Disclosure and Favoritism in Sequential Elimination Contests

ONLINE APPENDIX

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In this online appendix we collect the materials omitted from the main text of the paper.[1] The appendices are ordered according to where they are first referenced in the main text. Online Appendix A allows the contest organizer to randomize between full disclosure and no disclosure, and shows that partial disclosure is suboptimal. Therefore, it is without loss of generality to focus on the comparison between full disclosure and no disclosure in Proposition 1. Online Appendix B relaxes the restriction that the contest organizer must select the bias rule from those that induce a symmetric pure-strategy equilibrium, and demonstrates that Proposition 2 remains largely intact. Online Appendix C presents a three-player example to illustrate contest design with endogenous biases.

A Randomized Disclosure Schemes

We have assumed that the contest organizer chooses between full disclosure and no disclosure in the main text. In this section, we enrich the set of candidate disclosure schemes by allowing for partial disclosure. For the sake of simplicity, we employ and extend the setup in the baseline N-M two-stage model in Section 2.

Specifically, instead of full disclosure and no disclosure, the contest organizer now commits to a disclosure scheme indexed by $\mu \in [0, 1]$, where μ denotes the probability that the interim rankings are disclosed. Clearly, full transparency corresponds to $\mu = 1$ and full opacity corresponds to $\mu = 0$.

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[1]This note is not self-contained; it is the online appendix of the paper "Disclosure and Favoritism in Sequential Elimination Contests."

Denote the optimal disclosure scheme that maximizes the total effort and the expected winner's total effort by μ^* and μ^{**}, respectively. Standard technique leads to the following.

Proposition A1 ***(Suboptimality of Randomized Disclosure Schemes)*** *Consider a two-stage elimination contest. Suppose that Assumption 1 is satisfied and the contest organizer is allowed to randomize between full disclosure and no disclosure. Then $\mu^* \in \{0,1\}$ and $\mu^{**} \in \{0,1\}$.*

Proof. Fixing a disclosure rule $\mu \in [0,1]$, a symmetric pure-strategy equilibrium is characterized by the triple $(e_{1p}, e_{2p}, \hat{e}_{2p})$, where e_{1p} is a representative contestant's stage-1 effort, and e_{2p} (respectively, $\hat{e}_{2p}$) denotes the contestant's stage-2 effort when the interim rankings are disclosed (respectively, concealed). We use subscript p to indicate "partial disclosure."

Fixing $\mu \in [0,1]$, a contestant's stage-2 effort when the interim rankings are publicized is equal to that under transparency, i.e.,

$$e_{2p} = \frac{M-(M-1)r}{M^2}, \tag{A1}$$

and it remains to pin down $(e_{1p}, \hat{e}_{2p})$. Fixing the other contestants' effort profiles $(e_{1p}, \hat{e}_{2p})$, let a contestant choose $(e'_{1p}, \hat{e}'_{2p})$ for the following optimization problem:

$$\max_{\{\hat{e}'_{1p}, \hat{e}'_{2p}\}} \mu V \left[\sum_{m=1}^{M} P_m(e'_{1p}, \boldsymbol{e}_{1p}^{-i})\right] + (1-\mu)\left\{\left[\sum_{m=1}^{M} P_m(\hat{e}'_{1p}, \hat{\boldsymbol{e}}_{1p}^{-i})\right] \times \frac{(\hat{e}'_{2p})^r}{(\hat{e}'_{2p})^r + (M-1)(\hat{e}_{2p})^r} - \hat{e}'_{2p}\right\} - e'_{1p},$$

where V is defined in Equation (13). Analogously to the analysis in the proof of Lemmata 3 and 4, $(e_{1p}, \hat{e}_{2p})$ can be solved as follows:

$$\hat{e}_{2p} = \frac{(M-1)r}{NM}, \tag{A2}$$

and

$$e_{1p} = \frac{\left[\mu M V + (1-\mu)\right] r}{NM} \times \sum_{m=1}^{M}\left(1 - \sum_{g=0}^{m-1}\frac{1}{N-g}\right). \tag{A3}$$

