. Debt/GDP is normalized to 100 in 1979.
Source: Authors' calculations.

The Figure confirms the results in both our Figure 7 and Blanchard's Figures 5-6. With our interest rate measure, debt/GDP rises from 1979 to 2022 because r

usually exceeds g over this period. With Blanchard's measure of pre-tax interest rates, the ratio rises until 2002 and then falls, leaving it close to its 1979 level in the last few years. With Blanchard's after-tax interest rates, the ratio falls significantly from 1979 to 2022 because r is usually less than g .

For the analysis in this paper, the relevant interest rates are the rates paid by the government, not market yields. The rates paid by the government are the ones for which equation (1) for debt dynamics holds in the data.

The appropriate treatment of taxes is not obvious. Blanchard points out that taxes collected on the interest on government bonds reduce the debt. However, the issuance of government bonds crowds out capital, and the government loses the taxes it would have collected on the lost capital income. The relative sizes of the gain and loss in revenue is ambiguous. On the one hand, crowding out of capital by debt is likely to be less than one-for-one. On the other hand, the returns on capital are higher on average than the interest rate on debt (because of risk), so a dollar of capital produces more tax revenue than a dollar of debt. A natural baseline, we think, is to assume that debt has no net effect on tax revenue. In this case, the evolution of debt is determined by the pre-tax interest rate.

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