Online Appendix

Partial Specialization and Heterogeneous Task Assignments

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A Data Appendix

A.1 The PDII Variables

The PDII data collect a representative sample of US workers only once in 2008. Out of the 2,513 US adults interviewed, 1,333 provided information on wages, demographic characteristics, and occupations. Importantly, the PDII data currently stands as the sole available U.S. source for worker-level task assignments. The PDII data code worker-level task intensity in two ways. The first method involves asking workers about the frequency of performing certain tasks (e.g., using advanced math such as algebra or geometry). These variables are coded into five categories: (1) never; (2) less than a month; (3) monthly; (4) weekly; and (5) daily. The second method involves asking about the proportion of the workday used to perform tasks (e.g., managing or supervising workers), which is coded into four categories: (1) almost none; (2) less than half; (3) more than half; and (4) almost all.

Task intensity $\mathbf{T}_k^{G(i)}$. Following Autor and Handel (2013), I construct worker-level task intensity using the first component of principal components analysis, then transfer to percentile rankings to obtain $\mathrm{T}_k^{G(i)}$. Specifically, I use three variables for cognitive tasks, where the first component accounts for 59% of the variation. The social task is measured using one variable. Routine task intensity is calculated as the first component of four variables, where the first component accounts for 55% of the variation. Manual tasks are measured using one variable. These variables are as follows:

- Cognitive task intensity: (1) the frequency of using advanced mathematics tasks; (2) the frequency of problem-solving tasks requiring at least 30 minutes to find a good solution; and (3) the length of the longest document typically read as part of the job.
- Social task intensity: the proportion of workday managing or supervising other workers.
- Routine task intensity: (1) proportion of the workday spent performing short, repetitive tasks and complete absence of face-to-face interactions with (2) customers or clients, (3) suppliers or contractors, or (4) students or trainees.
- Manual task intensity: the proportion of the workday spent performing physical tasks (standing, operating machinery or vehicles, making or fixing things by hand).

Multiple Tasks. The data reveal that workers often perform multiple tasks in their jobs. For simplicity, I define a worker as *often* engaging in cognitive tasks if at least one of the following conditions is true: (1) spending at least 30 minutes solving problems once a week; (2) using advanced math (algebra, geometry, trigonometry, probability/statistics, or calculus) to solve problems at least once a week; (3) frequently reading documents that are more than 6 pages long. A worker is considered to *often* perform social tasks if they spend more than half of their workdays managing or supervising other workers. Routine task performance is defined as spending more than half of the workdays on repetitive tasks with minimal face-to-face interactions. A worker is classified as *often* performing manual tasks if they spend more than half of their workdays standing, operating machinery or vehicles, or manually making or fixing things. It appears that 78% of workers often perform at least two tasks, among which 41% often perform two tasks, 29% often perform three, and 8% workers often perform all four.

A.2 O*NET Variables

I follow Deming (2017) to measure cognitive (math) and social task intensities. I follow Acemoglu and Autor (2011) to measure routine and manual task intensities. As stated in Section 2, because the PDII does not permit a distinction between routine cognitive and routine manual tasks, I group these two into a single category, which is named routine tasks. Below provides the details

1. Cognitive (math) task intensity is the average of three variables: (1) mathematical reasoning ability (the ability to understand and organize a problem and then to select a mathematical method or formula to solve the problem), (2) mathematics knowledge (knowledge of numbers, their operations, and interrelationships including arithmetic, algebra, geometry, calculus, statistics, and their applications), and (3) mathematics skill (using mathematics to solve problems).

2. Social task intensity as the average of four variables: (1) social perceptiveness (being aware of others' reactions and understanding why they react the way they do), (2) coordination (adjusting actions in relation to others' actions), (3) persuasion (persuading others to approach things differently), and (4) negotiation (bringing others together and trying to reconcile differences).

3. Routine task intensity as the average of six variables: (1) the importance of repeating the same tasks (How important is repeating the same physical activities or mental activities over and over, without stopping, to performing this job?), (2) the importance of being exact or accurate (how important is being very exact or highly accurate in performing this job?), (3) structured verse unstructured work (to what extent is this job structured for the worker, rather than allowing the worker to determine tasks, priorities, and goals?), (4) pace determined by the speed of equipment (how important is it to this job that the pace is determined by the speed of equipment or machinery?), (5) controlling machines and processes (using either control mechanisms or direct physical activity to operate machines or processes), and (6) making repetitive motions.

4. Manual task as the average of four variables: (1) operating vehicles, mechanized devices, or equipment (running, maneuvering, navigating, or driving vehicles or mechanized equipment, such as forklifts, passenger vehicles, aircraft, or watercraft), (2) using hands to handle, control or feel objects, tools or controls, (3) manual dexterity (the ability to quickly make coordinated movements of one hand, a hand together with its arm, or two hands to grasp, manipulate, or assemble objects), and (4) spatial orientation (the ability to know one's location in relation to the environment, or to know where other objects are in relation to one's self).

For each task, I compute the first principal component using the variables specified. Subsequently, $\text{ptl}_{o,k}$ is derived by initially computing the percentile rankings of the first principal component across detailed SOC occupations. Then, I calculate the weighted average for 20 aggregate occupations in the baseline, weighted by hours, and for 40 occupations as a robustness check.

A.3 May/ORG CPS

I collect data from the Current Population Survey May and Outgoing Rotation Group samples (May/ORG CPS), which provide point-in-time measures of usual hourly or weekly earnings. The sample construction follows the approach outlined by Lemieux (2010) and Acemoglu and Autor (2011), where I limit the sample to individuals aged 21 to 60, excluding those in the military and self-employed. For hourly workers, their hourly earnings are directly reported in the May/ORG CPS. For other workers reporting weekly earnings, I compute their hourly earnings by dividing their usual weekly earnings by the hours worked in the previous week. Additionally, I adjust the top-coded earnings observations by a factor of 1.5. Following Autor et al. (2008), all earnings are measured in real terms and deflated by the PCE deflator. The dataset is available from David Autor's website, resulting in approximately 110,000 to 130,000 observations each year. To ensure consistency over time, I apply the occupation concordance developed by Autor and Dorn (2013) to create time-consistent occupation codes.

I start with the 30 broad occupational categories provided in OCC1990 (also referred to as BMV2019). Subsequently, I merge occupations with small employment sizes to obtain 20 occupations, while breaking down occupations with large employment sizes into separate groups to form 40 occupations. The baseline model estimation is conducted using 20 broad occupations, with additional results presented in Appendix E.2 using 40 occupation categories. Below, I list the occupations included in each classification.

20 Occupations. 1 "Executive Management" 2 "Management Related" 3 "STEM" 4 "Social Service, Lawyers" 5 "Education, Training, Library, legal support" 6 "Health Occupations" 7 "Technicians and Related Support" 8 "Financial Sales and Related Occupations" 9 "Retail Sales" 10 "Administrative Support" 11 "Housekeeping, Cleaning, Laundry" 12 "All Protective Service" 13 "Food Preparation and Service" 14 "Farm operators" 15 "Mechanics and Repairers" 16 "Construction" 17 "Precision production" 18 "Machine Operators, Assemblers, and Inspectors" 19 "Transportation and Material Moving" 20 "Handlers, Equipment Cleaners, and Helpers".

40 Occupations. 1 "Executive Management" 2 "Management Related" 3 "Architect" 4 "Engineer" 5 "Computer and Mathematics" 6 "Life, Physical, and Social Science" 7 "Health diagnosing occupations" 8 "Health assessment and treating, Therapists" 9 "Teacher postsecondary" 10 "Teacher except postsecondary" 11 "Librarians, Archivists, and Curators" 12 "Social Scientists and Urban Planners" 13 "Social, Recreation, and Religious Workers" 14 "Lawyers" 15 "Writers, Artists, Entertainers, and Athletes" 16 "Health Technologists and Technicians" 17 "Engineering and Related Technologists and Technicians" 18 "Sales Representatives, Finance and Business Services" 19 "Sales Representatives" 20 "Administrative Support" 21 "Information Clerks" 22 "Records Processing Occupations" 23 "Financial Records Processing Occupations" 24 "Duplicating, Mail, and Other Office Machine Operators" 25 "Material Recording, Scheduling, and Distributing Clerk" 26 "Adjusters and Investigators" 27 "Housekeeping, Cleaning, Laundry" 28 "All Protective Service" 29 "Food Preparation and Service" 30 "Health Service" 31 "Building, Grounds Cleaning and Maintenance" 32 "Personal Appearance" 33 "Child Care Workers" 34 "Farm operators" 35 "Mechanics and Repairers" 36 "Construction" 37 "Precision production" 38 "Machine Operators, Assemblers, and Inspectors" 39 "Transportation and Material Moving" 40 "Handlers, Equipment Cleaners, and Helpers".

B Additional Facts

B.1 Variance Decomposition

I demonstrate that the majority of both the level and the changes can be attributed to within-occupation variance, using alternative occupational classifications and composition-adjusted measures.

Table B.1: Log Wage Variance Decomposition Using OCCSOC Codes

	Within-Occupation Log Wages			**Within-Occupation Residual Log Wages (w/o states)**			**Within-Occupation Residual Log Wages (with states)**		
	1980	**2000**	**Changes**	**1980**	**2000**	**Changes**	**1980**	**2000**	**Changes**
2-digit	83.2%	77.0%	53.2%	92.4%	91.2%	85.4%	92.2%	91.2%	86.8%
3-digit	78.0%	73.2%	53.9%	89.5%	89.6%	89.8%	89.4%	89.5%	90.4%
4-digit	77.7%	73.0%	54.2%	89.2%	89.4%	90.4%	89.1%	89.4%	90.9%

Notes: The analysis is based on the May/ORC CPS 1980 and 2000 datasets. Each number represents the contribution of the within-component to the total log wage variance inequality, both in levels and changes. The changes are calculated as the changes in the within variance divided by the changes in the overall log wage variance. The first three columns are for log wages, the next three columns are for log wage residuals on 240 demographic dummies, and the last three columns are for log wage residuals on 240 demographic dummies and 51 states fixed effects. The results are derived from the 2, 3, and 4-digit OCCSOC occupation codes.

Table B.1 shows the results obtained using OCCSOC codes at varying levels of aggregation. Through the utilization of 2, 3, and 4-digit OCCSOC occupation codes, I observe that the within-component significantly contributes to both the levels and the changes. The first three columns pertain to log wages, the subsequent three columns relate to log wage residuals on 240 demographic dummies, and the last three columns correspond to log wage residuals on 240 demographic dummies and 51 states fixed-effects. These results are similar compared to those presented in Table 1, where within-occupation inequality emerges as a primary driver of overall inequality growth and exhibits greater importance in residual inequality growth.[38]

Composition-adjusted decomposition. I calculate the average and variance of the log wage within each occupation and demographic group and subsequently aggregate these measures across groups using constant weights averaged over 1980 and 2000. Specifically, I define the total composition-adjusted log wage variance as the sum of the between-occupation and within-occupation components:

$$B_t^a = \frac{1}{\bar{N}} \sum_o \bar{N}_o (\bar{w}_{ot}^a - \bar{w}_t^a)^2, \quad W_t^a = \frac{1}{\bar{N}} \sum_o \bar{N}_{ot} Var_{ot},$$

where $\bar{N}$ and $\bar{N}_o$ represent the overall employment and occupational employment averaging over 1980 and 2000. $\bar{w}_{ot}^a$ and $\bar{w}_t^a$ are the composition-adjusted average occupation and gross log wage, respectively, weighted by the average group-occupation employment between 1980 and 2000. $Var_{o,t}$ represents the

[38] The OCCSOC code is only available after the year 2000. To address this, I assigned the OCCSOC code to the CPS data in 1980 and 1990 using a crosswalk of OCCSOC to OCC1990.

composition-adjusted log wage variance in occupation o at time t, and it is computed as:

$$Var_{ot} = B_{ot} + W_{ot}, \tag{B.1}$$

$$B_{ot} = \sum_G \frac{\bar{N}_o^G}{\bar{N}_o}(\bar{w}_{ot}^G - \bar{w}_{ot}^a)^2, \qquad W_{ot} = \sum_G \frac{\bar{N}_o^G}{\bar{N}_o}\sum_{i\in G}(w_{it} - \bar{w}_{ot}^G)^2, \tag{B.2}$$

where $\bar{w}_{ot}^G$ is the average group-occupation level wage. Once again, the changes can be written as:

$$\Delta T_t^a = \Delta B_t^a + \Delta W_t^a. \tag{B.3}$$

Table B.2 displays the ratio of the composition-adjusted within-occupation variance to the overall log wage variance, both in levels and changes from 1980 to 2000. Across all occupation categories, the within-occupation components consistently dominate.

Table B.2: The Composition-Adjusted Within-Occupation Log Wage Variance Using OCC1990 and OCCSOC Codes.

	Within-Occupation Log Wages		
	1980	**2000**	**Changes**
A. OCC1990 Occupation Code			
20 Occupations	88.0%	87.1%	81.1%
30 Occupations	81.9%	80.1%	69.6%
40 Occupations	84.2%	82.3%	70.5%
383 Occupations	78.7%	78.5%	78.1%
B. OCCSOC Occupation Code			
2-digit	86.2%	84.5%	75.4%
3-digit	85.3%	82.4%	68.7%
4-digit	85.6%	82.9%	68.9%

Notes: The analysis utilizes the May/ORG CPS datasets from 1980 and 2000. Each number denotes the contribution of (composition-adjusted) within variance to the total log wage variance inequality, both in levels and changes. The changes are computed as the alterations in the within variance divided by the changes in the overall log wage variance. Panel A employs different OCC1990 codes, while Panel B utilizes OCCSOC codes.

B.2 Stylized Facts in Details

This section provides an extensive review for Facts 2, 3, and 4 discussed in Section 2.