It can be verified that contestants' participation constraints are satisfied under Assumption 1, and thus the effort profile specified by (A1)-(A3) constitutes a unique symmetric pure-strategy equilibrium. In addition, the total effort and the expected winner's total effort, which we denote by $TE_p(\mu)$ and $WE_p(\mu)$, respectively, are

$$TE_p(\mu) \equiv Ne_{1p} + \mu M e_{2p} + (1-\mu)N\hat{e}_{2p},$$

and

$$WE_p(\mu) \equiv e_{1p} + \mu e_{2p} + (1-\mu)\hat{e}_{2p}.$$

Note that e_{2p} and $\hat{e}_{2p}$ are independent of μ from Equations (A1) and (A2), and e_{1p} is linear in μ from Equation (A3). Therefore, $TE_p(\mu)$ and $WE_p(\mu)$ are convex in μ, implying that $\mu^* \in \{0,1\}$ and $\mu^{**} \in \{0,1\}$. This completes the proof. ■

By Proposition A1, randomization (i.e., $\mu \in (0,1)$) is always suboptimal regardless of the organizer's objective.

B Alternative Bidding Equilibria in Excessively Discriminatory Contests

The results in Section 4 are obtained under the condition that the organizer optimizes by choosing a bias rule $\hat{\boldsymbol{\delta}}$ from those that induce a symmetric pure-strategy equilibrium under opacity. In this section, we relax this restriction and show that Proposition 2 remains robust.

As seen from Lemma 2, when $r \leq 1/\Gamma_N(\hat{\boldsymbol{q}}^*)$, the constraint is nonbinding as all bias rules induce symmetric pure-strategy equilibria. The restriction, however, does limit the set of potential bias rules for optimization when r exceeds the cutoff. An increase in r encourages more aggressive bidding, which could cause the participation condition to break down and dissolve symmetric pure-strategy equilibrium, as in typical static contests. The comparison between transparency and opacity in Proposition 2 is immune to this result when the organizer aims to maximize total effort. To be more specific, Lemma 2(ii) states that there exists a bias rule that induces full rent dissipation in a symmetric pure-strategy equilibrium, in which case a total effort of 1 results. This indeed reaches the limit of the contest design, and no other mechanism could outperform it. The global optimality of opacity can therefore be established.

The same, however, cannot be said when the organizer's objective is to maximize the expected winner's total effort. With the restriction of a symmetric pure-strategy equilibrium, the expected winner's total effort is bounded above by $1/N$, which falls below the maximum under transparency from Proposition 2. In this case, more aggressive bias rules exist and they break down the symmetric pure-strategy equilibrium by violating the participation constraint. In general, multiple equilibria could arise under large r; for instance, there could exist symmetric mixed-strategy equilibria. Alternatively, there could exist *semi-symmetric equilibria* that resemble those in contests with endogenous entry depicted by Fu, Jiao and Lu (2015): In such equilibria, a subset of contestants play symmetric pure-strategy bidding among themselves, while the rest stay inactive by bidding zero with probability one. There may also exist many other types of asymmetric equilibria that involve various forms of randomization. These equilibria could lead to a greater expected winner's total effort than under the restriction of symmetric equilibrium in pure strategies. To see that, imagine a situation with $r > 1/\Gamma_N(\hat{\boldsymbol{q}}^*)$. In a restricted optimum, the contest ends up with an expected winner's total effort $1/N$. If the organizer instead sets a rule that breaks down this equilibrium and induces a semi-symmetric equilibrium with $N-1$ active contestants, the expected winner's total effort is then bounded by $1/(N-1)$ instead of $1/N$.

It is technically challenging to fully characterize these equilibria in our context. Recall that the dynamic linkage between stages dissolves in the contest under opacity, which leads contestants to behave as if they were choosing multiple actions simultaneously, i.e., $\hat{e}_1$ and $\hat{e}_2$, in a static contest. The literature provides little guidance in solving for asymmetric or mixed-strategy equilibria in imperfectly discriminatory contests that involve multi-dimensional strategies: In such a scenario, each contestant can randomize in either dimension, i.e., either $\hat{e}_1$ or $\hat{e}_2$. This is particularly challenging in our context because (i) the probability of winning in Tullock contests is discontinuous at the origin; and (ii) one's stage-2 outcome ultimately depends on stage-1 outcome, despite the dissolved dynamic linkage due to opacity. Moreover, the multiplicity of equilibria imposes conceptual limitations on contest design: It is difficult to predict the performance of the contest when the particular equilibrium to be played under a given bias rule remains ambiguous.