Fact 2: Within occupations, the job content in routine tasks has declined, while it has increased for cognitive and social tasks. Extending the method developed in Spitz-Oener (2006), APST2020 search keywords from 7 million newspaper job ads, and uses the changes in the frequency of word mentions as a measure of the demand changes within occupations. I measure the occupation-level relative task demand using the O*NET data in 2000, according to Equation (22). Using their data, I measure the changes in occupation-level relative task demand using the APST2020. Appendix Table B.5 reports the estimated occupation-level relative task demand for both years. It shows, within occupations, the relative demand declines in routine tasks but increases in cognitive and social tasks.

In the following, I detail facts on worker-level task assignments using PDII data.

Fact 3: Workers engage in multiple tasks, with task assignments varying significantly within the same occupation. There are multiple ways to show that workers frequently engage in multiple tasks. For simplicity, I use the definition provided in Appendix A.1. By that definition, it appears that 78% of workers often perform at least two tasks. Among them, 41% often perform two tasks, 29% often perform three tasks, and 8% of workers often perform all four tasks.

The task intensity varies systematically among workers with different demographic characteristics. I use the following regression to examine sorting on observable characteristics

$$T_k^{G(i)} = \sum_j \beta_j X_j^{G(i)} + \alpha_o + v^i, \qquad k \in \{\text{Cognitive, Social, Routine, Manual}\}, \tag{B.4}$$

where $T_k^{G(i)}$ is (group G) worker i's type-k task intensity (standardized to have a zero mean and a unit variance). $X_j^{G(i)}$ is observable characteristics j, including three education dummies, age, age square, gender, and occupational fixed effects. β_j picks up the task sorting across observable characteristics within occupations, relative to high school dropout (the omitted group). α_o is occupational fixed effects, and v^i is a residual.

Table B.3: Estimated Coefficients for Equation (B.4)

	Cognitive	**Social**	**Routine**	**Manual**
	(1)	(2)	(3)	(4)
HS graduate	0.215**	0.0906	-0.0785	-0.0478
	(0.103)	(0.126)	(0.104)	(0.0836)
Some college	0.332***	0.123	-0.249**	-0.0744
	(0.105)	(0.125)	(0.105)	(0.0854)
College and above	0.506***	0.227*	-0.398***	-0.405***
	(0.110)	(0.131)	(0.108)	(0.0940)
age	0.0122	0.0408***	-0.0325**	-0.0310***
	(0.0135)	(0.0149)	(0.0142)	(0.0118)
age^2	-0.000194	-0.000543***	0.000324*	0.000331**
	(0.000157)	(0.000173)	(0.000167)	(0.000140)
males	0.117**	0.121**	-0.197***	-0.0176
	(0.0555)	(0.0593)	(0.0577)	(0.0521)
N	1333	1333	1333	1333
R^2	0.390	0.172	0.290	0.483

Notes: All reduced-form equations are estimated using PDII data. In Panel A, the regression includes three education dummies, age, age square, gender, and occupational fixed effects. The omitted group is high school dropout females who are 41-60 years old. In Panel B, the regression includes four types of task intensity, and three education dummies, age, age square, gender, and occupational fixed effects. $N = 1333$ for all models. There are 21 2-digit and 76 of 3-digit occupations. Standard errors are reported in the parenthesis.

Table B.3 reports the estimates using each task as the dependent variable. I control the 3-digit OCC-SOC occupational fixed effect for all models. The task intensity varies systematically among workers with different demographic characteristics. Most notably, college graduates tend to perform more cognitive tasks but fewer routine and manual tasks in the same occupation. Compared to high school dropouts, they on average perform 0.51 of a standard deviation more cognitive tasks, 0.23 of a standard deviation more social tasks, and 0.4 of a standard deviation less for routine and manual tasks in the same occupation. The result for social tasks is statistically significant at 10% level. Task intensities also appear to vary across age and gender groups.

In all regressions, there are substantial variations unexplained by the observables, with R^2 ranging between 0.18 and 0.48. This indicates that task specialization also varies systematically among workers within the same demographic characteristics, suggesting idiosyncratic comparative advantages might play a role. To account for this fact, my model incorporates idiosyncratic skills and preferences, resulting in a closed-form expression for the equilibrium task allocation that varies by demographic characteristics and idiosyncratic skills. Consequently, I am able to leverage the empirical distribution of task assignments to estimate the distribution of log comparative advantages.

Fact 4: **Wages are determined by multidimensional skills and task assignments.** Empirical research has increasingly demonstrated that the returns to skills are multidimensional. See Heckman (1995), Heckman and Kautz (2012), and Deming (2017). Task assignments also contribute to disparities in earnings within occupations and demographic groups. To see this, I estimate the following regression:

$$\ln w^i = \sum_j \beta_j X_j^{G(i)} + \sum_k \gamma_k T_k^{G(i)} + \alpha_o + u^i, \tag{B.5}$$

where the dependent variable is log hourly wage and u_i is the unobserved idiosyncratic ability that affects earnings. This regression has been run in Autor and Handel (2013) with three task measures (cognitive, routine, and manual). I re-run their regressions with social task intensities as well.

Table B.4: Estimated Coefficients for of Wage Equation (B.5)

	(1)	(2)	(3)	(4)
Cognitive	0.104***	0.0738***	0.0815***	0.0693***
	(0.0195)	(0.0196)	(0.0187)	(0.0192)
Social	0.0262	0.00962	0.0328*	0.0162
	(0.0180)	(0.0182)	(0.0172)	(0.0177)
Routine	-0.0813***	-0.0626***	-0.0484***	-0.0374**
	(0.0162)	(0.0159)	(0.0155)	(0.0155)
Manual	-0.150***	-0.143***	-0.0979***	-0.104***
	(0.0178)	(0.0185)	(0.0174)	(0.0183)
N	1333	1333	1333	1333
R^2	0.427	0.516	0.494	0.556
2-digit Occup.	✓		✓	
3-digit Occup.		✓		✓
Demographic controls			✓	✓

Notes: All reduced-form equations are estimated using PDII data. In Panel A, the regression includes three education dummies, age, age square, gender, and occupational fixed effects. The omitted group is high school dropout females who are 41-60 years old. In Panel B, the regression includes four types of task intensity, and three education dummies, age, age square, gender, and occupational fixed effects. $N = 1333$ for all models. There are 21 2-digit and 76 of 3-digit occupations. Standard errors are reported in the parenthesis.

The first two columns of Table B.4 report the estimates using 2-digit and 3-digit different occupational fixed effects, respectively, without demographic characteristics. Cognitive task intensity is positively associated with earning variation within occupation, whereas the associations are negative for routine and manual tasks. Columns (3) and (4) report the estimates while controlling for demographic characteristics, the coefficients for cognitive, routine, and manual tasks fall but remain statistically significant. These findings reinforce the mechanism that changes in job content may have unequal effects on workers within the same occupation due to variations in their task exposure, and also provide suggestive evidence that earnings are influenced by multiple task sources.

In some specifications, social tasks show a positive and statistically significant coefficient at a 10% confidence level. Given the recent emphasis on the increasing return to social skills, I also incorporate social skills and tasks in the model.

Table B.5: Occupation Relative Task Demand ($\lambda_{o,k}$) in 1980 and 2000

Occupation	Cognitive	Social	Routine	Manual
Year 1980				
Executive Management	0.211	0.188	0.508	0.093
Management Related	0.178	0.111	0.648	0.063
STEM	0.197	0.107	0.594	0.102
Social Service, Lawyers	0.177	0.299	0.450	0.074
Education, Training, Library, legal support	0.231	0.212	0.474	0.083
Health Occupations	0.145	0.171	0.522	0.162
Technicians and Related Support	0.165	0.131	0.590	0.114
Financial Sales and Related Occupations	0.196	0.160	0.514	0.130
Retail Sales	0.147	0.191	0.535	0.126
Administrative Support	0.128	0.138	0.643	0.091
Housekeeping, Cleaning, Laundry	0.061	0.092	0.549	0.298
All Protective Service	0.124	0.143	0.469	0.264
Food Preparation and Service	0.050	0.137	0.570	0.243
Farm operators	0.064	0.074	0.472	0.391
Mechanics and Repairers	0.081	0.052	0.459	0.407
Construction	0.089	0.058	0.464	0.390
Precision production	0.099	0.072	0.567	0.263
Machine Operators, Assemblers, and Inspectors	0.043	0.020	0.638	0.299
Transportation and Material Moving	0.042	0.073	0.577	0.308
Handlers, Equipment Cleaners, and Helpers	0.023	0.021	0.560	0.396
Year 2000				
Executive Management	0.376	0.408	0.125	0.092
Management Related	0.408	0.308	0.205	0.079
STEM	0.425	0.278	0.176	0.121
Social Service, Lawyers	0.301	0.497	0.107	0.094
Education, Training, Library, legal support	0.407	0.365	0.117	0.110
Health Occupations	0.273	0.328	0.195	0.204
Technicians and Related Support	0.328	0.235	0.272	0.165
Financial Sales and Related Occupations	0.368	0.364	0.133	0.134
Retail Sales	0.275	0.322	0.232	0.171
Administrative Support	0.275	0.266	0.319	0.140
Housekeeping, Cleaning, Laundry	0.128	0.216	0.276	0.380
All Protective Service	0.224	0.287	0.202	0.288
Food Preparation and Service	0.104	0.313	0.280	0.303
Farm operators	0.153	0.205	0.252	0.389
Mechanics and Repairers	0.196	0.147	0.247	0.410
Construction	0.210	0.159	0.246	0.385
Precision production	0.236	0.199	0.303	0.262
Machine Operators, Assemblers, and Inspectors	0.128	0.070	0.428	0.374
Transportation and Material Moving	0.094	0.182	0.307	0.417
Handlers, Equipment Cleaners, and Helpers	0.067	0.073	0.371	0.490

C Technical Appendix

C.1 Derivation for the Time Allocation

The derivation for the probability of choosing k at time t follows the standard deviation of the discrete choice model.

$$\begin{aligned}
\Pi_{k|o,t} =& \mathbf{Prob}\big(k = \{\operatorname{argmax}_j p_{o,j}\nu_j^G \varepsilon_{o,j,t}^i\}\big) = \int_0^\infty \mathbf{Prob}\big(\varepsilon_{o,j,t} < \frac{p_{o,k}v_k^G x}{p_{o,j}v_j^G} \ \&\ \varepsilon_{o,k,t}^i = x, \forall j\big) dF(x) \\
=& \int_0^\infty \Big[\prod_{j\neq k} \mathbf{Prob}(\varepsilon_{o,j,t} < \frac{p_{o,k}v_k^G x}{p_{o,j}v_j^G})\Big] dF(x) \\
=& \int_0^\infty \Big[\prod_{j\neq k} \mathbf{Prob}\big(\varepsilon_{o,j,t} < \frac{p_{o,k}v_k^G x}{p_{o,j}v_j^G}\big)\Big](-\theta)x^{-\theta-1}\exp(-x^{-\theta})dx \\
=& \int_0^\infty (-\theta)x^{-1-\theta}\exp\big(-\sum_{j\neq k}\frac{p_j^\theta v_j^\theta}{p_k^\theta v_k^\theta x^\theta}\big)\exp(-x^{-\theta})dx \\
=& \int_0^\infty (-\theta)x^{-1-\theta}\exp\big(-x^{-\theta}\sum_j \frac{p_j^\theta v_j^\theta}{p_k^\theta v_k^\theta}\big)dx = \int_0^\infty (-\theta)x^{-1-\theta}\exp\big(-x^{-\theta}\Pi_{k|o}^{-1}\big)dx \\
=& \int_0^\infty (-1-\theta)x^{-\theta}\exp\big(-x^{-\theta}\Pi_{k|o}^{-1}\big)dx = \int_0^\infty \Pi_{k|o}\exp\big(-y\big)dy = \Pi_{k|o}.
\end{aligned} \tag{C.1}$$

The second equality holds because $dF(x) = (-\theta)(-x^{-\theta-1})\exp(-x^{-\theta})$. The final equality holds by $y = x^{-\theta}\Pi_{k|o}^{-1}$ and $dy = (-\theta)x^{-\theta-1}\Pi_{k|o}^{-1}dx$.

Since ε_o^i follows an i.i.d. Fréchet distribution across occupations, the derivation for the occupational probability in Equation (11) can be obtained analogously.

C.2 Derivation for the CES Utility Aggregator

The utility of worker i who has skill ν^G of working at occupation o is given in equation (4). $V_o^i(\nu^G)$ can be written as

$$V_o^i(\nu^G) = \Big(\int_{[0,1]} \big(\varepsilon_{o,k,t}\mathbb{1}_{k,t}\big)^{\frac{\sigma-1}{\sigma}} dt\Big)^{\frac{\sigma}{\sigma-1}} = \Big(\sum_k \int_{\Omega_k} \varepsilon_{o,k,t}^{\frac{\sigma-1}{\sigma}} dt\Big)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1. \tag{C.2}$$

$\Omega_k = \big\{t \in [0,1] | k = \operatorname{argmax} p_{o,k}\nu_k^G \varepsilon_{o,k,t}^i\big\}$ is the collection of t where k task is chosen. Next, let me solve each integral. Specifically,

$$\begin{aligned}
\Big(\int_{\Omega_k} \varepsilon_{o,k,t}^{\frac{\sigma-1}{\sigma}} dt\Big)^{\frac{\sigma}{\sigma-1}} =& \int_0^\infty x^{\frac{\sigma-1}{\sigma}}\Big[\prod_{j\neq k}\mathbf{Prob}(\varepsilon_{o,j,t} < \frac{p_{o,k}v_k^G x}{p_{o,j}v_j^G})\Big]dF(x) \\
=& \int_0^\infty x^{\frac{\sigma-1}{\sigma}}\Big[\prod_{j\neq k}\mathbf{Prob}\big(\varepsilon_{o,j,t} < \frac{p_{o,k}v_k^G x}{p_{o,j}v_j^G}\big)\Big](-\theta)x^{-\theta-1}\exp(-x^{-\theta})dx \\
=& \int_0^\infty (-\theta)x^{-\frac{1}{\sigma}-\theta}\exp\big(-\sum_{j\neq k}\frac{p_j^\theta v_j^\theta}{p_k^\theta v_k^\theta x^\theta}\big)\exp(-x^{-\theta})dx \\
=& \int_0^\infty (-\theta)x^{-\frac{1}{\sigma}-\theta}\exp\big(-x^{-\theta}\sum_j\frac{p_j^\theta v_j^\theta}{p_k^\theta v_k^\theta}\big)dx = \int_0^\infty (-\theta)x^{-\frac{1}{\sigma}-\theta}\exp\big(-x^{-\theta}\Pi_{k|o}^{-1}\big)dx \\
=& \int_0^\infty (-\theta)x^{-\frac{1}{\sigma}-\theta}\exp\big(-x^{-\theta}\Pi_{k|o}^{-1}\big)dx = \int_0^\infty y^{\frac{1}{\theta\sigma}-\frac{1}{\theta}}\Pi_{k|o}^{\frac{1}{\theta\sigma}-\frac{1}{\theta}+1}\exp\big(-y\big)dy \\
=& \Pi_{k|o}^{\frac{1}{\theta\sigma}-\frac{1}{\theta}+1}\Gamma\big(1-\frac{1}{\theta}+\frac{1}{\theta\sigma}\big).
\end{aligned} \tag{C.3}$$

The second equality holds because $dF(x) = (-\theta)(-x^{-\theta-1})\exp(-x^{-\theta})$. The final equality holds by $y = x^{-\theta}\Pi_{k|o}^{-1}$ and $dy = (-\theta)x^{-\theta-1}\Pi_{k|o}^{-1}dx$. The gamma function is $\Gamma(x) = \int_0^\infty y^{x-1}\exp(-y)dy$. Therefore, one can obtain the CES utility aggregator as

$$V_o^i(\nu^G) = \Gamma\big(1 - \frac{1}{\theta} + \frac{1}{\theta\sigma}\big)\big(\sum_k \Pi_{k|o}^{\frac{1}{\theta\sigma}-\frac{1}{\theta}+1}\big). \tag{C.4}$$

Two limiting cases are worth noting. The first case is when σ goes to 1, in which case

$$V_o^i(\nu^G) = \Gamma\big(1\big)\big(\sum_k \Pi_{k|o}\big) = 1. \tag{C.5}$$

The second case is when σ goes to infinity, then

$$V_o^i(\nu^G) = \Gamma(1 - \frac{1}{\theta})\sum_k \Pi_{k|o}^{\frac{\theta-1}{\theta}}. \tag{C.6}$$

Note the gamma function equals 1 when it is evaluated at 1.

C.3 The Existence and Uniqueness of Equilibrium

Below I show the existence and uniqueness, in which the proof relies heavily on Alvarez and Lucas (2007) and Allen and Arkolakis (2015). Denote $\mathbb{P}$ as the vector of task prices and define the excess labor demand function as

$$D_{o,k}(\mathbb{P}) = L_{o,k}^{\text{demand}} - L_{o,k}^{\text{supply}},$$

where the demand is defined in Equations (14) and (15) and the labor supply is defined in Equation (13). Following Alvarez and Lucas (2007), I verify the following six conditions hold, which ensures the existence and the uniqueness of a vector $\mathbb{P}$ such that $D_{o,k}(\mathbb{P}) = 0$:

1. $D_{o,k}(\mathbb{P})$ is continuous in $\mathbb{P}$, which holds immediately from the functional form of labor supply and demand.
2. $D_{o,k}(\mathbb{P})$ is homogeneous of degree zero. For any $\alpha > 0$,

$$\begin{aligned}
D_{o,k}(\alpha\mathbb{P}) &= L_{o,k}^{\text{demand}}(\alpha\mathbb{P}) - L_{o,k}^{\text{supply}}(\alpha\mathbb{P}) \\
&= \frac{\lambda_{o,k}A_o(\alpha\mathbb{P})^\rho}{\alpha p_{o,k}}\big[P_o(\alpha\mathbb{P})\big]^{1-\rho}\big[P(\alpha\mathbb{P})\big]^{\rho-1}Y(\alpha\mathbb{P}) - \sum_G \int N^G\nu_k^G\Pi_o(\alpha\mathbb{P},\nu^G)\Pi_{k|o}(\alpha\mathbb{P},\nu^G)dF_\nu^G \\
&= \frac{\lambda_{o,k}A_o(\alpha\mathbb{P})^\rho}{\alpha p_{o,k}}\alpha^{1-\rho}\big[P_o(\mathbb{P})\big]^{1-\rho}\alpha^{\rho-1}\big[P(\alpha\mathbb{P})\big]^{\rho-1}\alpha Y(\mathbb{P}) - \sum_G \int N^G\nu_k^G\Pi_o(\mathbb{P},\nu^G)\Pi_{k|o}(\mathbb{P},\nu^G)dF_\nu^G \\
&= L_{o,k}^{\text{demand}}(\mathbb{P}) - L_{o,k}^{\text{supply}}(\mathbb{P}) \\
&= D_{o,k}(\mathbb{P}).
\end{aligned}$$

   The third equality holds because $\Pi_{o,k}(\mathbb{P},\nu^G)$, $\Pi_{k|o}(\mathbb{P},\nu^G)$, and $A_o(\alpha\mathbb{P})$ are homogeneous of degree zero and $Y(\mathbb{P})$ and $P_o(\mathbb{P})$ are homogeneous of degree one in $\mathbb{P}$.

3. For all $\mathbb{P} > 0$, it is true that

$$\begin{aligned}
\sum_{o,k} p_{o,k} D_{o,k}(\mathbb{P}) &= \sum_{o,k} p_{o,k} L^{\text{demand}}_{o,k}(\mathbb{P}) - \sum_{o,k} p_{o,k} L^{\text{supply}}_{o,k}(\mathbb{P}) \\
&= \sum_{o,k} \lambda_{o,k} P_o^{1-\rho} A_o^{\rho} P^{\rho-1} Y - \sum_{o,k} p_{o,k} \sum_G \int N^G \nu_k^G \Pi_{k|o}\big(\mathbb{P}, \nu^G\big) \Pi_o\big(\mathbb{P}, \nu^G\big)\, dF_\nu^G \\
&= \sum_o \lambda_{o,k} Y_o - \sum_o Y_{o,k} = 0 \\
&= \sum_o \lambda_{o,k} Y_o - \sum_o \lambda_{o,k} Y_o = 0.
\end{aligned}$$

The second equality holds by the definition of aggregate labor supply and demand. The third equality holds because of perfect competition, i.e., the total output is the sum of the value added of all tasks by workers. The fourth equality holds because $\lambda_{o,k}$ corresponds to expenditure share under Cobb-Douglas production function.

4. For all $\mathbb{P}$, there is a uniform lower bound. For a specific pair (o,k),

$$\begin{aligned}
D_{o,k}(\mathbb{P}) &\geqslant -\sum_G \int N^G \nu_k^G \Pi_{k|o}\big(\mathbb{P}, \nu^G\big) \Pi_o\big(\mathbb{P}, \nu^G\big) dF_\nu^G \geqslant -\sum_G N^G \int \nu_k^G dF_\nu^G \\
&= -\sum_G N^G \exp(\mu_k^G + \frac{\sigma_k^2}{2}).
\end{aligned}$$

The second inequality holds because $\Pi_o\big(\mathbb{P}, \nu^G\big) \leqslant 1$ and $\Pi_{k|o}\big(\mathbb{P}, \nu^G\big) \leqslant 1$. The equality holds because $\int \nu_k^G dF_\nu^G = \exp(\mu_k^G + \frac{\sigma_k^2}{2})$, which is the expected value of log normal distribution. The uniform lower bound can be set as the $-\sum_G N^G \exp(\mu_k^G + \frac{\sigma_k^2}{2}) < 0$.

5. The following limit holds for any pair (o,k):

$$\lim_{p_{o,k} \to 0} D_{o,k}(\mathbb{P}) = \infty > 0.$$

Following Alvarez and Lucas (2007), there exists at least an equilibrium if conditions 1-5 hold. To ensure the equilibrium is unique, we need the gross substitution property below.

6. Pick an occupation o' and task k' (either $o' \neq o$ or $k' \neq k$, or both are different), it is straightforward to show that $\frac{\partial L^{\text{demand}}_{o,k}}{\partial p_{o',k'}} > 0$ and $-\frac{\partial L^{\text{supply}}_{o,k}}{\partial p_{o',k'}} > 0$. Then

$$\frac{\partial D_{o,k}(\mathbb{P})}{\partial p_{o',k'}} > 0.$$

Since $D_{o,k}(\mathbb{P})$ is homogeneous of degree zero in $\mathbb{P}$, this implies

$$\nabla D_{o,k}(\mathbb{P}) \cdot \mathbb{P} = 0.$$

Combining these two results, it must be the case that

$$\frac{\partial D_{o,k}(\mathbb{P})}{\partial p_{o,k}} < 0.$$

Therefore, the gross substitution property holds.

Conditions 1-6 ensure the equilibrium is unique.

C.4 The Structural Estimation

I estimate the model structurally using the SMM estimator by the following steps.

Solving for General Equilibrium (Inner Problem). The inner problem consists of three loops. Before going into each loop, I simulate the following skill distribution and take some initial guesses. First, I draw $R = 300$ pseudo individuals $(\ln z^G_{r,1}, \ln z^G_{r,2}, \ln z^G_{r,3}, \ln \nu^G_{r,4})$ from a multivariate normal distribution given in (21) for each group G. The G group has a mass of N^G. Here, r refers to the rth pseudo individual. Second, I take an initial guess of $p^t_{o,k} = [1, ..., 1]$, where the t index represents the number of iterations.

1. Calibrating $\tau^G_o(\Theta, \varpi)$. The first loop calibrates $\tau^G_o(\Theta, \varpi)$ such that the model and the data agree in terms of group-occupation employment shares, Π^G_o. The iteration methods apply contract mapping as used in Berry (1994).

- For each pseudo individual, compute the occupational probability using Equation (11), where P_o is given by (3); compute the time allocation to each task k using Equation (8); and compute the wage profile using Equation (9).
- Compute the aggregate group-occupation employment shares as

$$\Pi^{\text{model}}_{G,o} = \frac{1}{R}\sum_{r=1}^{R} \Pi_o(\nu^{G,r}).$$

- Stop the procedure if $\max_{G,o} |\Pi^{\text{model}}_{G,o} - \Pi^{\text{data}}_{G,o}| < 10^{-7}$. Otherwise, update:

$$\ln \tau^{t+1}_{G,o} = \ln \tau^t_{G,o} + \big(\ln \Pi^{\text{data}}_{G,o} - \ln \Pi^{\text{model}}_{G,o}\big). \tag{C.7}$$

2. Solving $p_{o,k}(\Theta, \varpi)$. In the second loop, given Υ (the set of parameters prior obtained), and a guess of Θ and of A_o, I apply the contraction mapping algorithm (Alvarez and Lucas, 2007) to solve $p_{o,k}(\Theta, \varpi)$ numerically as follows

- Compute the aggregate efficiency units of labor supply according to Equation (13).
- Compute the labor demand according to Equation (14), where the occupational output and the total outputs are computed using Equations (15) and (16), respectively.
- Compute the excess labor demand as:

$$Z_{o,k}\big(\{p^t_{o,k}\}\big) = L^{\text{demand}}_{o,k} - L^{\text{supply}}_{o,k}. \tag{C.8}$$

  Stop the procedure if $\max_{o,k} Z_{o,k}\big(\{p^t_{o,k}\}\big) < 10^{-5}$. Otherwise, update:

$$p^{t+1}_{o,k} = p^t_{o,k} + \alpha_1 \cdot \frac{\sqrt{Z_{o,k}(\{p^t_{o,k}\})^2}}{\max\left\{L^{\text{demand}}_{o,k}, L^{\text{supply}}_{o,k}\right\}}, \quad \alpha_1 \in (0,1). \tag{C.9}$$

3. Calibrating $A_o(\Theta, \varpi)$. In the third loop, given the solved $\tau^G_o(\Theta, \varpi)$ and $p_{o,k}(\Theta, \varpi)$, I use contraction mapping to solve $A_o(\Theta, \varpi)$ that target the average occupational wages as follows

- Using the solved $p_{o,k}(\Theta, \varpi)$ and the wages defined in equation (9), I compute the average occupation-level wages as

$$\overline{W}^{\text{Model}}_o = \frac{\sum_G \int N^G \cdot \Pi_o(\nu^G) \cdot W_o(\nu^G)\, dF^G_\nu}{\sum_G \int N^G \cdot \Pi_o(\nu^G)\, dF^G_\nu} \tag{C.10}$$

- Compute the sum of occupation-level wage gap $\sqrt{\sum_o(\overline{W}_o^{\text{Model}} - \overline{W}_o^{\text{Data}})^2}$, and stop if the gap smaller than 10^{-5}. Otherwise, update:

$$A_o^{t+1} = A_o^t + \alpha_2 \frac{\sqrt{(\overline{W}_o^{\text{Model}} - \overline{W}_o^{\text{Data}})^2}}{\max\left\{\overline{W}_o^{\text{Model}}, \overline{W}_o^{\text{Data}}\right\}}, \quad \alpha_2 \in (0,1). \tag{C.11}$$