Despite the limitations, our result does not lose its bite when we allow for the aforementioned semi-symmetric equilibria and consider bias rules that could induce semi-symmetric equilibria. Imagine an equilibrium that involves $N' \in \{3, \ldots, N-1\}$ active contestants under opacity. The contest in this equilibrium is essentially equivalent to an alternative N'-contest in a symmetric pure-strategy equilibrium. As a result, enlarging the set of eligible bias rules to allow for these semi-symmetric equilibria is no different than letting the contest organizer *shortlist* the contestants—i.e., excluding $N - N'$ contestants and inviting the rest to participate in a two-stage contest—while optimizing over the set of bias rules that induce a symmetric pure-strategy equilibrium. We then consider an alternative optimization problem: Under a given disclosure policy, the organizer sets the optimal number of participants, and chooses the optimal bias rule accordingly over the set of candidate rules that induce a symmetric pure-strategy equilibrium.

Lemma A1 $\Gamma_N(\hat{\boldsymbol{q}}^*)$ *strictly increases with* N.

Proof. To highlight the fact that $\hat{\boldsymbol{q}}^*$ depends on N, let us denote the optimal winning probabilities for the case of N contestants by $\hat{\boldsymbol{q}}_N^* := (\hat{q}_{1N}^*, \ldots, \hat{q}_{NN}^*)$. To prove the lemma, it suffices to show that $\Gamma_N(\hat{\boldsymbol{q}}_N^*) < \Gamma_{N+1}(\hat{\boldsymbol{q}}_{N+1}^*)$. Let $\hat{\boldsymbol{q}}_{N+1}^{\star} = (\hat{q}_{1N}^*, \ldots, \hat{q}_{NN}^*, 0)$. It follows from

Equation (28) that

$$\Gamma_{N+1}(\hat{\boldsymbol{q}}^{\star}_{N+1}) - \Gamma_N(\hat{\boldsymbol{q}}^{*}_{N}) = \sum_{m=1}^{N}\left\{\left[\left(1-\sum_{g=0}^{m-1}\frac{1}{N+1-g}\right)-\left(1-\sum_{g=0}^{m-1}\frac{1}{N-g}\right)\right]\times \hat{q}^{*}_{mN}\right\}$$
$$= \frac{1}{N+1}\times\sum_{m=1}^{N}\left[\frac{m}{N-m+1}\times\hat{q}^{*}_{mN}\right] > 0.$$

Therefore, we have that $\Gamma_{N+1}(\hat{\boldsymbol{q}}^{*}_{N+1}) \geq \Gamma_{N+1}(\hat{\boldsymbol{q}}^{\star}_{N+1}) > \Gamma_N(\hat{\boldsymbol{q}}^{*}_{N})$. This completes the proof. ■

By Lemma A1, the cutoff $\frac{1}{\Gamma_N(\hat{\boldsymbol{q}}^*)}$ strictly decreases when the number of participants increases. When fewer participants are involved, a symmetric pure-strategy equilibrium is more likely to emerge. In other words, the organizer, when narrowing the pool, ends up with additional freedom in choosing the bias rule. Note that $\Gamma_3(\hat{\boldsymbol{q}}^*) = \frac{11}{12} < 1$, which implies that if the organizer invites only three participants, she can induce a symmetric pure-strategy equilibrium for any contest rule under Assumption 2. The following result can then be obtained.

Proposition A2 *Fix $N \geq 4$ and $r \in (0,1]$. Suppose that the contest organizer is allowed to shortlist contestants and select $N' \in \{3,\ldots,N\}$ of them for the competition. When the contest organizer is able to set the bias rule for the second-stage competition, she always prefers transparency to opacity if she aims to maximize the expected winner's total effort.*

We do not have to lay out a formal proof, as the logic is straightforward. Suppose that the optimum under opacity requires N participants. Then the optimum under opacity is outperformed by that under transparency by Proposition 2. Suppose otherwise that it requires $N' \in \{3,\ldots,N-1\}$ participants, which demands that the organizer shortlist. The optimum is still outperformed by that under transparency: The organizer, under transparency, can shortlist the same number N' of participants and set the optimal bias rule accordingly, which again generates a greater expected winner's total effort by Proposition 2. We thus restore the optimality of transparency in a broader setting.