SMM Estimation for Θ (Outer Problem). I then compute $\overline{W}_{G,o}^{\text{model}}$ and $\text{Var}_{G,o}^{\text{model}}$. I use gradient-based methods that search for the SMM estimator

$$\widehat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \left[\Psi_o^G(\Theta; \mathcal{F}(\Theta))\right]\Omega\left[\Psi_o^G(\Theta; \mathcal{F}(\Theta))\right]', \tag{C.12}$$

where $\Psi_o^G(\Theta; \mathcal{F}(\Theta)) = \left[\overline{W}_{G,o}^{\text{model}} - \overline{W}_{G,o}^{\text{data}}, \text{Var}_{G,o}^{\text{model}} - \text{Var}_{G,o}^{\text{data}}\right]$ is the vector of targeted moments. I denote $\mathcal{F}(\Theta) = \{\tau_o^G(\Theta, \varpi), p_{o,k}(\Theta, \varpi), A(\Theta, \varpi), \varpi\}$. Ω is the weighting matrix, with diagonal element being the size of group-occupation employment. I compute the model-predicted average and variance of wages, and the occupational employment as

$$\overline{W}_{G,o}^{\text{model}} = \frac{1}{R}\sum_{r=1}^{R} W_o\Big(\nu_r^G, \Theta; \mathcal{F}(\Theta)\Big)\Pi_o\Big[\nu_r^G, \Theta; \mathcal{F}(\Theta)\Big], \tag{C.13}$$

$$\text{Var}_{G,o}^{\text{model}} = \frac{1}{R-1}\sum_{r=1}^{R}\Big(W_o(\nu_r^G, \Theta; \mathcal{F}(\Theta)) - \overline{W}_{G,o}^{\text{model}}\Big)^2 \Pi_o\Big[\nu_r^G, \Theta; \mathcal{F}(\Theta)\Big], \tag{C.14}$$

$$\Pi_o^{\text{model}} = \frac{1}{R}\sum_{r=1}^{R}\Pi_o(\nu^r, \Theta, \mathcal{F}(\Theta)), \qquad \Pi_o(\nu^r, \Theta; \mathcal{F}(\Theta)) = \frac{\Big[W_o(\nu_r^G, \Theta; \mathcal{F}(\Theta))V_o^i(\nu^G)\tau_o^G(\Theta)\Big]^{\vartheta}}{\sum_o\Big[W_o(\nu_r^G, \Theta; \mathcal{F}(\Theta))V_o^i(\nu^G)\tau_o^G(\Theta)\Big]^{\vartheta}} \tag{C.15}$$

The standard gradient-based method (using Fmincon in Matlab) is used to search for parameters Θ.

C.5 Estimating Alternative Models in Section 6.2

I detail how I estimate T_1 in Section 6.2 when setting $\tilde{\nu}_1^G = T_1$ and $\tilde{\nu}_\ell^G = \nu_\ell^G, \ell = 2,3,4$. A similar procedure applies to other skills. Using the baseline estimated skill parameter $\{\mu_z^G, \mu_\nu^G, \Sigma_z^G, \Sigma_{z\nu}^G, \Sigma_\nu^G\}$, I draw the joint skill distribution of social, routine, and manual skills $\{\upsilon_2^G, \upsilon_3^G, \upsilon_4^G\}$.

Solving for General Equilibrium. Given a guess of T_1^0, the same baseline parameter estimates of θ, ϑ, ρ, A_0, $\lambda_{o,k}$, and τ_o^G, I compute aggregate task demand and supply for occupation and apply the contraction mapping algorithm (Alvarez and Lucas, 2007) to solve the task price $p_{o,k}(T_1^0)$ where $L_{o,k}^{\text{supply}} = L_{o,k}^{\text{demand}}$. The iteration follows the same as step 1 of SMM estimator described in Section C.4.

Update T_1. Given $p_{o,k}(T_1^0)$, the second step then computes the log average wage and updates T_1^0 until the predicted log wage and the data agree. Specifically, I compute the average log wages as

$$\overline{W}^{\text{Model}} = \frac{\sum_o\sum_G\int N^G\cdot\Pi_o(\nu^G)\cdot W_o(\nu^G)\,dF_\nu^G}{\sum_o\sum_G\int N^G\cdot\Pi_o(\nu^G)\,dF_\nu^G} \tag{C.16}$$

After that, I compute the gap between the data and model as $\sqrt{\sum_o(\overline{W}^{\text{Model}} - \overline{W}^{\text{Data}})^2}$, and stop if the gap smaller than 10^{-5}. Otherwise, update:

$$T_1^{t+1} = T_1^t + \alpha_2\frac{\sqrt{(\overline{W}^{\text{Model}} - \overline{W}^{\text{Data}})^2}}{\max\left\{\overline{W}^{\text{Model}}, \overline{W}^{\text{Data}}\right\}}, \quad \alpha_2 \in (0,1). \tag{C.17}$$

C.6 The Variance-Covariance of SMM Estimator

According to Wooldridge (2010), the asymptotic variance-covariance matrix can be obtained as

$$\Big[\Phi'\Omega\Phi\Big]^{-1}\Big[\Phi'\Omega H\Omega\Phi\Big]\Big[\Phi'\Omega\Phi\Big]^{-1} \tag{C.18}$$

where Φ is the gradient matrix of $\Psi_o^G\big(\Theta; p_{o,k}(\Theta), A_o(\Theta)\big)$ with respect to parameters Θ defined as

$$\Phi = \begin{bmatrix} \frac{\partial(\overline{W}_{G,o}^{\text{model}} - \overline{W}_{G,o}^{\text{data}})}{\partial\Theta} \\ \frac{\partial(S_{G,o}^2)^{\text{model}} - (S_{G,o}^2)^{\text{data}}}{\partial\Theta} \\ \frac{\partial\Pi_{G,o}^{\text{model}} - \Pi_{G,o}^{\text{data}}}{\partial\Theta} \end{bmatrix}_{\Theta_0}.$$

The gradient matrix is evaluated at parameter values Θ_0, which are the SMM estimator. Ω, again, is the diagonal matrix with each element being the size of group-occupation employment. H is the variance-covariance of the moment condition (evaluated at the truth).

$$H = \text{Var}\Big[(\Psi_o^G)' \times \Psi_o^G\Big]_{\Theta_0}, \tag{C.19}$$

where Ψ_o^G is the shorthand for $\Psi_o^G\big(\Theta; \mathcal{F}(\Theta)\big)$. The asymptotic variance of the SMM estimators is then the diagonal elements.

D The Fitness of Two Alternative Models

This section evaluates the fit of two alternative models: (1) a model with common task assignments across workers, and (2) the Roy model.

D.1 Model # 1: Common task assignments across workers

First, I study a model that deviates from the baseline by assuming common task assignments across workers. Once entering an occupation, all workers allocate the same fraction of time to each task. Since the task assignment is the same across all workers, the scale is thus absorbed by the price. The wage equation is

$$W_o(\nu^G) = \sum_k p_{o,k} \times \nu_k^G. \tag{D.1}$$

The production function and occupational choice follow the same as the baseline model. Specifically, the production functions follow Equations (1) and (2). The log absolute and comparative advantage schedules follow a joint normal distribution defined in (21). Workers choose one occupation that maximizes their utility. The share of workers choosing occupation o is, again, given in Equation (11). I re-estimate parameter Θ by adopting the two-step SMM procedure and targeting the same sets of moments using the 2000 CPS. Using the estimated skills, I then solve the equilibrium in 1980. Again, A_o is calibrated to match average occupational wages and τ_o^G matches group-occupation employment

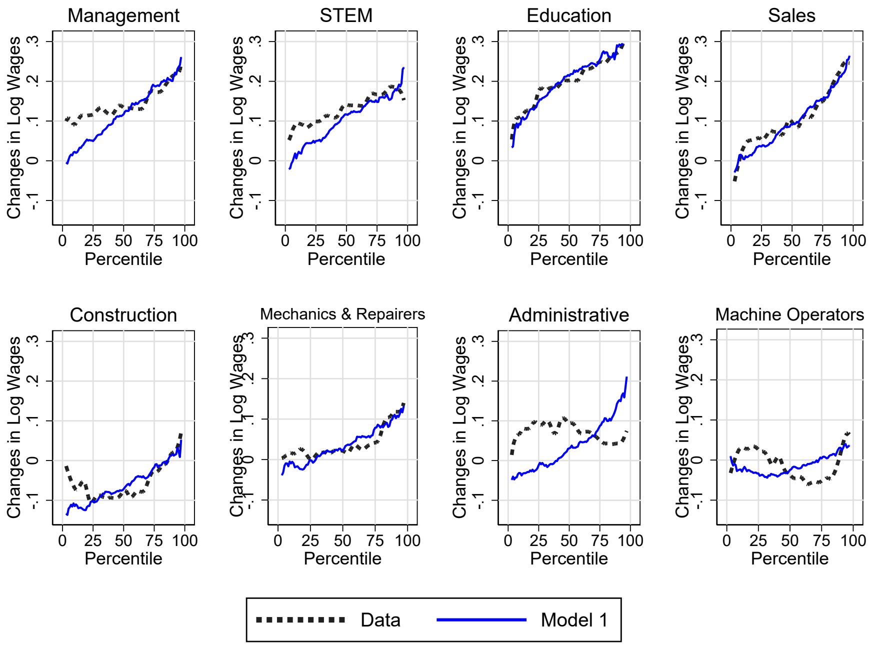

Figure D.1: Changes in Occupational Log Wage Percentiles, 1980-2000: Model and Data

In Equation (D.1), wage still has multiple sources and $\Delta \ln p_{o,k}$ can generate unequal responses within occupations. Figure D.1 shows the model predicts nearly linear wage changes for most occupations. For occupations such as management, STEM, Sales, and Construction, the predicted wage changes replicate the top end but fail to match the smooth changes at the middle to bottom end. For the overall distribution, the model fits less well for the middle-to-bottom end (see Figure D.3).

Table D.1: The SMM Estimates of Structural Parameters, A Model with Common Task Assignments

Education and Age Groups	$\mu^G_{z_1}$	$\mu^G_{z_2}$	$\mu^G_{z_3}$	$\mu^G_{z_4}$	$\mathbf{Cov}(\ln z^G_2, \ln \nu^G_4)$	$\mathbf{Cov}(\ln z^G_2, \ln \nu^G_4)$	$\mathbf{Cov}(\ln z^G_3, \ln \nu^G_4)$	$\mathbf{Var}(\ln \nu^G_4)$
				Females				
HS dropouts 21-41 years Old	-.34 (.016)	-.06 (.211)	.101 (.269)	-.06 (.824)	-.13 (.036)	-.06 (.053)	-.12 (.023)	.156 (.094)
HS dropouts 41-60 years Old	0 (0)	0 (0)	0 (0)	0 (0)	-.21 (.055)	-.13 (.033)	-.18 (.042)	.244 (.005)
HS grad 21-41 years Old	.048 (.012)	.003 (.073)	.548 (.134)	-.11 (.230)	-.05 (.013)	-.07 (.028)	-.16 (.011)	.197 (.041)
HS grad 41-60 years Old	.292 (.005)	-.09 (.028)	.557 (.068)	-.04 (.039)	-.10 (.011)	-.08 (.027)	-.10 (.017)	.158 (.116)
Some college 21-41 years Old	.332 (.238)	-.73 (.004)	-.00 (.143)	.238 (.454)	-.12 (.046)	-.06 (.042)	-.20 (.069)	.250 (.105)
Some college 41-60 years Old	.356 (.010)	-.45 (.003)	-.04 (.087)	.309 (.089)	-.20 (.164)	-.11 (.110)	-.31 (.273)	.406 (.366)
College and above 21-41 years Old	.358 (.307)	-.03 (.141)	-.81 (.002)	.494 (.374)	-.22 (.351)	-.22 (.482)	-.31 (.569)	.471 (.107)
College and above 41-60 years Old	.493 (.038)	.234 (.024)	-.86 (.013)	.523 (.017)	-.20 (.201)	-.13 (.423)	-.32 (.538)	.423 (.394)
				Males				
HS dropouts 21-41 years Old	-.48 (.006)	-.34 (.060)	-.13 (.070)	.230 (.438)	.020 (.003)	-.01 (.007)	-.01 (.004)	.069 (.060)
HS dropouts 41-60 years Old	-.47 (.003)	-.45 (.016)	.217 (.102)	.293 (.230)	-.04 (.006)	.008 (.020)	-.04 (.008)	.083 (.008)
HS grad 21-41 years Old	.190 (.195)	-.75 (.016)	.273 (.310)	.301 (.768)	.046 (.147)	.046 (.093)	-.15 (.062)	.191 (.097)
HS grad 41-60 years Old	.301 (.032)	-.73 (.009)	.356 (.165)	.380 (.239)	-.02 (.045)	.034 (.100)	-.04 (.038)	.108 (.141)
Some college 21-41 years Old	.345 (.189)	-.74 (.022)	.041 (.070)	.420 (.380)	-.04 (.075)	.043 (.163)	-.09 (.031)	.154 (.158)
Some college 41-60 years Old	.455 (.286)	-.63 (.026)	.253 (.239)	.505 (.620)	-.07 (.115)	-.00 (.253)	-.12 (.045)	.182 (.263)
College and above 21-41 years Old	.511 (.005)	1.05 (.561)	.385 (.051)	.013 (.606)	-.17 (.273)	-.11 (.664)	-.15 (.168)	.327 (.781)
College and above 41-60 years Old	.477 (.039)	.867 (.131)	-.07 (.003)	.382 (.140)	-.21 (.190)	-.18 (.561)	-.21 (.186)	.371 (.657)

Notes: Standard errors are reported in parenthesis. Appendix C.6 details how I construct the standard errors of the parameter estimates.

In the baseline model, heterogenous task sorting introduces wage convexity in comparative advantage. When the task assignment is common, the model needs to load a higher skill dispersion in order to fit the steep wage growth at the upper tail, worsening the model's fit at the bottom end. Appendix Table D.1 reports the SMM estimates for skill parameters. It shows the estimated parameters of μ_ν^G and Σ_ν^G are generally larger than the baseline estimates. Thus, wage convexity results from heterogeneous task sorting in generating steep occupational wage growth at the top end.

D.2 Model # 2: The Roy Model

I structurally estimate a Roy model where there is one skill price for each occupation. I assume the production function follows Equation (1). In contrast to the baseline model, all skills are paid at the same price, $p_{o,k} = P_o, \forall k$. Workers choose one occupation that maximizes their utility given in Equation (4). ε_o^i and τ_o^G are the same as before. The probability of choosing occupation o follows Equation (11). Workers' earning has only one source and equals $W(\nu^G) = p_o \times \nu^G$.

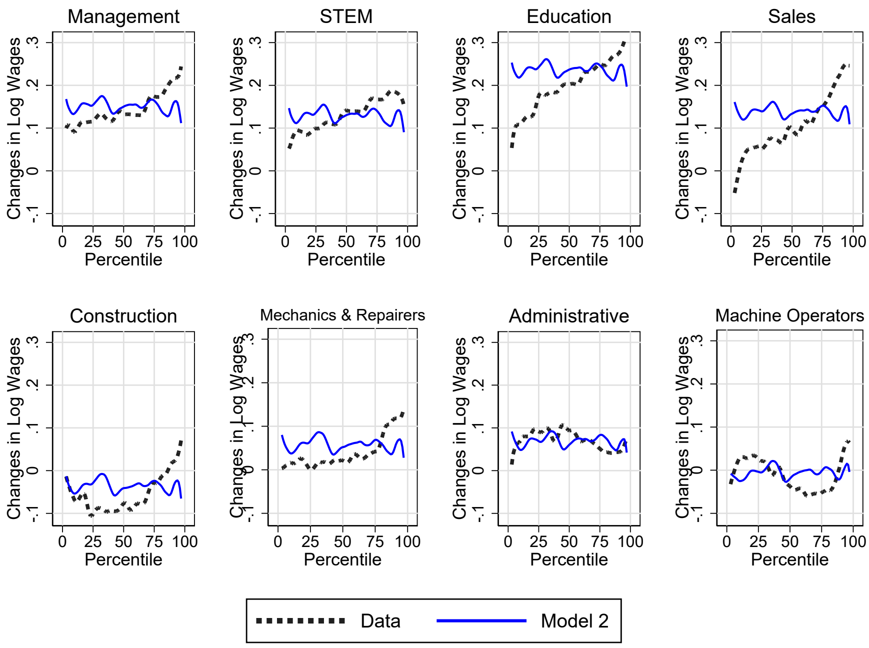

Figure D.2: Changes in Occupational Log Wage Percentiles, 1980-2000: Data and the Roy Model

Every worker draws a single-dimensional skill, where the log skill, $\ln \nu^G$, follows a normal distribution $\mathcal{N}(\mu^G, \sigma^G)$.[39] While the literature commonly uses Roy models with Fréchet skills (Lagakos and Waugh, 2013, Hsieh et al., 2019), I assume normally-distributed skill to draw direct comparisons with the baseline model. Irrespective of the skill distribution, these models all imply that demand shocks equally affect workers within-occupation in partial equilibrium.

I adopt the two-step SMM procedure to estimate μ^G and σ^G, and assign the same values to ρ and ϑ as in the baseline model. I target the same sets of moments using the May/ORG CPS 2000: group-occupation employment shares and the mean and variance of log wages. I then solve the equilibrium

[39] Although I assume the idiosyncratic skill is invariant across occupations, it is equivalent to a Roy model of occupation-specific draws, where $\ln \nu_o^G$ follows a normal distribution $\mathcal{N}(\mu_o \mu^G, \mu_o^2 \sigma^G)$. These two setups are equivalent as μ_o is absorbed in A_o.

for 1980. Note that demand changes only involve the changes in A_o. I calibrate A_o to match average occupational wages for each year. Figure D.2 shows that the model predicts limited inequality responses both within-occupation. On the aggregate, the predicted wage changes are more flat than the observed wage changes, See Figures D.3.

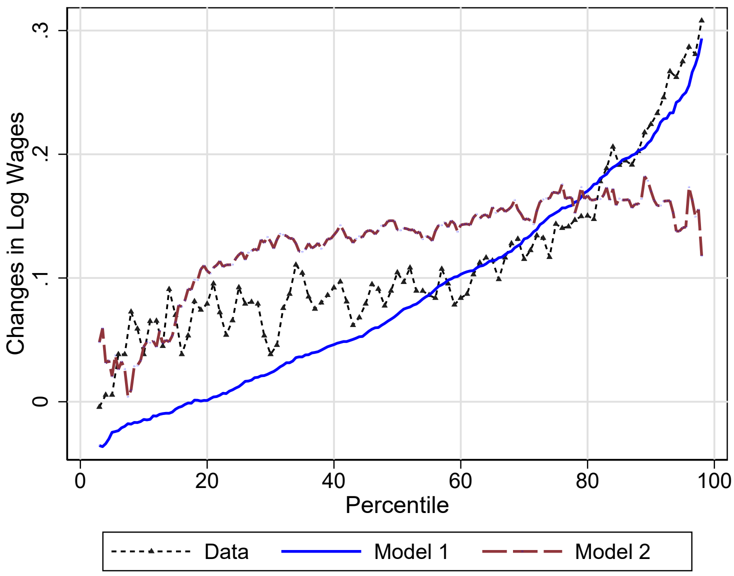

Figure D.3: Changes in the Log Wage By Percentiles: Alternative Models.

E Additional Quantitative Results

Section 6 indicates that changes in $\lambda_{o,k}$ are the primary drivers of the observed increase in within-occupation inequality, with skill heterogeneity playing crucial roles in shaping the unequal responses. This section conducts sensitivity analysis across various model specifications and parameter values to assess the robustness of the results.

Appendix E.1 presents quantitative results under different demand elasticities, while Appendix E.2 presents results under alternative occupation aggregation. Appendix E.3 explores scenarios where there is perfect substitution in task-utility over time, with σ approaching infinity. Appendix E.4 uses an alternative measure for $\lambda_{o,k}$ based on the frequency of words mentioned. Appendix E.5 presents quantitative results under different values of θ. Appendix E.6 summarizes the findings.

To conserve space, I only report the between and within-occupation decomposition results. Results on the percentile wage gap and between and within-group inequalities are available upon request.

E.1 The Demand Elasticities

I provide results under different demand elasticity values: the elasticity of substitution across occupations (ρ) and the elasticity of substitution across tasks within occupations. The value of demand elasticity affects the calibrated value A_o. For this analysis, I keep $\lambda_{o,k}$, occupation choice parameters (ϑ, σ), and the covariance Σ_z^G the same as in the baseline. I structurally re-estimate the model parameters Θ, A_o, τ_o^G, and $p_{o,k}$ through a two-step Simulated Method of Moments (SMM) procedure. This involves targeting the same sets of moments using the 2000 Current Population Survey (CPS) data and solving the equilibrium for the year 2000. Using the estimated skills, I subsequently solve the equilibrium for 1980. Once more, A_o is calibrated to align with average occupational wages, while τ_o^G is adjusted to match group-occupation employment.

E.1.1 The Elasticity of Substitution Across Occupations

I consider two scenarios: $\rho = 1.8$ as the lower bound based on BMV2019, and $\rho = 4$ as the upper bound. Table E.1 presents the decomposition results for the two scenarios. The findings are threefold. First, for all shocks, the between-occupation variances vary with ρ, but the within-occupation variances are insensitive to different values. This is because ρ affects the equilibrium through Equation (15) and has no impact on the relative task prices. Second, the calibrated A_o depends on the value of ρ. A higher value of ρ leads to more evenly distributed changes in A_o across occupations, resulting in a smaller between-occupation response. The effects of changes in A_o increase between-occupation variance by 0.036 when $\rho = 1.8$, but by only 0.006 when $\rho = 4$. Third, the other two shocks, namely the changes in $\lambda_{o,k}$ and labor composition, are measured independently of ρ. Changes in $\lambda_{o,k}$ affect the occupation price P_o, which acts as occupation-specific demand shifts in Equation (15). A higher value of ρ corresponds to a larger magnitude of shock, leading to a more significant between-occupation response. For labor composition changes, a higher value of ρ results in less pronounced responses in occupation prices P_o, thereby generating smaller between-occupation responses.

Table E.1: The Between and Within Occupation Inequality with Different Values of ρ

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
		$\rho = 1.8$			
Between	0.025	0.036	0.016	-0.009	0.017
Within	0.029	0.001	0.023	0.003	-0.002
Total	0.054	0.037	0.039	-0.007	0.015
		$\rho = 4$			
Between	0.025	0.006	0.028	-0.007	0.003
Within	0.029	0.002	0.024	0.003	-0.001
Total	0.054	0.008	0.053	-0.004	0.002

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

E.1.2 The Elasticity of Task Substitution Within Occupation.

Recalling the baseline Cobb-Douglas technology, which implies a unit elasticity of substitution across tasks, I extend the model by introducing a CES task aggregator within-occupation that allows this elasticity to deviate from one. In replacing Equation (2), I assume the occupational output is produced using the following CES technology:

$$Q_o = \left[\sum_k \lambda_{o,k} L_{o,k}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{E.1}$$

where $\lambda_{k,o}$ measures the k-task intensity in occupation o. η measures the elasticity of substitution across tasks within-occupation. Firms' profit maximization implies that the price per unit of occupational output is given by:

$$P_o = \left[\sum_k \lambda_{o,k}^{\eta} p_{o,k}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{E.2}$$

The supply side remains unchanged from the baseline model. In equilibrium, the task demand at occupation o becomes:

$$L_{o,k}^{\text{demand}} = \frac{1}{p_{o,k}^{\eta}} \lambda_{o,k}^{\eta} P_o^{\eta-1} Y_o = \frac{1}{p_{o,k}^{\eta}} \lambda_{o,k}^{\eta} P_o^{\eta-\rho} A_o^{\rho} Y. \tag{E.3}$$

where Y_o follows Equation (15) and Y follows Equation (16), respectively. Under the CES production function, $\lambda_{o,k}$ does not precisely capture the expenditure share but is positively associated with it. Specifically, for occupation o, the expenditure share on the k-task is given by $\frac{\lambda_{o,k}^{\eta} p_{o,k}^{1-\eta}}{\sum_\ell \lambda_{o,\ell}^{\eta} p_{o,\ell}^{1-\eta}}$. As η approaches 1, $\lambda_{o,k}$ converges to the expenditure share.

To the best of my knowledge, the value of η has not been estimated in the literature. Therefore, Table E.2 presents the results obtained using a wide range of values. Specifically, when $\eta = 2$, changes in $\lambda_{o,k}$ account for 72% $(0.021/0.029)$ of the observed increase in within-variance. This proportion declines to 62% $(0.018/0.029)$ when $\eta = 4$. In the most conservative scenario with $\eta = 10$, changes in $\lambda_{o,k}$ still account for 52% $(0.015/0.029)$ of the observed increase. The intuition behind these results is as follows: when η is larger, changes in $\lambda_{o,k}$ lead to greater adjustments within occupations but fewer changes in occupational employment. As a result, the response in relative task prices $(p_{o,k})$ is smaller, while the response in occupational prices (P_o) is larger. This leads to a smaller increase in within-variance but a larger increase in between-variance.

Table E.2: The Between and Within Occupation Inequality with Different Values of η

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
			$\rho = 3$ **and** $\eta = 2$		
Between	0.025	0.006	0.033	-0.007	0.007
Within	0.029	0.003	0.021	0.003	-0.001
Total	0.054	0.009	0.054	-0.004	0.006
			$\rho = 3$ **and** $\eta = 4$		
Between	0.025	0.001	0.040	-0.007	0.009
Within	0.029	0.005	0.018	0.003	-0.003
Total	0.054	0.007	0.058	-0.005	0.006
			$\rho = 3$ **and** $\eta = 10$		
Between	0.025	-0.001	0.047	-0.007	0.014
Within	0.029	0.007	0.015	0.002	-0.006
Total	0.054	0.006	0.062	-0.005	0.008

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

E.2 Alternative Occupation Aggregation

The baseline decomposition results are derived from 20 occupational categories. Here, I present quantitative results under an alternative 40 occupation classification. Appendix A.3 provides lists of occupations included in each classification.[40]

For each model, I maintain $\lambda_{o,k}$, ϑ, σ, and the covariance Σ_z^G the same as in the baseline. I then structurally re-estimate the model parameters Θ, A_o, τ_o^G, and $p_{o,k}$ through a two-step Simulated Method of Moments (SMM) procedure, targeting the same sets of moments using the 2000 Current Population Survey (CPS) data and solving the equilibrium for the year 2000. Using the estimated skills, I subsequently solve the equilibrium for 1980. As before, A_o is calibrated to align with average occupational wages, while τ_o^G is adjusted to match group-occupation employment.

Table E.3 presents the results. Having more disaggregated occupational groups predicts a broader between-occupation inequality of changes in A_o. The changes in $\lambda_{o,k}$ still emerge as the primary contributor to the growth of within-occupation variance.

[40] I base on 30 broad occupational categories provided in OCC1990, which have been employed in BMV2019, and then break down occupations with large employment sizes into separate groups.