C Three-player Example of Optimal Contest Design with Endogenous Biases

As stated in Section 4, we can establish a correspondence between contestants' efforts and winning probabilities in equilibrium. This further allows us to rewrite design objectives, total effort and the expected winner's total effort, as functions of the equilibrium winning probability distribution. Our optimization approach lets the organizer choose equilibrium winning probability distribution to maximize reformulated objective functions. Table A1 summarizes the equilibrium winning probability distribution in the optimal contest under transparency for the case of $N = 3$.

Transparency	q_1^*	q_2^*	q_3^*	$TE^{RT}(\boldsymbol{q}^*)$
$\frac{63-3\sqrt{241}}{50} < r \leq 1$	$\frac{8r^2+9r-72}{3(7r^2-36)}$	$\frac{r^2+45r-90}{6(7r^2-36)}$	$\frac{25r^2-63r+18}{6(7r^2-36)}$	$\frac{r(5r^3+33r^2+171r-621)}{126r^2-648}$
$0 < r \leq \frac{63-3\sqrt{241}}{50}$	$\frac{15-5r}{2(12-5r)}$	$\frac{9-5r}{2(12-5r)}$	0	$\frac{r(25r^2-170r+273)}{288-120r}$
Transparency	q_1^{**}	q_2^{**}	q_3^{**}	$WE^{RT}(\boldsymbol{q}^{**})$
$0 < r \leq 1$	$\frac{21-5r}{36-10r}$	$\frac{15-5r}{36-10r}$	0	$\frac{r(25r^2-230r+513)}{1296-360r}$

Table A1: Optimal Equilibrium Winning Probabilities under Transparency in Three-Player Contests.

By Table A1, when $r > \frac{63-3\sqrt{241}}{50} \approx 0.3285$, the optimal contest involves three active players in the second stage—i.e., $q_1^* > q_2^* > q_3^* > 0$—and the equilibrium winning distribution can be induced by a bias rule $(\delta_1^*, \delta_2^*, \delta_3^*) = \left(\frac{1}{1-q_1^*}(q_1^*)^{\frac{1-r}{r}}, \frac{1}{1-q_2^*}(q_2^*)^{\frac{1-r}{r}}, \frac{1}{1-q_3^*}(q_3^*)^{\frac{1-r}{r}}\right)$. When $r \leq \frac{63-3\sqrt{241}}{50} \approx 0.3285$, the optimal contest involves two active players in the second stage—i.e., $q_1^* > q_2^* > q_3^* = 0$—and the equilibrium winning distribution can be induced by a bias rule $(\delta_1^*, \delta_2^*, \delta_3^*) = \left(\frac{1}{1-q_1^*}(q_1^*)^{\frac{1-r}{r}}, \frac{1}{1-q_2^*}(q_2^*)^{\frac{1-r}{r}}, 0\right)$.

Table A2 summarizes the equilibrium winning probability distribution in the optimal contest under opacity.

Opacity	$\hat{q}_1^*$	$\hat{q}_2^*$	$\hat{q}_3^*$	$TE^{RT}(\hat{\boldsymbol{q}}^*)$	$WE^{RT}(\hat{\boldsymbol{q}}^*)$
$0 < r \leq 1$	$\frac{5}{8}$	$\frac{3}{8}$	0	$\frac{91}{96}r$	$\frac{91}{298}r$

Table A2: Optimal Equilibrium Winning Probabilities under Opacity in Three-Player Contests.

Although the bottom-ranked contestant has zero chance of winning the prize in the optimum, he is uninformed of his status and continues to exert effort in the second stage. The optimal equilibrium winning probability distribution simultaneously maximizes the total effort exerted in the overall contest and the expected winner's total effort, and is independent of the discriminatory power of the contest technology (i.e., r).

References

Fu, Qiang, Qian Jiao, and Jingfeng Lu, "Contests with endogenous entry," *International Journal of Game Theory*, 2015, *44* (2), 387–424.