Table E.3: The Between and Within Occupation Inequality Under 40 Occupations

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
Between	0.028	0.020	0.016	-0.008	0.000
Within	0.026	0.001	0.024	0.000	-0.000
Total	0.054	0.021	0.040	-0.008	0.000

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

E.3 Alternative Values of σ

I present results for an alternative model wherein the task utilities are perfect substitutes over time—σ approaches infinity. The only difference from the baseline model lies in the expression for occupational choice (11), where (recall $V_o^i(\nu^G) = 1$ in the baseline model)

$$V_o^i(\nu^G) = \Gamma(1 - \frac{1}{\theta}) \sum_k \Pi_{k|o}^{\frac{\theta-1}{\theta}}. \tag{E.4}$$

Again, I maintain $\lambda_{o,k}$, ϑ, and the covariance Σ_z^G the same as in the baseline. I structurally re-estimate the model parameters Θ, A_o, τ_o^G, and $p_{o,k}$ through a two-step Simulated Method of Moments (SMM) procedure, targeting the same sets of moments using the 2000 Current Population Survey (CPS) data and solving the equilibrium for the year 2000. Using the estimated skills, I subsequently solve the equilibrium for 1980. As before, A_o is calibrated to align with average occupational wages, while τ_o^G is adjusted to match group-occupation employment.

Table E.4 reports the results. The results are very much the same as the baseline, suggesting that the values of σ have a small impact on the results. The reason is that the value of σ only affects the equilibrium results through $V_o^i(\nu^G)$ and the occupational choice in equation (11). As much of the variation in occupational choice is captured by τ_o^G, the results are insensitive to the value of σ.

Table E.4: The Between and Within Occupation Inequality When σ Goes to Infinity

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
Between	0.025	0.014	0.026	-0.008	0.007
Within	0.029	0.001	0.025	0.002	-0.001
Total	0.054	0.015	0.050	-0.006	0.006

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

E.4 Alternative Measures of $\lambda_{o,k}$

The baseline model quantifies $\lambda_{o,k}^{2000}$, the relative task demand within an occupation, using the ratio of task percentile rankings, following Autor, Levy and Murnane (2003). This section presents quantitative results using an alternative approach for $\lambda_{o,k}^{2000}$, utilizing the ratio in the frequency of task-related words collected by APST2020. Specifically,

$$\lambda_{o,k}^{2000} = \frac{F_{o,k}^{2000}}{\sum_{k'} \mathbf{F}_{o,k'}^{2000}}. \tag{E.5}$$

Compared to the baseline measure using the O*NET, this alternative measure tends to assign significantly higher weights to cognitive and social-related tasks, which are more likely to be mentioned in job ads (Also see APST2020). Same as the baseline model, I measure changes in task demand by utilizing changes in the frequency of task-related words and compute task shares in 1980 as:

$$\lambda_{o,k}^{1980} = \frac{\lambda_{o,k}^{2000} \times \left(F_{o,k}^{2000} \Big/ F_{o,k}^{1980}\right)}{\sum_{k'} \lambda_{o,k'}^{2000} \times \left(F_{o,k'}^{2000} \Big/ F_{o,k'}^{1980}\right)}, \tag{E.6}$$

where $\lambda_{o,k}^{2000}$ is measured according to equation (E.5). Indeed, task-related words that are mentioned more frequently in job advertisements suggest an increase in demand for those tasks.

Again, I maintain $\lambda_{o,k}$, ϑ, and the covariance Σ_z^G the same as in the baseline. I structurally re-estimate the model parameters Θ, A_o, τ_o^G, and $p_{o,k}$ through a two-step Simulated Method of Moments (SMM) procedure, targeting the same sets of moments using the 2000 Current Population Survey (CPS) data and solving the equilibrium for the year 2000. Using the estimated skills, I subsequently solve the equilibrium for 1980. As before, A_o is calibrated to align with average occupational wages, while τ_o^G is adjusted to match group-occupation employment.

Table E.5: The Between and Within Occupation Inequality with Alternative $\lambda_{o,k}$

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
Between	0.025	0.042	-0.005	-0.008	0.004
Within	0.029	-0.007	0.025	-0.005	-0.016
Total	0.054	0.035	0.020	-0.013	-0.012

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

Compared to the baseline result, Table E.5 shows that this alternative measure tends to find a larger between-occupation response to the changes in A_o. In short, while the alternative measure tends to have a different interpretation of what drives the between-occupation inequality changes, both measures indicate that the changes in $\lambda_{o,k}$ are the primary driver of rising inequality within occupations.[41]

E.5 Within-Occupation Task Adjustments

The extent to which workers can adjust across tasks would impact the price response. The intuition is as follows: when task supply is elastic, changes in relative task demand result in significant adjustments in task assignment, leading to small changes in relative task prices. Consequently, the effect on inequality is small. On the other hand, with inelastic supply, the response of relative task prices is more pronounced, which in turn amplifies the inequality response.

[41]The differences in the prediction for between-occupation inequality is driven by the systematic difference between the alternative and the baseline O*NET measure, as the former assigns significantly greater weights to cognitive and social tasks. For this reason, I prefer the baseline measure which uses job ads and word frequency only to measure changes in task demand within occupations, aligning with the emphasis of APST2020.

To quantify the effects of task adjustments, I examine two alternative scenarios: one with elastic supply ($\theta = 10$) and another with inelastic supply ($\theta = 1.1$). Unlike the exercises in Appendices E.1-E.4, the objective here is to isolate the effect of within-occupation task adjustments. This exercise is comparative statics by nature, involving a change in a specific model parameter while keeping the rest of the parameters exactly the same as the baseline. Specifically, I maintain the demand parameters (A_o, $\lambda_{o,k}$, ρ) and occupation choice parameters (σ, τ_o^G, ϑ) the same as the baseline estimates, while setting θ at a different value. However, it should be noted that this would result in a model-predicted wage that does not align with the data in 2000. To maintain consistency, I set the skills as $\widetilde{\nu_k^G} = C\nu_k^G$, where ν_k^G follows the skill distribution estimated in the baseline. C is a constant estimated such that the predicted average log wage matches the data while solving the general equilibrium in 2000. Importantly, the inclusion of C does not alter the relative skill distribution, and thus it has no impact on the inequality results. Using the estimated skill $\widetilde{\nu_k^G}$, I then solve for the equilibrium in 1980.

Table E.6: The Between and Within Occupation Inequality Responses with Different Values of θ

	Data	A_o	$\lambda_{o,k}$	N^G	Residual
	(1)	(2)	(3)	(4)	(5)
	Elastic supply, $\theta = 10$				
Between	0.025	0.015	0.024	-0.007	0.006
Within	0.029	0.002	0.023	0.003	-0.001
Total	0.054	0.017	0.047	-0.004	0.005
	Inelastic supply, $\theta = 1.1$				
Between	0.025	0.014	0.026	-0.009	0.006
Within	0.029	0.001	0.027	0.000	-0.001
Total	0.054	0.015	0.053	-0.009	0.004

Notes: The impacts in Columns (2)-(5) are derived by assessing what would have occurred in 1980 if each shock was adjusted to its respective level in 2000.

Table E.6 shows that the effects of changes in A_o and labor composition are insensitive to the values of θ, but the effects of changes in $\lambda_{o,k}$ do. When the supply of tasks is elastic, the effects of changes in $\lambda_{o,k}$ on all three variance terms become smaller. Conversely, when the supply of tasks is inelastic, the effects become more pronounced. Notably, in the case of elastic supply, changes in $\lambda_{o,k}$ account for 79% ($0.023/0.029$) of the within-occupation inequality, which rises to 90% ($0.026/0.029$) when the supply of tasks is inelastic.

E.6 Summary

This section conducts sensitivity analysis across various model specifications and parameter values. Through these exercises, I assess the robustness of my findings to different values of demand elasticities, task supply elasticity θ, varying σ, and alternative measures of $\lambda_{o,k}$. I focus on the decomposition of between- and within-occupation inequality. While the quantitative results vary across cases, the main message remains consistent: across all scenarios, I find that changes in $\lambda_{o,k}$ are the primary driver of rising inequality, accounting for at least 52% ($0.015/0.029$) of the rising inequality within-occupation. See Appendix Table E.2 for the least value.

F The Targeted Moments

The wage distribution in 2000. Table F.1 compares the observed and predicted average log wages by four education groups (Columns 1 and 2), as well as the log wage variance (Columns 3 and 4). The model fits the data closely in most cases, which is expected since the skill distribution is estimated by targeting the group-occupation wages. Additionally, Figure F.2 shows that the model closely replicates the overall log wage distribution, and Figure F.3 demonstrates that the model fits the wage distribution well for disaggregated education and age groups in 2000.

Table F.1: The Average and Variance of Log Wages in 2000: Model vs. Data

	Average Log Wage		**Variance Log Wage**	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
HS dropouts	2.349	2.418	0.148	0.156
HS grad	2.627	2.654	0.193	0.187
Some college	2.756	2.777	0.220	0.210
College and above	3.134	3.155	0.289	0.247

Notes: The wages are in real terms, for which I deflate the hourly wages by the PCE price deflator.

Group-level occupational employment. Figure F.1 plots the observed group-level occupational shares on the x-axis against the predicted shares on the y-axis for 1980 and 2000, respectively. As τ_o^G is calibrated to fit exactly the group-occupation employment shares in general equilibrium, the model fits the data perfectly for both years.

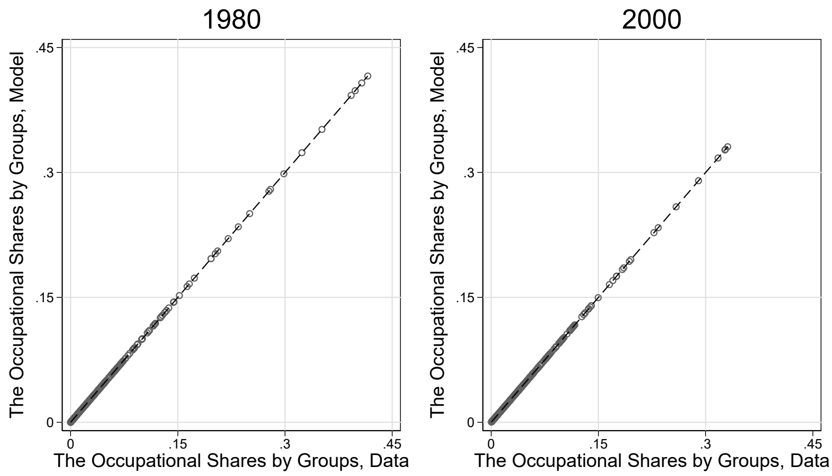

Figure F.1: The Fit of Occupational Shares: Data vs. Model

r refers to the rth pseudo-individual, and $\Pi_o(\cdot)$ is the function of probability choice defined in Equation (11). The predicted group-level occupational shares are estimated as follows:

$$\Pi_o^G = \int \Pi_o(\nu^G) dF_\nu^G = \frac{1}{R}\sum_{r=1}^{R} \Pi_o(\nu^{G,r}).$$

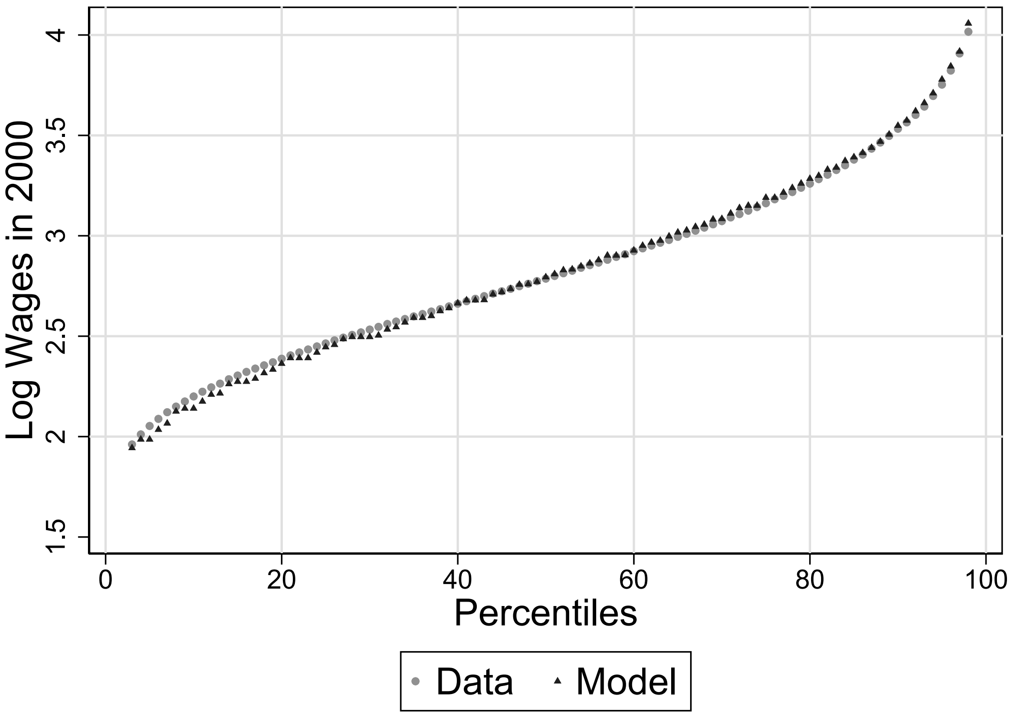

Figure F.2: The Percentile Values of Log Wage in 2000 (Targeted Moments)

Notes: This figure plots the percentile on the horizontal axis and the value of log wage on the vertical axis.

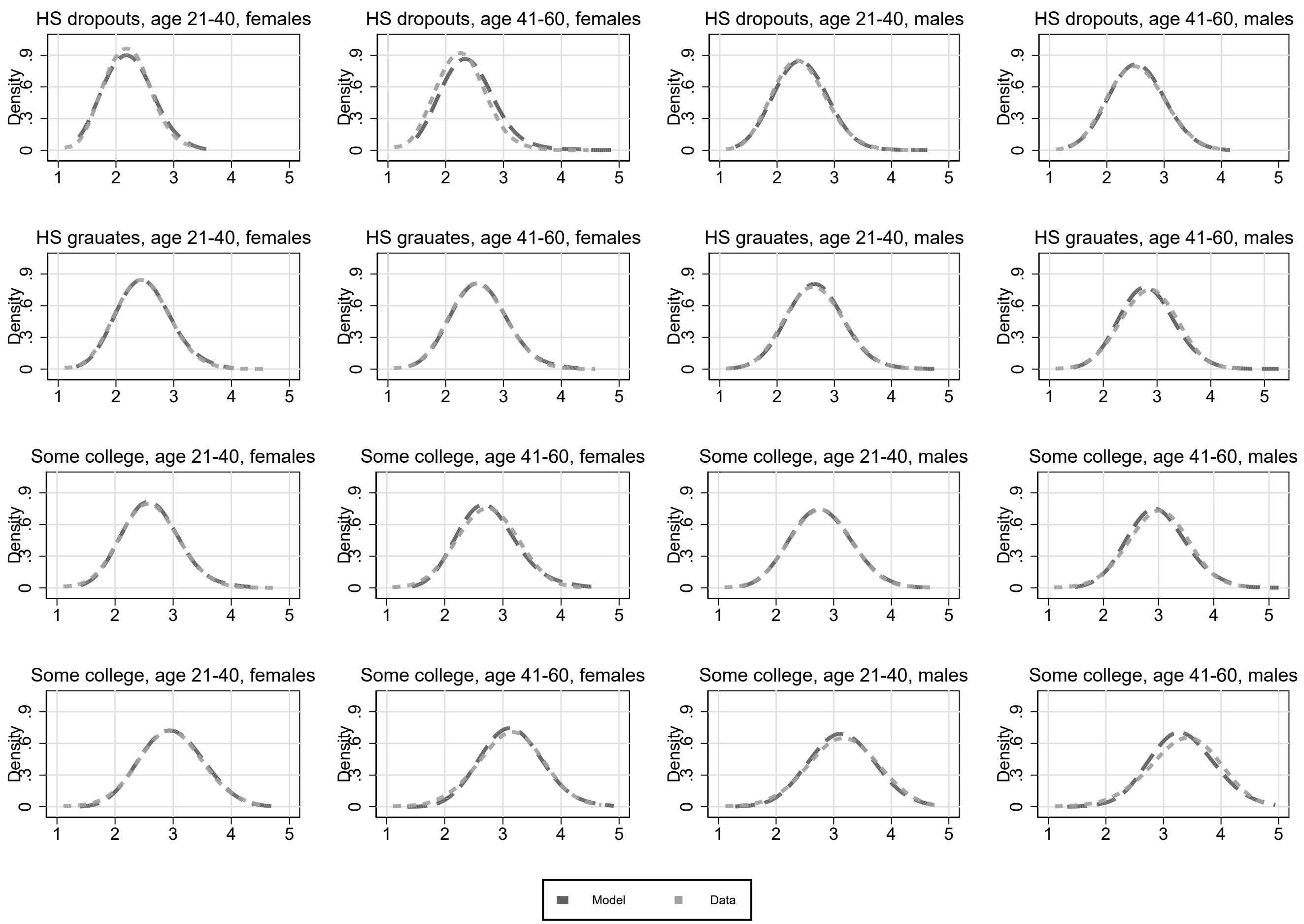

Figure F.3: The Kernel Density of the Empirical and Predicted Log Wages by Groups in 2000 (Targeted Moments)

Notes: I use the Epanechnikov kernel. To obtain a clear visualization in the log wage support, I set the bandwidth to be the default optimal bandwidth choice under normal density.

G Tables and Figures

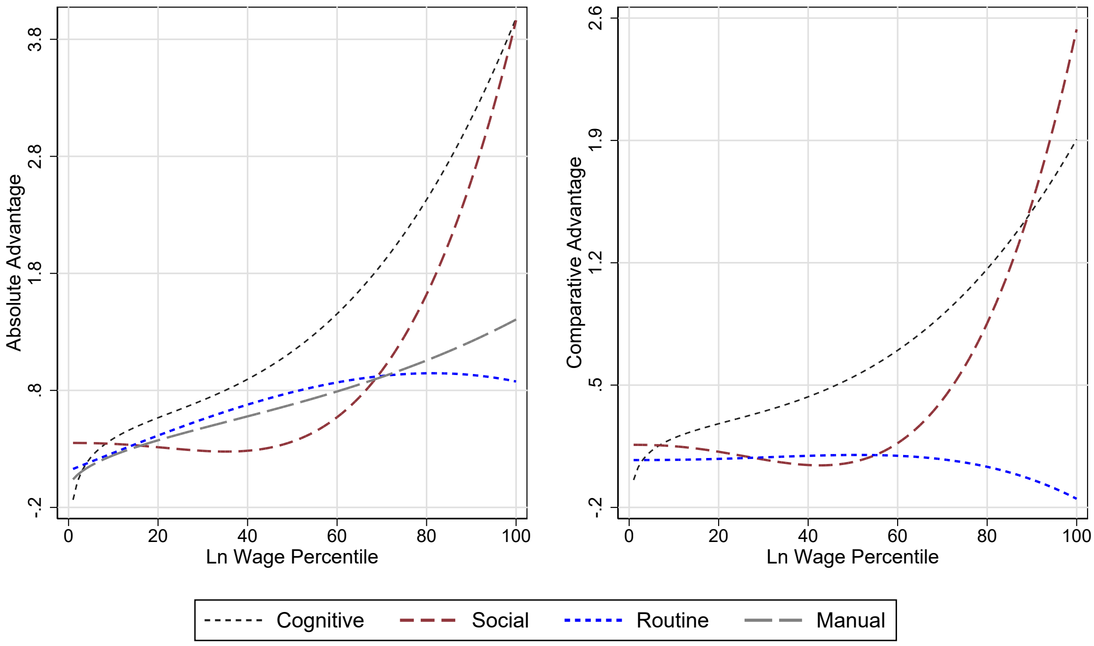

Figure G.1: The Skills and Comparative Advantages Along the Wage Distribution

Notes: The left panel plots a polynomial best-fit line of absolute advantage for each skill (y-axis) against the log wage percentile (x-axis). It shows that while there is a monotonic increasing relationship between each type of skill and the log wages, the slopes are generally steeper for cognitive and social skills than for routine and manual skills. The right panel plots the polynomial best-fit line of comparative advantage for cognitive, social, and routine, against the log wage percentile. Manual skill is used as the normalization.

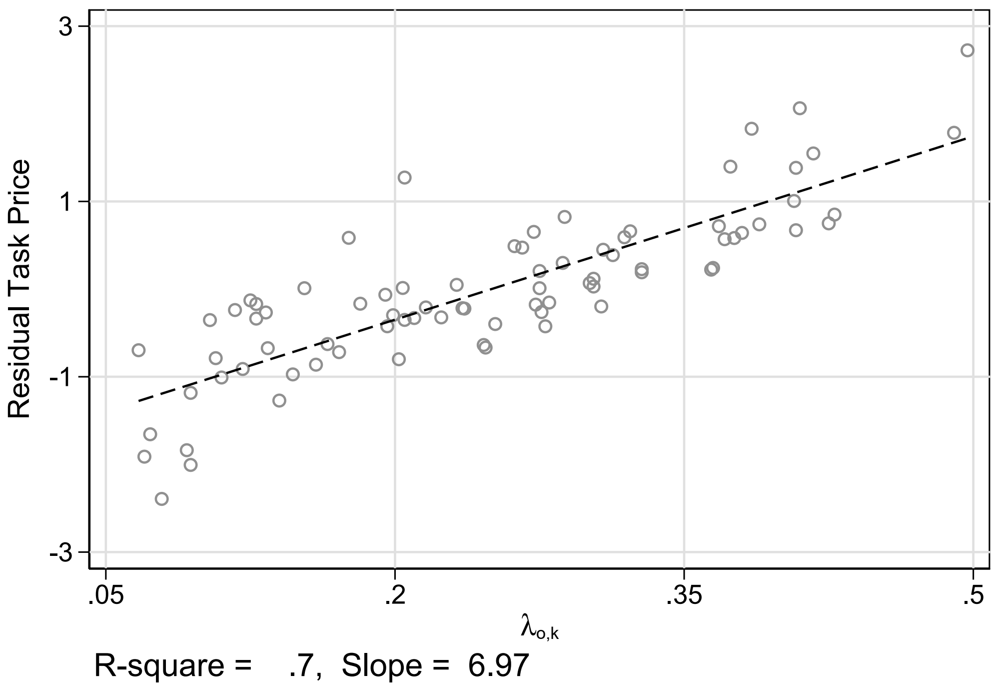

Figure G.2: Task Price Residuals and the Occupational Task Intensity, $\lambda_{o,k}$

Notes: The y-axis is the residual of estimated equilibrium task prices $p_{o,k}$ on 4 task dummies. The residual is used to ensure that each type of skill (as well as the prices) has the same average. The x-axis is the relative task demand in 2000, reported in Table B.5. The data are cross-sectional relations in 2000.

Figure G.3: The Mean of Task Shares ($\times 100$) for Other 12 Occupations: Data vs. Model

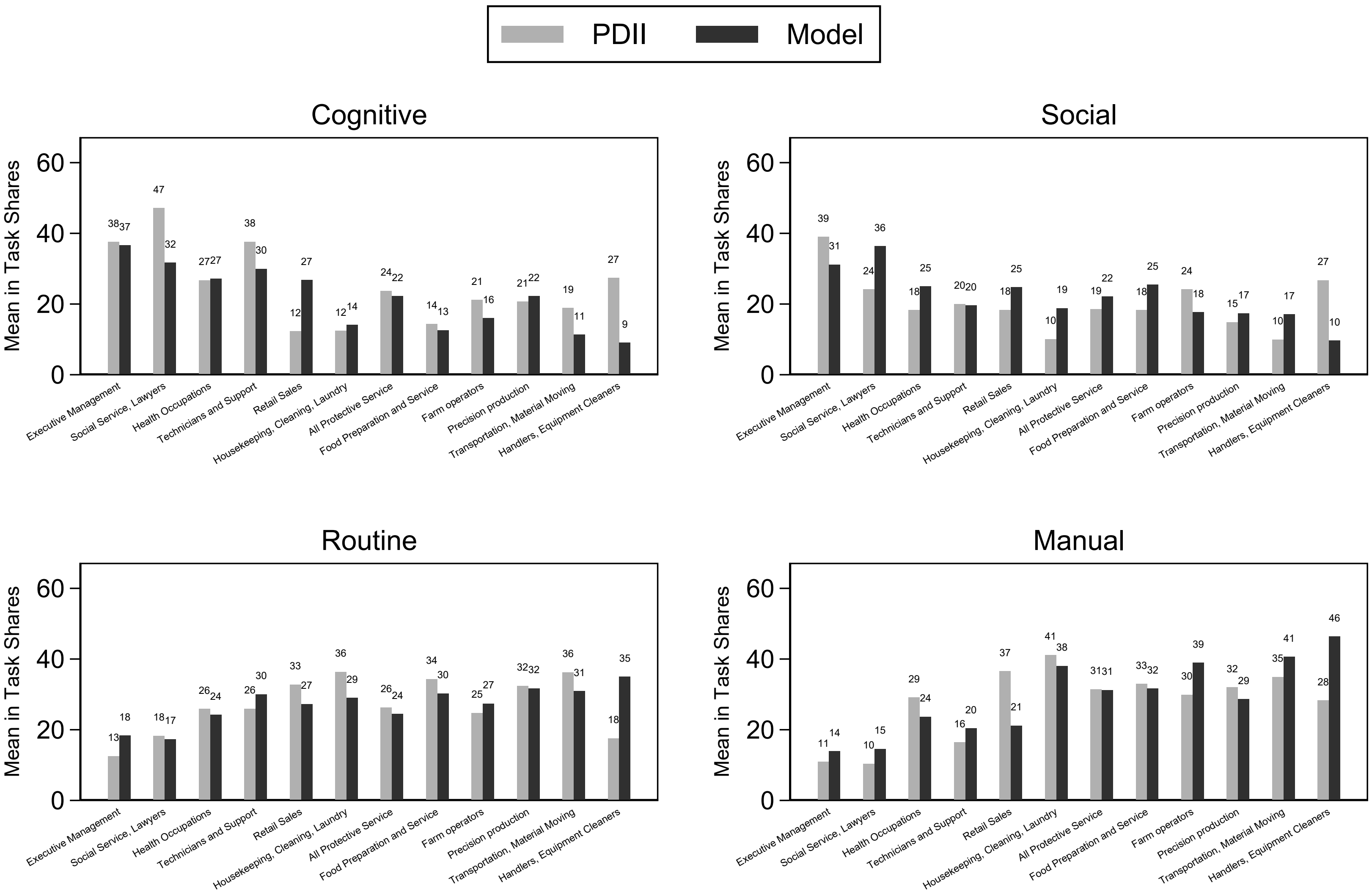

Figure G.3: The Variance of Task Shares ($\times 100$) for Other 12 Occupations: Data vs. Model (Continued)

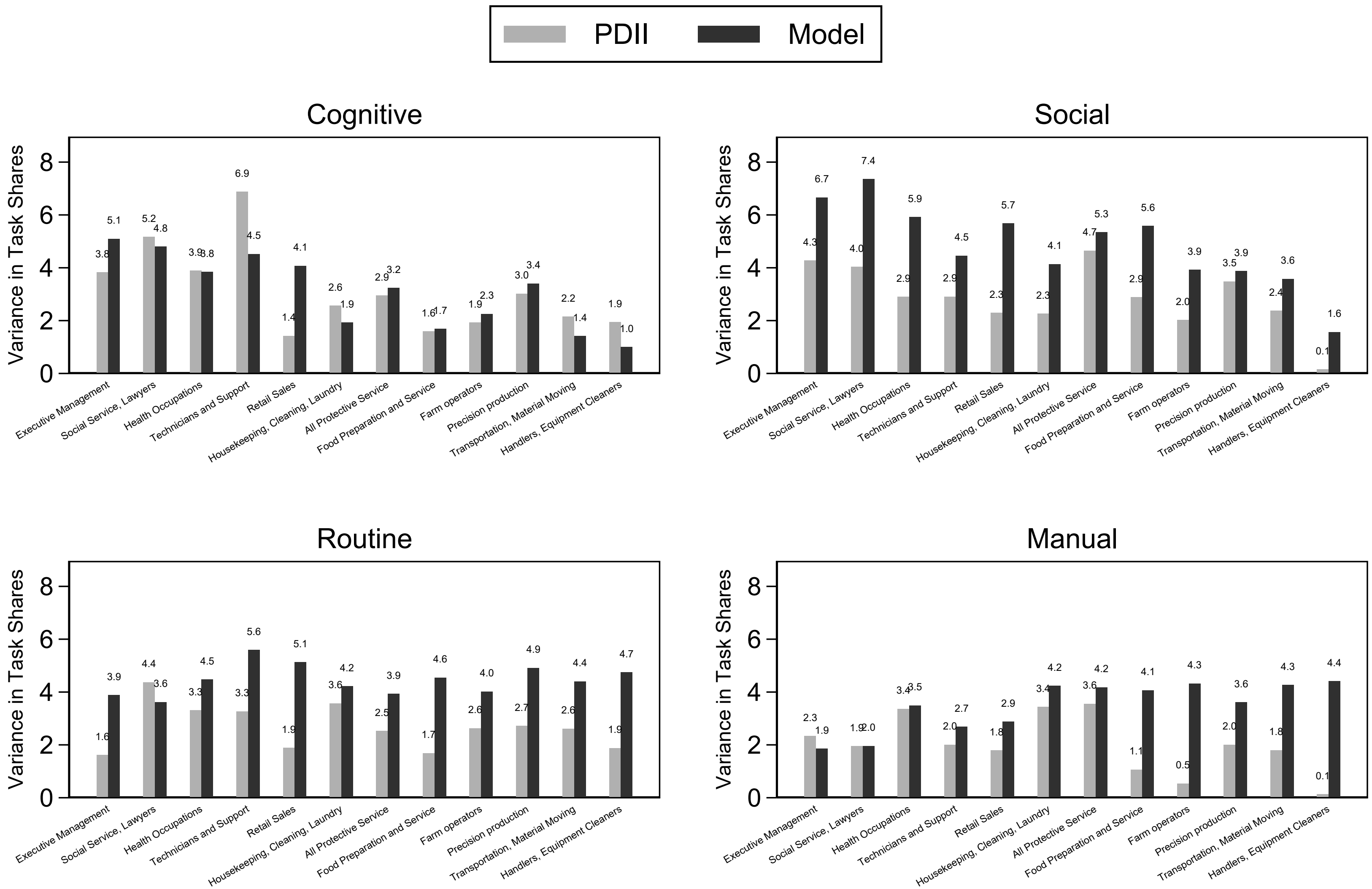

Table G.1: The Variance-Covariance Matrix of $\ln \widetilde{\Pi_{k|o}^{G(i)}} - \ln \widetilde{\Pi_{4|o}^{G(i)}}$ by Groups

	Variance			**Covariance**		
	Cognitive	*Social*	*Routine*	*Cog-Soc*	*Cog-Rou*	*Soc-Rou*
High School or Less, 41-60 years Old, Females	2.00	2.99	1.35	.497	.320	.325
High School or Less, 41-60 years Old, Males	1.34	2.26	1.20	.451	.314	.197
High School or Less, 21-41 years Old, Females	2.38	2.27	1.21	.306	.368	.345
High School or Less, 21-41 years Old, Males	1.59	2.04	1.22	.219	.253	.074
Some College or Above, 41-60 years Old, Females	1.73	2.86	2.48	.560	.484	.390
Some College or Above, 41-60 years Old, Males	1.52	2.73	1.70	.648	.500	.418
Some College or Above, 21-41 years Old, Females	1.53	2.62	1.77	.586	.497	.415
Some College or Above, 21-41 years Old, Males	1.73	2.31	1.47	.459	.452	.260

Notes: The first three columns report the variance of $\ln \widetilde{\Pi_{k|o}^{G(i)}} - \ln \widetilde{\Pi_{4|o}^{G(i)}}$. The last three columns display the correlation coefficient for cognitive-social, cognitive-routine, and social-routine. Manual skills are used as the normalization for constructing comparative advantages.

Table G.2: Breusch Pagan Test Statistic.

	High School or Less				**Some College or Above**			
	41-60 Years Old		**21-40 Years Old**		**41-60 Years Old**		**21-40 Years Old**	
	Females	**Males**	**Females**	**Males**	**Females**	**Males**	**Females**	**Males**
A. Cognitive tasks								
P-value	.12	.885	.08	.384	.29	.078	.358	.92
F-stat	1.344	.698	1.69	1.097	1.103	1.299	1.08	.699
Degree of Freedom	40	39	22	37	61	63	49	52
B. Social tasks								
P-value	.924	.134	0	.593	.393	.279	.628	.033
F-stat	.666	1.357	4.27	.924	1.044	1.11	.916	1.54
Degree of Freedom	40	39	22	37	61	63	49	52
C. Routine tasks								
P-value	.91	.003	.763	.661	.071	.136	.36	0
F-stat	.684	2.101	.745	.872	1.308	1.224	1.078	2.32
Degree of Freedom	40	39	22	37	61	63	49	52

Notes: Panels A, B, and C display the Breusch-Pagan test statistic for cognitive, social, and routine tasks, respectively. Each column is a specific education-age-gender group. The tests are performed by 3 tasks $\times$ 8 groups. The null hypothesis is that variance of $\ln \widetilde{\Pi_{k|o}^{G(i)}} - \ln \widetilde{\Pi_{4|o}^{G(i)}}$ is constant across occupations.

Table G.3: The SMM Estimates of Structural Parameters, Baseline Model

Education and Age Groups	$\mu^G_{z_1}$	$\mu^G_{z_2}$	$\mu^G_{z_3}$	$\mu^G_{z_4}$	**Cov**$(\ln z^G_1, \ln \nu^G_4)$	**Cov**$(\ln z^G_2, \ln \nu^G_4)$	**Cov**$(\ln z^G_3, \ln \nu^G_4)$	**Var**$(\ln \nu^G_4)$
				Females				
HS dropouts 21-41 years Old	-.34 (.009)	-.05 (.085)	.035 (.060)	-.03 (.270)	-.16 (.012)	-.14 (.035)	-.15 (.016)	.131 (.023)
HS dropouts 41-60 years Old	0 (0)	0 (0)	0 (0)	0 (0)	-.24 (.004)	-.29 (.012)	-.20 (.002)	.209 (.000)
HS grad 21-41 years Old	-.12 (.005)	.065 (.019)	.462 (.010)	-.03 (.022)	-.00 (.045)	-.12 (.053)	-.20 (.016)	.221 (.033)
HS grad 41-60 years Old	.107 (.001)	-.00 (.025)	.510 (.007)	-.00 (.040)	-.17 (.028)	-.17 (.078)	-.17 (.011)	.193 (.071)
Some college 21-41 years Old	.384 (.010)	-.78 (.000)	.040 (.004)	.315 (.023)	-.15 (.008)	-.07 (.020)	-.22 (.013)	.241 (.025)
Some college 41-60 years Old	.367 (.004)	-.44 (.006)	-.02 (.007)	.320 (.016)	-.15 (.018)	-.15 (.033)	-.24 (.020)	.205 (.064)
College and above 21-41 years Old	.538 (.058)	-.25 (.012)	-.79 (.000)	.475 (.075)	-.22 (.010)	-.21 (.025)	-.29 (.036)	.305 (.026)
College and above 41-60 years Old	.571 (.048)	-.10 (.023)	-.62 (.000)	.537 (.071)	-.31 (.022)	-.25 (.035)	-.35 (.020)	.406 (.006)
				Males				
HS dropouts 21-41 years Old	-.42 (.000)	-.20 (.028)	-.23 (.016)	.263 (.126)	-.01 (.003)	-.09 (.006)	-.01 (.005)	.091 (.035)
HS dropouts 41-60 years Old	-.41 (.000)	-.39 (.022)	.121 (.066)	.316 (.200)	-.07 (.012)	-.02 (.040)	-.07 (.011)	.109 (.017)
HS grad 21-41 years Old	.002 (.046)	-.53 (.035)	.183 (.123)	.357 (.381)	.005 (.056)	-.00 (.022)	-.12 (.011)	.145 (.114)
HS grad 41-60 years Old	-.18 (.001)	-.64 (.019)	.437 (.075)	.419 (.139)	.032 (.098)	.076 (.215)	-.09 (.052)	.129 (.105)
Some college 21-41 years Old	.364 (.056)	-.80 (.007)	.060 (.007)	.439 (.121)	-.05 (.074)	.066 (.176)	-.09 (.017)	.133 (.085)
Some college 41-60 years Old	.401 (.009)	-.54 (.009)	.310 (.010)	.481 (.017)	-.09 (.077)	.028 (.218)	-.17 (.029)	.180 (.076)
College and above 21-41 years Old	.875 (.009)	.895 (.022)	.466 (.010)	-.11 (.005)	-.20 (.019)	-.23 (.096)	-.20 (.024)	.344 (.086)
College and above 41-60 years Old	.807 (.001)	.622 (.045)	-.06 (.006)	.316 (.048)	-.25 (.033)	-.30 (.115)	-.29 (.041)	.372 (.095)

Notes: Standard errors are reported in parenthesis. Appendix C.6 details how I construct the standard errors of the parameter estimates.

Table G.4: (%) Percentage Changes in Task Prices by Occupations Between 1980 and 2000

Occupation	Cognitive	Social	Routine	Manual
Executive Management	35.3	52.1	-37.6	11.4
Management Related	44.4	59.5	-28.7	20.7
STEM	38.6	55.3	-34.6	14.4
Social Service, Lawyers	42.5	48.8	-29.5	29.3
Education, Training, Library, legal support	39.7	47.8	-32.4	25.9
Health Occupations	47.3	58.0	-10.8	31.6
Technicians and Related Support	39.3	45.1	-15.7	26.0
Financial Sales and Related Occupations	15.1	31.4	-55.9	-9.1
Retail Sales	33.4	40.4	-20.3	19.8
Administrative Support	35.9	41.6	-20.0	21.9
Housekeeping, Cleaning, Laundry	24.2	37.1	-31.5	8.2
All Protective Service	33.9	47.1	-19.9	16.8
Food Preparation and Service	29.9	44.5	-24.2	13.5
Farm operators	31.9	46.5	-25.5	4.7
Mechanics and Repairers	34.1	49.9	-23.3	7.0
Construction	24.0	39.2	-32.9	-3.4
Precision production	27.5	42.6	-29.4	-1.8
Machine Operators, Assemblers, and Inspectors	40.3	56.8	-17.4	12.5
Transportation and Material Moving	22.6	37.2	-32.8	7.3
Handlers, Equipment Cleaners, and Helpers	27.2	45.0	-29.8	1.0

Table G.5: Comparison Between PDII and O*NET

Occupation	Cognitive		Social		Routine		Manual	
	PDII	O*Net	PDII	O*Net	PDII	O*Net	PDII	O*Net
Executive Management	0.375	0.376	0.390	0.408	0.126	0.125	0.109	0.092
Management Related	0.411	0.408	0.265	0.308	0.208	0.205	0.116	0.079
STEM	0.494	0.425	0.214	0.278	0.226	0.176	0.066	0.121
Social Service, Lawyers	0.472	0.301	0.242	0.497	0.183	0.107	0.104	0.094
Education, Training, Library, legal support	0.375	0.407	0.142	0.365	0.208	0.117	0.276	0.110
Health Occupations	0.267	0.273	0.183	0.328	0.259	0.195	0.291	0.204
Technicians and Related Support	0.376	0.328	0.200	0.235	0.260	0.272	0.164	0.165
Financial Sales and Related Occupations	0.366	0.368	0.256	0.364	0.215	0.133	0.163	0.134
Retail Sales	0.123	0.275	0.183	0.322	0.328	0.232	0.366	0.171
Administrative Support	0.241	0.275	0.126	0.266	0.425	0.319	0.209	0.140
Housekeeping, Cleaning, Laundry	0.124	0.128	0.101	0.216	0.364	0.276	0.412	0.380
All Protective Service	0.237	0.224	0.185	0.287	0.263	0.202	0.315	0.288
Food Preparation and Service	0.144	0.104	0.183	0.313	0.343	0.280	0.330	0.303
Farm operators	0.211	0.153	0.242	0.205	0.248	0.252	0.299	0.389
Mechanics and Repairers	0.324	0.196	0.153	0.147	0.210	0.247	0.313	0.410
Construction	0.254	0.210	0.206	0.159	0.217	0.246	0.322	0.385
Precision production	0.207	0.236	0.149	0.199	0.324	0.303	0.320	0.262
Machine Operators, Assemblers, and Inspectors	0.121	0.128	0.082	0.070	0.439	0.428	0.358	0.374
Transportation and Material Moving	0.189	0.094	0.100	0.182	0.363	0.307	0.349	0.417
Handlers, Equipment Cleaners, and Helpers	0.274	0.067	0.267	0.073	0.176	0.371	0.283	0.490

Notes: This table compares the average task shares in each occupation as obtained from the PDII data with those estimated using the O*NET.